Corporate Finance:
Agency models of capital structure

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Jensen and Meckling, JFE 1976

The agency cost of outside equity
The investment model

- **The timing:**

  
  Period 1
  
  The firm is established by an entrepreneur who has R dollars and needs to invest I

  Period 2
  
  The investment yields V(I), R-I is consumed as perks

- V'(I) > 0 > V''(I) and V(0) = 0

- V'(0) = \infty and V'(\infty) = 0 (interior sol’n)

- The entrepreneur’s payoff: \( U(I) = V(I) + (R-I) \)
Internal financing

- F.O.C for the entrepreneur’s problem:

\[ U'(I) = V'(I) - 1 = 0 \quad \Rightarrow \quad V'(I^*) = 1. \]
Internal financing

- The entrepreneur’s payoff at the optimum:

\[ U(\text{I}^*) = V(\text{I}^*) - I^* + R \]

- But what if \( I^* > R \)? In that case the entrepreneur must have external financing.
Debt financing

- The entrepreneur issues debt with face value D to raise I*- R upfront
- The entrepreneur’s payoff:

\[ U(I, D) = V(I) - D + R + (I^* - R) - I \]

- \( U(I, D) \) is maximized at I* (the firm invests optimally)
- Debt is safe: ex post the firm has \( V(I^*) > I^* > I^*-R \) \( \Rightarrow D^* = I^*-R \)
- The entrepreneur’s payoff:

\[ U(I^*, D^*) = V(I^*) - D^* + R + (I^* - R) - I^* \]

\[ = V(I^*) - I^* + R \]
Equity financing with commitment

- The entrepreneur issues equity with equity participation 1-\(\alpha\) (the entrepreneur keeps \(\alpha\)) and commits to invest \(I^*\)

- To raise \(I^*-R\):

\[
(1-\alpha) V(I^*) = I^*-R \quad \Rightarrow \quad \alpha^* = \frac{I^*-R}{V(I^*)}.
\]

- The entrepreneur’s payoff:

\[
U(I^*, \alpha^*) = \alpha^* V(I^*) + E^* + R - I^*
\]

Ex post payoff

Perks

\[
= V(I^*) - I^* + R
\]
Equity financing without commitment

- The entrepreneur issues equity with equity participation $1-\alpha$, but cannot commit to invest $I^*$

- After receiving $E$, but before choosing $I$, the entrepreneur’s payoff:

$$U(I, \alpha) = \alpha V(I) + E + R - I$$

Ex post payoff Perks
Equity financing without commitment

- F.O.C for the entrepreneur’s problem:
  \[ U'(I, \alpha) = \alpha V'(I) - 1 = 0 \; \Rightarrow \; \alpha V'(I^{**}) = 1. \]

- We get underinvestment
- \( \alpha \downarrow \) (more outside equity) \( \Rightarrow \) \( I^* \downarrow \) (more underinvestment)
The agency cost of outside equity

- The entrepreneur’s payoff:
  \[ U(I^{**}, \alpha) = \alpha V(I^{**}) + E^{**} - I^{**} + R \]
  \[ = \alpha V(I^{**}) + (1 - \alpha) V(I^{**}) - I^{**} + R \]
  \[ = V(I^{**}) - I^{**} + R \]

- The entrepreneur’s payoff with commitment:
  \[ U(I^*, \alpha) = V(I^*) - I^* + R \]

- By revealed preferences (and since \( I^{**} < I^* \)):
  \[ V(I^*) - I^* + R > V(I^{**}) - I^{**} + R \]

- The entrepreneur bears the cost of underinvestment (outside investors break even)
The optimal choice of $\alpha$

- How does $\alpha$ affect $U(I^{**},\alpha)$?

$$\frac{\partial U(I^{**},\alpha)}{\partial \alpha} = \left(\frac{1}{\alpha} V'(I^{**}) - 1\right) \frac{\partial I^{**}}{\partial \alpha} > 0$$

- The entrepreneur will raise $\alpha$ up to the point where

$$I^{**} = R + \left(1 - \alpha\right)V(I^{**})$$

- $E^{**} = 0$ at $\alpha = 0$ (since then $I^{**} = 0$) and at $\alpha = 1 \Rightarrow E^{**}$ is inverse U-shaped
The optimal choice of $\alpha$

- The entrepreneur’s budget constraint:

- The relevant sol’n is with the maximal $\alpha$
The effort model

- The timing:
  
  **Period 1**
  
  The firm is established by an entrepreneur who has $R$ dollars and needs to invest $I$ and exert effort, $e$

  **Period 2**
  
  The effort yields $V(e)$, the cost of effort is $e$ (no perks)

- $V'(e) > 0 > V''(e)$ and $V(0) = 0$

- $V'(0) = \infty$ and $V'(\infty) = 0$ (interior sol’n)

- The entrepreneur’s payoff: $U(e) = V(e) - e$
Internal financing

F.O.C for the entrepreneur’s problem:

\[ U'(e) = V'(e) - 1 = 0 \implies V'(e^*) = 1 \]
Internal financing

- The entrepreneur’s payoff at the optimum:

\[ U(e^*) = V(e^*) - e^* \]

- But what if I > R?

- Debt financing: debt is safe so D* = I - R
Equity financing without commitment

- The entrepreneur issues equity with equity participation $1-\alpha$

- After receiving $E$, but before choosing $e$, the entrepreneur’s payoff is:

$$U(e, \alpha) = \alpha V(e) - e$$
Equity financing without commitment

- F.O.C for the entrepreneur’s problem:

\[ U'(e, \alpha) = \alpha V'(e) - 1 = 0 \implies \alpha V'(e^{**}) = 1 \]

- We get underinvestment
The agency cost of outside equity

- The entrepreneur’s payoff:
  \[ U(e^{**}, \alpha) = \alpha V(e^{**}) - e^{**} \]

- The entrepreneur’s payoff with commitment:
  \[ U(e^*, \alpha) = \alpha V(e^*) - e^* \]

- By revealed preferences (and since \( e^{**} < e^* \)):
  \[ \alpha V(e^*) - e^* > \alpha V(e^{**}) - e^{**} \]

- The entrepreneur bears the cost of underinvestment (outside investors break even)
The optimal choice of $\alpha$

- How does $\alpha$ affect $U(e^{**}, \alpha)$?

\[
\frac{\partial U(e^{**}, \alpha)}{\partial \alpha} = V(e^{**}) + \left(\alpha V'(e^{**}) - 1\right) \frac{\partial e^{**}}{\partial \alpha} > 0
\]

- The entrepreneur will raise $\alpha$ up to the point where

\[
I = R + \underbrace{(1-\alpha)V(e^{**})}_{E^{**}}
\]

- $E^{**} = 0$ at $\alpha = 0$ (since then $e^{**} = 0$) and at $\alpha = 1 \Rightarrow E^{**}$ is inverse U-shaped
The optimal choice of $\alpha$

- The entrepreneur’s budget constraint:

- The relevant sol’n is with the maximal $\alpha$
Jensen and Meckling, JFE 1976

The asset substitution problem
A simple example

- Consider a box with two sealed envelopes: one with $100 and the with $0

- You can either pick one envelop from the box or receive $70 for sure – what would you do?

- Now suppose you owe someone $50 out of your gains

- If you take $70, your payoff is $70 - $50 = $20

- If you pick one of the sealed envelopes then you either have $100 - $50 = $50, or you pick the empty envelop and your payoff is 0 because you cannot pay the $50 ⇒ your expected payoff is ($0+$50)/2 = $25

- The lottery is better even though its NPV is only $50
The model

- Two projects:
  - Safe project with return \( Z \)
  - Risky project with return \( X \sim [0, \infty) \)

- The firm has debt with face value \( D \)

- The management is perfect agent for equityholders – the agency problem is between equityholders and debtholders
Payoffs under the two projects

- Equityholders’ payoff with the risky project:
  \[ Y_R = \int_{D}^{\infty} (X - D) dF(X) \]

- Equityholders’ payoff with the safe project:
  \[ Y_S = \text{Max}\{Z - D, 0\} \]

- The firm surely chooses the risky project if \( Z \leq D \) \( \Rightarrow \) assume that \( Z > D \). Hence
  \[ Y_S = Z - D \]
Comparing the two projects

- The safe project is better iff:

\[ Y_S > Y_R \iff Z > X^C(D) = D + \int_D (X - D) dF(X) \]

- Properties of \( X^C(D) \):

\[
\frac{\partial X^C(D)}{\partial D} = 1 - \int_D dF(X) = 1 - (1 - F(D)) = F(D)
\]

\[
X^C(0) = \int_0^\infty X dF(X) = \hat{X}
\]
Choice of projects with leverage

- $X^C(D) > \hat{X}$ for all $D > 0$

- A leveraged firm prefers the risky project even when it is inefficient.
Myers, JFE 1977

The debt overhang problem
The model

- The timing:
  - Period 1
    - The firm is established by an entrepreneur who issues debt with face value D
  - Period 1.5
    - The entrepreneur learns the return, X, from a project that costs I and decides whether to invest
  - Period 2
    - X is realized and debt is paid

- In period 1 it is common knowledge that $X \sim [0, \infty)$
- Absent debt, the firm invests iff $X \geq I$. The value of the firm:

\[
V(0) = \int_{I}^{\infty} (X - I) dF(X)
\]
Illustrating – All-equity firm
Short-term debt (due at period 1.5)

- Suppose debt has to be paid before it is time to invest

- If \( X - I \geq D \), the firm will invest

- If \( X - I < D \), the firm will not invest and will go bankrupt. The debtholders will invest provided that \( X \geq I \)

- The firm invests iff \( X \geq I \Rightarrow \) investment is efficient
The value of the firm with short-term debt

- The value of debt:
  \[ B(D) = \int_{I}^{D+I} (X - I) dF(X) + \int_{D+I}^{\infty} D dF(X) \]

- The value of equity:
  \[ E(D) = \int_{D+I}^{\infty} (X - D - I) dF(X) \]

- The total value of the firm:
  \[ V(D) = \int_{I}^{D+I} (X - I) dF(X) + \int_{D+I}^{\infty} (X - I) dF(X) \]
  \[ = \int_{I}^{\infty} (X - I) dF(X) \]
Illustrating the debt overhang problem

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Long-term debt

- Suppose debt has to be paid after it is time to invest

- If $X - I \geq D$, the firm will invest

- If $X - I < D$, the firm will not invest and will go bankrupt. The debtholders get a firm with no investment opportunities
The value of the firm with long-term debt

- The value of debt:
  \[ B(D) = \int_{D+I}^{\infty} DdF(X) \]

- The value of equity:
  \[ E(D) = \int_{D+I}^{\infty} (X - D - I)dF(X) \]

- The total value of the firm:
  \[ V(D) = \int_{D+I}^{\infty} (X - I)dF(X) \]

- The value is lower than under short-term debt. This is the debt overhang problem
Illustrating the debt overhang problem
Berkovitch and Kim, JF 1990

Overinvestment and underinvestment
The model

- **The timing:**

  - **Period 0:** The firm is established by an entrepreneur who issues debt with face value $D$.
  - **Period 1:** The cash flow $X$ is realized and the firm invests $I$.
  - **Period 2:** Investment yields $zg(I)$, $D$ is due.

- $g'(I) > 0 > g''(I)$ and $g(0) = 0$
- $g'(0) = \infty$ and $g'(\infty) = 0$ (interior sol’n)
- $z \sim [0, \infty)$
Excess funds: $X > I$

- Cash flow at the end of period 2:
  \[ zg(I) + X - I \]

- The firm is solvent iff
  \[ zg(I) + X - I \geq D \quad \Rightarrow \quad z \geq z_1 = \frac{D + I - X}{g(I)} \]

- The expected value of equity:
  \[ E_1(D) = \int_{z_1}^{\infty} (zg(I) + X - D - I) dF(z) \]
Illustrating – excess funds

\[(zg(I)+X-I)f(z)\]

\[Df(z)\]

\[E_1(D)\]

\[(X-I)f(z)\]
Investment with excess funds

- F.O.C for investment:

\[
\frac{\partial E_1(D)}{\partial I} = \int_{z_1}^{\infty} (zg'(I) - 1) dF(z) = g'(I) \int_{z_1}^{\infty} zdF(z) - (1 - F(z_1)) = 0
\]

- Rewriting:

\[
g'(I) = \frac{1}{\int_{z_1}^{\infty} zdF(z)} \equiv \frac{1}{E(z \mid z \geq z_1)} \cdot \frac{1}{1 - F(z_1)}
\]
Deficit: \( X < I \)

- The firm covers the deficit by issuing extra equity
- Cash flow at the end of period 2: \( zg(I) \)
- The firm is solvent iff
  \[
  zg(I) \geq D \quad \Rightarrow \quad z \geq z_2 \equiv \frac{D}{g(I)}
  \]
- The expected value of equity:
  \[
  E_2(D) = \int_{z_2}^{\infty} (zg(I) - D)dF(z) - \left( I - X \right)
  \]
  Extra funds to cover the deficit
Illustrating – deficit

\[ (X-I)f(z) \]

\[ (X-I)f(z) \]

\[ E_2(D) \]

\[ Df(z) \]

\[ zg(I)f(z) \]
Investment with deficit

- F.O.C for investment:

\[
\frac{\partial E_2(D)}{\partial I} = \int_{z_2}^{\infty} zg'(I)dF(z) - 1
\]

\[
= g'(I)\int_{z_2}^{\infty} zdF(z) - 1 = 0
\]

- Rewriting:

\[
g'(I) = \frac{1}{\int_{z_2}^{\infty} zdF(z)}
\]
The value of the firm with excess funds

\[ V_1(D) = \int_{z_1}^{\infty} (zg(I) + X - D - I) dF(z) \]

\[ = \int_{0}^{z_1} (zg(I) + X - I) dF(z) + \int_{z_1}^{\infty} D dF(z) \]

\[ = \int_{0}^{\infty} (zg(I) + X - I) dF(z) \]

\[ = \hat{z}g(I) + X - I \]
The value of the firm with deficit

\[ V_2(D) = \int_{z_2}^{\infty} (zg(I) - D) dF(z) - (I - X) + \int_{0}^{z_2} zg(I) dF(z) + \int_{z_2}^{\infty} D dF(z) \]

\[ = \int_{0}^{\infty} zg(I) dF(z) - (I - X) \]

\[ = \hat{z}g(I) + X - I \]

- The value of the firm is exactly as in the case of excess funds
Efficient investment

- F.O.C for investment:
  \[
  \frac{\partial V(D)}{\partial I} = \hat{z}g'(I) - 1 = 0 \quad \Rightarrow \quad g'(I) = \frac{1}{\hat{z}}
  \]

- Excess funds:
  \[
  g'(I) = \frac{1}{\int_{z_1}^{\infty} zdF(z)} \equiv \frac{1}{E(z | z \geq z_1)} < \frac{1}{\hat{z}}
  \]

- Deficit:
  \[
  g'(I) = \frac{1}{\int_{z_2}^{\infty} zdF(z)} > \frac{1}{\hat{z}}
  \]
Comparison

Excess funds ⇒ overinvestment
Deficit ⇒ underinvestment