### **TOPIC 6: CAPITAL STRUCTURE AND THE MARKET FOR CORPORATE CONTROL**

# 1. Introduction

## 2. The free rider problem

In a classical paper, Grossman and Hart (*Bell J.*, 1980), show that there is a fundamental problem with takeovers. If each of the existing shareholders holds a small amount of shares then no takeover will ever take place. Grossman and Hart refer to this as the "free-rider" problem. To understand the nature of the free rider problem, consider a firm whose current value is X. If we normalize the number of outstanding shares to 1, X also represents the market value of each outstanding share. Now suppose that a raider comes along and wants to take over the firm. If the raider is successful and buys enough shares (say more than half of the outstanding shares), he gains control over the firm and can enhance its value by R, so the value of the firm becomes X+R. In order to buy shares, the raider needs to make a tender offer to the current shareholders that specifies a the price that they would get for each share they tender. For simplicity, suppose that the offer is conditional on being successful: that is the raider will buy shares at the specified price provided that enough shares are tendered to ensure that he can take over the firm. Otherwise the raider buys no shares and pays nothing. Obviously the price that the price is X+P where P is the premium on each share above its current value. How big should P be in order to induce shareholders to tender their shares?

To answer this question, consider an individual shareholder who holds a fraction  $\alpha$  of the firm's

equity. If the shareholder does not tender his shares, his expected payoff is

$$\boldsymbol{\alpha} \left( \boldsymbol{\gamma}_1 (\boldsymbol{X} + \boldsymbol{R}) + (1 - \boldsymbol{\gamma}_1) \boldsymbol{X} \right) = \boldsymbol{\alpha} (\boldsymbol{X} + \boldsymbol{\gamma}_1 \boldsymbol{R}), \tag{1}$$

where  $\gamma_1$  is the probability that the takeover succeeds given that the shareholder did not tender his shares. That is, with probability  $\gamma_1$  the takeover succeeds and the value of the shares is X+R and with probability 1- $\gamma_1$  the takeover fails and the value of the shares is X. If the shareholder tenders his shares, his expected payoff is

$$\boldsymbol{\alpha}\left(\boldsymbol{\gamma}_{2}(\boldsymbol{X}+\boldsymbol{P}) + (1-\boldsymbol{\gamma}_{2})\boldsymbol{X}\right) = \boldsymbol{\alpha}(\boldsymbol{X}+\boldsymbol{\gamma}_{2}\boldsymbol{P}), \tag{2}$$

where  $\gamma_2$  is the probability that the takeover succeeds given that the shareholder tenders his shares.<sup>1</sup> Obviously,  $\gamma_2 > \gamma_1$  since if the shareholder tenders his share there is a higher likelihood that the raider will be able to buy enough shares to gain control over the firm. Comparing the expected payoffs of the shareholder when he does not tender his share and when he does, it is clear that tendering shares is an optimal strategy if and only if:

$$\gamma_2 \boldsymbol{P} \geq \gamma_1 \boldsymbol{R}. \tag{3}$$

However if each shareholder holds a negligible amount of shares, the difference between  $\gamma_2$  and  $\gamma_1$  is also negligible so the takeover can be successful if and only if  $P \ge R$ . This means that the raider should pay the whole increase in firm value to shareholder, implying in turn that his payoff is 0 since he now owns shares worth X+R each but pays for each share X+R. So if the takeover involves some transaction costs or requires an effort on the part of the raider, his payoff from taking over the firm is negative and hence he is better off not making the tender offer in the first place. The problem is referred to as the free-rider problem since each shareholders is better-off not tendering his own shares unless  $P \ge R$  because he fails to take into account his own impact on the likelihood of a takeover and instead free rides on the

<sup>&</sup>lt;sup>1</sup> Note that absent the assumption that the tender offer is conditional on being successful, the payoff of a tendering shareholder would have been  $\alpha P$  regardless of whether the takeover is successful or not.

willingness of other shareholders to submit shares and enable the raider to take over the firm. In other words, so long as 0 < P < R, each shareholder wants other shareholders to tender their shares to ensure that the takeover is successful but wishes to hold on to his own share and obtain the full increase in firm value.

#### 2.1 Dilution as a solution to the free rider problem

A possible solution to the free-rider problem, offered by Grossman and Hart (1980), is to write a corporate charter that allows the raider, once he takes over the firm, to dilute the share value of the non-tendering shareholders. This can be achieved by either allowing the raider to pay himself excessive salary, or issue to himself new shares at below their value, or sell assets or output at below their cost to another firm owned by the raider. In any event, whatever form it takes, the dilution make it undesirable for small shareholders to hold on to their shares as they are not going to enjoy the full increase in firm value due to the takeover.

To see how dilution works, suppose that the corporate charter allows the raider to dilute the value of the firm after he takes it over by  $\phi$ . Given a premium P, a shareholder who tenders his shares gets the same expected payoff as in equation (2), i.e.,  $\alpha(X+\gamma_2P)$ . If the shareholder does not tender, his expected payoff is  $\alpha(X+\gamma_1(R-\phi))$ . This payoff is similar to that in equation (1) except that R is replaced by R- $\phi$ . Assuming as before that individual shareholders are small and have no influence on the likelihood that the takeover will succeed (i.e.,  $\gamma_1 = \gamma_2$ ), it follows that tendering shares is an optimal strategy if and only if

$$\boldsymbol{P} \geq \boldsymbol{R} - \boldsymbol{\phi}. \tag{4}$$

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This inequality implies that the optimal strategy for the raider's is to offer a price,

$$\boldsymbol{P}^* = \boldsymbol{R} - \boldsymbol{\Phi}. \tag{(5)}$$

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It should be noted that P\* could be less than X in which case the equilibrium is supported by the "pessimistic" belief of each shareholder that other shareholders are going to tender their shares and hence it is better to tender and get X+P than not to tender and get X+R- $\phi$ . Although there is another (arguably more reasonable) equilibrium in which shareholders believe that others will not tender if P < X and hence not tendering is a best-response given this belief, I choose to consider the pessimistic beliefs case since this simplifies the exposition (otherwise P\* = Max{R- $\phi$ ,X}).

Now let C be the cost that the raider needs to incur in order to initiate a takeover. Since the premium P\* ensures that all shareholders tender their shares, the raider's payoff is:

$$X + R - P^* - C = X - C + \phi.$$
(6)

This equation implies that a takeover will take place if and only if  $C < X+\phi$ .

Now suppose that when the corporate charter is written, the identity of the raider, and hence the value of C, is yet unknown. Assume that the founders of the firm are interested in maximizing the market value of equity when the firm is just established and they believe that C will be drawn from some distribution function F(C) on the interval  $[0, \infty)$ . Since the condition for a takeover is  $C < X+\phi$ , the ex ante probability that the firm will be taken over is  $F(X+\phi)$ . Thus, the market value of equity when the firm is just established is

$$V(\phi) = F(X + \phi)(X + P^*) + (1 - F(X + \phi))X = X + F(X + \phi)(R - \phi).$$
<sup>(/)</sup>

The founders of the firm choose  $\phi$  so as to maximize V( $\phi$ ). Let  $\phi^*$  be the optimal value of dilution. When choosing  $\phi^*$ , the founders of the firm face the following tradeoff: as  $\phi$  grows towards R, the likelihood of a takeover increases towards 1 since no shareholder would wish to retain his shares knowing that after the takeover his holdings in the firm can be diluted to 0. This is the benefit from raising  $\phi$ . On the other hand, when  $\phi$  grows, the market value of each share falls since shareholder realize that given that  $\phi$  is high, the premium that a raider would offer in a tender offer (if it takes place) is small. The first order condition for an interior solution for  $\phi^*$  is given by:

$$V'(\mathbf{\phi}) = f(X + \mathbf{\phi})(\mathbf{R} - \mathbf{\phi}) - F(X + \mathbf{\phi}) = 0.$$
<sup>(8)</sup>

The first term in this condition is the marginal benefit from  $\phi$  associated with the increase in the likelihood of a takeover while the second term is the marginal cost of  $\phi$  due to the reduction in the price that shareholders get in case of a successful takeover.

To examine the properties of  $\phi^*$ , note first that if R > F(X)/f(X), the optimal dilution factor  $\phi^*$ will be positive since in that case, V'(0) > 0. Moreover, note that  $\phi^*$  cannot exceed R since then the market value of each share will fall to 0. Hence, assuming that R > F(X)/f(X),  $\phi^*$  will be between 0 and R.

Second, as R grows (the founders of the firm expect a "better" raider), the marginal benefit of  $\phi$  grows and hence the founders will choose a higher dilution factor  $\phi^*$ . This result is intuitive since an increase in R means that the founders are more eager to see their firm being taken over by a raider and hence they will try to make it easier for a raider to succeed by allowing him to dilute the non tendering shareholders to a greater extent.

Third, we can ask how  $\phi$  is affected by the status quo value of the firm, X. Fully differentiating the first order condition for  $\phi^*$  with respect to  $\phi$  and X we obtain:

$$\frac{\partial \boldsymbol{\Phi}^*}{\partial X} = -\frac{f'(X + \boldsymbol{\Phi}^*)(\boldsymbol{R} - \boldsymbol{\Phi}^*) - f(X + \boldsymbol{\Phi}^*)}{V''(\boldsymbol{\Phi}^*)}.$$
(9)

Since  $V''(\phi^*) < 0$  by the second order condition for maximization, the sign of the derivative depends on the sign of the numerator. Using the first order condition it follows that

$$\boldsymbol{R} - \boldsymbol{\phi} = \frac{F(X + \boldsymbol{\phi})}{f(X + \boldsymbol{\phi})}.$$
(10)

Substituting for  $R-\phi$  in equation (9) yields:

$$\frac{\partial \boldsymbol{\Phi}^*}{\partial X} = -\frac{f'(X + \boldsymbol{\Phi}^*)F(X + \boldsymbol{\Phi}^*) - f^2(X + \boldsymbol{\Phi}^*)}{V''(\boldsymbol{\Phi}^*)f(X + \boldsymbol{\Phi}^*)}.$$
(11)

The sign of this expression depends on the sign of the derivative of f(.)/F(.). If f(.)/F(.) is increasing at  $X+\phi^*$  then a slight increase in X will lead to an increase in  $\phi^*$  and if f(.)/F(.) is decreasing at  $\phi^*$ , a slight increase in X will lead to a decrease in  $\phi^*$ . Hence, we do not have a sharp prediction about the change in  $\phi^*$  when the status quo value of the firm changes as  $\phi^*$  may either increase or decrease.

### 2.2 Toeholds as a solution to the free-rider problem

#### 2.3 The case of nonatomistic shareholders

In the paper "To the Raider goes the Surplus? A reexamination of the Free-Rider Problem," Holmstrom and Nalebuff (*JEMS*, 1992) show that the free-rider problem of Grossman and Hart goes away if we assume that shareholders are nonnegligeble. In fact they show that in that case there exist a mixed strategy equilibrium in which the raider can capture a positive surplus....

### 3. Outside equity and takeovers

One of the first papers to explore the linkage between capital structure and competition in the market for corporate control is "Managerial Control of Voting Rights: Financing Policies and the Market for Corporate Control" by René Stultz (1988, *JFE*). The paper shows that by issuing equity to outsiders, a firm can influence the likelihood and the conditions of a future takeover.

Consider the following simplified version of Stultz (1988). An entrepreneur establishes a firm in

period 0 and needs to raise K dollars from external funds for investment. The cash flow of the firm in period 1 if the entrepreneur retains control over the firm is X where X > K. However before period 1 begins, a raider shows up and may take over the firm. For simplicity, assume that the cash flow under the raider is also X. Although the raider does not enhance the cash flow of the firm, he may nonetheless wishes to take it over because he obtains a private, non pecuniary, benefit of control  $B_r$  by running the firm. In period 0 when the firm is just established, the entrepreneur knows in advance that the raider will show up but he still does not know at that point how large  $B_r$  will be. To capture this uncertainty, let us assume that the entrepreneur believes that  $B_r$  is drawn from a distribution function  $F(B_r)$  on the interval  $[0, \infty)$ .

In order to take over the firm, the raider needs to buy at least half of the outstanding shares. A main assumption in the model is that the entrepreneur never sells his own shares to the raider, presumably because he also obtains non pecuniary benefit of control,  $B_e$ , that are lost once he loses control over the firm. Therefore, in order to take over the firm, the raider needs to buy enough shares from outsiders to ensure that eventually he will control half of the firm's shares. Outsiders are willing to sell their shares only if they get a large enough price for each share.

On the face of it, it seems that since the cash flow of the firm is independent of who controls it, we do not have a free raider problem as in Grossman and Hart (1980, *Bell J.*) and so each shareholder will sell his shares for X. Hence, if we normalize the number of outstanding shares to 1 (so that X is also the value per share), it seems that the price of the shares should be X and the raider should be able to buy as many shares as he wants at that price. However, Stultz assumes that outsiders are heterogeneous in the sense that different shareholders have different reservation values for shares. The difference in the reservation values across different shareholders could be due for example to different levels of capital gains taxes that different shareholders have to pay when they sell their shares. This heterogeneity of shareholders gives raise to an increasing supply function of shares, S(P), that describes the fraction of

outsider shareholders (who control in total a fraction 1- $\alpha$  of the firm's equity) who are willing to sell their shares when they are offered a price of X+P, where P is a "premium" on shares above their current value (note that since the number of shares was normalized to 1, P is the total premium and the per share premium). The minimal P that the raider has to offer in order to obtain half of the firm's equity is given implicitly by:

$$(1-\alpha)S(\boldsymbol{P}) = \frac{1}{2}.$$
 (12)

Let  $P(\alpha)$  be the solution to equation (12).  $P(\alpha)$  is illustrated in Figure 1 under the assumption that S(P) is linear, although in general it need not be linear. Since S(P) is increasing, it is clear that in order to buy 1/2 of the firm's equity the raider needs to pay more than X which is the minimal price that has to be paid to induce shareholders to tender shares. Moreover, since an increase in  $\alpha$  shifts the supply function of shares to the left, it leads to an increase in  $P(\alpha)$ . Hence,  $P'(\alpha) > 0$ . The intuition for this is that since the raider needs to buy 1/2 of the firm's shares, then if  $1-\alpha$  is small, there is simply insufficient supply of "cheap" shares and this forces the raider to also buy "expensive" shares from investors who require a large P to tender their shares. Since the raider cannot discriminate among investors, the raider has to offer the same high P to everybody and this leads to a large premium. In other words, the more equity is held by the entrepreneur, the less equity is left for outsiders, so in order to take over the firm, the premium has to be relatively large.

Since the raider must offer a premium of  $P(\alpha)$  in order to gain control over the firm, his payoff if he takes over the firm is  $B_r - P(\alpha)$ . Given this payoff, it is clear that the raider is willing to take over the firm if and only if  $B_r > P(\alpha)$ . Since ex ante  $B_r$  is a random variable, the likelihood of a successful takeover in period 0, before the raider shows up is



Figure 1: The minimal premium that ensures that the raider buys 1/2 of the firm's equity

$$Prob(P(\alpha) < B_r) = 1 - F(P(\alpha)).$$
<sup>(13)</sup>

When the entrepreneur decides how much equity to issue he wants to maximize his expected payoff which is the sum of money he gets when he first issues equity (i.e., the market value of equity) and the benefits of control he gets in period 1 if the firm is not taken over by the entrepreneur. In order to write this expected payoff, let us first consider the market value of equity. Assuming that the capital market is perfectly competitive, the market value of equity,  $E(\alpha)$ , will be equal to the expected payoff of shareholders. Hence,

$$E(\alpha) = (1 - \alpha)[(1 - F(P(\alpha)))(X + P(\alpha)) + F(P(\alpha))X]$$
  
=  $(1 - \alpha)[X + (1 - F(P(\alpha)))P(\alpha)].$  (14)

With probability 1-F(P( $\alpha$ )) the firm is taken over and the market value of each share is P( $\alpha$ ). With probability F(P( $\alpha$ )), there is no takeover and the value of each share is X. Alternatively, the value of the firm is always at least X, and with probability 1-F(P( $\alpha$ )) the shareholders may get a premium P( $\alpha$ ) per share. Given E( $\alpha$ ), the expected payoff of the entrepreneur is:

$$Y(\alpha) = F(P(\alpha))B_e + \alpha X + (1 - \alpha)[X + (1 - F(P(\alpha)))P(\alpha)]$$
  
= X + F(P(\alpha))B\_e + (1 - \alpha)(1 - F(P(\alpha)))P(\alpha). (15)

The first term here is the cash flow of the firm which the entrepreneur captures since the market is competitive. The second term represents the expected value of the entrepreneur's benefits of control from running the firm in period 1. This event occurs with probability  $F(P(\alpha))$  which is the probability that the firm is not taken over by the raider. The third term in  $Y(\alpha)$  is the value of equity when it is sold in the market in period 0.

The entrepreneur chooses  $\alpha$  with the objective of maximizing Y( $\alpha$ ). Let  $\alpha^*$  be the optimal equity fraction that the entrepreneur decides to retain in period 0. The first order condition for an interior solution for  $\alpha^*$  is given by:

$$Y'(\alpha) = f(.)P'(\alpha)B_e - (1 - F(.))P(\alpha) + (1 - \alpha)(1 - F(.) - f(.)P(\alpha))P'(\alpha) = 0.$$
<sup>(16)</sup>

The first term in the first order condition is the marginal benefit of retaining more equity and it is associated with the decline in the probability that a takeover will take place and the entrepreneur will lose his benefits of control,  $B_e$ . The second term is the marginal (opportunity) cost of holding more shares. This cost arises because the entrepreneur can sell shares on the market when the firm is just established and get a price of  $(1-F(.))P(\alpha)$  per share. The last term in the first order condition is either positive or negative and it represents the marginal change in the value of equity. This change is due to the fact that an increase in  $\alpha$  leads to an increase in the premium  $P(\alpha)$  that the raider pays on shares if he takes over the firm. This leads to a marginal gain 1-F(.) (which is the probability that the premium is paid) but also to a marginal cost  $-f(.)P(\alpha)$ , that reflects the decline in the probability that the raider will be willing to take over the firm and offer the premium  $P(\alpha)$ .

When  $\alpha > 1/2$  the raider can never take over the firm because there are simply not enough shares that can be bought in the capital market to give him control over the firm. Hence, as  $\alpha$  approaches 1/2, the premium needed to ensure a takeover, P( $\alpha$ ), approaches infinity and the probability that there will not be takeover, F(P( $\alpha$ )) approaches 1. Thus, evaluated at  $\alpha = 1/2$ ,

$$Y'(1/2) = B_e - \frac{P(1/2)}{2} = -\infty.$$
(17)

It therefore follows that at the optimum,  $\alpha^* < 1/2$ , implying that it is never optimal for the entrepreneur to retain more than half of the firm shares. In other words, when the firm is just established, the entrepreneur will sell more than half of the firm's equity to outsiders in order to ensure that there is a positive probability that the firm will be taken over in period 1 by the raider. Thus the model has the interesting feature that although ex post, after the equity is issued, the entrepreneur does not want the takeover to succeed, ex ante, before he sells shares on the market, he wants a takeover to succeed with a positive probability since this allows him to get a higher price for the shares he sells on the market.

This higher price reflects that expectation of outsiders to get a premium from the raider in period 1 if a takeover takes place.

### 4. Leverage and takeovers

In this section which is based on the paper "Capital Structure and the Market for Corporate Control: The Defensive Role of Debt Financing," by Ronen Israel (*JF*, 1991) we are going to see that an entrepreneur can also affect the likelihood of a takeover and his gain from a takeover if it occurs by using debt. This model then complements the Stultz model that we saw earlier in that here the focus is on debt financing rather than equity financing. We are going to examine a simplified version of the model that nonetheless carries the main message.

Like in Stultz (1988), a firm is established by an entrepreneur in period 0 and can be taken over by a raider in period 1. For simplicity, let's normalize the cash flow of the firm under the entrepreneur to 0 and assume that the cash flow under the raider is R > 0. Hence, a takeover generates a surplus of R. In period 0 when the firm is just established, the entrepreneur still does not know the identity of the raider and hence the exact value of the surplus R. Suppose however that the entrepreneur believes that R will be drawn from a distribution function F(R) with a support  $[0, \infty)$ . If the raider decides to take over the firm, R becomes common knowledge and given the value of R, the raider and the entrepreneur bargain over the firm's shares. Suppose that the bargaining leads to a division of the surplus generated by the takeover, R, between the entrepreneur and the raider in proportions  $\gamma$  and 1- $\gamma$ , where  $\gamma$  is the entrepreneur's shares in the surplus while the raider's share is 1- $\gamma$ . In what follows I shall refer to  $\gamma$  and 1- $\gamma$  as the bargaining powers of the entrepreneur and the raider. If no takeover takes place, the cash flow of the firm remains 0 and the payoff of the raider and the entrepreneur is 0.

To analyze the model, let's first examine the benchmark case in which the firm is all-equity. Since the purpose of the model is to study the impact of debt financing on takeovers, we shall assume that the equity is held exclusively by the entrepreneur. This assumption ensures that we do not have to face the free-raider problem once again (since the focus here is on debt financing it is best to avoid the problem in the current context). Since a takeover generates a positive surplus, the raider will always take over the firm and will split the resulting surplus R with the entrepreneur. Given their bargaining powers, the entrepreneur's payoff is  $\gamma R$  and the raider's payoff is  $(1-\gamma)R$ . Therefore, the expected payoff of the entrepreneur in period 0 before he learns the value of R (which also represents the market value of the firm in period 0 given that it has 0 debt) is

$$V(0) = \int_{0}^{\infty} \gamma R dF(R) = \gamma \hat{R}, \qquad (18)$$

where  $\hat{R}$  is the mean value of R.

Now, suppose that in period 0, after the firm is established and before the entrepreneur learns the value of R, the firm issues debt with face value D to be repaid at the end of period 1. Given D, the value of equity if there is a takeover is R-D. Clearly, the raider will taker the firm if and only if R > D, otherwise a bankruptcy is inevitable so the raider is better-off not buying the firm at all. Although a takeover can still generate a surplus R, the entire surplus is captured by debtholders so the raider, who is going to become a shareholder, has no incentive to take over the firm. Thus, if  $R \le D$ , the raider does not take over the firm, and as a result, the cash flow remains 0. Hence, the firm goes bankrupt for sure and the payoff of the entrepreneur, the debtholders, and the raider is 0. If however R > D, the raider will take over the firm since after paying off the debtholders there is still a positive surplus that the raider and the entrepreneur can split.

As before, the surplus is divided between the entrepreneur and the raider in proportions  $\gamma$  and 1- $\gamma$ . Hence, if a takeover takes place, the payoff of the entrepreneur is  $\gamma(R-D)$  and the raider's payoff is (1- $\gamma$ )(R-D). Now, when the firm sells debt on the capital market in period 0, debtholders realize that they will get nothing if  $R \le D$  and will be paid in full otherwise. Since the debtholders know that R is drawn from a cumulative distribution function F(R), the market value of debt in period 0 is

$$\boldsymbol{B}(\boldsymbol{D}) = \int_{\boldsymbol{D}}^{\infty} \boldsymbol{D} dF(\boldsymbol{R}).$$
(19)

Likewise, the market value of equity in period 0 is

$$E(\boldsymbol{D}) = \int_{\boldsymbol{D}}^{\infty} \gamma(\boldsymbol{R} - \boldsymbol{D}) dF(\boldsymbol{R}).$$
(20)

Since the entire firm belongs to the entrepreneur, he gets both the value of debt and the value of equity. Hence his expected payoff in period 0 is equal to the market value of the firm and is given by:

$$V(D) = E(D) + B(D) = \int_{D}^{\infty} (\gamma (R - D) + D) dF(R).$$
(21)

Let  $D^*$  be the optimal debt level from the entrepreneur's point of view. The entrepreneur chooses  $D^*$  in order to maximize V(D). As can be seen, the advantage of issuing debt is that once the firm is taken over, debtholders receive D in full. Since the entrepreneur captures this amount when he issues debt in period 0, it means that the by issuing debt, the entrepreneur lowers the amount of money that he needs to split with the raider when takeover takes place. However, doing so is costly since the higher is D, the lower is the probability that the raider would wish to take over the firm. Thus, the choice of D\* involves a tradeoff between the likelihood that a takeover will take place and the entrepreneur's payoff once a takeover takes place.

The first order condition for an interior solution for D\* is given by

$$V'(D) = \int_{D}^{\infty} (1 - \gamma) dF(R) - Df(D)$$
  
=  $(1 - \gamma)(1 - F(D)) - Df(D) = 0.$  (22)

The first term in the first order condition is the marginal benefit from issuing debt. This benefit arises since an additional dollar of debt raises the entrepreneur's payoff in the event of a takeover from  $\gamma(R-(D+1))+(D+1)$ , i.e., by 1- $\gamma$ . This amount has to be multiplied by 1-F(D) which is the probability that a takeover actually takes place. The second term in the first order condition is the marginal cost of debt associated with the reduction in the probability that a takeover takes place. When there is no takeover, the entrepreneur looses the amount of D because this is the amount that debtholders would have paid for debt in states of nature in which a takeover takes place. Note that at D = 0, the marginal cost of a takeover vanishes. Since in the absence of debt, the firm is always taken over by the entrepreneur, then F(0) = 0, so the marginal benefit of debt at D = 0 is equal to  $(1-\gamma)(1-F(0)) = 1-\gamma > 0$ . Hence, D\* > 0, implying that the firm will always issues a positive amount of debt.

To examine the properties of D\*, let's rewrite the first order condition for D\* as follows:

$$1 - \gamma = DH(D) = \frac{Df(D)}{1 - F(D)},$$
(23)

where H(D) is the hazard rate of the distribution of R. Since  $1-\gamma$  is a constant, the second order condition for a unique interior solution for D\* requires that DH(D) increases with D and attains a value that exceeds  $1-\gamma$  as D increases to infinity. Now it is easy to see that as the bargaining power of the entrepreneur,  $\gamma$ , increases, D\* decreases. The intuition is that an increase in  $\gamma$  means that the entrepreneur gets a larger fraction of the surplus from a takeover and hence his benefit from issuing debt is smaller. Another way to look at this result is as follows: issuing debt puts the firm at a risk that R will not be large enough relative to D to induce the raider to take over the firm. As  $\gamma$  increases, the entrepreneur becomes more eager to see that a takeover succeeds (his share in the resulting surplus is bigger) so he lowers D in order to enhance the chances that a takeover will take place. Indeed, when  $\gamma = 1$ , the entrepreneur gets the entire surplus from a takeover and hence he will not issue debt at all.

## 5. Leverage and managerial entrenchment

Another interesting model that examines the connection between leverage and the market for corporate control is Zwiebel (1996). The main idea in Zwiebel's model is as follows: once the firm hires a manager, the manager becomes entrenched and the only way to remove him is either through bankruptcy (in which case the firm is also liquidated) or through a takeover. The reason for initiating a takeover is that the manager in Zwiebel's model is empire builder and likes to invest as much as possible, even if investment is inefficient. In and of itself, a takeover does not create value: once a takeover occurs, the firm does not invest anymore and simply keeps its cash flow intact. Hence, the benefit from a takeover comes from the fact that it stops the incumbent manager from making inefficient investments and hence can boost the expected value of the firm. Realizing that the possibility of making inefficient investments will create an incentive to take over the firm, the manager (who likes to run the firm) has an incentive to commit to refrain from making inefficient investments. Problem is, the choice of investment in non-contractible. Hence, in order to credibly commit to invest optimally, the manager issues debt. The advantage of issuing debt is that once the firm is leveraged, inefficient investments raise the likelihood of bankruptcy. Since the manager does not want to lose his job, he will not make inefficient investments once the firm is leveraged. The implication is that leverage is a way to commit the manager to efficient investments and hence, when the firm is leveraged there is no need to initiate takeovers.

To see the relationship between leverage and takeovers, let's examine in more detail a simplified version of Zwiebel's model. There are 2 periods (in the original model there are 3 periods). The sequence of events in each period i is as follows:

- (i) The manager chooses the face value of debt,  $D_i$ . This debt is due at the end of the period.
- (ii) A raider may takeover the firm if the takeover raises the firm's value by at least e. That is, e is the cost of takeover.
- (iii) The manager learns whether the firm has a good project that yields return r or a bad project that yields return -r. The probability that the project is good is t and the probability that the project is bad is 1-t. The probability t that the project is good is interpreted as the manager's type. After learning about the type of project that the firm faces, the manager decides whether to invest or not. If the manager invests, the earnings of the firm are y+r if the project is good and y-r if the project is bad. If the manager does not invest, the earnings of the firm are y.
- (iv) The firm's earnings are realized and the firm pays off its debt. If the firm cannot pay off its debt, the firm is liquidated.

The manager gets a positive utility from running the firm and additional utility from investing. Hence, the manager will always invest unless this investment will surely lead to a takeover or bankruptcy in which case he looses his job.

The model is solved by backward induction. In period 2, the manager will always invest if there is no takeover since bankruptcy can occur only at the end of the period at which point the model ends anyway. In other words, in period 2 the manager has nothing to lose from investing. Given the proceeds from issuing debt with face value  $D_2$  at the beginning of period 2,  $B(D_2)$ , the expected earnings of the firm if the manager is in place are therefore

$$\boldsymbol{B}(\boldsymbol{D}_2) + t(y+r) + (1-t)(y-r). \tag{24}$$

If the manager is replaced, the expected earnings of the firm are given by

$$\boldsymbol{B}(\boldsymbol{D}_{2}) + \boldsymbol{y}. \tag{25}$$

Comparing the two expressions, it follows that a takeover boosts the expected earnings of the firm by r(1-2t). Since the cost of takeover is e, a takeover will take place provided that

$$r(1-2t) \ge e, \quad \Rightarrow \quad t \le \frac{1-E}{2}, \tag{26}$$

where  $E \equiv e/r$ . That is, takeover will take place only if the manager's type is low. If the manager's type is high, there will not be takeover and hence the manager will then invest regardless of whether the investment is efficient.

Next, let's consider period 1. If  $t \le (1-E)/2$ , then the manager realizes that he will be replaced by a takeover in period 2 anyway and hence, he has nothing to lose from investing in period 1 even if this is inefficient. Moreover, the manager has no incentive to commit to refrain from investing inefficiently, since a takeover in period 2 is inevitable. However, since the manager will surely invest even if this is inefficient, then the raider's faces the same problem as in period 2: by taking over the firm in period 1, the raider can stop the inefficient investment in period 1 and hence can boost the expected earnings of the firm by r(1-2t). But since by assumption,  $t \le (1-E)/2$ , then r(1-2t)  $\ge rE \equiv e$ , so a takeover in period 1 is profitable. Hence, whenever  $t \le (1-E)/2$ , a takeover will take place already in period 1.

If t > (1-E)/2, then there is no takeover in period 2. Hence, given the proceeds from issuing debt with face value  $D_1$  at the beginning of period 1,  $B(D_1)$ , the expected earnings of the firm if the manager is in place are

$$\boldsymbol{B}(\boldsymbol{D}_{1}) + 2[t(y+r) + (1-t)(y-r)].$$
(27)

This expression reflects the fact that since there is no takeover in period 2, the manager will have two chances to invest: once in period 1 and once in period 1. On the other hand, if the manager is replaced in period 1, then there is no investment in periods 1 and 2, so the expected earnings of the firm are

$$\boldsymbol{B}(\boldsymbol{D}_1) + 2\boldsymbol{y}. \tag{28}$$

The last two expressions imply that a takeover in period 1 will boost the expected earnings of the firm by 2r(1-2t). Since the cost of takeover is e, a takeover will take place in period 1 provided that

$$2r(1-2t) \ge e, \quad \Rightarrow \quad t \le \frac{1-\frac{E}{2}}{2}. \tag{29}$$

Note that a takeover will take place in period 1 for a larger set of values of t than in period 2 since  $(1-E/2)^2 > (1-E)^2$ . The intuition for this is that a takeover in period 1 prevents the manager from making inefficient investments in both periods 1 and 2 and hence is more valuable than a takeover merely in period 2.

As we saw, there is no takeover in period 1 if the manager's type is t > (1-E/2)2. Hence, in this case, the manager will invest for sure in period 1. On the other hand, if the manager's type is  $t \le (1-E/2)2$ , then there is a takeover in period 2 and the manager will lose his job. But, since the manager does not want to be replaced, he has an incentive to commit to refrain from making inefficient investments. As mentioned above, this can be done by issuing debt. To see how, suppose that the manager issues at the beginning of period 1 debt with face value y-r < D<sub>1</sub> < y, and distributes the proceeds as dividends. Then, if the firm makes an inefficient investment, its cash flow at the end of period 2 will be y-r < D<sub>2</sub>. As a result, the firm goes bankrupt at the end of period 1 and the manager will lose his job. In that case, the manager gets benefits only in period 1 but no benefits in period 2, and is therefore better off not investing inefficiently since then he gets the benefit from running the firm in both periods 1 and 2 plus the benefits from investing in period 2. Hence, when y-r < D<sub>2</sub> < y, the manager has an incentive to invest efficiently and therefore the raider has no incentive to takeover the firm. In other words, whenever  $t \le (1-E/2)2$ , debt with face value y-r < D<sub>2</sub> < y will deter takeover in period 1. Notice that since D<sub>2</sub> < y and since the manager invests efficiently, the firm's debt is safe in the sense that the firm will be able to fully repay

it at the end of period 1.

To summarize, there are three cases to consider, depending on the manager's type. Case (i): t > (1-E/2)2. Here the firm issues no debt and there is no takeover in period 1. Case (ii):  $(1-E)2 \le t \le (1-E/2)2$ . In this case, the firm issue in period 1 debt with face value  $y-r < D_1 < y$ , and distributes the proceeds as dividends. The firm's debt deters takeover. Case (iii): t < (1-E)2. Takeover in period 1 occurs no matter whether the firm has or does not have debt.