Corporate Finance: Agency models of capital structure

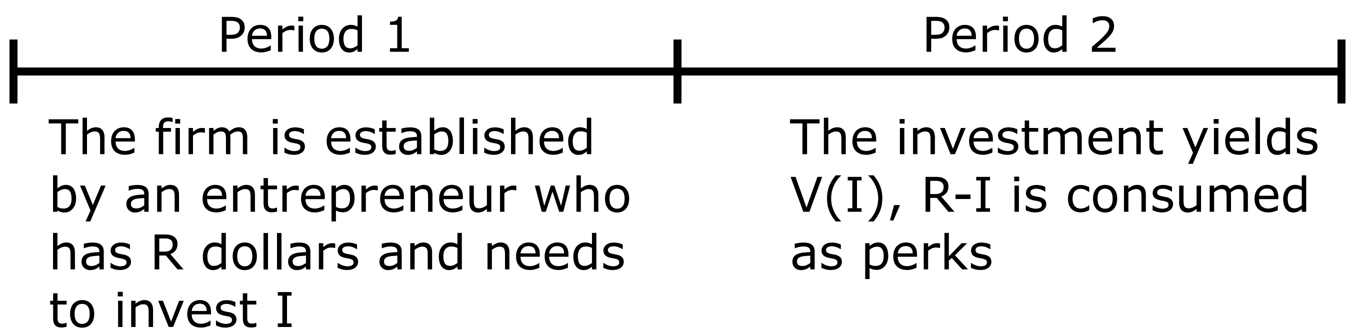
Yossi Spiegel
Recanati School of Business

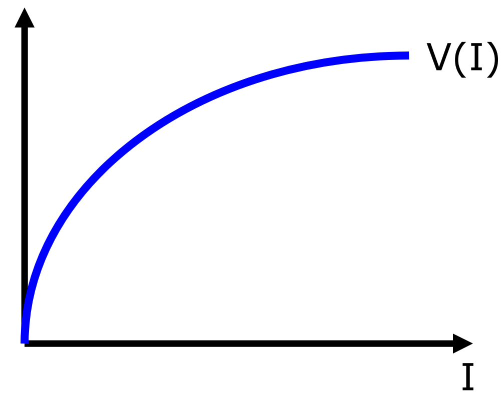
Jensen and Meckling, JFE 1976

The agency cost of outside equity

The investment model

- The timing:



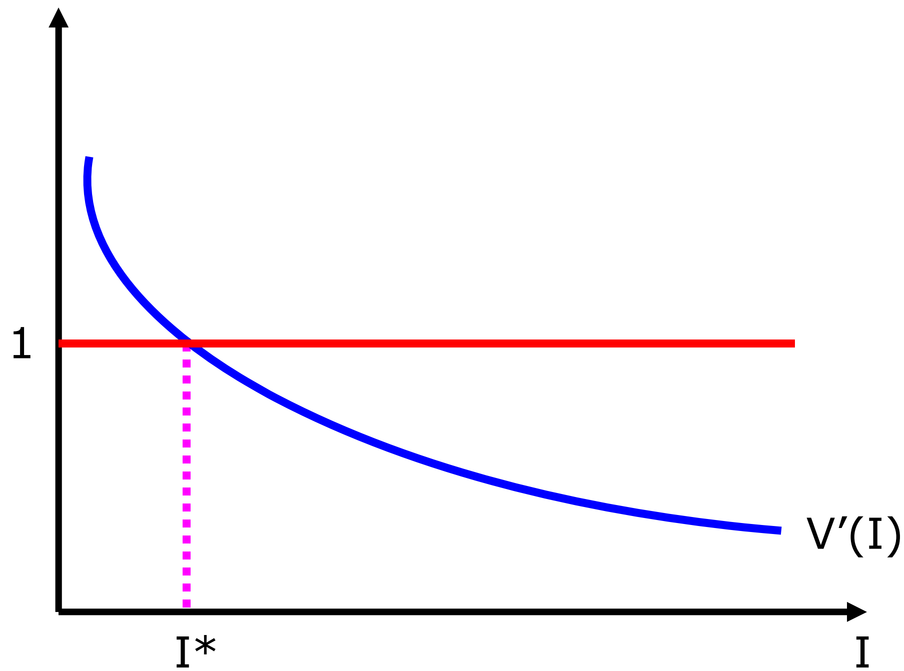


- $V'(I)>0>V''(I)$ and $V(0) = 0$
- $V'(0) = \infty$ and $V'(\infty) = 0$ (interior sol'n)
- The entrepreneur's payoff: $U(I) = V(I) + (R-I)$

Internal financing

- F.O.C for the entrepreneur's problem:

$$U'(I) = V'\left(I\right) - 1 = 0 \quad \Rightarrow \quad V'(I^*) = 1.$$



Internal financing

- The entrepreneur's payoff at the optimum:

$$U(I^*) = V\left(I^*\right) - I^* + R$$

- But what if I* > R? In that case the entrepreneur must have external financing

Debt financing

- The entrepreneur issues debt with face value D to raise I*- R upfront
- The entrepreneur's payoff:

$$U(I,D) = \underbrace{V(I) - D}_{\text{Payoff ex post}} + \underbrace{\overbrace{R + (I^* - R)}^{\text{Cash flow ex ante}} - I}_{\text{Perks}}$$

- U(I,D) is maximized at I* (the firm invests optimally)
- Debt is safe: ex post the firm has V(I*) > I* > I*-R ⇒ D* = I*-R
- The entrepreneur's payoff:

$$\begin{aligned} U(I^*, D^*) &= V(I^*) - D^* + R + (I^* - R) - I^* \\ &= V(I^*) - I^* + R \end{aligned}$$

Equity financing with commitment

- The entrepreneur issues equity with equity participation 1-α (the entrepreneur keeps α) and commits to invest I*
- To raise I*-R:

$$\underbrace{(1-\alpha)V(I^*)}_{E^*} = I^* - R \quad \Rightarrow \quad \alpha^* = 1 - \frac{I^* - R}{V(I^*)}.$$

- The entrepreneur's payoff:

$$U(I^*, \alpha^*) = \underbrace{\alpha^* V(I^*)}_{\text{Ex post payoff}} + \underbrace{E^* + R - I^*}_{\text{Perks}}$$

$$= V(I^*) - I^* + R$$

Equity financing without commitment

- The entrepreneur issues equity with equity participation 1-α, but cannot commit to invest I*

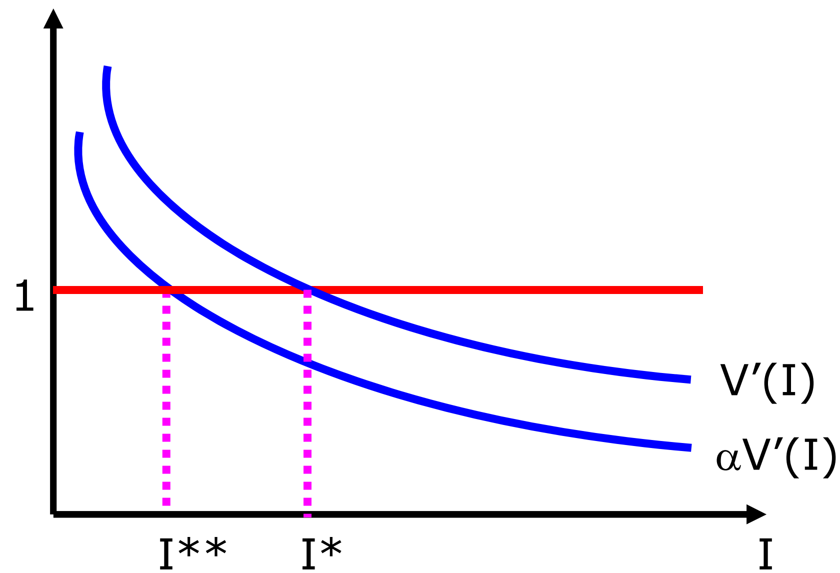
- After receiving E, but before choosing I, the entrepreneur's payoff:

$$U(I,\alpha) = \underbrace{\alpha V(I)}_{\text{Ex post payoff}} + \underbrace{E + R - I}_{\text{Perks}}$$

Equity financing without commitment

- F.O.C for the entrepreneur's problem:

$$U'(I,\alpha)=\alpha V'\left(I\right)-1=0 \quad \Rightarrow \quad \alpha V'(I^{**})=1.$$



- We get underinvestment
- α↓ (more outside equity) ⇒ I*↓ (more underinvestment)

The agency cost of outside equity

- The entrepreneur's payoff:

$$U(I^{**},\alpha) = \alpha V\left(I^{**}\right) + E^{**} - I^{**} + R$$
$$= \alpha V(I^{**}) + \left(1-\alpha\right)V(I^{**}) - I^{**} + R$$
$$= V(I^{**}) - I^{**} + R$$

- The entrepreneur's payoff with commitment:

$$U(I^*,\alpha) = V(I^*) - I^* + R$$

- By revealed preferences (and since I** < I*):

$$V(I^*) - I^* + R > V(I^{**}) - I^{**} + R$$

- The entrepreneur bears the cost of underinvestment (outside investors break even)

The optimal choice of α

- How does α affect U(I**,α)?

$$\frac{\partial U(I^{**},\alpha)}{\partial \alpha} = \underbrace{\left(\overbrace{V'(I^{**})}^{1/\alpha} - 1 \right)}_{(+)} \underbrace{\frac{\partial I^{**}}{\partial \alpha}}_{(+)} > 0$$

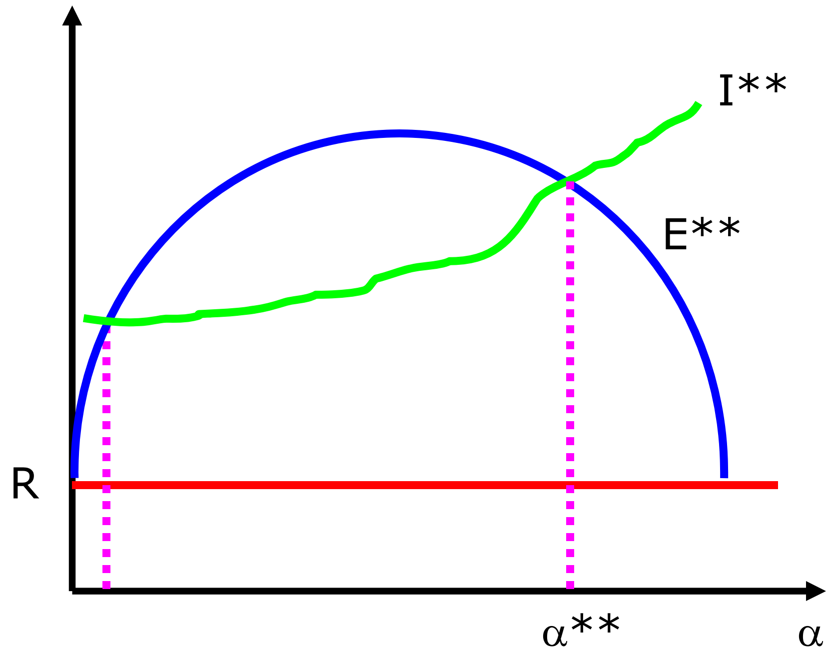
- The entrepreneur will raise α up to the point where

$$I^{**} = R + \underbrace{(1-\alpha)V(I^{**})}_{E^{**}}$$

- E**=0 at α = 0 (since then I**=0) and at α = 1 $\Rightarrow$ E** is inverse U-shaped

The optimal choice of α

- The entrepreneur's budget constraint:



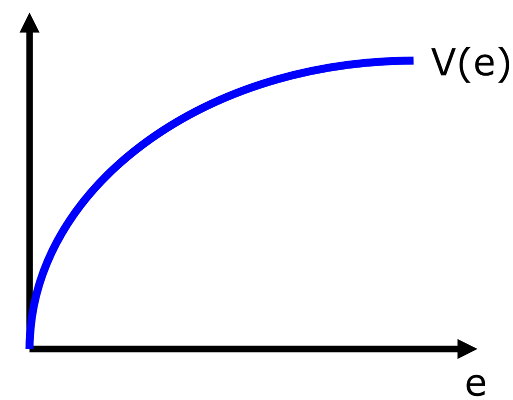
- The relevant sol'n is with the maximal α

The effort model

- The timing:

Period 1: The firm is established by an entrepreneur who has R dollars and needs to invest I and exert effort, e

Period 2: The effort yields V(e), the cost of effort is e (no perks)

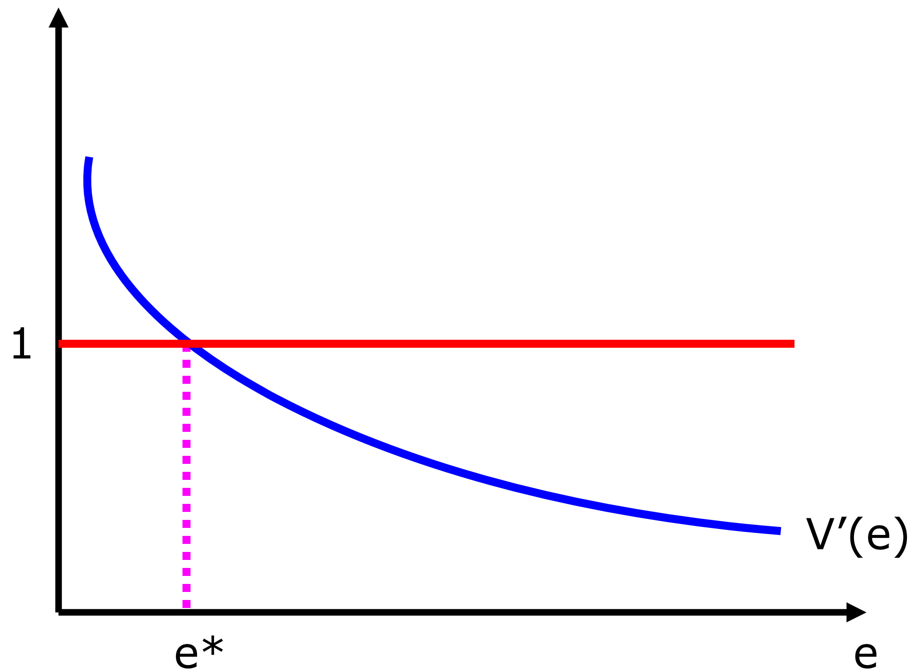


- $V'(e) > 0 > V''(e)$ and $V(0) = 0$
- $V'(0) = \infty$ and $V'(\infty) = 0$ (interior sol'n)
- The entrepreneur's payoff: $U(e) = V(e) - e$

Internal financing

- F.O.C for the entrepreneur's problem:

$$U'(e) = V'(e) - 1 = 0 \quad \Rightarrow \quad V'(e^*) = 1$$



Internal financing

- The entrepreneur’s payoff at the optimum:

$$U(e^*) = V(e^*) - e^*$$

- But what if I > R?

- Debt financing: debt is safe so D*=I-R

Equity financing without commitment

- The entrepreneur issues equity with equity participation 1-α

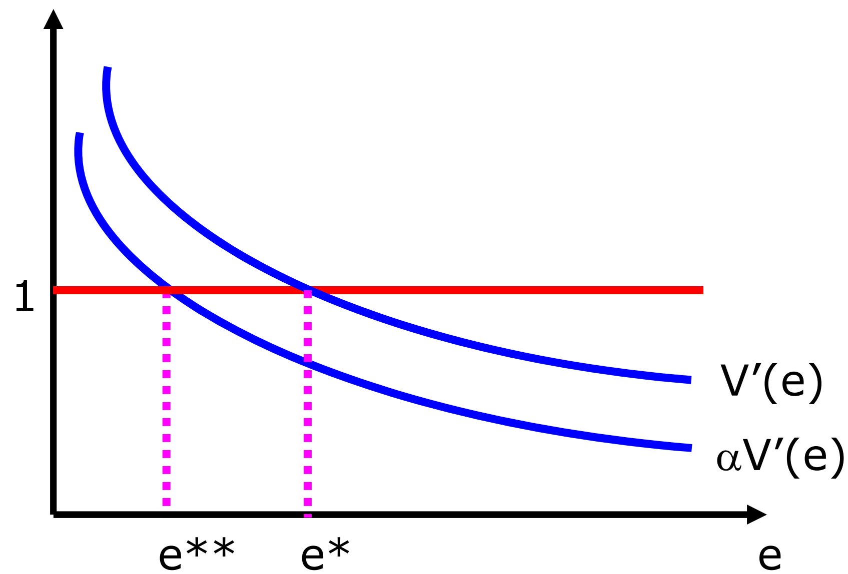
- After receiving E, but before choosing e, the entrepreneur's payoff is:

$$U(e,\alpha) = \alpha V(e) - e$$

Equity financing without commitment

- F.O.C for the entrepreneur's problem:

$$U'(e,\alpha) = \alpha V'(e) - 1 = 0 \quad \Rightarrow \quad \alpha V'(e^{**}) = 1$$



- We get underinvestment

The agency cost of outside equity

- The entrepreneur's payoff:

$$U(e^{**},\alpha)=\alpha V\left(e^{**}\right)-e^{**}$$

- The entrepreneur's payoff with commitment:

$$U(e^{*},\alpha)=\alpha V\left(e^{*}\right)-e^{*}$$

- By revealed preferences (and since e** < e*):

$$\alpha V(e^{*})-e^{*}>\alpha V(e^{**})-e^{**}$$

- The entrepreneur bears the cost of underinvestment (outside investors break even)

The optimal choice of α

- How does α affect U(e**,α)?

$$\frac{\partial U(e^{**},\alpha)}{\partial \alpha} = V(e^{**}) + \underbrace{\left(\alpha V'(e^{**}) - 1\right)}_{=0} \frac{\partial e^{**}}{\partial \alpha} > 0$$

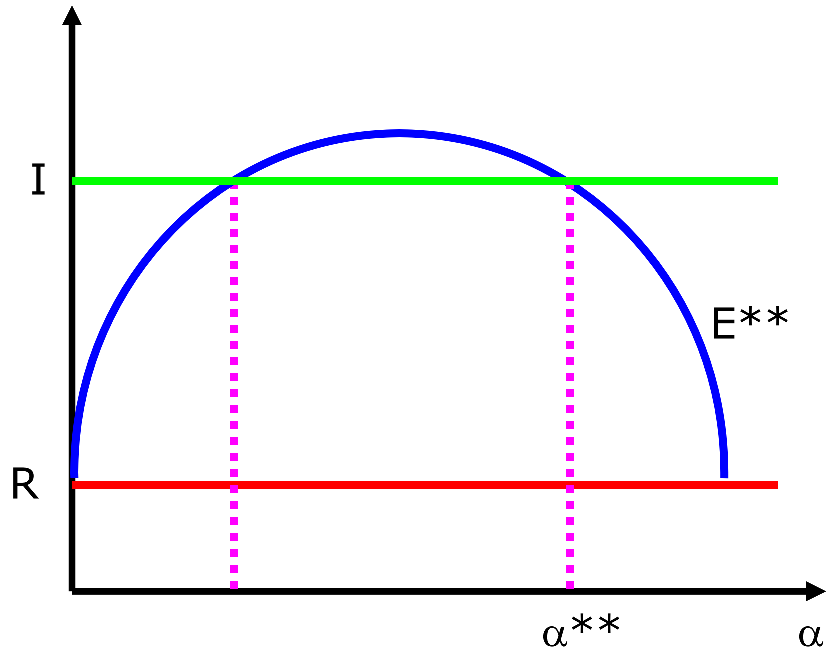
- The entrepreneur will raise α up to the point where

$$I = R + \underbrace{(1-\alpha) V\left(e^{**}\right)}_{E^{**}}$$

- E**=0 at α = 0 (since then e**=0) and at α = 1 $\Rightarrow$ E** is inverse U-shaped

The optimal choice of α

- The entrepreneur's budget constraint:



- The relevant sol'n is with the maximal α

Jensen and Meckling, JFE 1976

The asset substitution problem

A simple example

- Consider a box with two sealed envelopes: one with \$100 and the with \$0
- You can either pick one envelop from the box or receive \$70 for sure – what would you do?
- Now suppose you owe someone \$50 out of your gains
- If you take \$70, your payoff is \$70 - \$50 = \$20
- If you pick one of the sealed envelops then you either have \$100 - \$50 = \$50, or you pick the empty envelop and your payoff is 0 because you cannot pay the \$50 ⇒ your expected payoff is (\$0+\$50)/2 = \$25
- The lottery is better even though its NPV is only \$50

The model

- Two projects:
  - Safe project with return Z
  - Risky project with return $X \sim [0, \infty)$

- The firm has debt with face value D

- The management is perfect agent for equityholders – the agency problem is between equityholders and debtholders

Payoffs under the two projects

- Equityholders' payoff with the risky project:

$$Y_R = \int_D^{\infty} \left(X - D\right) dF(X)$$

- Equityholders' payoff with the safe project:

$$Y_S = Max\left\{Z - D, 0\right\}$$

- The firm surely chooses the risky project if $Z \leq D \Rightarrow$ assume that $Z > D$. Hence

$$Y_S = Z - D$$

Comparing the two projects

- The safe project is better iff:

$$Y_S > Y_R \quad \Leftrightarrow \quad Z > X^C(D) \equiv D + \int_D^\infty (X - D)\,dF(X)$$

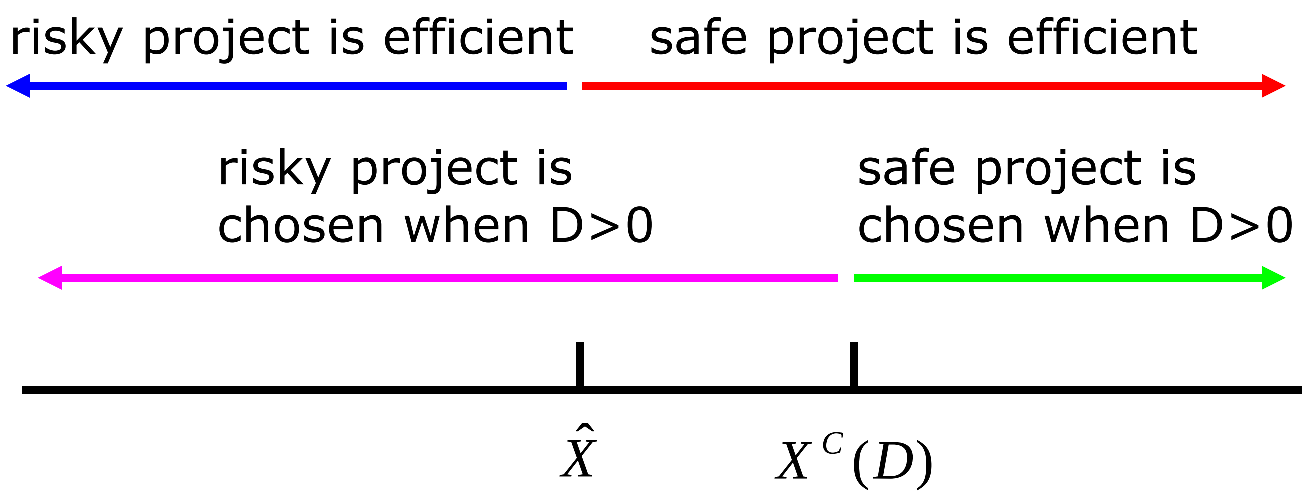
- Properties of $X^C(D)$:

$$\frac{\partial X^C(D)}{\partial D} = 1 - \int_D^\infty dF(X) = 1 - (1 - F(D)) = F(D)$$

$$X^C(0) = \int_0^\infty X\,dF(X) = \hat{X}$$

Choice of projects with leverage

- $X^C(D) > \hat{X}$ for all D > 0



- A leveraged firm prefers the risky project even when it is inefficient

Myers, JFE 1977

The debt overhang problem

The model

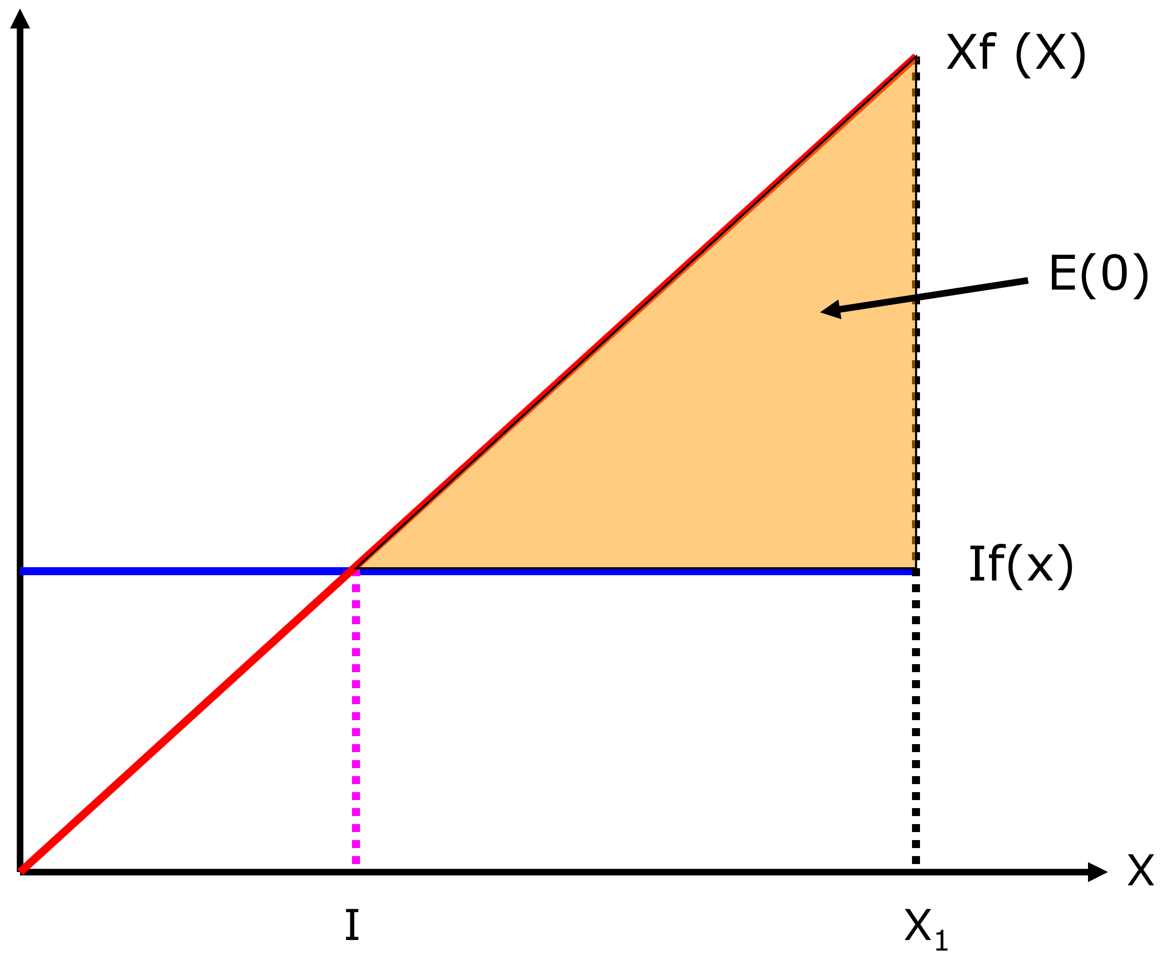
- The timing:

Period 1	Period 1.5	Period 2
The firm is established by an entrepreneur who issues debt with face value D	The entrepreneur learns the return, X, from a project that costs I and decides whether to invest	X is realized and debt is paid

- In period 1 it is common knowledge that $X \sim [0, \infty)$
- Absent debt, the firm invests iff $X \geq I$. The value of the firm:

$$V(0) = \int_{I}^{\infty} \left(X - I \right) dF(X)$$

Illustrating – All-equity firm



Short-term debt (due at period 1.5)

- Suppose debt has to be paid before it is time to invest
- If $X - I \geq D$, the firm will invest
- If $X - I < D$, the firm will not invest and will go bankrupt. The debtholders will invest provided that $X \geq I$
- The firm invests iff $X \geq I \Rightarrow$ investment is efficient

The value of the firm with short-term debt

- The value of debt:

$$B(D) = \int_{I}^{D+I} (X - I)\, dF(X) + \int_{D+I}^{\infty} D\, dF(X)$$

- The value of equity:

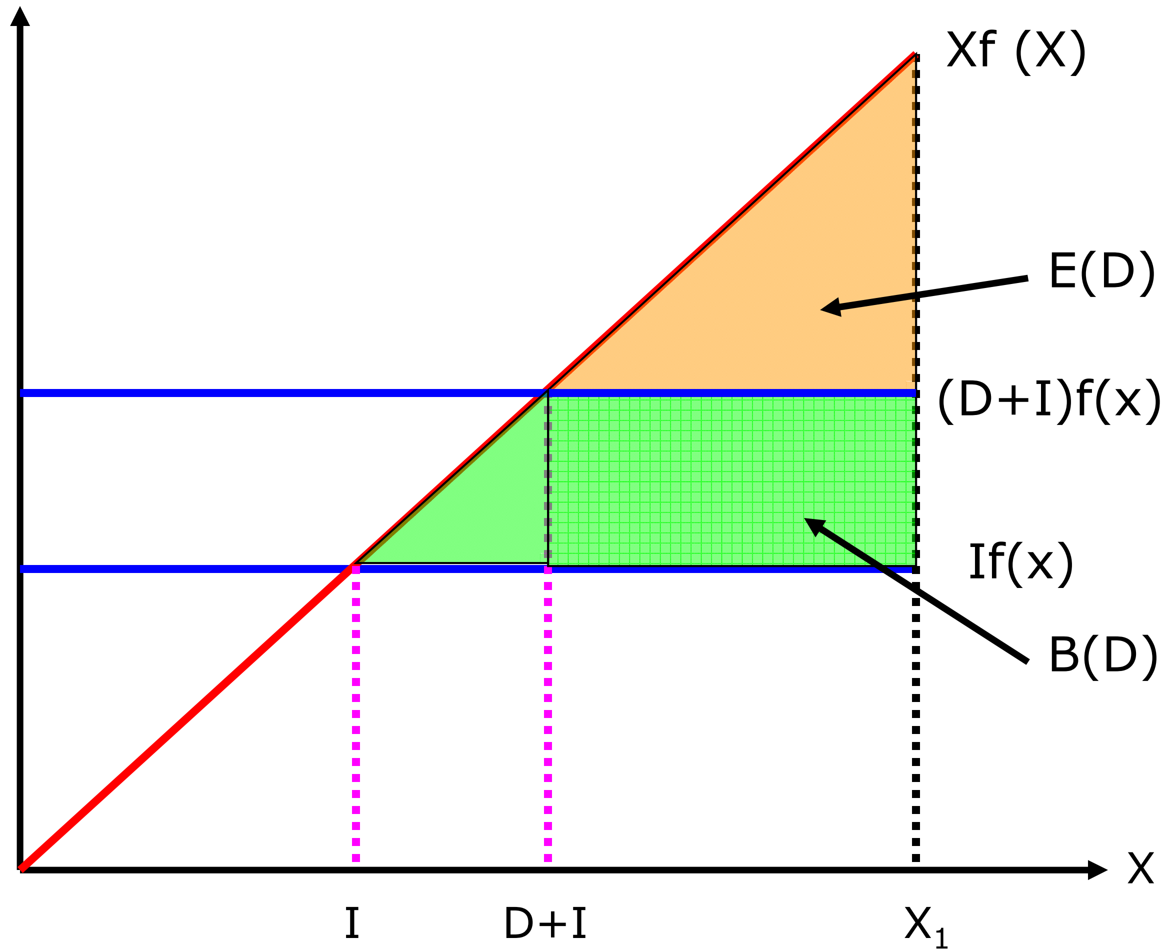
$$E(D) = \int_{D+I}^{\infty} (X - D - I)\, dF(X)$$

- The total value of the firm:

$$V(D) = \int_{I}^{D+I} (X - I)\, dF(X) + \int_{D+I}^{\infty} (X - I)\, dF(X)$$

$$= \int_{I}^{\infty} (X - I)\, dF(X)$$

Illustrating the debt overhang problem



Long-term debt

- Suppose debt has to be paid after it is time to invest
- If $X - I \geq D$, the firm will invest
- If $X - I < D$, the firm will not invest and will go bankrupt. The debtholders get a firm with no investment opportunities

The value of the firm with long-term debt

- The value of debt:

$$B(D) = \int_{D+I}^{\infty} D\,dF(X)$$

- The value of equity:

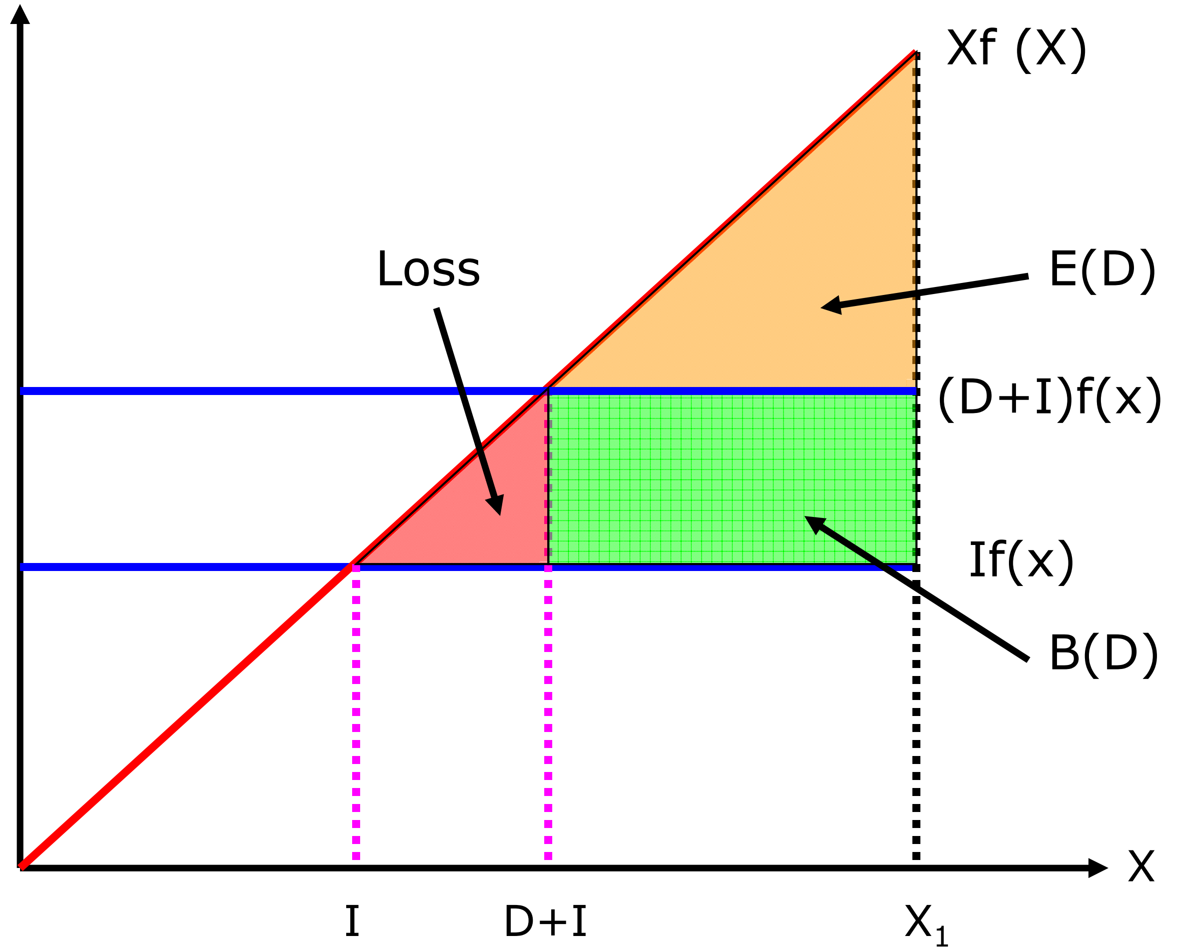
$$E(D) = \int_{D+I}^{\infty} \left(X - D - I\right) dF(X)$$

- The total value of the firm:

$$V(D) = \int_{D+I}^{\infty} \left(X - I\right) dF(X)$$

- The value is lower than under short-term debt. This is the debt overhang problem

Illustrating the debt overhang problem



Berkovitch and Kim, JF 1990

Overinvestment and underinvestment

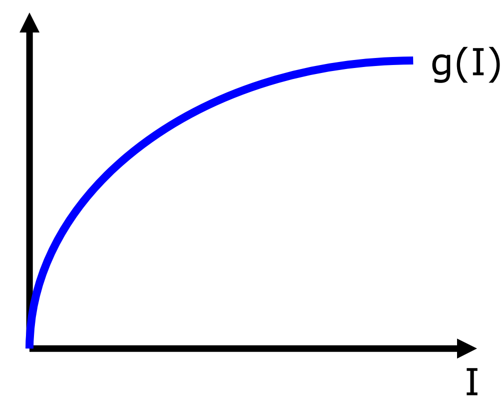
The model

- The timing:

Period 0: The firm is established by an entrepreneur who issues debt with face value D

Period 1: The cash flow X is realized and the firm invests I

Period 2: Investment yields $zg(I)$, D is due



- $g'(I) > 0 > g''(I)$ and $g(0) = 0$
- $g'(0) = \infty$ and $g'(\infty) = 0$ (interior sol'n)
- $z \sim [0, \infty)$

Excess funds: X > I

- Cash flow at the end of period 2:

$$zg(I) + X - I$$

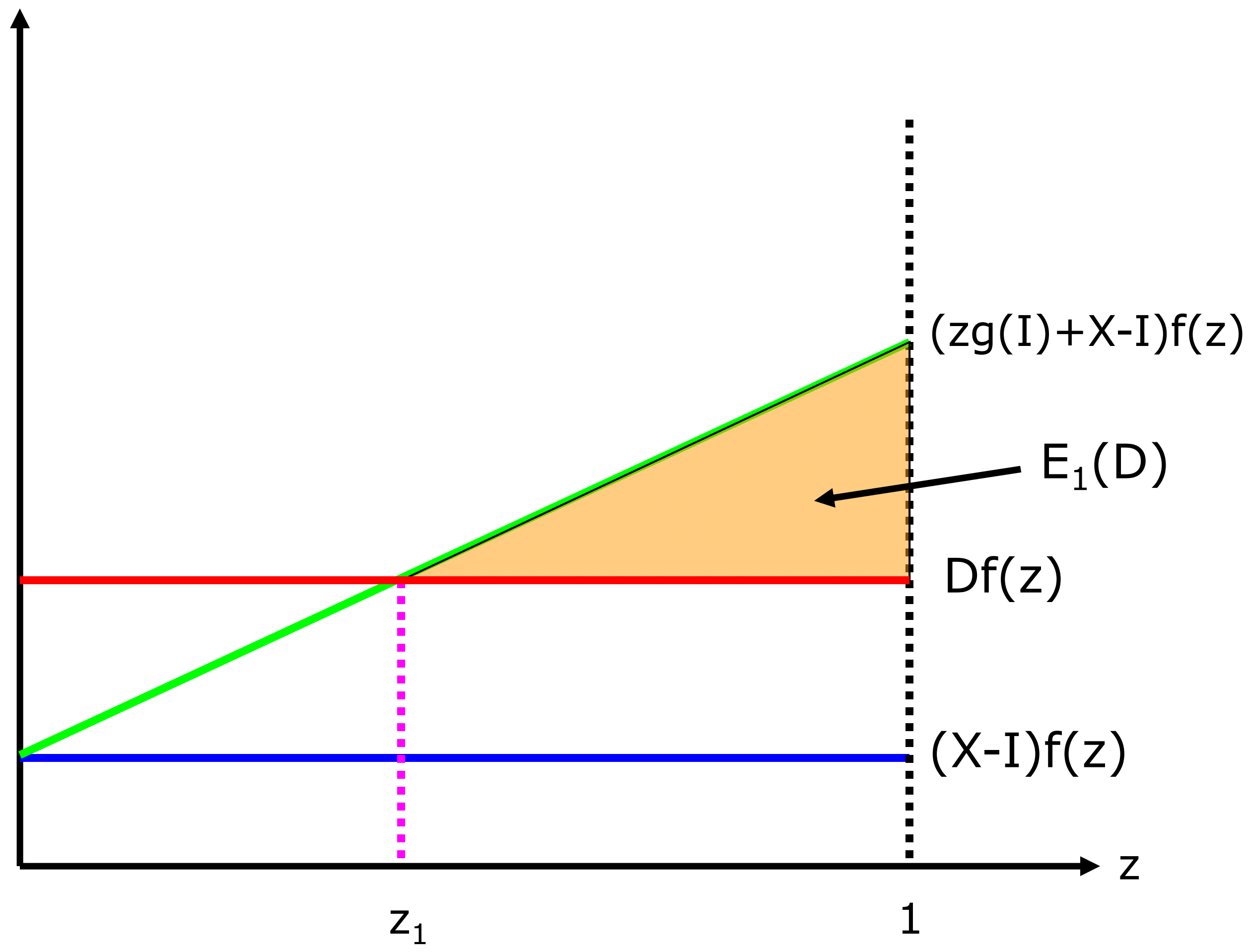
- The firm is solvent iff

$$zg(I) + X - I \geq D \quad \Rightarrow \quad z \geq z_1 \equiv \frac{D + I - X}{g(I)}$$

- The expected value of equity:

$$E_1(D) = \int_{z_1}^{\infty} \left(zg(I) + X - D - I \right) dF(z)$$

Illustrating – excess funds



Investment with excess funds

- F.O.C for investment:

$$\frac{\partial E_1(D)}{\partial I} = \int_{z_1}^{\infty} \left(z g'(I) - 1 \right) dF(z)$$

$$= g'(I) \int_{z_1}^{\infty} z dF(z) - \left(1 - F(z_1) \right) = 0$$

- Rewriting:

$$g'(I) = \frac{1}{\dfrac{\int_{z_1}^{\infty} z dF(z)}{1 - F(z_1)}} \equiv \frac{1}{E\left(z \mid z \geq z_1 \right)}$$

Deficit: X < I

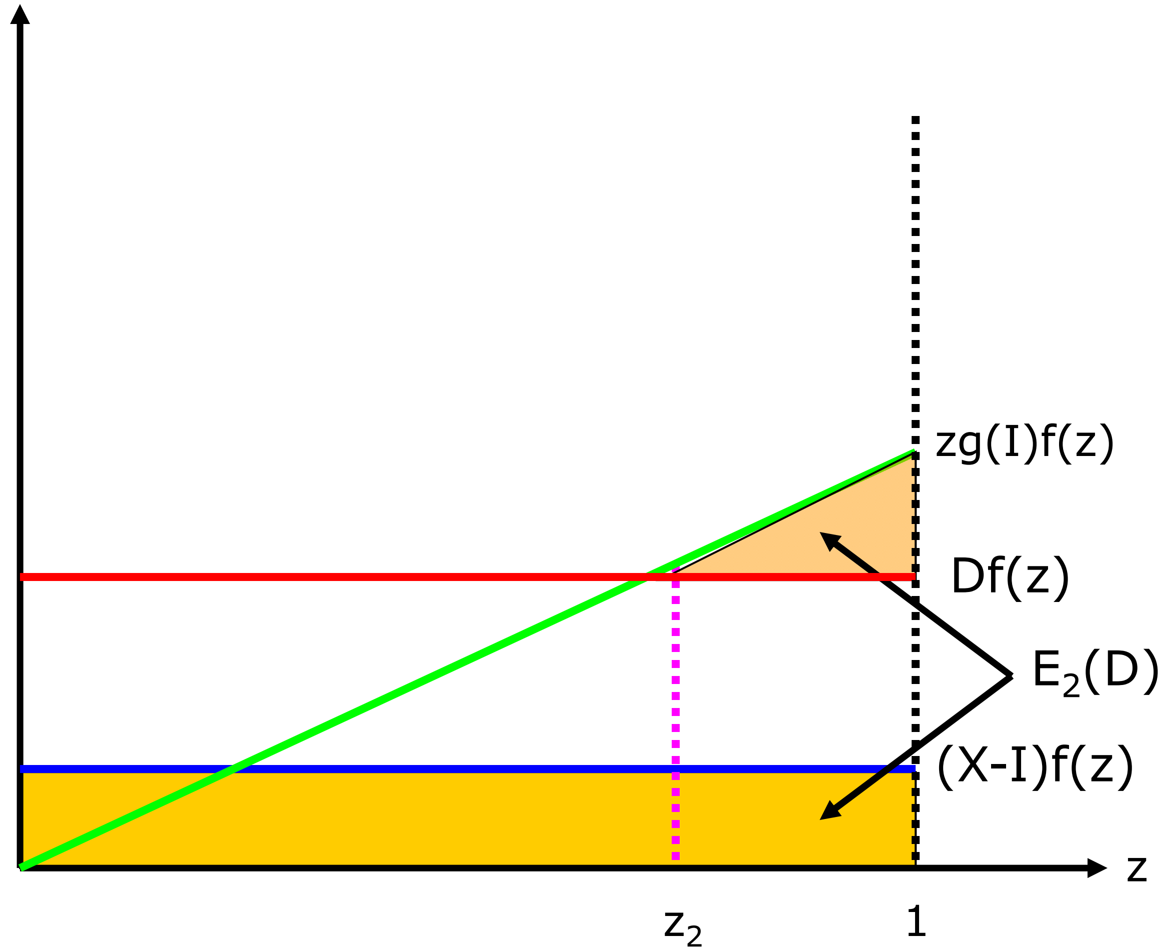
- The firm covers the deficit by issuing extra equity
- Cash flow at the end of period 2: zg(I)
- The firm is solvent iff

$$zg(I) \geq D \quad \Rightarrow \quad z \geq z_2 \equiv \frac{D}{g(I)}$$

- The expected value of equity:

$$E_2(D) = \int_{z_2}^{\infty} \left(zg(I) - D\right) dF(z) - \underbrace{(I - X)}_{\text{Extra funds to cover the deficit}}$$

Illustrating – deficit



Investment with deficit

- F.O.C for investment:

$$\frac{\partial E_2(D)}{\partial I} = \int_{z_2}^{\infty} zg'(I)dF(z) - 1$$

$$= g'(I)\int_{z_2}^{\infty} zdF(z) - 1 = 0$$

- Rewriting:

$$g'(I) = \frac{1}{\int_{z_2}^{\infty} zdF(z)}$$

The value of the firm with excess funds

$$V_1(D) = \underbrace{\int_{z_1}^{\infty} \left(zg(I) + X - D - I \right) dF(z)}_{\text{Equity}}$$

$$+ \underbrace{\int_{0}^{z_1} \left(zg(I) + X - I \right) dF(z) + \int_{z_1}^{\infty} D\, dF(z)}_{\text{Debt}}$$

$$= \int_{0}^{\infty} \left(zg(I) + X - I \right) dF(z)$$

$$= \hat{z} g(I) + X - I$$

The value of the firm with deficit

$$V_2(D) = \underbrace{\int_{z_2}^{\infty} \big(zg(I) - D\big) dF(z) - \big(I - X\big)}_{\text{Equity}} + \underbrace{\int_{0}^{z_2} zg(I) dF(z) + \int_{z_2}^{\infty} D dF(z)}_{\text{Debt}}$$

$$= \int_{0}^{\infty} zg(I) dF(z) - \big(I - X\big)$$

$$= \hat{z}g(I) + X - I$$

- The value of the firm is exactly as in the case of excess funds

Efficient investment

- F.O.C for investment:

$$\frac{\partial V(D)}{\partial I} = \hat{z}g'(I) - 1 = 0 \quad \Rightarrow \quad g'(I) = \frac{1}{\hat{z}}$$

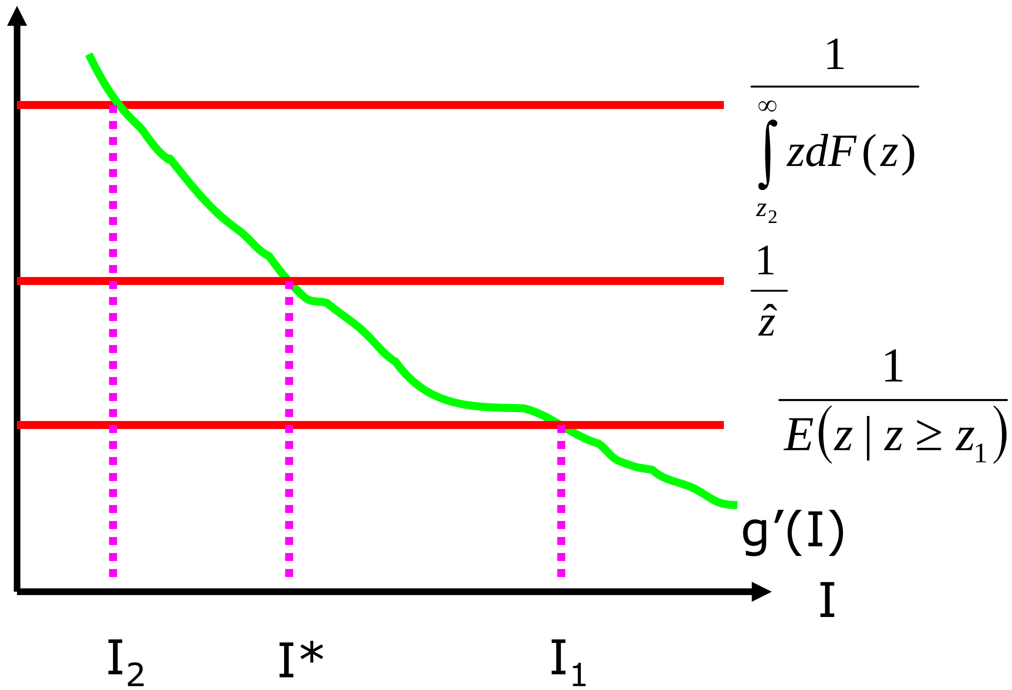
- Excess funds:

$$g'(I) = \frac{1}{\frac{\int_{z_1}^{\infty} z dF(z)}{1 - F(z_1)}} \equiv \frac{1}{E\left(z \mid z \geq z_1\right)} < \frac{1}{\hat{z}}$$

- Deficit:

$$g'(I) = \frac{1}{\int_{z_2}^{\infty} z dF(z)} > \frac{1}{\hat{z}}$$

Comparison



- Excess funds ⇒ overinvestment
- Deficit ⇒ underinvestment