# Corporate Finance: The trade-off theory

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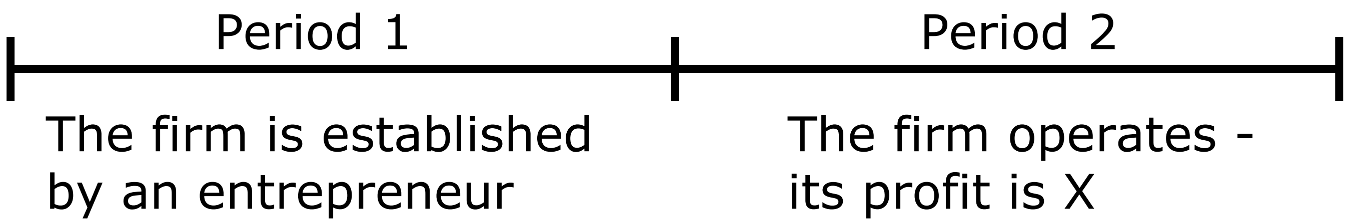
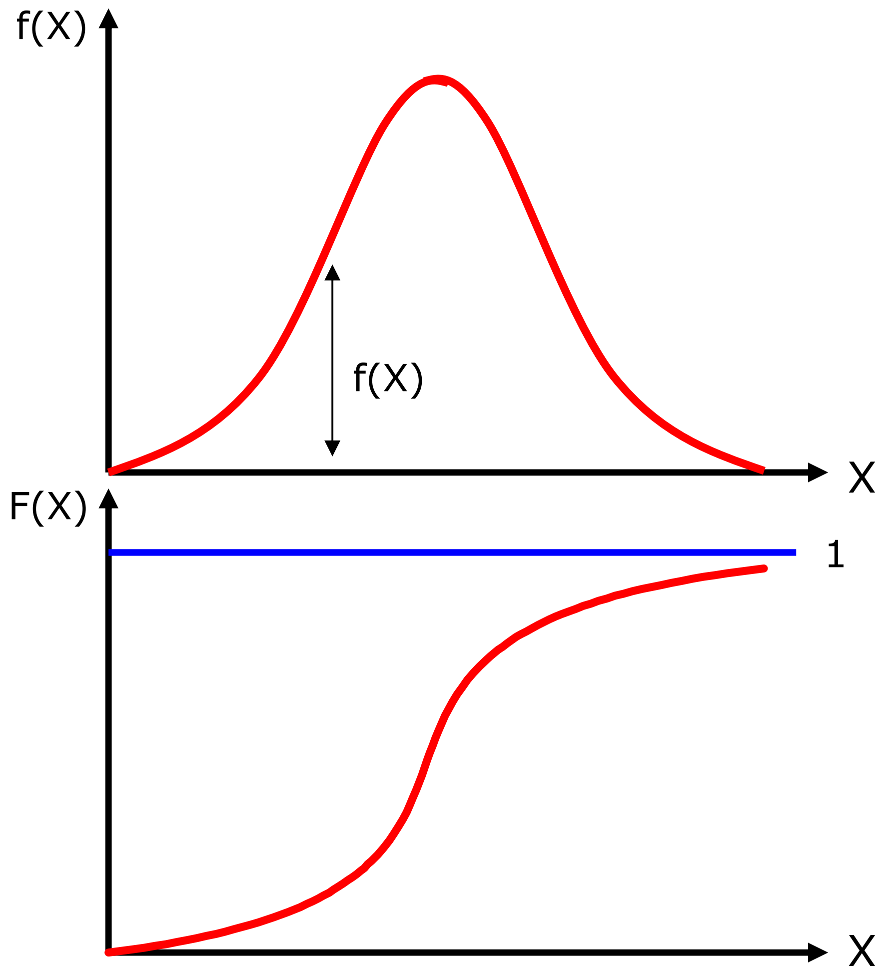
# The main assumptions

- The timing:



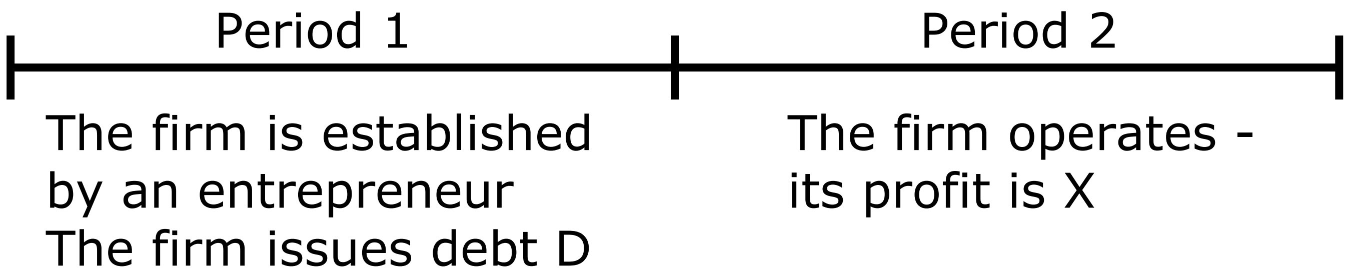

- The entrepreneur wishes to maximize the firm’s value
- $X \sim [X_0, X_1]$; dist. function f(X) and CDF F(X)
- The mean earnings are $\hat{X}$

# The dist. function and CDF



# Risky debt

- The timing:




- The face value of debt is D
- Debt is paid in full if $X \geq D$
- If $X < D$, the firm goes bankrupt
- Fixed cost of bankruptcy is $C < X_0$ (to avoid uninteresting complications)
- An obvious alternative is proportional costs: $C = c(D-X)^+$

# Taxation

- Corporate tax is $t_c$
- Debt is fully deductible
- Interest rate is r (r is the alternative that investors can get for their money)

# All-equity firm: D = 0

- The value of the firm:

$$\underbrace{V(0)(1+r)}_{\text{Alternative cost}} = \underbrace{\int_{X_0}^{X_1} [X - t_c X] dF(X)}_{\text{Payoff from investing in the firm}}$$

$$V(0) = \frac{1}{1+r} \int_{X_0}^{X_1} [X - t_c X] dF(X)$$

$$= \frac{(1-t_c)\hat{X}}{1+r}$$

# Riskless debt: D < $X_0$

- Equity: $$E(D) = \frac{1}{1+r} \int_{X_0}^{X_1} \left[ X - D - (X - D) t_c \right] dF(X)$$ $$= \frac{(1 - t_c)(\hat{X} - D)}{1+r}$$
- Debt: $$B(D) = \frac{D}{1+r}$$
- Total value: $$V(D) = E(D) + B(D) = \frac{(1 - t_c)\hat{X} + t_c D}{1+r}$$

# Riskless debt: Implications

- Total value: $$V(D) = \frac{(1 - t_c)\hat{X} + t_c D}{1 + r}$$
- $t_c = 0 \Rightarrow$ Debt is irrelevant $\Rightarrow$ M&M 1958
- $t_c > 0 \Rightarrow$ The firm will be all-debt $\Rightarrow$ M&M 1963
- Debt benefits the firm by providing a tax shield

# Risky debt: D > $X_0$
# Kraus and Litzenberger, *JF* 1973

- Equity: $$E(D)=\frac{1}{1+r}\int_{D}^{X_1}\left[X-D-(X-D)t_c\right]dF(X)$$
- Debt: $$B(D)=\frac{1}{1+r}\int_{X_0}^{D}\left[X-C\right]dF(X)+\frac{1}{1+r}\int_{D}^{X_1}D\,dF(X)$$
- Total value:

$$V(D)=\frac{1}{1+r}\int_{X_0}^{D}\left[X-C\right]dF(X)+\frac{1}{1+r}\int_{D}^{X_1}\left[X-(X-D)t_c\right]dF(X)$$

$$=\frac{\hat{X}-CF(D)}{1+r}-\frac{1}{1+r}\int_{D}^{X_1}(X-D)t_c\,dF(X)$$

# First-order conditions

$$V'(D) = -\frac{Cf(D)}{1+r} + \frac{1}{1+r}(D-D)t_c f(D) + \frac{1}{1+r}\int_D^{X_1} t_c dF(X)$$

$$= -\frac{Cf(D)}{1+r} + \frac{1}{1+r}t_c(1-F(D)) = 0$$

- Simplifying:

$$Cf(D) = t_c(1-F(D)) \quad \Rightarrow \quad H(D) \equiv \frac{f(D)}{1-F(D)} = \frac{t_c}{C}$$

- Properties of H(D): $H'(D)>0$, $H(X_1)=\infty$

# Hazard rate – Uniform dist.

- $F(X)=(X-X_0)/(X_1-X_0)$

- $f(x) = 1/(X_1-X_0)$

$$H(D) \equiv \frac{f(X)}{1-F(X)} = \frac{\dfrac{1}{X_1 - X_0}}{1 - \dfrac{X - X_0}{X_1 - X_0}} = \frac{1}{X_1 - X}$$

$\Rightarrow$ H'(X)>0

# Hazard rate – Exponential dist.

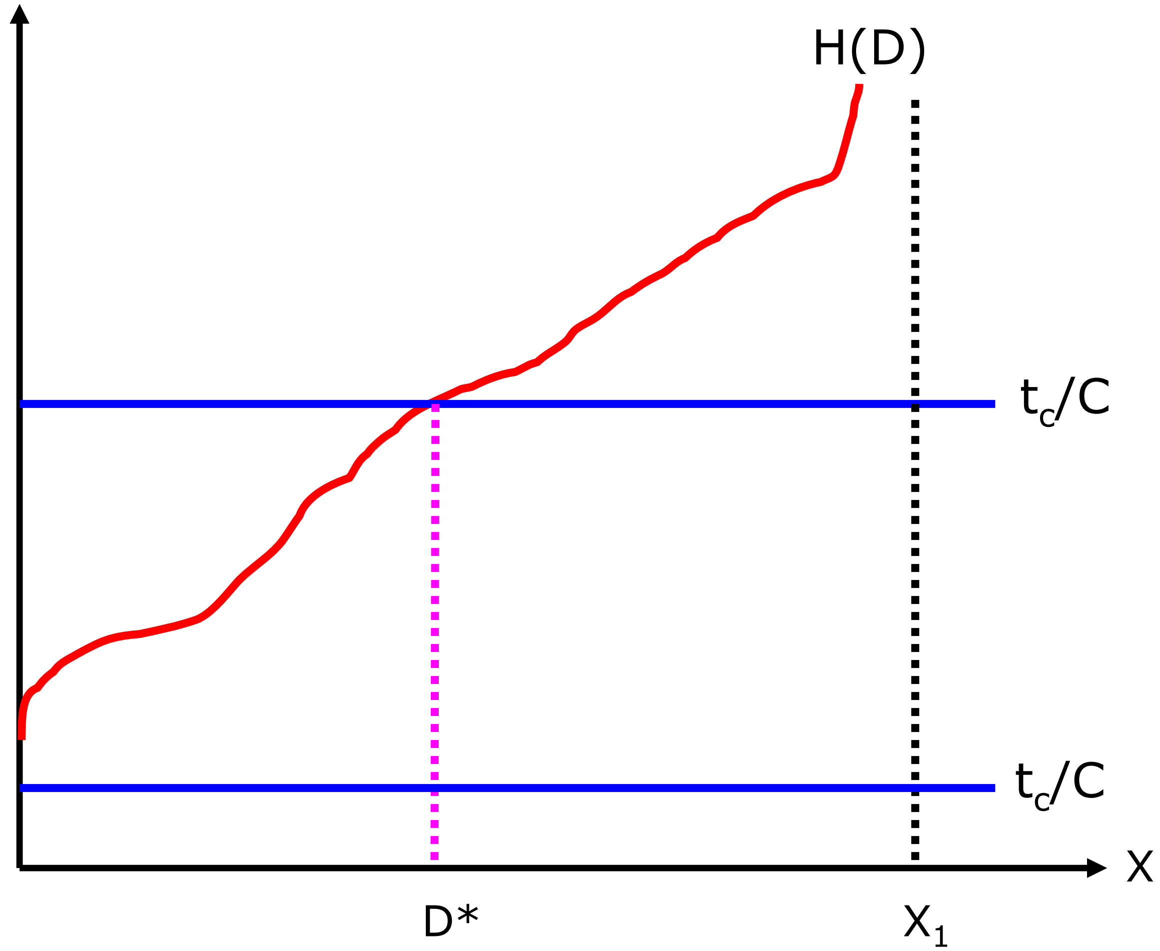
- $F(X)=1-e^{-\lambda x}$

- $f(x) = \lambda e^{-\lambda x}$

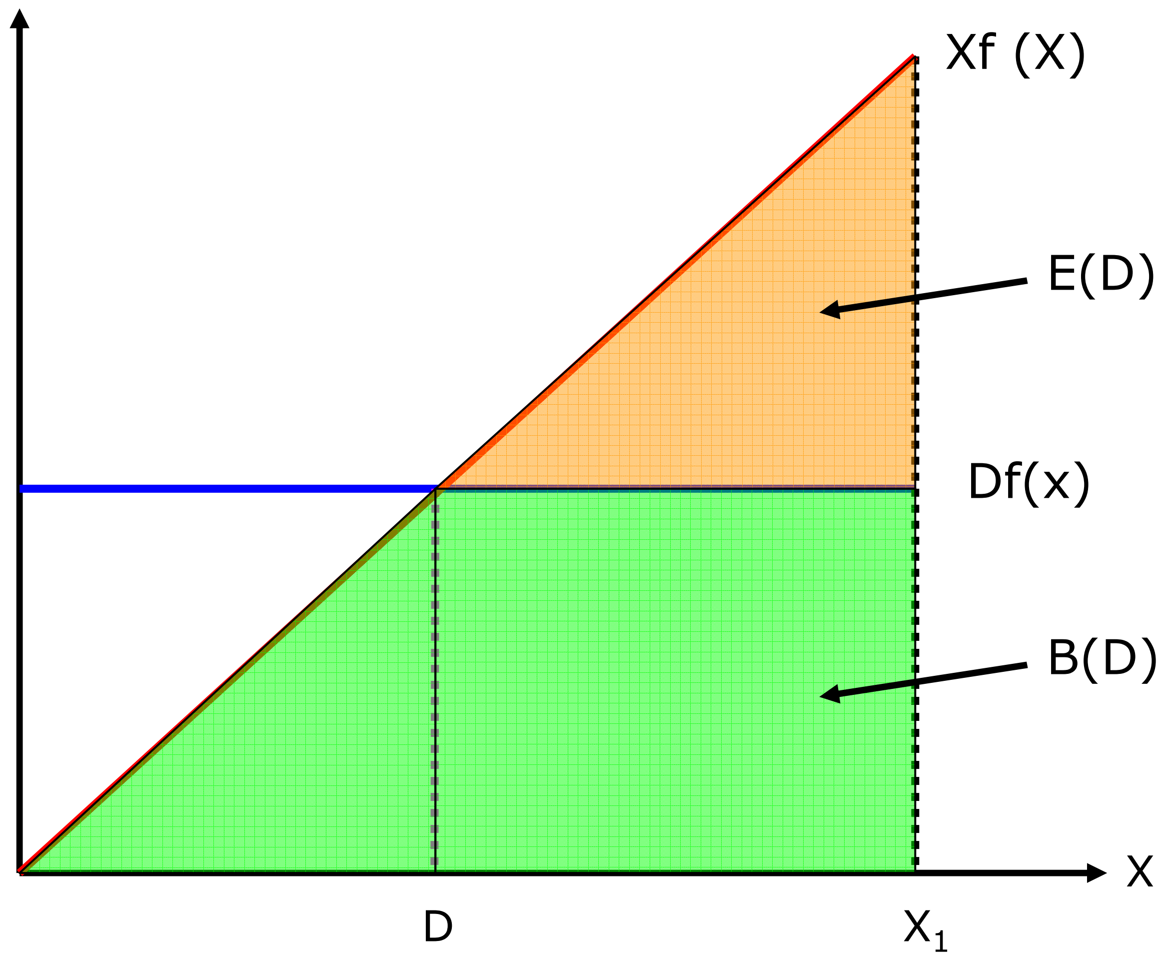
$$H(D) \equiv \frac{f(X)}{1-F(X)} = \frac{\lambda e^{-\lambda x}}{1-\left(1-e^{-\lambda x}\right)} = \lambda$$

$\Rightarrow$ H'(X)=0

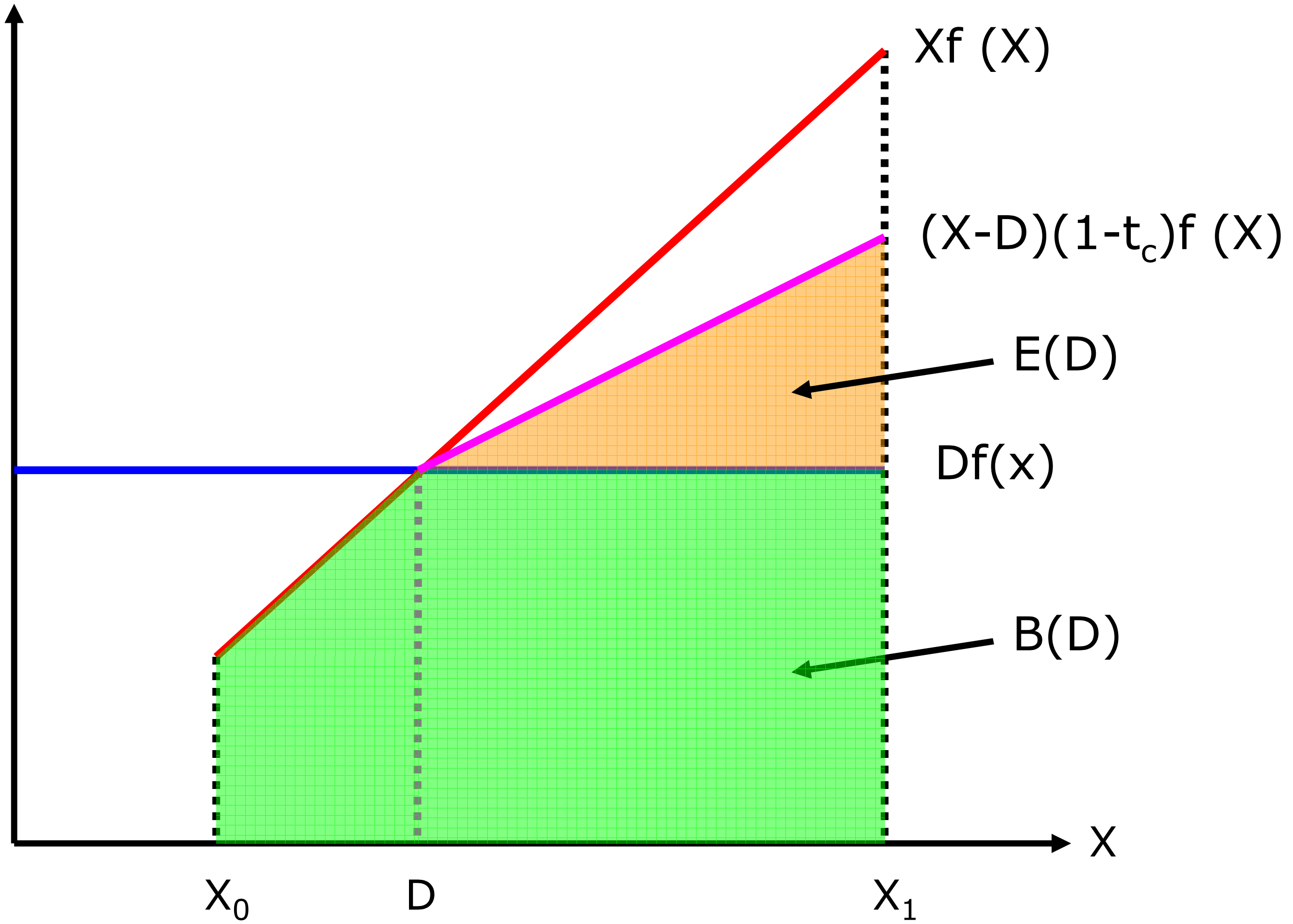
# Illustrating the first-order conditions



# Illustrating M&M 1958 ($C = t_c = 0$)

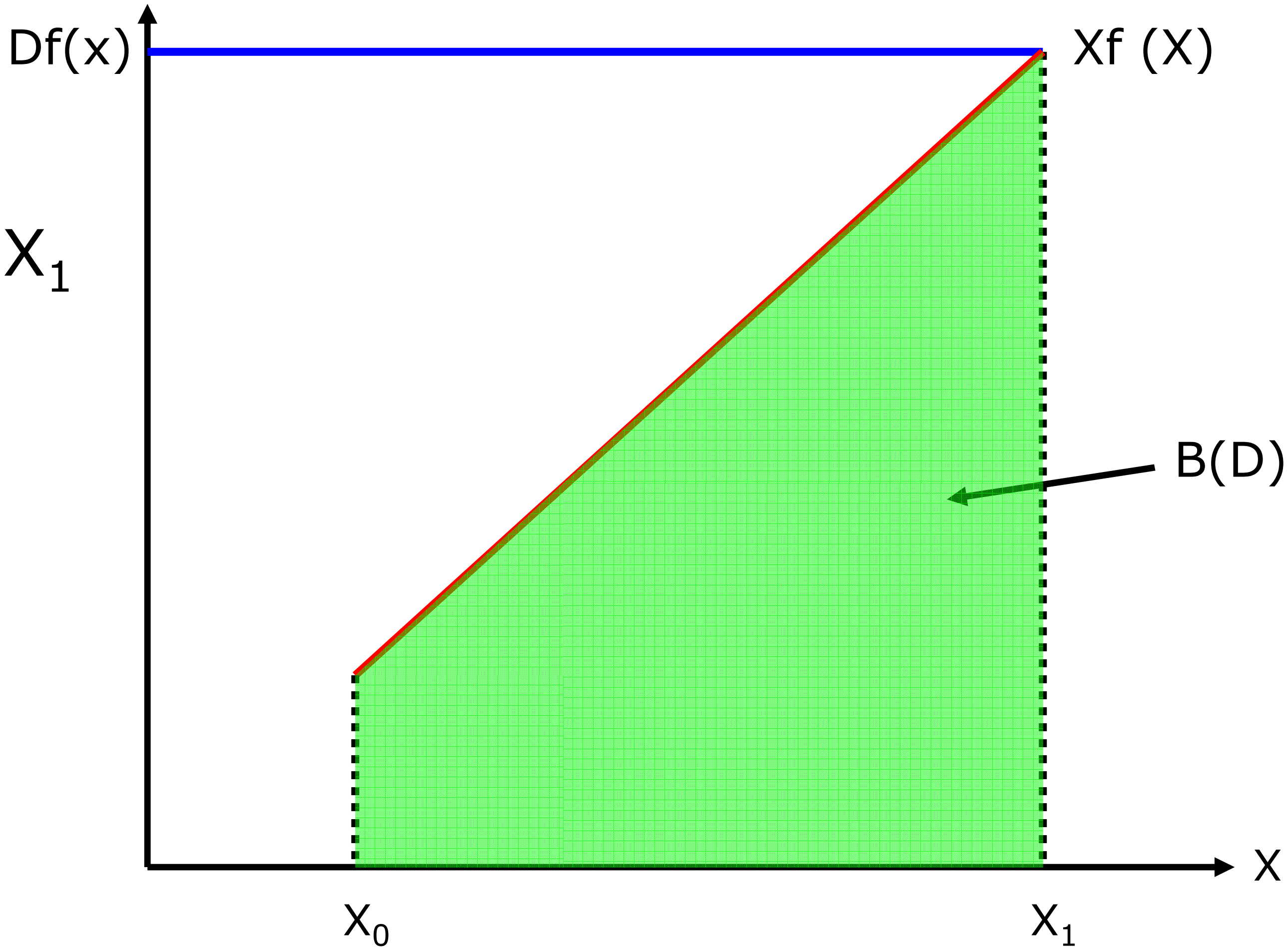


# Illustrating M&M 1963 ($C = 0, t_c > 0$)

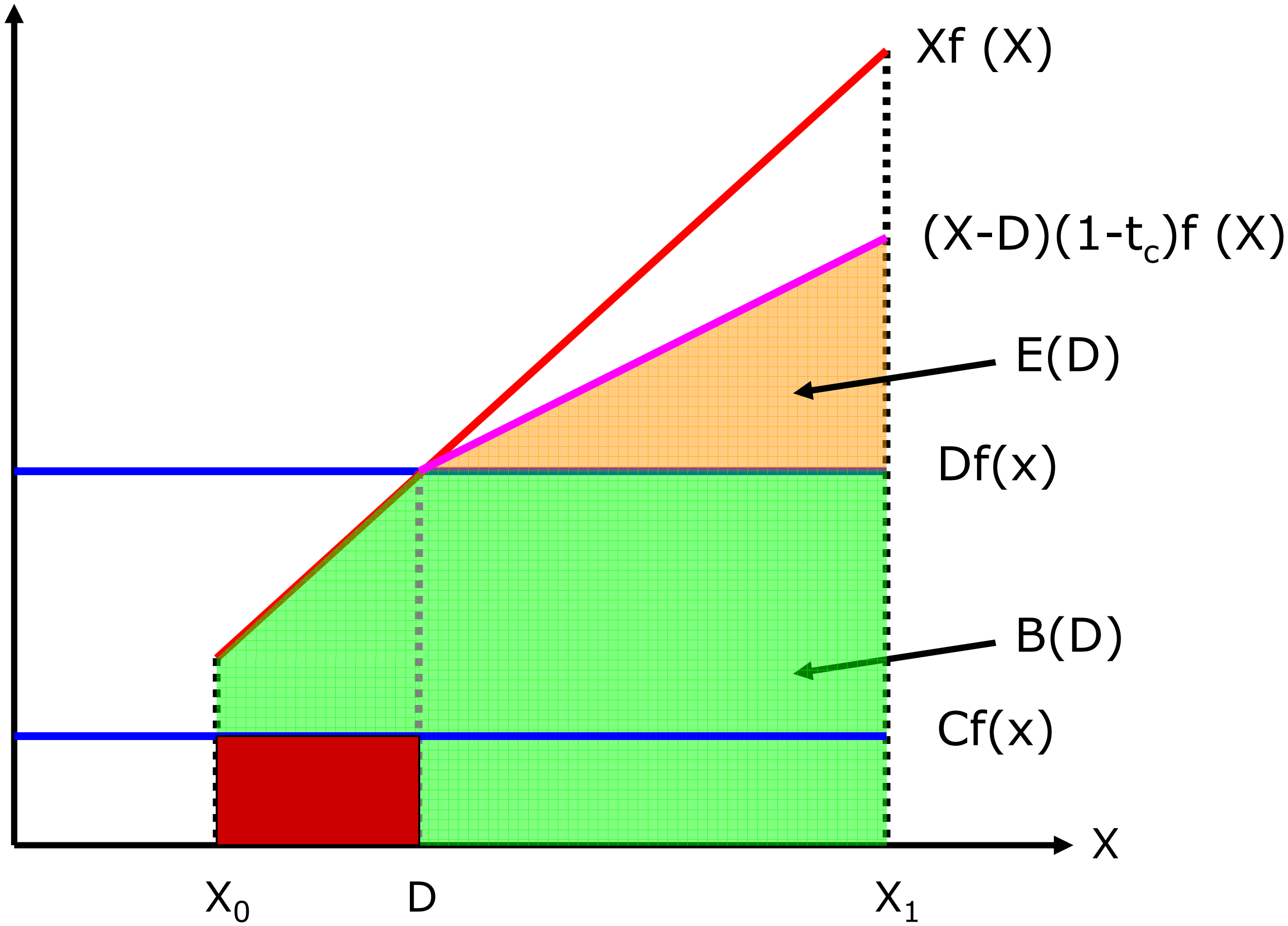


# Illustrating M&M 1963 ($C = 0, t_c > 0$)

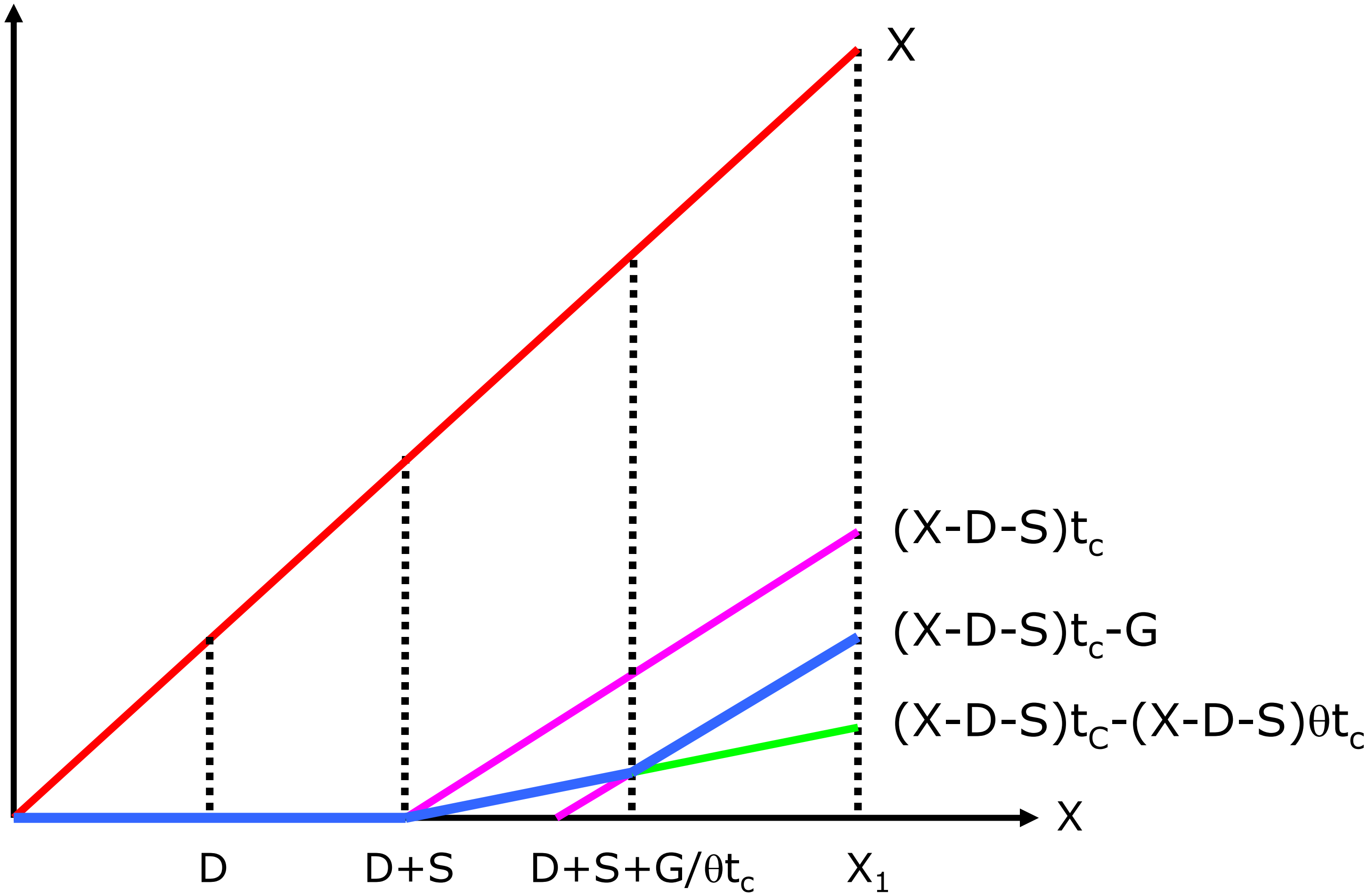
$D^* = X_1$



# Illustrating K&L 1973 ($C > 0$, $t_c > 0$)



# DeAngelo and Masulis, *JFE* 1980
# Tax code



# DeAngelo and Masulis, *JFE* 1980 Payoffs

| State of nature | Debt-holders | Equity-holders | IRS | Total |
|---|---|---|---|---|
| $X < D$ | X | 0 | 0 | X |
| $D < X < D+S$ | D | X-D | 0 | X |
| $D+S < X < D+S+G/\theta t_c$ | D | $X-D-t_c(X-D-S)+\theta t_c(X-D-S)$ | $t_c(1-\theta)(X-D-S)$ | X |
| $X > D+S+G/\theta t_c$ | D | $X-D-t_c(X-D-S)+G$ | $t_c(X-D-S)-G$ | X |

# DeAngelo and Masulis, *JFE* 1980
# Equity and debt values

$$E(D)=\frac{1}{1+r}\int_{D}^{D+S}[X-D]dF(X)+\frac{1}{1+r}\int_{D+S}^{D+S+G/\theta t_c}[X-D-t_c(1-\theta)(X-D-S)]dF(X)$$

$$+\frac{1}{1+r}\int_{D+S+G/\theta t_c}^{X_1}[X-D-t_c(X-D-S)+G]dF(X)$$

$$=\frac{1}{1+r}\int_{D}^{X_1}[X-D]dF(X)-\frac{1}{1+r}\int_{D+S}^{X_1}t_c(X-D-S)dF(X)$$

$$+\frac{1}{1+r}\int_{D+S}^{D+S+G/\theta t_c}t_c\theta(X-D-S)dF(X)+\frac{1}{1+r}\int_{D+S+G/\theta t_c}^{X_1}G\,dF(X)$$

$$B(D)=\frac{1}{1+r}\int_{X_0}^{D}X\,dF(X)+\frac{1}{1+r}\int_{D}^{X_1}D\,dF(X)$$

# DeAngelo and Masulis, *JFE* 1980
# Firm value

$$V(D) = \frac{1}{1+r}\int_{X_0}^{X_1} X dF(X) - \frac{1}{1+r}\int_{D+S}^{X_1} t_c (X - D - S) dF(X)$$

$$+ \frac{1}{1+r}\int_{D+S}^{D+S+G/\theta t_c} t_c \theta (X - D - S) dF(X) + \frac{1}{1+r}\int_{D+S+G/\theta t_c}^{X_1} G dF(X)$$

$$V'(D) = \frac{1}{1+r}\int_{D+S}^{X_1} t_c dF(X) - \frac{1}{1+r}\int_{D+S}^{D+S+G/\theta t_c} t_c \theta dF(X) = 0$$

$$\Rightarrow \quad 1 - F(D+S) = \theta\left[F(D+S+G/\theta t_c) - F(D+S)\right]$$