TOPIC 2: TAX-BASED THEORIES OF CAPITAL STRUCTURE

1. Introduction

Thus far we ignored personal taxes altogether and showed that in the presence of corporate taxes, firms would be interested in issuing as much debt as they possibly can. However, in a world with personal taxes, it is reasonable to assume that investors will be interested in <u>after personal tax</u> cash flows. Consequently, the tax advantage of debt at the corporate level should be traded-off against the possible tax disadvantage of debt at the personal level. Our discussion on this topic will follow Miller's influential paper "Debt and taxes" (*JF*, 1977). In this paper Miller considers the trade-off between the tax advantage of debt at the corporate level and the tax disadvantage at the personal level and examines the implications of this trade-off for optimal capital structure. Miller's conclusion is that so long as there is a continuum of investors with varying marginal rates of personal taxes, capital structure will be irrelevant, because in equilibrium, the cost of financing a project with debt and financing it with equity (and indeed the cost of any mix of debt and equity) will be the same. This conclusion restores the implications of the original M&M's Proposition 1 despite the fact that taxes are taken into account.

When Miller wrote his paper in 1977, personal tax on interest income in the U.S., t_D, used to be higher than personal tax on equity, t_E, since the t_D was equal to the tax rate on ordinary income, while t_E was a combination of tax on dividend yield (which was equal to the ordinary tax rate) and capital gains (which was much lower than ordinary rate). Today in the U.S., the tax rates on all sources of income are the same. Nonetheless, capital gains still have a tax advantage because the tax on capital gains is paid only when the gain is realized. Thus, if an investor does not need his money right away, he can defer the realization well into the future and thereby enjoy in the meantime a higher wealth. Moreover, investors can choose to realize their capital gains in periods where they have capital losses and thereby lower their total tax bill. The ability to choose when to realize capital gains and thereby defer the corresponding taxes into the future means that even today, t_E is effectively lower than t_D. In fact, Miller cites conventional

wisdom on capital gain taxes as indicating that "10 years of tax deferral is almost as good as exemption" (p. 270). He then argues that this disadvantage of debt at the personal level should be traded-off against the tax advantage of debt at the corporate tax level.

To examine the implications of corporate and personal taxation for optimal capital structure, suppose that there are two firms with identical cash flows that differ only with respect to their capital structures: firm U is all-equity, and firm L is leveraged. Each firm generates each period a random cash flow, $\tilde{X}$, whose expectation is $\hat{X}$. Assume that the interest rate on tax-exempt bonds is r_0, and the before-tax yield on (riskless) corporate bonds is r. Now, we shall maintain Assumptions (A1)-(A3) and (A5) from the discussion on M&M, but replace Assumption (A4) with the following assumptions:

(A4)   The corporate tax rate for all firms is t_C, the personal tax rate on interest income is t_D, and the personal tax rate on equity, which is an average of tax on dividend and capital gains, is t_E.

Given this new assumption we are ready to state the following proposition:

Proposition (Miller): *Given Assumptions (A1)-(A5),*

$$V_L = V_U + \left[1 - \frac{(1 - t_C)(1 - t_E)}{1 - t_D} \right] D. \tag{1}$$

Proof: The proof uses the familiar no-arbitrage arguments. Consider an investor with a fraction α of firm U. His per-period net future cash flow is $\alpha\tilde{X}(1 - t_C)(1 - t_E)$, and the value of his portfolio is αV_U. Now, suppose that the investor adopts the following investment strategy:

"Sell your holdings in firm U and buy instead a fraction α of firm L's equity,

and a fraction $\beta \equiv \alpha(1 - t_C)(1 - t_E)/(1 - t_D)$ of firm L's debt."

Let's examine the per-period payoff associated with the strategy: the per-period net payoff from equity

is $\alpha(\tilde{X}-rD)(1-t_C)(1-t_E)$, and the net payoff from debt is $\beta rD(1-t_D)$. Hence the total per-period net payoff of the investor is:

$$
\begin{aligned}
&\alpha\big(\tilde{X}-rD\big)(1-t_C)(1-t_E) \;+\; \beta\, rD(1-t_D) \\[6pt]
=\; &\alpha\big(\tilde{X}-rD\big)(1-t_C)(1-t_E) \;+\; \alpha\,\frac{(1-t_C)(1-t_E)}{1-t_D}\, rD\,(1-t_D) \qquad\qquad (2)\\[6pt]
=\; &\alpha\,\tilde{X}\,(1-t_C)(1-t_E).
\end{aligned}
$$

Since this is exactly equal to the payoff from holding a fraction α of U's equity, the two investment strategies should cost the same, otherwise there will be arbitrage opportunities. Hence,

$$
\begin{aligned}
\alpha\,V_U \;=\; &\alpha\big(V_L - D\big) \;+\; \beta\,D \\[6pt]
=\; &\alpha\big(V_L - D\big) \;+\; \alpha\,\frac{(1-t_C)(1-t_E)}{1-t_D}\,D \qquad\qquad (3)\\[6pt]
=\; &\alpha\,V_L \;-\; \alpha\left[1 \;-\; \frac{(1-t_C)(1-t_E)}{1-t_D}\right]D.
\end{aligned}
$$

Rearranging this expression yields the expression in the proposition. ■

Several remarks on the proposition are in order: first, if there are no taxes at all so that $t_D = t_E = t_C = 0$, then $V_L = V_U$, which is exactly the result of M&M1. And, if there are no personal taxes, so that $t_D = t_E = 0$, then $V_L = V_U + t_C D$, which is exactly the result of M&M1 with corporate taxation. Therefore Miller's proposition can be viewed as a generalization of M&M to a world with personal taxes.

Second, if $(1-t_C)(1-t_E) = (1-t_D)$, then $V_L = V_U$. To understand this result, note that $1-t_D$ is the after-tax interest income on every dollar of debt, while $(1-t_C)(1-t_E)$ is the after-tax income from dividends or and capital gains. If $(1-t_C)(1-t_E) = (1-t_D)$, then the after-tax incomes from debt and equity are the same, so investors should be indifferent to the firm's capital structure meaning that the firm will have nothing to gain by using one type of securities rather than another.

Third, Miller's proposition implies that debt is preferred to equity if and only if:

$$(1 - t_D) > (1 - t_C)(1 - t_E). \tag{4}$$

This implies in turn that if t_D, t_E and t_C were the same for all individuals and all firms, then all firms would be either all-debt if $V_L > V_U$ or all-equity financed if $V_L < V_U$. In other words, firms would never wish to use a mix of equity and debt in their capital structures. The intuition for this is straightforward: If the firm were all-debt, debtholders would receive the entire cash flow, $\tilde{X}$; since this payout is in the form of interest, the taxable income of the firm would be $\tilde{X}-\tilde{X} = 0$. At a personal level, however, investors would have to pay a tax of t_D per each dollar they receive, so their net payoff in this case would be $\tilde{X}(1-t_D)$. On the other hand, if the firm were all-equity, the payoff of investors (all of whom are now equityholders) would consist of either dividends (if the firm pays dividends) or capital gains (if the firm retain earnings) and would therefore be equal to the after-tax earnings of the firm. Since there are no interest payments to deduct, the taxable income of the firm would be $\tilde{X}$, and the net income to distribute to equityholders would be $\tilde{X}(1 - t_C)$. On this payout, each investor would have to pay a personal tax $\tilde{X}(1 - t_C)t_E$, so their net payoff would be $\tilde{X}(1 - t_C)(1-t_E)$. Comparing this payoff with the payoff when the firm is all-debt, $\tilde{X}(1-t_D)$, leads to the conclusion about the capital structure of the firm.

2. The case where personal tax rates vary across individuals

Now suppose (more realistically) that the personal tax rate differs across individuals. To simplify the analysis in this case, assume in addition that $t_E = 0$. This assumption will allow us to emphasize the personal tax disadvantage of debt (note however that this assumption is not crucial and can be easily relaxed). Now, Assumptions (A4) has to be modified as follows:

(A4) The corporate tax rate for all firms is t_C, the personal tax rate on equity is 0, so that $t_E = 0$, and

the personal tax rates on interest income, t_D, varies from 0 for tax-exempt organizations (e.g.,

universities and some international investors), to high rates such that for individuals with high income, $t_D > t_C$.

Apart from the modification of Assumption (A4) we shall add the following assumptions:

(A6)   Firms generate a certain cash flow and their debt is riskless.

(A7)   There is a continuum of individual investors, indexed by t_D^i, each of which wishes to invest B dollars. Tax exempt investors wish to invest B_0 dollars.

(A8)   Borrowing on personal account to buy tax-free bonds and shortselling is not permitted.

Assumption (A6) simplifies the analysis since, given that $t_E = 0$, equity and tax-free bonds become perfect substitutes. This means that investors have only two alternatives: they can either invest in corporate bonds, or they can invest in equity or tax-free bonds (the latter are, say, bonds issued by municipalities, who in the U.S. can issue bonds that pay tax-free interest rate). Since equity and tax-free bonds are perfect substitutes, their ROR's must be the same, and we shall denote it by r_0. Assumptions (A7) will help us to describe the supply of money in the market for corporate bonds. The importance of Assumption (A8) will become clear below.

Given our assumptions, we will analyze the equilibrium in the market for corporate bonds. The relevant good in this market is corporate bonds measured in dollar terms. The relevant price of these bonds is the <u>before tax</u> interest rate on corporate bonds, r. To characterize the equilibrium, we must specify first the demand and supply curves for funds in the form of corporate bond.

The supply of funds in the market for corporate bonds

The supply of funds in the market for corporate bonds comes from investors. To describe the supply curve for funds, we must hold as fixed the rate of return on tax-free bonds and equity, r_0. Given r_0, individual i will supply funds to the firm by buying its corporate bonds (rather than buying tax-free bonds

and equity) if and only if

$$r(1 - t_D^i) > r_0. \tag{5}$$

That is, individual i prefers corporate bonds provided that the tax rate on interest income is sufficiently small. Now, we can define the marginal investor, indexed by m, who is indifferent between the two alternatives by the following equation:

$$r(1 - t_D^m) = r_0, \quad \Leftrightarrow \quad t_D^m = 1 - \frac{r_0}{r}. \tag{6}$$

All investors such that $t_D^i < t_D^m$ (e.g., non-for-profit organizations) will buy corporate bonds, while all investors with $t_D^i > t_D^m$ (e.g., high income individuals) will buy tax-free bonds and equity.

In order to determine the supply of funds for firms who issue corporate bonds, note that tax-exempt investors will invest all their money, B_0, in corporate bonds. In order to raise more than B_0 by issuing corporate bonds, firms must attract individuals with higher personal tax rates. This can be done by increasing the before-tax interest rate, r, above r_0. As r increases, $t_D^m = 1 - r_0/r$ increases, so more investors choose to buy corporate debt. Given Assumption (A7), the supply of funds that firms who issue corporate bonds face, is given by

$$B^S(r) = \begin{cases} 0 & \textit{if } r < r_0, \\ B_0 & \textit{if } r = r_0, \\ B_0 + B\left(1 - \dfrac{r_0}{r}\right) & \textit{if } r > r_0. \end{cases} \tag{7}$$

Figure 1 shows the supply of funds in the market for corporate debt, holding r_0 constant. The supply curve starts at r_0 and is horizontal up to B_0. From this point on, it increases in r as more investors migrate from the market for equity and tax-free bonds.
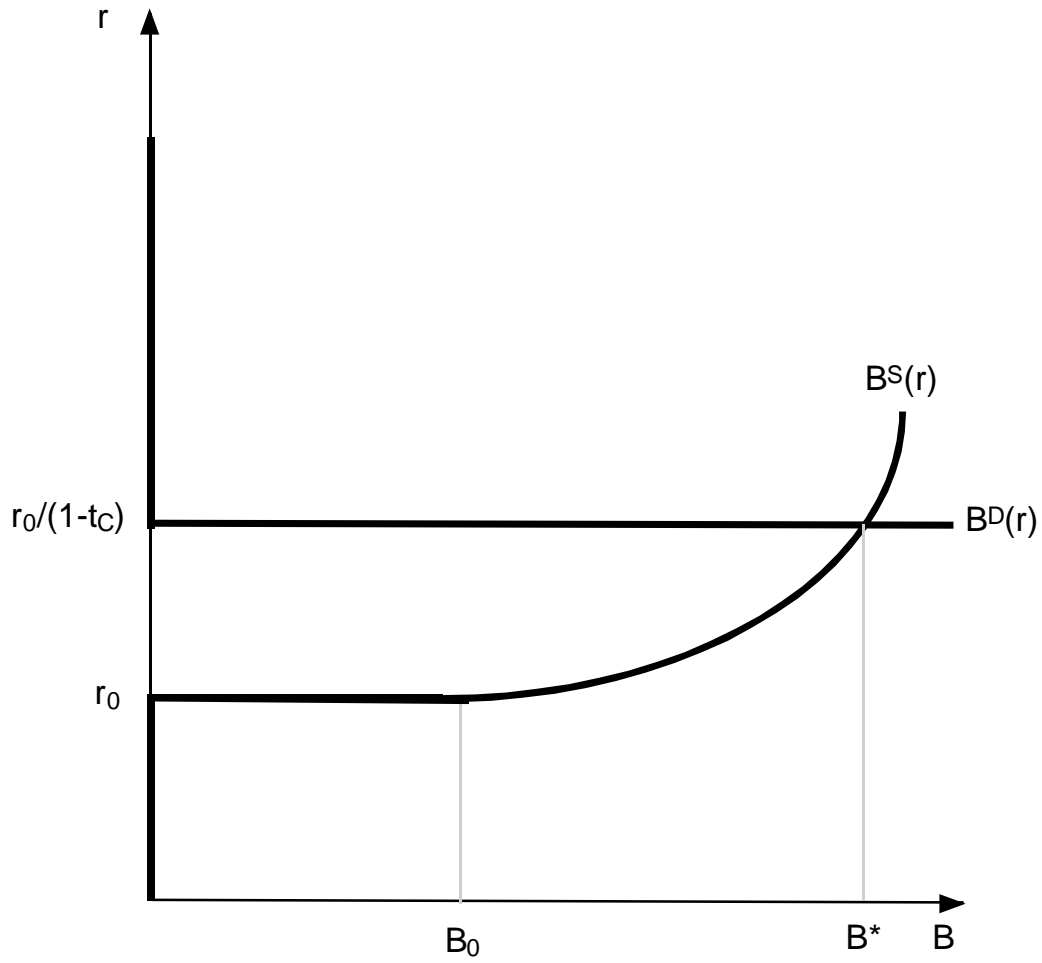
Figure 1: Miller Equilibrium

The demand for funds in the market for corporate bonds

To describe the demand curve for funds raised by issuing corporate bonds, we must hold as fixed the rate of return on equity, r_0, since firms can also raise funds by issuing equity. Now, one dollar of equity costs the firm r_0 since this is the return that the firm needs to promise equityholders in order to induce them to buy its shares. Since debt payments are tax deductible, one dollar of debt costs the firm $r(1-t_C)$. That is, the firm pays the investor r and receive a tax shield of t_C per each dollar of debt, so the net cost of debt from the firm's perspective is less than r. Since firms wish to minimize the cost of capital, they will issue only equity if $r_0 < r(1-t_C)$ (equity is cheaper), will issue only corporate bonds if $r_0 > r(1-t_C)$ (bond are cheaper), and will be indifferent between equity and corporate bonds if $r_0 = r(1-t_C)$ (equity and bonds cost the same). Thus, the demand for funds in the form of corporate debt, shown in Figure 1 is given by

$$\boldsymbol{B}^D(r) = \begin{cases} \infty & \text{if } r \geq \dfrac{r_0}{1-t_C}, \\[3mm] 0 & \text{if } r < \dfrac{r_0}{1-t_C}. \end{cases} \tag{8}$$

That is, the demand curve is perfectly elastic at $r = r_0/(1-t_C)$, which is the return that leaves equity and corporate bonds equally costly from the firm's perspective.

Miller equilibrium

Since we assume that the capital market is perfectly competitive, the equilibrium in Miller's model (or, Miller equilibrium as it is sometimes referred to in the literature) is achieved when demand and the supply curves intersect, i.e., when $B^D(r) = B^S(r)$. As Figure 1 shows, this happens on the horizontal part of the demand curve where $r = r_0/(1 - t_C)$. Since at this point the cost of equity and the cost of debt are the same, it follows that in equilibrium, each firm is just indifferent to its capital structure. That is, if a firm needs to raise an additional dollar, it will be completely indifferent between raising by issuing equity or by issuing corporate bonds.

Substituting the equilibrium interest rate in $B^S(r)$, reveals that the equilibrium quantity of corporate bonds is $B_0 + B(1 - 1/(1-t_C))$. Further, substituting the equilibrium interest rate in equation (6), reveals that in equilibrium, the marginal investor m is such that

$$t_D^m = 1 - \frac{r_0}{\left(\dfrac{r_0}{1-t_C}\right)} = t_C. \tag{9}$$

That is, in a Miller equilibrium, the personal tax rate of the marginal investor, t_D^m, is equals to the corporate tax rate, t_C.

To sum up, the main implications of Miller's model are as follows. First, firms should be completely indifference to their capital structure. Moreover, since the values of debt and equity are determined by the <u>marginal</u> investor, it follows from equation (1) that in a Miller equilibrium,

$$V_L = V_U + \left[1 - \frac{(1-t_c)(1-t_E)}{1-t_D^m}\right] D. \tag{10}$$

But, since by assumption $t_E = 0$, and since $t_D^m = t_C$, it follows that in a Miller equilibrium, $V_L = V_U$. Hence, firm value is independent of capital structure exactly as in M&M1 without taxation. This result verifies what we have found before: individual firms have no optimal capital structure.

Second, although individual firm are indifferent to their capital structures, the model gives rise to a unique equilibrium level of corporate bonds at the aggregate level. This is in contrast with M&M1 without taxation where the invariance to capital structure holds not only at the individual firm level but also in the aggregate.

Third, in a Miller equilibrium, we have a tax clientele: individuals with high personal tax rate $t_D^i > t_D^m = t_C$ hold equity and tax-free bonds, while individuals with low personal tax rate $t_D^i < t_D^m = t_C$ hold corporate bonds.

Let us now discuss a few problems with Miller's model. First, in reality, $t_D = t_E$ contrary to what

Miller assumed. In addition, $t_D < t_C$, since the highest tax on ordinary income is 33%, while corporate

tax is 34%. Hence, the corporate tax advantage of debt always exceeds the personal tax disadvantage of

debt, so firms should issue as much debt as possible.

Second, implicit in the analysis is the assumption that personal interest expenses are tax-deductible

(otherwise individuals cannot use a "homemade" leverage and exploit arbitrage opportunities that are

required to equate price in Miller's proposition). In practice however this is not always the case.

Third, if assumption (A8) is violated (as is reasonable to expect), the following tax arbitrage can

take place: investors with $t_D^i < t_C$, can borrow one dollar at an interest rate r (recall our assumption of

competitive capital markets) and invest the money in equity or tax-free bonds which yield an ROR equal

to r_0. After taxes, the interest rate on the loan is $r(1-t_D^i) < r(1-t_C) = (r_0/(1-t_C))(1-t_C) = r_0$ so those investors

can make money. Investors with $t_D^i > t_C$ can shortsell equity or tax-free bonds and receive a return of r_0,

and then buy corporate bonds which yield an after tax return of $r(1-t_D^i) > r(1-t_C) = (r_0/(1-t_C))(1-t_C) = r_0$.

For a Miller equilibrium to exist it must be the case that Assumption (A8) which effectively rules out "tax

arbitrages" holds.

3. Empirical evidence on debt and taxes

A key implication of Miller's model is that in equilibrium, $r = r_0/(1-t_C)$, or $1-t_C = r_0/r$. The model can

therefore be tested by checking whether in the data, r_0/r is indeed equal to (or more precisely, not

significantly different than) $1-t_C$. This hypothesis is indeed tested by Trczinka in "The pricing of

tax-exempt bonds and the miller hypothesis," (*JF*, 1982) by using data from the yields on utilities'

corporate bonds for the period 1970-1979. Specifically, he tests whether the ratio of the monthly averages

of the yields on newly issued municipal bonds (which are tax free) and utilities' bonds is significantly

different than 0.52 which was equal to $1-t_C$ at the time. After adjusting for risk (i.e., making sure that the

municipal bonds and utilities bonds that he uses bear the same risks) he finds the following:

Ratio	Mean r_0/r	Number of Observations
30 Yr. prime municipal bonds	0.516	117
Long-term Aaa utilities bonds	(9.5)	
20 Yr. prime municipal bonds	0.57	117
Long-term Aaa utilities bonds	(10.1)	
10 Yr. prime municipal bonds	0.608	45
10 Yr. Aa utilities bonds	(8.24)	
30 good municipal bonds	0.48	117
Long-term Aa utilities bonds	(9.14)	
20 good municipal bonds	0.54	117
Long-term Aa utilities bonds	(10.5)	
10 good municipal bonds	0.54	81
10 Yr. Aa utilities bonds	(11.5)	
30 Yr. medium municipal bonds	0.446	117
Long-term A utilities bonds	(7.8)	
20 Yr. medium municipal bonds	0.487	117
Long-term A utilities bonds	(8.56)	
10 Yr. medium municipal bonds	0.526	103
10 yr. A utilities bonds	(11.2)	

*   The numbers in parentheses are standard deviations.

Trczinka finds that none of these estimates are statistically different from 0.52 which is what Miller's model predicts. Thus, Trczinka's results support Miller's predictions that the cost of equity and the cost of debt are the same from the firms point of view. The implication is that firms should be indifferent to their capital structures.

A more recent paper that examines the impact of taxes on capital structure is "Taxes and Capital structure: Evidence from Firm's Response to the Tax Reform Act of 1986," by Givoly et al. (*RFS*, 1992). They examine the responses of firms to the U.S. Tax Reform Act (TRA) of 1986 which drastically

changed the tax regime. In particularly, the TRA lowered the corporate tax rate, thereby reducing the value of tax shields to firms. This leads to the hypothesis that the value of the tax shields of firms that initially had high marginal effective corporate tax rates was lowered by more than the value of the tax shields of firms that initially had low marginal effective corporate tax rates. Consequently, the former type of firm should lower their leverage by more than firms of the second type. Thus, Givoly et al. run the following regression:

$$\Delta LEV_t = \alpha_t + \beta_1 ETR_t + \ldots + \beta_6 SIZE_t + \beta_7 BRISK_t + \beta_8 TBQ_t + \epsilon_t, \tag{11}$$

where ΔLEV_t is the change in leverage between t and t-1 (leverage is measured as the ratio of the book value of long term debt to the book value of long term debt plus the book value of equity), ETR_t is the effective corporate tax rate at time t (measured as the ratio of actual taxes paid over the 10-year period preceding period t plus the discounted deferred taxes accumulated over that period, to the pretax income over the period), $SIZE_t$ is the size of the firm measured as the natural logarithm of total firm value, $BRISK_t$ is the riskiness of the firm's income, TBQ_t is a measure of bankruptcy costs (measured as the ratio of the book value of tangible assets to the market value of equity)[1], and ε_t is an error term. (The right side of the regression contains additional explanatory variables that are unimportant for our purposes).

The results show that β_1 was negative and significant (at the 0.01 level) for t = 1987, but not for t = 1986, t = 1985, and t = 1984. Specifically, the value of β_1 was -0.115, implying that a 1% increase in the effective corporate tax rate leads to an 11.5% reduction in leverage. This is consistent with the hypothesis that the TRA induced firms with higher effective corporate tax rates to reduce their leverage by more than firms with low effective corporate tax rates. This result provides evidence for the relevance of corporate taxation for the capital structure of firms: corporate taxes create tax shields and they induce firms to issue debt. The results also show that β_6 - β_8 were also negative and significant for 1984-1987.

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[1] This measure is intended to capture the liquidation value of the firm and also reflect the potential loss of growth opportunities in the case of a bankruptcy.

This is consistent with the hypothesis that firms with higher risks and higher bankruptcy costs issue less debt, and that firms tend to use less debt and more retained earnings as they grow.

4. The tradeoff between tax shields and bankruptcy costs

One of the first models to examine the implications of risky debt in a world with corporate taxes and bankruptcy costs is "A State-Preferences Model of Optimal Financial Leverage" by Kraus and Litzenberger (*JF*, 1973). To examine their model, we shall consider a two-period model, in which the firm is established by an entrepreneur in period 1 and operates once in period 2. The earnings of the firm at the end of period 2 are represented by a random variable $\tilde{X}$ distributed over the interval $[X_0, X_1]$ according to a cumulative distribution function $F(\tilde{X})$. The mean earnings of the firm are given by

$$\hat{X} = \int_{X_0}^{X_1} \tilde{X}\, dF(\tilde{X}),$$

(12)

where $dF(\tilde{X}) \equiv f(\tilde{X})d\tilde{X}$. When the firm is just established, it issues risky debt with face value D. The assumption that debt is risky means that its face value, D, exceeds X_0 so there are states of nature at which the firms does not have enough money to repay its debt obligation in full. To simplify matters, let us assume in addition that $D < X_1$ so that the firm never issues debt to point where it goes bankrupt for sure (again note that promising debtholders a payment of more than X_1 is not credible since the debtholders realize that at most they are going to get back X_1; hence, there is no real difference between the promise to pay X_1 and the promise to pay more than X_1). After the earnings are realized at the end of period 2, the firm is liquidated and its securityholders are paid according to their respective claims.

Given a debt obligation D, the firm is able to repay its debt in full if and only if $\tilde{X} \geq D$. The firm then remains solvent: it pays D to debtholders and its equityholders are the residual claimants, receiving a payment of $\tilde{X} - D$. When $\tilde{X} < D$, the firm declares bankruptcy, and debtholders become the residual

claimants, receiving a payment of $\tilde{X}$. Equityholders are protected by limited liability so their payoff is

0. We shall continue to maintain the usual assumptions (e.g., that the capital market is competitive and

that all investors can diversify risks completely so they behave as if they were risk neutral). However the

usual assumption that bankruptcy is costless will now be replaced by the assumption that bankruptcy

imposes extra fixed costs C, on debtholders due, among other things, to legal fees and to transaction costs

associated with transferring ownership to debtholders. To simplify matters we shall assume that $C < X_0$.

This ensures that debtholders never have to pay the bankruptcy costs out of their own pocket. Absent this

assumption we will have to deal with the question of what happens when debtholders are also protected

by limited liability and refuse to pay C out of their pockets. This problem however is not very interesting

and probably not very important from an empirical standpoint so it is best to avoid it. When $C < X_0$, it

is understood that debtholders need to pay C (say to lawyer who deal with bankruptcy procedures) in order

to recover the firm's earnings , $\tilde{X}$, so their payoff is always positive since $\tilde{X} - C \geq X_0 - C > 0$.

As for taxes, we shall assume that the corporate tax rate is t_C and that D is fully tax deductible

(i.e., D is all interest and does not include any principal payments). This latter assumption is the

equivalent in the two-period model of the assumption that the firm issues consol bonds (perpetual debt)

in an infinite model. Moreover, we shall assume that the debtholders have an absolute priority over the

tax authorities when the firm goes bankrupt. In other words, the firm pays taxes only when it remains

solvent.

Now, let us consider the all-equity benchmark. The market value of an all-equity firm in period

1 is equal to the market value of equity given by

$$V(0) \;=\; \frac{1}{1+r} \int_{X_0}^{X_1} \left[\tilde{X} \;-\; t_C \tilde{X} \right] dF(\tilde{X}) \;=\; \frac{\hat{X}(1-t_C)}{1+r}. \tag{13}$$

That is, the market value of the firm is equal to the present value of its after-tax earnings.

Next we consider the case where the firm issues debt. We begin by considering safe debt such

that $D < X_0$. This debt is safe since it can be paid back to debtholders in all states of nature so the firm

never goes bankrupt. Since the capital market is perfectly competitive, the market value of equity must

adjust to provide equityholders with a return r on their investment, so

$$E(D) = \frac{1}{1+r} \int_{X_0}^{X_1} \left[\tilde{X} - t_C(\tilde{X} - D) - D \right] dF(\tilde{X}) = \frac{(\hat{X} - D)(1 - t_C)}{1+r}. \qquad (14)$$

Since debt is safe, the market value of the firm's debt is equal to the present value of the face value of

debt:

$$B(D) = \frac{\int_{X_0}^{X_1} D dF(\tilde{X})}{1+r} = \frac{D}{1+r}. \qquad (15)$$

Again, the entrepreneur cares only about the combined value of equity and debt as he sales both types of

securities on the capital market. Together, equation (14) and (15) imply that the market value of the firm

in period 1 is

$$V(D) = E(D) + B(D) = \frac{\tilde{X}(1 - t_C) + t_C D}{1+r} = V(0) + \frac{t_C D}{1+r}. \qquad (16)$$

The difference between V(D) and V(0) is the present value of the tax shield. This tax shield is due to the

fact that by issuing debt, the firm can deduct debt payments from its tax bill. Since the entrepreneur puts

in his pocket the money raised by selling the equity and debt on the capital market, the reduction in the

tax bill is a net gain for him. Equation (16) is the equivalent of M&M Proposition 1 with taxes in our

two-period model. The conclusion is as in M&M: the firm should issue as much safe debt as possible.

Here the upper limit on safe debt is X_0, since once $D > X_0$, debt becomes risky and the analysis has to

be modified accordingly. Hence we found that the optimal debt level, D*, is <u>at least</u> X_0. We now to find

out whether D* > X_0.

When the firm issues risky debt such that $X_0 < D < X_1$, the market value of its equity in period

1 adjusts again to provide equityholders with a return r on their investment, so

$$E(D) = \frac{1}{1+r} \int_{D}^{X_1} \left[\tilde{X} - t_C(\tilde{X} - D) - D \right] dF(\tilde{X}). \tag{17}$$

The right side of the equation represents the discounted value of the firm's expected earnings, net of corporate taxes and debt payment over states of nature in which the firm remains solvent. Similarly, the market value of the firm's debt is such that debtholders earn the risk-free rate on their investment,

$$\boldsymbol{B(D)} = \frac{1}{1+r} \left[\int_{D}^{X_1} \boldsymbol{D} dF(\tilde{X}) + \int_{X_0}^{D} (\tilde{X} - C) dF(\tilde{X}) \right]. \tag{18}$$

The first term on the right side of the equation represents the discounted value of the expected return to debtholders over states of nature in which they are paid in full. The second term represents the discounted value of the firm's expected revenues over states of nature in which the firm goes bankrupt, debtholders become the residual claimants, and incur a cost C.

Combining (17) and (16) and using equation (13) reveals that the market value of the firm in period 1 is

$$\begin{aligned}
V(D) &= E(D) + \boldsymbol{B(D)} \\
&= \frac{1}{1+r} \left[\int_{D}^{X_1} \left[\tilde{X} - t_C(\tilde{X} - D) - D \right] dF(\tilde{X}) + \int_{D}^{X_1} \boldsymbol{D} dF(\tilde{X}) + \int_{X_0}^{D} (\tilde{X} - C) dF(\tilde{X}) \right] \\
&= \frac{1}{1+r} \left[\tilde{X}(1 - t_C) + \int_{X_0}^{D} \left[t_C \tilde{X} - C \right] dF(\tilde{X}) + \int_{D}^{X_1} t_C D dF(\tilde{X}) \right] \\
&= V(0) + \frac{1}{1+r} \left[\int_{X_0}^{D} t_c \tilde{X} dF(\tilde{X}) - CF(D) + t_C D(1 - F(D)) \right].
\end{aligned} \tag{19}$$

The difference between V(D) and V(0), given by the expression inside the square brackets, represents the tax benefits associated with debt payments, net of the present value of bankruptcy costs. The first expression inside the square brackets represents the tax savings that arise because the firm does not need

to pay taxes while in bankruptcy. The third term is the present value of the tax shield that debt payments

provide in states of nature in which the firm is solvent. Finally, the second expression is the expected cost

of bankruptcy, which is the cost of bankruptcy when it occurs, C, times the probability of bankruptcy

which is given by the probability that $\tilde{X} \leq D$.

Let D* be the debt level that maximizes the value of the firm. At this debt level, the difference

between V(D) and V(0) is maximized. Assuming that $D* > X_0$, the first order condition for D* is given

by:

$$V'(D) = \frac{1}{1+r} \left[\left(t_C D - C \right) f(D) + t_C (1 - F(D)) - t_C D f(D) \right]$$

$$= \frac{1}{1+r} \left[t_C (1 - F(D)) - C f(D) \right] = 0.$$

(20)

To find out whether the first order condition is sufficient for a maximum we need to look at the second

derivative of V(D):

$$V''(D) = \frac{-t_C f(D) - C f'(D)}{1+r}.$$

(21)

If V''(D) < 0, which happens when f'(D) is either positive or at least not too negative, then V(D) is a

concave function that attains a unique maximum point which is characterized by equation (20).[2] If V''(D)

> 0, then the solution to equation (20) provides a minimum point rather than a maximum point. Clearly

then we shall assume in what follows that V''(D) < 0 so that equation (20) is a sufficient (not only

necessary) condition for a maximum.

We now need to verify that indeed D* > 0. To this end let us evaluate V'(D) at $D = X_0$. We

already saw that the firm issues debt with a face value which is at least X_0. Therefore if $V'(X_0) > 0$ then

the value of the firm will only increase if it would increase its debt level beyond X_0 so clearly, this would

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[2] Note that when $F(\tilde{X})$ is uniform, $f'(\tilde{X}) = 0$ so V''(D) < 0.

mean that $D^* > X_0$. On the other hand, if we will find that $V'(X_0) < 0$ then the firm would be better-off limiting its debt level to X_0, i.e., issue the maximal amount of safe debt. Differentiating $V(D)$ with respect to D and evaluating at $D = X_0$ yields:

$$V'(X_0) = \frac{t_C(1-F(X_0)) - Cf(X_0)}{1+r} = \frac{t_C - Cf(X_0)}{1+r}, \tag{22}$$

where the second equality follows since $F(X_0) = 0$. Equation (22) implies that $D^* > X_0$, provided that C is smaller than $t_C/f(X_0)$. The intuition for this is simple: if C is very large even the first dollar of risky debt is too costly for the firm. Hence the firm is willing to issue risky debt only if the cost of bankruptcy is not too large. If $C > t_C/f(X_0)$, then the firm will not issue risky debt. Together with the fact that $D \geq X_0$, this implies that $D^* = X_0$: the firm will simply issue debt to point where it just becomes risky but will not cross that level.

Assuming that $C < t_C/f(X_0)$, $D^* > X_0$, so D^* is characterized by equation (20). After simplification this condition becomes:

$$t_C(1-F(D)) = Cf(D). \tag{23}$$

The left side of the equation is the marginal benefit from debt associated with the tax shield provided by debt conditional on the firm remaining solvent (an event which occurs with probability (1-F(D)). The left side of the equation is the marginal cost of debt associated with the increase in the probability of bankruptcy which leads to a marginal increase in the expected cost of bankruptcy. Note that absent bankruptcy costs, i.e., when $C = 0$, then $F(D^*) = 1$, or $D^* = X_1$: the firm issues the maximal amount of debt possible as M&M predict when corporate taxation is present. Note also that in the absence of corporate taxation, i.e., when $t_C = 0$, the firm has no benefit from issuing debt so the firm will never issue risky debt. Hence $D^* \leq X_0$ (the firm of course has nothing to gain by issuing a positive level of debt but so long as $D \leq X_0$, it has nothing to lose either since it never goes bankrupt in this case).

To examine how the optimal debt level varies with the bankruptcy costs and the corporate tax rate,

let us write equation (23) as follows:

$$\frac{t_C}{C} = \frac{f(D)}{1 - F(D)} \equiv H(D), \tag{24}$$

where H(.) is the hazard rate of the distribution of cash flows. The hazard rate is a positive function but in general it can take many different shapes. Nonetheless for most standard continuous distributions (e.g., uniform, exponential, normal, lognormal, logistic), the hazard rate is an increasing function. Hence, if this property is satisfied, the firm will issue more debt when the ratio t_C/C gets larger.[3] In particularly, the firm will issue more debt when the corporate tax rate is higher and when the cost of bankruptcy is lower. It should be emphasized however that this need not always be the case as the hazard rate could be decreasing in which case these comparative statics result may not hold.

To illustrate this tradeoff, assume for simplicity that $F(\tilde{X})$ is uniform on the interval [0, X], so $F(\tilde{X}) = \tilde{X}/X$ and $f(\tilde{X}) = 1/X$. Assume further that $t_C X > C$. Then, the first order condition for D* becomes

$$t_C\left(1 - \frac{D}{X}\right) = \frac{C}{X} \Rightarrow D^* = X - \frac{C}{t_C}. \tag{25}$$

Equation (25) shows that the optimal debt level for the firm increases with t_C and decreases with C. When $t_C X = C$, debt confers no advantage on the firm as the benefit associated with the tax shield of interest payments is completely offset on the margin by the cost of bankruptcy. As t_C increases, so does the firm's debt. At the limit when $t_C = 1$, $D^* = \tilde{X} - C$, so the firm's expected coverage ratio (the fraction of debt payments that is covered by expected earnings is $(1 - C/\tilde{X})$.

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[3] An increasing hazard rate also ensures that the second order condition for a maximum is satisfied (it is not a necessary condition though). To see why note that $V'' = -t_C f - C f'$. But, if the hazard rate is increasing, then $H' = (f'(1-F)+f^2)/(1-F) > 0$, or, $f' > -f^2/(1-F) > 0$. Hence $V'' = -t_C f - C f' < -t_C f + C f^2/(1-F) = Cf[-t_C/C-H] = 0$, where the last equality follows from equation (14). Hence, the second order condition for a maximum is satisfied.

5. The impact of risky debt in the presence of non-debt tax shields

The Kraus and Litzenberger model shows that when the firm chooses its capital structure, it needs to tradeoff the tax benefits of debt against the expected costs of bankruptcy. One unsatisfactory aspect the model is that the source for the cost of bankruptcy is not specified. In particularly it is not obvious in the model whether the cost of bankruptcy are direct (e.g., payments to lawyers and legal fees) or indirect (due say to the interruption of normal business operations). In a widely cited article, Warner (1977) estimated the direct costs of bankruptcy (e.g., legal fees), and found them to be very small. In his sample of 11 railroad companies that went bankrupt between 1933 and 1955, the average direct costs of bankruptcy were merely 1.5% of firms value. Such a small cost should imply that firm need to have very high debt levels since the benefits from tax shields would dominate the cost of bankruptcy.

In a response to this criticism, DeAngelo and Masulis (*JFE*, 1980) consider a more realistic model in which the firm has besides the debt tax shields non-debt tax shields such as depreciation allowance and past loses that can be carried over, and in addition it has a tax credit that reduces the tax bill. Now in bankruptcy, the benefits from these tax shields and tax credit are lost and this adds to the cost of bankruptcy. Their model then claims that the capital structure of the firm should reflect the tradeoff between the tax-shields of interest payments and the probability that bankruptcy will lead to a loss of non-debt tax shields.

Instead of reviewing their model directly, let us consider Han's (*JF*, 1986) model which is a simplified version of DeAngelo and Masulis. The model is similar to Kraus and Litzenberger (1973) except that now corporate taxes are modeled in more detail. Specifically, it is assumed that the firm has non-debt tax shields (allowances that reduce the taxable income), S, and a tax credit G, (a reduction in the tax bill) that are completely lost if the firm goes bankrupt. The firm, however, can fully use the tax credit only if it does not exceed a fraction θ of the gross tax liability, so the actual tax credit is $\text{Min}\{G, \theta t_C(\tilde{X}-D-S)\}$. Hence, when the firm is solvent, its tax bill is $\text{MAX}\{t_C(\tilde{X}-D-S)-G, t_C(1-\theta)(\tilde{X}-D-S), 0\}$.

Given these assumptions, the payoffs of equityholders, debtholders, and the IRS are shown in the following table:

Table 1: payoffs in the DeAngelo and Masulis model

State of nature	Debtholders	Equityholders	IRS	Total
$\tilde{X} < D$	$\tilde{X}$	0	0	$\tilde{X}$
$D < \tilde{X} < D+S$	D	$\tilde{X} - D$	0	$\tilde{X}$
$D+S < \tilde{X} < D+S+G/\theta t_C$	D	$\tilde{X}-D-t_C(1-\theta)(\tilde{X}-D-S)$	$t_C(1-\theta)(\tilde{X}-D-S)$	$\tilde{X}$
$\tilde{X} > D+S+G/\theta t_C$	D	$\tilde{X}-D-t_C(\tilde{X}-D-S)+G$	$t_C(\tilde{X}-D-S)-G$	$\tilde{X}$

Again let us start by establishing the all-equity benchmark. To simplify matters, let's assume that the non-debt tax shield and tax credit are such that $S + G/\theta t_C < X_0$. Given this assumption, the market value of an all-equity firm in period 1 is given by

$$V(0) = \frac{1}{1+r} \int_{X_0}^{X_1} \left[\tilde{X} - t_C(\tilde{X} - S) + G\right] dF(\tilde{X}) = \frac{\hat{X}(1-t_C) + t_C S + G}{1+r}. \qquad (26)$$

That is, the market value of the firm is the present value of its after-tax earnings plus the present value of the non-debt tax shields. The assumption that $S + G/\theta t_C < X_0$ ensures that S and G are always fully utilized.

When the firm issues risky debt such that $X_0 < \tilde{X} < X_1$, the market value of equity in period 1 adjusts again to provide equityholders with a return r on their investment, so

$$E(D) = \frac{1}{1+r}\left[\int_{D}^{D+S} \left[\tilde{X}-D\right]dF(\tilde{X}) + \int_{D+S}^{D+S+\frac{G}{\theta t_C}} \left[\tilde{X}-t_C(1-\theta)(\tilde{X}-D-S)-D\right]dF(\tilde{X}) \right.$$

$$\left. + \int_{D+S+\frac{G}{\theta t_C}}^{X_1} \left[\tilde{X}-t_C(\tilde{X}-D-S)+G-D\right]dF(\tilde{X}) \right]$$

$$= \frac{1}{1+r}\left[\int_{D}^{X_1}[\tilde{X}-D]dF(\tilde{X}) - \int_{D+S}^{X_1} t_C(\tilde{X}-D-S)dF(\tilde{X}) + \int_{D+S}^{D+S+\frac{G}{\theta t_C}} \theta t_C(\tilde{X}-D-S)dF(\tilde{X}) + \int_{D+S+\frac{G}{\theta t_C}}^{X_1} G dF(\tilde{X}) \right].$$

$$\tag{27}$$

The first term on the right side of the equation represents the discounted value of the firm's expected earnings, net of debt payments over states of nature in which the firm remains solvent. The second term represents the present value of gross tax payments over all states of nature in which the tax bill is positive. The third and forth term represent the present value of the tax credit (the third term represent states of nature for which the tax credit is only partially utilized, whereas the forth term represents states for which it is fully utilized). Similarly, the market value of the firm's debt is such that debtholders earn the risk-free rate on their investment,

$$B(D) = \frac{1}{1+r}\left[D(1-F(D)) + \int_{X_0}^{D} \tilde{X}dF(\tilde{X}) \right]. \tag{28}$$

The first term on the right side of the equation represents the discounted value of the expected return to debtholders over states of nature in which they are paid in full. The second term represent the discounted value of the firm's expected revenues over states of nature in which the firm goes bankrupt, debtholders become the residual claimants.

Combining (27) and (28) and using equation (26) reveals that the market value of the firm in period 1 is

$$V(D) = E(D) + B(D)$$

$$= \frac{1}{1+r}\left[\int_D^{X_1} [\tilde{X} - D]dF(\tilde{X}) - \int_{D+S}^{X_1} t_C(\tilde{X}-D-S)dF(\tilde{X}) \right.$$

$$\left. + \int_{D+S+\frac{G}{\theta t_C}}^{X_1} GdF(\tilde{X}) + D(1-F(D)) + \int_{X_0}^{D} \tilde{X}dF(\tilde{X}) \right] \tag{29}$$

$$= V(0) + \frac{1}{1+r}\left[\int_{X_0}^{D+S} t_C(\tilde{X} - S)dF(\tilde{X}) + \int_{D+S}^{X_1} t_C D dF(\tilde{X}) - \int_{X_0}^{D+S+\frac{G}{\theta t_C}} GdF(\tilde{X}) \right].$$

The difference between V(D) and V(0) is the present value of tax shields. The firm will issue debt to the point where this difference is maximized. Let D* be the debt level that maximizes the value of the firm. The first order condition for D* is:

$$V'(D) = \int_{D+S}^{X_1} t_C dF(\tilde{X}) - Gf\left(D+S+\frac{G}{\theta t_C}\right) = 0, \tag{30}$$

or after rearranging terms,

$$t_C(1 - F(D+S)) = Gf\left(D+S+\frac{G}{\theta t_C}\right). \tag{31}$$

The left side of the equation is the marginal benefit from debt associated with the tax shields provided by debt whenever the firm's earnings are high enough to ensure that the firm indeed pays corporate taxes (an event which occurs with probability (1-F(D+S)). The right side of the equation is the marginal cost of debt associated with the potential loss of the tax credit. The firm receives this credit only when its earnings net of debt payments are sufficiently high. But as D increases, the likelihood that earnings net of debt payments will be high becomes smaller and this leads to a loss in expected tax credit. The marginal loss is captured by the right side of the equation.

To illustrate the tradeoff that the firm faces at the optimum, assume again that $F(\tilde{X})$ is uniform on the interval [0, X], and assume that $G < \theta t_C(X\text{-}S\text{-}D)$. Then, the first order condition for D* becomes

$$\theta t_C\left(1 - \frac{D+S}{X}\right) = \frac{G}{X} \quad \Rightarrow \quad D^* = X - S - \frac{G}{\theta t_C}. \tag{32}$$

That is, the optimal debt level for the firm increases with the corporate tax rate, t_C, and decreases with the non-debt tax shields, S, and the tax credit, G. Equation (32) shows that debt and non-debt tax shields and tax credits are substitutes and this substitution the driving force behind the capital structure of the firm.