Tel Aviv University
Faculty of Management

Corporate Finance - Yossi Spiegel

Problem set 4

Problem 1

The following question is based on Myers and Majluf (*JFE*, 1984). An entrepreneur establishes a firm in period 1 and needs to raise I dollars to invest in a project, where I < 1. The firm operates only once in period 2 and is then liquidated. The cash flow of the firm in period 2 equals X + 1 if the entrepreneur invests in the project, and X if he does not, where X is a random variable distributed uniformly on the interval [0, 1]. Suppose that the capital market competitive, and the realization of X is known only to the entrepreneur; outside investors know only the distribution of X but not its actual value.

(a) Prove that the entrepreneur will invest optimally if he can use debt financing.

(b) Suppose that the entrepreneur needs to finance I by issuing equity. Write out the condition that ensures that he will wish to finance the project and prove that this condition is more likely to hold when X is small.

(c) Now let X_H be the highest realization of X for which the entrepreneur chooses to invest. Given X_H, rewrite the condition that ensures that the entrepreneur will finance the project by issuing equity and prove that if $I \leq 3/4$, the entrepreneur will always invest regardless of the realization of X.

(d) Now suppose that I = 4/5. Which types will invest and which types will not?

Problem 2

The following question is based on Ross (*Bell J.*, 1977). The earnings of a firm are given by a random variable, X, distributed uniformly over the interval [0, T], where T = 3 if the firm is of type L and T = 5 if the firm is of type H. Before X is realized, the firm can issue debt with face value D to be paid after X is realized. If X < D, the firm becomes financially distressed and its manager suffers a personal loss of C (due say to a loss in reputation, or due the risk of being fired). If $X \geq D$, the firm has sufficient resources to pay its debt in full and the manager loses nothing. The objective of the manager is to maximize his own payoff, which is equal to the market value of the firm, net of his private cost of financial distress.

(a) Assume that T is common knowledge. Compute the market value of the firm, V, as a function of D (recall that V is the sum of the value of equity and the value of debt). Does V depend on D? Explain your answer.

(b) Using your answer to (a), write down the manager's payoff as a function of D and show that D = 0 maximizes this expression.

(c) Now assume that only the manager knows whether T = 3 or T = 5. Outside investors only know that it is either L or H but not its true value. Draw a Figure in the (D, V) space (i.e., D is on the X axis and Y on the Y axis) that shows all the combinations of D and V that give the manager of an L-type the same payoff as in (b).

(d) What is the maximum amount of debt, D*, that an L-type firm would be willing to issue in order to try and fool outside investors into believing that its type is H?

(e) Explain why, under the Dom criterion, an H-type can signal his type by issuing debt with face value D*, and show that it is better-off doing so than issuing no debt at all in which case outside investors would believe that its type is L.

(f) What are the empirical implications of the model? In particular, what is the correlation between debt and firm value, and how does debt vary with C and with the difference between H and L? Which empirical regularities can be explained by this model? (See my notes on empirical evidence on capital structure that appears in the course's webpage, week 2 of the course).

(g) Suppose that ex ante, investor believe that the firm is L-type with probability α. Compute the set of pooling equilibria under the Dom criterion.

Problem 3

Consider the following variant of Bhattacharya (*Bell J.* 1979). A firm is established in period 0 and operates in period 1. The cash flow of the firm in period 1, $\tilde{X}$, is drawn from a uniform distribution on the interval [0, X]. In period 0, the manager commits the firm to a dividend payment, y, that will be paid out in period 1 after $\tilde{X}$ is realized. If in period 1, y > $\tilde{X}$, the firm becomes financially distressed and bears a fixed cost C > 0. The objective of the manager when choosing y is to maximize the expression $W = bV_0 + (1-b)V_1$, where $0 < b < 1$, V_0 is the expected value of the firm in period 0, and V_1 is the expected value of the firm in period 1. Assume that dividends are taxed at personal income tax t, while capital gains are tax free.

(a) Suppose that the upper bound of the support of the firm's cash flow, X, is common knowledge. Explain why in that case, $V_0 = V_1$. Express V_0 as a function of y and compute the value of y that maximizes V_0. Explain the intuition for your answer.

(b) Now suppose that only the manager knows the value of X, while outside investors only know that it is either equal to H or L, with H > L. Once outside investors observe y, they revise their belief on X. What is the amount of dividend that the manager of an L-type firm would commit to pay in a separating equilibrium? Explain why.

(c) Draw a figure with y on the horizontal axis and V_0 on the vertical axis that shows all the combinations of y and V_0 that give the manager of an L-type firm the same payoff as in (a).

(d) Draw in the same figure the period 0 value of a firm that is believed to be an H-type firm.

(e) What is the maximum amount of dividend, y*, that a manager of an L-type firm would be willing to pay in order to fool investors into believing that the firm is an H-type?

(f) Show your answer in the figure.

(g) Explain why, under the Dom criterion, the manager of an H-type firm can signal his type by choosing y*, and show that he is indeed better-off doing so than not paying any dividend and being believed by investors to be a manager of an L-type firm.

(h) What are the empirical implications of the model? In particular, what is the correlation between dividends and firm value and how does y* change with the cost of distress, C, the tax rate t, the degree of information asymmetry (i.e., H-L), and the degree of managerial myopia, b?

(i) Let $A = V_H(y^*) - V_L(0)$ be the difference between the equilibrium values of an H-type and an L-type firms, and let A/y* be a measure of the bang-for-the-buck effect of dividend payments. How does A/y* vary with t? What does it mean in words?

Problem 4

The following is a simple agency theory of dividends, based on Bernheim and Wantz (*AER*, 1995). A firm is established in period 0 and operates in period 1. The cash flow of the firm in period 1 is X. Given X, the firm's manager chooses a dividend payment, y. The retained earnings of the firm, X-y, are invested and yield a return of $(X-y)^{1/2}$. Dividend payments are taxed at a personal tax rate t, while capital gains are tax free. Hence, given y, the cum dividend value of the firm is $V(y) = (X-y)^{1/2} + (1-t)y$. The objective of the manager when he chooses y is to maximize the sum of V(y) and his own benefits of control which are equal to $b(X-y)^{1/2}$, where b is a positive parameter.

(a) Compute the first order condition for the manager's problem, and explain it carefully.

(b) Verify that the second order condition is satisfied, find a condition that ensures that y* > 0, and assuming that this condition holds, solve for the optimal dividend payment, y*.

(c) How is y* affected by t, b, and X? Explain the intuition.

(d) Given y* compute the value of the firm V(y*).

(e) Now assume that b is private information for the manager but it is common knowledge that b is drawn from a uniform distribution on the unit interval. What is the expected value of the firm, EV(y*), before the dividend payments are made?

(f) Let A(y) = V(y)-EV(y*) be the change in firm value due to dividend payments, and define A'(y*) (the derivative of A(y) evaluated at y*) as the marginal bang-for-the-buck, i.e., the change in firm value due to the last dollar of dividend. How does A'(y*) vary with t?

(g) Contrast this result with the corresponding result in Bhattacharya (1979).