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Corporate Finance - Yossi Spiegel

Solution to Problem set 1

Problem 1

See the paper.

Problem 2

(a) The market value of equity is given by

$$E(D) = (1-t_E)\int_D^{100} (\tilde{X} - D)(1-t_C)dF(\tilde{X})$$

and the market value of debt is given by

$$B(D) = (1-t_D)\left[\int_0^D (1-b)\tilde{X}dF(\tilde{X}) + \int_D^{100} DdF(\tilde{X})\right]$$

The total value of the firm, V(D), is the sum of E(D) and B(D).

(b) When we differentiate V(D) we get

$$V'(D) = \underbrace{-(1-t_E)(1-t_C)\int_D^{100} dF(\tilde{X})}_{\text{Decrease in the value of equity}} + \underbrace{(1-t_D)\left[(1-b)Df(D) - Df(D) + \int_D^{100} dF(\tilde{X})\right]}_{\text{Change in the value of debt}}$$

$$= -(1-t_E)(1-t_C)(1-F(D)) + (1-t_D)[-bDf(D) + 1 - F(D)]$$

$$= \left[\underbrace{(1-t_D)}_{\substack{\text{Net income}\\ \text{from debt}}} - \underbrace{(1-t_E)(1-t_C)}_{\text{Net income from equity}}\right]\underbrace{(1-F(D))}_{\text{Prob. of solvency}} - \underbrace{b(1-t_D)Df(D)}_{\substack{\text{Increase in the ex.}\\ \text{cost of bankruptcy}}}$$

We can also write the first order condition

$$V'(D) = (1-F(D))b(1-t_D)\left[\frac{(1-t_D)-(1-t_E)(1-t_C)}{b(1-t_D)} - \frac{Df(D)}{1-F(D)}\right]$$

$$= (1-F(D))b(1-t_D)\left[\frac{(1-t_D)-(1-t_E)(1-t_C)}{b(1-t_D)} - DH(D)\right],$$

where H(D) is the hazard rate of the distribution of firm earnings.

(c) For D* = 0, it must be the case that V'(D) < 0 for all D > 0. This will be the case if $(1-t_D) < (1-t_E)(1-t_C)$: the net income from debt is less than the net income from equity.

(d) For D to be maximial, it must be that V'(D) > 0 for all D. Clearly, this cannot be the case if H(D) is increasing because then H(D) approaches infinity when D approaches 100 (the upper bound of the support). If H(D) is constant, then V'(D) > 0 for all D if the square bracketed term is positive at the largest D possible which is 100. This requires that b will be small.

(e) If there exists an interior solution then,

$$\frac{(1-t_D)-(1-t_E)(1-t_C)}{b(1-t_D)} = DH(D).$$

Assuming that H(D) is increasing, the comparative statics results depend on the left hand side of the equality. Hence, D* will increase when t_C is larger (more need for tax shields), when t_E is larger (equity is less attractive), and when t_D is smaller (debt is "cheaper").