Supplementary material for the paper

Price and non-price restraints when retailers are vertically differentiated

by

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December 27, 2002

This note contains the proof of Proposition 3 and presents the numerical solutions of the model in the imperfect customer restrictions case that are reported in Section 5.2.

Proof of Proposition 3: Given a uniform wholesale price, w, and since θ is distributed uniformly on the interval $[0,\bar{\theta}]$, the retailers' profits are:

$$\pi_H(w) = \left(1 - \frac{\theta_H}{\bar{\theta}}\right)(p_H - c_H - w), \tag{B-1}$$

and

$$\pi_L(w) = \begin{cases} \left(\dfrac{\theta_H - \theta_L}{\bar{\theta}}\right)(p_L - c_L - w), & p_H > p_L/\gamma, \\ 0, & p_H \le p_L/\gamma. \end{cases} \tag{B-2}$$

Given w, the two retailers simultaneously choose p_H and p_L to maximize their respective profits. Let the Nash equilibrium prices be $p_H(w)$ and $p_L(w)$.

Now, suppose that both retailers operate in the market, i.e., $p_H(w) > p_L(w)/\gamma$. Then, using equation (3), $p_H(w)$ and $p_L(w)$ are defined by the following best-response functions:

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$$\frac{\partial \pi_H(w)}{\partial p_H} = \left(1 - \frac{\theta_H}{\bar{\theta}}\right) - \frac{1}{(1-\gamma)\bar{\theta}}\left(p_H - c_H - w\right) = 0, \tag{B-3}$$

and

$$\frac{\partial \pi_L(w)}{\partial p_L} = \frac{\theta_H - \theta_L}{\bar{\theta}} - \left[\frac{1}{(1-\gamma)\bar{\theta}} + \frac{1}{\gamma\bar{\theta}}\right]\left(p_L - c_L - w\right) = 0. \tag{B-4}$$

To facilitate the analysis, we will characterize the Nash equilibrium in terms of θ_H and θ_L that are induced by p_H and p_L rather than directly by p_H and p_L. Equation (3) indicates that whenever $p_H > p_L/\gamma$, then $p_H = \gamma\theta_L+(1-\gamma)\theta_H$ and $p_L = \theta_L\gamma$. Substituting these expressions in (B-3) and (B-4) and solving, yields

$$\theta_H(w) = \frac{(2-\gamma)c_H - c_L + (1-\gamma)(2-\gamma)\bar{\theta} + (1-\gamma)w}{(1-\gamma)(4-\gamma)},$$

$$\theta_L(w) = \frac{\gamma c_H + 2c_L + \gamma(1-\gamma)\bar{\theta} + (2+\gamma)w}{\gamma(4-\gamma)}. \tag{B-5}$$

Equation (2) implies that both Q_H and Q_L are positive (i.e., both retailers are active) only if $\theta_H(w) > \theta_L(w)$. From (B-5) it follows that $\theta_H(w) > \theta_L(w)$ only if

$$w < w^{**} \equiv \frac{\gamma(1-\gamma)\bar{\theta} + \gamma c_H - (2-\gamma)c_L}{2(1-\gamma)}. \tag{B-6}$$

M sets the wholesale price, w, to maximize his revenue from wholesale:

$$\pi(w) = \left(Q_H(w) + Q_L(w)\right)w, \tag{B-7}$$

where $Q_H(w)$ and $Q_L(w)$ are given by (2), evaluated at $\theta_H(w)$ and $\theta_L(w)$. Substituting from (B-5) into (B-7) and rearranging terms, M's profit is

$$\pi(w) = \frac{\left(3\gamma\bar{\theta} - \gamma c_H - 2c_L - (2+\gamma)w\right)w}{\gamma(4-\gamma)\bar{\theta}}. \tag{B-8}$$

Differentiating this expression and evaluating the derivative at w = w**, we obtain:

$$\begin{aligned}\pi'(w^{**}) &= \frac{(2+2\gamma-\gamma^2)c_L - 3\gamma c_H + \gamma(1-\gamma)^2\bar{\theta}}{\gamma(4-\gamma)(1-\gamma)\bar{\theta}} \\ &> \frac{(2+2\gamma-\gamma^2)\gamma c_H - 3\gamma c_H + \gamma(1-\gamma)^2\bar{\theta}}{\gamma(4-\gamma)(1-\gamma)\bar{\theta}} \\ &= \frac{(1-\gamma)(\bar{\theta} - c_H)}{(4-\gamma)\bar{\theta}} > 0,\end{aligned} \tag{B-9}$$

where the first inequality follows because by assumption, $c_L > \gamma c_H$, and the last inequality follows because by assumption $\bar{\theta} > c_H$. Noting from (B-8) that $\pi(w)$ is strictly concave, it follows that it is never optimal to set $w \leq w^{**}$, so in equilibrium L is effectively foreclosed.

When H is the sole provider of M's product, its profit is given by (B-1) with $p_H = \theta_H$. The optimal choice of H is given by

$$\theta_H(w) = \frac{\bar{\theta} + c_H + w}{2}. \tag{B-10}$$

Since M deals only with H, M's profit is

$$\pi(w) = \left(1 - \frac{\theta_H(w)}{\bar{\theta}}\right)w = \frac{\left(\bar{\theta} - c_H - w\right)w}{2\bar{\theta}}. \tag{B-11}$$

This expression is maximized at $w^* = (\bar{\theta}-c_H)/2$. The assumptions that $\gamma < 1$ and $c_H < c_L/\gamma$, ensure that $w^* > w^{**}$. Given w*, the lowest type that is served is $\theta_H = \theta_H(w^*) = (3\bar{\theta}+c_H)/4$. Since $F(\theta)$ is uniform on the interval $[0,\bar{\theta}]$, Lemma 1 shows that under vertical integration, the lowest type that is served is $\theta^* = (\bar{\theta}+c_H)/2$ which is above $(3\bar{\theta}+c_H)/4$ since $c_H < \bar{\theta}$. Hence M sells less than in vertical integration case. ■

Numerical solution of the model under imperfect customer restrictions:

The numerical solution is based on the following assumptions:

(i) The distribution of θ is uniform on the interval [0,1]

(ii) The distribution of $\tilde{\varepsilon}$ is uniform on the interval $[-\varepsilon,\varepsilon]$

Figure 3 presents M's profit, $\pi(z_{CR})$, for $c_L = 1/8$, $c_H = 1/4$, $\gamma = 3/8$, and 4 different values of ε: 0, 0.05, 0.1, and 0.15.[1] When $\varepsilon = 0$, we are back in the perfect CR case, so $z_{CR}^* = \theta_{CR}$. The horizontal line, marked π_H^*, represents M's profit when H is an exclusive distributor. The Figure shows that although $\pi(z_{CR}^*)$ is always above π_H^*, $\pi(z_{CR}^*)$ decreases with ε. Moreover, z_{CR}^* also decreases with ε. Hence, as the signal z becomes less informative about θ, CR become less profitable and M gives H a larger segment of the market. Consequently, Figure 4 shows that as ε increases, H's sales increase and L's sales and M's aggregate sales decrease. The figure also compares the sales of H, L, and M with their corresponding sales when M deals exclusively with H (this case is denoted by *). The figure shows that even when CR are imperfect, H sells less, L sells more, and M sells more than they do in the case where H is an exclusive distributor.

The effect of imperfect CR on consumers and on welfare is shown in Figures 5 and 6. These figures present consumers' surplus and social welfare as functions of ε for $c_L = 1/8$, $c_H = 1/4$, $\gamma = 3/8$ and were obtained by raising θ from 0 to 0.15 in steps of 0.0001 and solving the model numerically each time. The perfect CR case corresponds to $\varepsilon = 0$. In each figure we also show consumers' surplus and welfare when M deals exclusively with H (again, this case is denoted by *). Figure 5 shows that under imperfect CR, consumers are better off than under perfect CR although they are worse off than in the case where L is foreclosed. Moreover, consumers' surplus is increasing with ε, so although M's aggregate sales fall with ε, the fact that more consumers are served by H who provides more customer services than L, imply that overall consumers become better-off. Figure 6 shows that relative to the case where H is an exclusive distributor, CR is welfare enhancing when ε is small but welfare decreasing otherwise. This

[1] Qualitatively, the picture does not change when we use other values of c_L, c_H, γ, and ε, such that $c_L < c_H < c_L/\gamma < 1$ and $c_L/\gamma < \theta^*$.

supports the conclusion from Proposition 7 that CR may or may not be socially desirable and therefore should be considered under the rule of reason.

Figure 3: The manufacturer's profit under imperfect CR, as a function of z_{CR}

(c_L = 1/8, c_H = ¼, γ = 3/8)

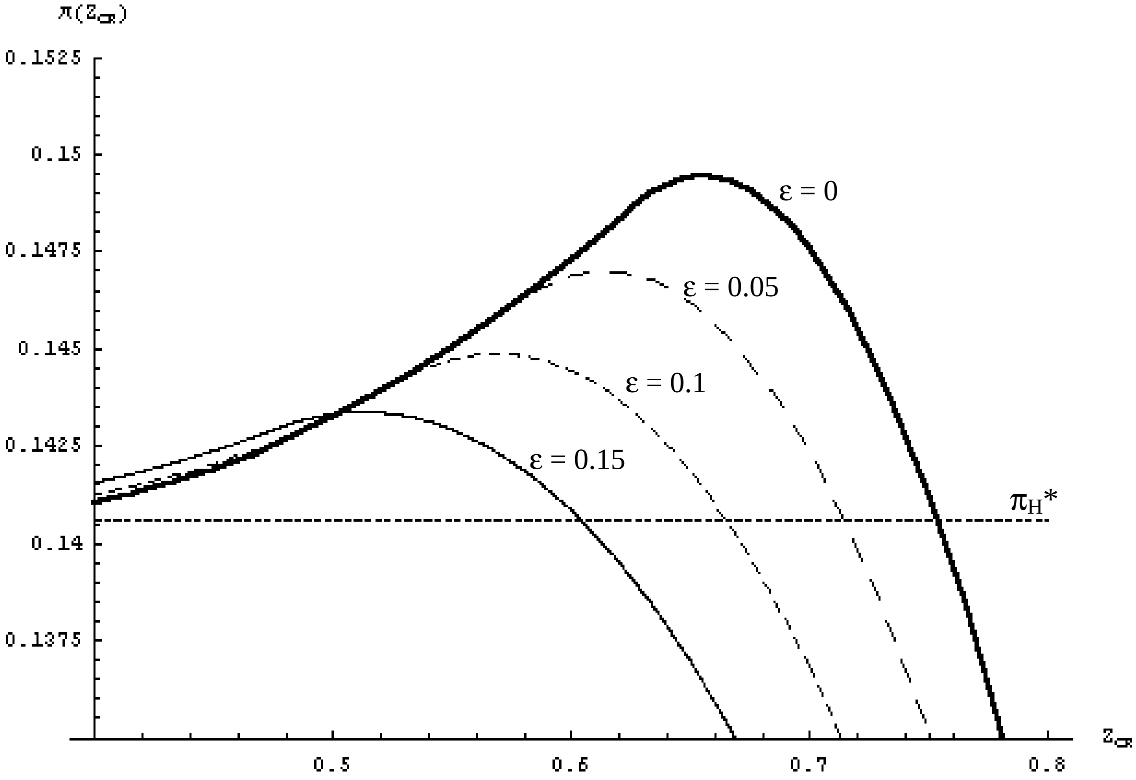

Figure 4: Q_L, Q_H, and the total quantity, Q, as functions of ε

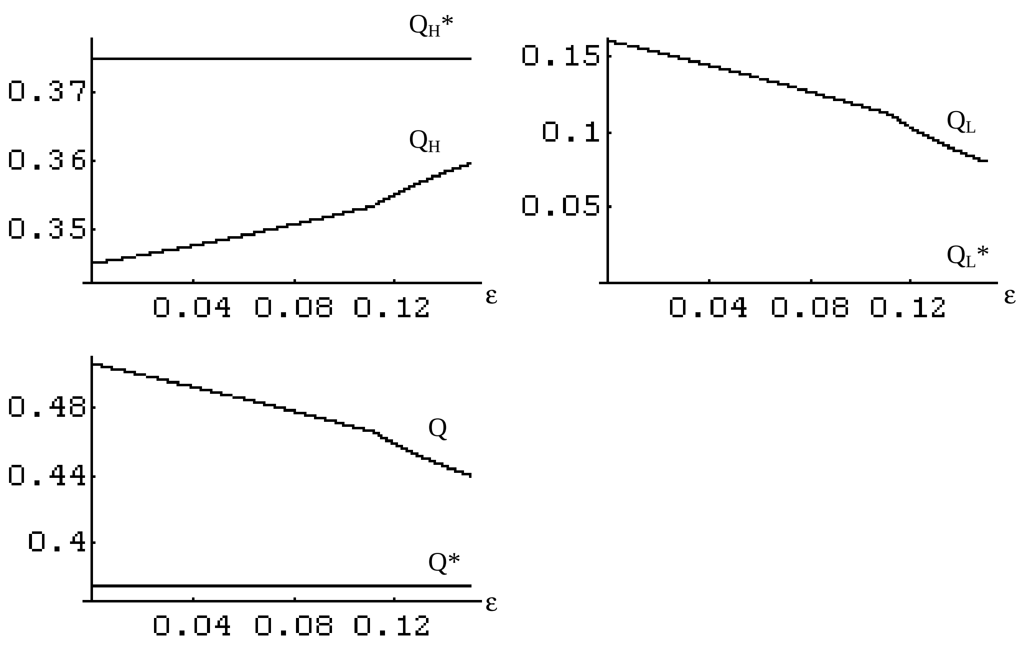

Figure 5: Consumers' surplus as a function of ε

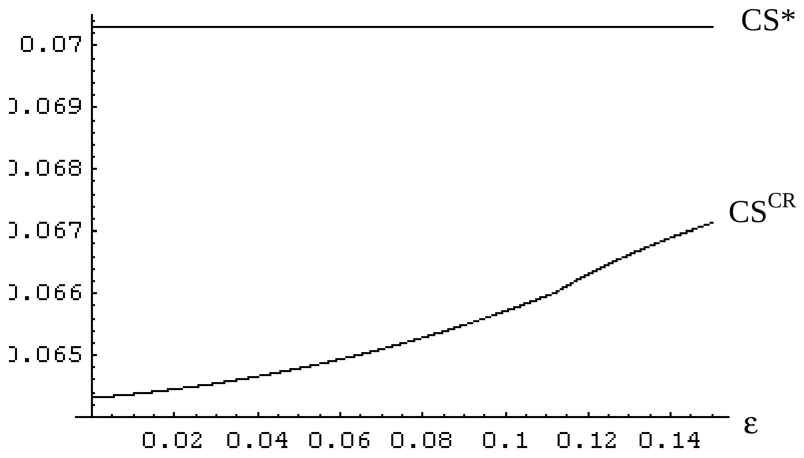

Figure 6: Social welfare as a function of ε