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Solution to Problem set 3

Problem 1

(a) According to the arrangement, the bank forgives the old debt, B, and instead gives the firm in period 1 the amount I+qD-Y to ensure that it is solvent and can invest if this amount is positive or it receives this amount in period 1 if this amount is negative (in a sense then the firm forgives only B-(Y-I-qD) in period 1). Since the firm goes bankrupt for sure in period 2, only the bank receives in period 2 the entire expected cash flow. Now consider the case in which the bank is pari passue with the public debt in period 1: in that case, the bank receives the amount YB/(B+D) if the firm is liquidated in period 1. Hence, the condition that ensures that the bank agrees to lend money to the firm is given by

$$\underbrace{\hat{X}}_{\substack{\text{Ex. payoff}\\ \text{in period 2}}} - \underbrace{(I+qD-Y)}_{\substack{\text{Funds that the bank pays}\\ \text{or receives in period 1}}} \geq \underbrace{\frac{YB}{B+D}}_{\substack{\text{The bank's payoff if the firm}\\ \text{is liquidated in period 1}}} .$$

We can now rewrite this condition as

$$\underbrace{\hat{X}-I}_{NPV} = qD - Y + \frac{YB}{B+D}$$
$$= qD - \frac{YD}{B+D} \equiv qD - L_D .$$

This expression is the same as equation (5) in the paper (the only difference is that Gertner and Scharfstein denote the expected payoff of the firm by $\overline{X}$). Note that qD - L_D is the subsidy that debtholders receive when the firm restructures its bank debt.

If the bank is senior in period 1, then it receives the amount min{Y,B} if the firm is liquidated in period 1. Hence, the condition that ensures that the bank agrees to lend money to the firm is given by

$$\underbrace{\hat{X}}_{\substack{\text{Ex. payoff}\\ \text{in period 2}}} - \underbrace{(I+qD-Y)}_{\substack{\text{Funds that the bank pays}\\ \text{or receives in period 1}}} \geq \underbrace{\min\{Y,B\}}_{\substack{\text{The bank's payoff if the firm}\\ \text{is liquidated in period 1}}} .$$

We can now rewrite this condition as

$$\hat{X} - I = qD - Y + \min\{Y,B\}$$
$$= qD - \max\{Y-B,0\}.$$

This expression is the same as equation (6) in the paper. Note that qD – max{Y-B,0} is the subsidy that debtholders receive now when the firm restructures its bank debt since max{Y-B,0} is the value of debt if the firm is liquidated in period 1.

And, if the bank is junior in period 1, then it receives the amount max{Y-D,0} if the firm is liquidated in period 1. Hence, the condition that ensures that the bank agrees to lend money to the firm is given by

$$\underbrace{\hat{X}}_{\substack{\text{Ex. payoff} \\ \text{in period 2}}} - \underbrace{\left(I + qD - Y\right)}_{\substack{\text{Funds that the bank pays} \\ \text{or receives in period 1}}} \geq \underbrace{\max\{Y - D, 0\}}_{\substack{\text{The bank's payoff if the firm} \\ \text{is liquidated in period 1}}} .$$

We can now rewrite this condition as

$$\begin{aligned} \hat{X} - I &= qD - Y + \max\{Y - D, 0\} \\ &= qD - \min\{Y, D\}. \end{aligned}$$

This expression is the same as equation (6) in the paper. Here the subsidy that debtholders receive is min{Y,D}.

(b) Clearly,

$$\min\{Y,D\} > L_D > \max\{Y-B,0\}.$$

To see why, note first that if Y < D, then min{Y,D} = Y > L_D (since L_D is a fraction of Y). If Y > D, then min{Y,D} = D, and

$$\begin{aligned} D - L_D &= D - \frac{YD}{D+B} \\ &= \frac{D\left(D + B - Y\right)}{D+B} > 0, \end{aligned}$$

where the inequality follows because Y < D+B (otherwise the firm is not financially distressed). Similarly, notice that if Y < B, then max{Y-B,0} = 0 so clearly L_D > max{Y-B,0}. If Y > B, then max{Y-B,0} = Y-B, and

$$\begin{aligned} L_D - \max\{Y - B, 0\} &= \frac{YD}{D+B} - (Y - B) \\ &= \frac{YD - (Y - B)(D + B)}{D + B} \\ &= \frac{B(D + B - Y)}{D + B} > 0, \end{aligned}$$

where the inequality follows once again because Y < D+B.

The implication is that the more senior the bank's debt is, the higher is the threshold for the NPV of the project, $\hat{X}$ - I, above which the firm invests. Hence, if there is an underinvestment problem, seniority makes it worse. On the other hand, if there is overinvestment problem, seniority alleviates it. The intuition is that the more senior the bank's debt is, the more reluctant is the bank to invest since it gets paid a larger amount in period 1 and hence the smaller is the bank's incentive to reach a deal with the firm. Hence, investment has to be more attractive to induce the bank to reach a deal when its debt is more senior. Another way to look at it is that the more senior the bank's debt is, the smaller the subsidy that goes to debtholders so the less costly investment becomes.

(c) Since the firm goes bankrupt for sure in period 2 and since the new public debtholders are senior, their expected payoff in period 2 is equal to the entire expected cash flow in period 2. Since the new debtholders need to give the firm I+B+qD-Y in period 1 to ensure that the firm is solvent, the condition that ensures that the firm can issue the new senior public debt is given by

$$\underbrace{\hat{X}}_{\text{Ex. payoff in period 2}} - \underbrace{\left(I + B + qD - Y\right)}_{\substack{\text{Funds provided by new} \\ \text{debtholders in period 1}}} \geq 0.$$

Using the fact that $L_D+L_B = Y$, we can rewrite this condition as:

$$\underbrace{\hat{X} - I}_{\text{NPV}} \geq qD + B - Y = q_D - L_D + B - L_B,$$

where q_D - L_D is the subsidy that debtholders receive and B - L_B is the subsidy that the bank receives. The seniority of the existing bank's debt does not matter since both the bank and the existing debtholders are receiving a subsidy and both do not need to make any concessions (i.e., they are completely passive).

(d) Clearly, qD - max{Y-B,0} $\leq$ qD + B – Y. Hence, the firm adopts here an even higher threshold for the project's NPV: the project should now be higher than the threshold when the firm restructures its bank debt and replaces it with a senior bank debt and the existing bank debt is senior to the existing public debt. The intuition is that now, the bank gets paid in full without having to make any concessions. Hence only if the NPV is high enough to pay the bank in full can the firm issue new public debt and invest.

Problem 2

(a) To determine the range of values of R for which we have a soft budget constraints (SBC) problem we need to solve the maximization problem of the first creditor when the bad project is refinanced. To this end we need to solve the problem:

$$\underset{a}{Max}\, \pi(a) = aR - 2a^2 - 1$$

Recalling that a $\leq$ 1, it follows that the value of a which maximizes the problem is a* = Min{R/4, 1}. We will now consider two cases:

Case 1: R $\leq$ 4. In this case, a* = R/4 $\leq$ 1, so the payoff of the first creditor if he refinances the project is

$$\pi(a^*) = a^* R - 2(a^*)^2 - 1 = \frac{R^2}{8} - 1.$$

Now, $\pi(a^*) > 0$ if $R > (8)^{1/2} = 2.82$. Hence, whenever $2.82 < R \leq 4$, then $\pi(a^*) > 0$, and we have an SBC problem.

Case 2: R > 4: Now, a* = 1, so the payoff of the first creditor if he refinances the project is

$$\pi(1) = R - 2 - 1.$$

This expression is positive since R > 4 and hence we have an SBC problem.

Altogether then, we have an SBC problem if and only if R is above 2.82.

(b) If the first creditor is small, then the extra dollar for refinancing the project has to be raised from a second creditor. This creditor requires a payment C such that $\hat{a}C = 1$. Given this expression, the maximization problem of the first creditor is

$$\underset{a}{Max}\,\pi(a) = a\left(R - \frac{1}{\hat{a}}\right) - 2a^2 - 1$$

The first order condition for this problem is

$$R - \frac{1}{\hat{a}} - 4a = 0.$$

(c)+(d) In equilibrium, $\hat{a} = a$. Hence, the equilibrium condition is

$$R - \frac{1}{a} - 4a = 0.$$

Note that 1/a + 4a is a U-shaped function with a minimum at a = 1/2, where its value is 4. Hence, the above equation has a solution only if $R \geq 4$. Otherwise, the left hand side of the equation is negative for all a, so the first creditor will simply not exert any effort. In this case, we do not have an SBC problem because the entrepreneur will anticipate that a bad project will not be refinanced.

Next assume that $R \geq 4$. Then, the above equation has two solutions

$$a_1^* = \frac{R - \sqrt{R^2 - 16}}{8} \qquad a_2^* = \frac{R + \sqrt{R^2 - 16}}{8}.$$

From my power point slides you should note that a_2* is a stable equilibrium while a_1* is not. Let's focus on the stable equilibrium and let's assume it is below 1. For instance, if R = 4 then a_1* = a_2* = 0.5. Given a_2*, the payoff of the first creditor if he refinances the project is

$$\pi(a_2^*) = a_2^*\left(R - \frac{1}{a_2^*}\right) - 2\left(a_2^*\right)^2 - 1 = \frac{R^2 - 8 + R\sqrt{R^2 - 16}}{16} - 1.$$

An SBC problem exists if $\pi(a_2^*) > 0$ but not if $\pi(a_2^*) > 0$. If we solve $\pi(a_2^*) = 0$ (which is easy if you set $R^2 = x$ and solve for x), then $\pi(a_2^*) < 0$ for all $R < (18)^{1/2} = 4.24$ and $\pi(a_2^*) > 0$ for R > 4.24.

(e) We have an SBC problem if R > 2.82. Having a small creditor solves the SBC problem when R < 4.24 since then a small creditor will not refinance a bad project while a big creditor will. When R > 4.24 we have an SBC either way.