INVESTMENT IN A NEW TECHNOLOGY AS A SIGNAL OF FIRM VALUE UNDER REGULATORY OPPORTUNISM

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We examine the question of whether a regulated firm that makes a long-term investment in infrastructure can credibly signal its private information regarding the future demand for its output to the capital market. We show that necessary conditions for a separating equilibrium in which the magnitude of investment signals high future demand may include a low degree of managerial myopia, large variability of future demand, a lenient regulatory climate, and low sunk cost. Our model suggests that in estimating valuation models of regulated firms it is important to separate firms into two groups: firms for which a separating equilibrium is likely to obtain and firms for which the equilibrium is likely to be pooling. The market value of a firm in the first group is positively correlated with its level of investment, but uncorrelated with the level of actual demand, whereas for the second group the opposite holds.

1. Introduction

Regulated firms make large investments in infrastructure. For example, in 1990, investment in new plant and equipment in the U.S. public-utilities sector totaled 65.91 billion dollars and accounted for ap-

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proximately 12.34 percent of total business expenditure for new plant and equipment. Often, it takes a long time for investments in infrastructure to become operational and produce products or services. This is especially so if firms invest in a new market that does not yet exist. A case in point is the ongoing large investments by telecommunication firms in the emerging market for information services (the information superhighway).

The securities of large regulated firms are typically traded on the capital market and are often widely held. The market values of these firms reflect, among other things, the capital market's assessment of the future success of their investments. In general, due to extensive studies that they conduct prior to investing, firms are better informed than the capital market about the prospects of their investments. This raises the question: Can regulated firms credibly convey their private information to the capital market and thereby affect their current market values? Moreover, to what extent does the effect of investment on market values depend on the future behavior of regulators?

To address these questions, we model the interaction between the firm, the capital market, and the regulator as a three-period model. In the first period, the firm privately learns whether the future demand for a new service will be high or low. Based on this information, the firm chooses how much to invest in the new service. In the second period, the firm's securities are priced in the capital market according to the capital market's belief about the future demand for the new service. This belief is based on the size of the firm's investment, which the capital market interprets as a signal for the firm's private information. Finally, in the third period, the demand for the firm's service is realized, and based on this information the regulator chooses the price of the service. The assumption that the regulator sets the price of the new service after the firm has already made a (partially) irreversible investment reflects the lack of regulatory commitment to prices that characterizes the regulatory framework both in the U.S. and in Britain. This in turn provides regulators with an incentive to behave opportunistically by setting prices that ignore the firm's sunk cost of investment. Our analysis shows that regulatory opportunism affects the firm's ability to use investment as a credible signal for future value. Our paper, therefore, emphasizes the interaction between invest-

1. Source: Department of Commerce, Bureau of the Census.
2. Among the New York Times's list of favorite stocks, which reports the fifteen issues with the most shareholders, ten are stocks of regulated utilities (AT&T, the seven RBOCs, GTE, and PGE).
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investment, rate regulation, and the valuation of the firm in the capital market.

We focus attention on equilibria that satisfy the belief-based refinement of undefeated equilibrium due to Mailath et al. (1993). This refinement rules out equilibria that are supported by "unreasonable" beliefs of the capital market. Its main advantage is that it requires out-of-equilibrium beliefs to be "globally" consistent, thereby avoiding the logical problems inherent in other belief-based refinements. We show that the existence and properties of undefeated equilibria in our model depend critically on; the degree to which the firm's management cares about future profits, the range of possible demand realizations, the regulatory climate, and the degree to which new investment requires sunk costs. More specifically, we show that high investment in a new technology is more likely to increase the regulated firm's market value if the firm's management is not too myopic, if the range of possible demand realizations is large, if the regulatory climate is sufficiently lenient, and if investment does not require too much sunk cost. When these conditions fail, the level of investment in a new technology conveys no private information to the capital market and therefore has no effect on the current market value of the firm. These results suggest that any attempt to estimate the effect of investment on the market value of regulated firms must take into account the regulatory climate in which the firm operates and the degree to which the firm is exposed to the risk of regulatory opportunism.

Regulatory opportunism under asymmetric information is also explored by Banks (1992) in the context of regulatory auditing, Besanko and Spulber (1992) in the context of cost-reducing investment, and Spiegel and Spulber (1993) in the context of optimal capital structure. The current paper differs from the first two papers in that here, the firm signals its private information to the capital market rather than to a regulator. Moreover, unlike Banks (but like Besanko and Spulber), the current paper assumes that the firm signals its private information by choosing a level of investment rather than a proposed regulated price. The main difference between the current paper and the third paper is that Spiegel and Spulber assume that the level of investment is fixed and so the firm signals its private information by choosing a mix of equity and debt to finance this investment. Moreover, in all three papers, the firm's private information is about its cost, whereas in the current paper, the firm has private information about the future demand for its output.

By now, it is well known that regulatory opportunism may induce firms to underinvest (see, e.g., Spulber, 1989, Ch. 20). However, as Besanko and Spulber (1992) show, the underinvestment problem
can be mitigated if regulators are less informed than the firm.\textsuperscript{3} Our analysis, in contrast, shows that the underinvestment problem can be mitigated even when by the time that investment is completed, regulators are as informed as the firm. This is because firms that expect a high future demand for their services may wish to distort their investment levels upward to signal their private information to the capital market.\textsuperscript{4} Our analysis, however, shows that this result need not hold in general, since cases exist in which high-demand firms can signal their private information to the capital market without having to distort their investment levels. Moreover, we also show that when the model admits pooling equilibria, high-demand firms distort their investment level downward, thereby exacerbating the underinvestment problem. Thus, in general, the presence of uninformed investors has an ambiguous effect on the underinvestment problem.

The rest of the paper is organized as follows. The basic three-period model is presented in Section 2. The equilibrium under full information is characterized in Section 3. Section 4 defines the equilibrium concept under asymmetric information and characterizes the equilibrium outcomes. In Section 5, the properties of the equilibria are examined and empirical implications are derived. Concluding remarks are offered in Section 6.

2. THE BASIC MODEL

We present a model of rate regulation that examines the interaction between the firm's investment, the regulator's pricing strategy, and the capital market's valuation of the firm. The sequence of events is shown in Figure 1. There are three periods. In period 1, the regulated firm, whose securities are publicly traded on the capital market, develops a new technology that enables it to offer a new service. The firm privately learns whether the future demand for the new service will be low or high, and it then decides how much to invest in the new technology.\textsuperscript{5} In period 2, the capital market observes the firm's investment level (but not the level of future demand), and based on this

\textsuperscript{3} Other means of alleviating the underinvestment problem include repeated regulation (Salant and Woroch, 1992), regulatory bureaucracy (Sappington, 1986), and debt financing by the firm (Spiegel, 1994).

\textsuperscript{4} The result that firms may overinvest in long-term projects when the capital market is imperfectly informed about their productivity is also obtained by Bebchuk and Stole (1993). However, unlike this paper, they do not consider regulated firms that are exposed to the risk of regulatory opportunism, and they restrict attention to separating equilibria.

\textsuperscript{5} The analysis does not change if instead of learning the true realization of the demand parameter, the firm only receives an unbiased signal on this parameter.
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The firm invests \( k \) in a new technology. A capital market observes \( k \) and determines the firm's service becomes common knowledge. The demand for the firm's service becomes common knowledge. The regulator sets the price of the service.

**FIGURE 1. THE SEQUENCE OF EVENTS**

information and the market's expectation about the regulatory process that follows in period 3, the market value of the firm is determined. At the beginning of period 3, the new technology becomes operational and the level of demand becomes common knowledge. After learning whether demand is low or high, the regulator establishes the regulated price, taking the firm's investment level as given. Finally, the firm provides the service and its profit is realized.

The sequential structure of the model reflects the lack of regulatory commitment to rates that characterizes the regulatory framework both in the U.S. and in Britain. It also reflects the fact that in reality, rates are adjusted more often than the firm's investments. In the U.S., regulatory commissions cannot commit to rates because, historically, courts gave them a great deal of leeway in choosing rates. According to the Supreme Court in the landmark Hope Natural Gas case of 1944, a regulatory agency is "not bound to the use of any single formula or combination of formulae in determining rates." Moreover, in the United Railways case of 1930, the Supreme Court stated that "What will formulate a fair return in a given case is not capable of exact mathematical demonstration." In Britain, the agencies established to regulate the newly privatized public utilities were given wide discretion in setting rates.

We assume that the demand for the new service is perfectly inelastic in the relevant range, and we normalize it to one unit. Let \( V(k, \theta) \) be the total willingness of consumers to pay for the new service, where \( k \) is the firm's investment level, and \( \theta \) is a demand parameter, which we will refer to as the firm's type. For example, \( k \) may be

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8. For example, the telecommunication act of 1984 allows the Director General of Telecommunications to act "in a manner he considers best calculated."
thought of as representing investment in the quality of the service, e.g., investment in service reliability, or investment in increasing the number of different applications that the service may have. The demand parameter can take on two values, low ($\theta^l$) or high ($\theta^h$), so the set of possible firm’s types is $\Theta = \{\theta^l, \theta^h\}$. Assume that $V(k, \theta)$ is increasing and concave in $k$, i.e., $V_k(k, \theta) > 0 > V_{kk}(k, \theta)$, where subscripts denote partial derivatives. In addition, assume that for all $k$, $V(k, \theta^h) > V(k, \theta^l)$ and $V_k(k, \theta^h) > V_k(k, \theta^l)$, so that both the total and marginal benefits from investment increase with the demand parameter. To ensure an interior solution for the firm’s problem, assume that $\lim_{k \to 0} V_k(k, \theta) = \infty$, and $\lim_{k \to \infty} V_k(k, \theta) = 0$, $\theta \in \Theta$.

Investment is firm-specific: once it is installed (but before it is used in production), its value in alternative uses becomes $sk$, where $0 \leq s < 1$. Thus, $(1 - s)k$ is the firm’s sunk cost of investment, while $sk$ is the avoidable cost of investment. Once production takes place, the firm incurs an operating cost, $c$. To simplify the analysis, assume that $V(0, \theta^h) > V(0, \theta^l) > c$, so that the new service is profitable provided that $k$ is not too large. Using $p$ to denote the regulated price, the firm’s period 3 actual profit is $\pi(p, k) = p - c - sk$. The payoff of consumers is represented by consumers’ surplus, $CS(p, k, \theta) = V(k, \theta) - p$.

According to the Supreme Court’s decision in the Hope Natural Gas case of 1944, “The fixing of ‘just and reasonable’ rates, involves a balancing of the investor’s and the consumers’ interests,” which should result in rates that are “within a range of reasonableness.” To capture this balancing of interests, we assume that the regulator chooses the regulated price $p$ with the objective of maximizing the expression $W(p, k, \theta) = CS(p, k, \theta)^{1-\gamma} \pi(p, k)^{\gamma}$, where $0 < \gamma < 1$. Using the definitions of consumer’s surplus and firm profits, the regulator’s objective function becomes

$$W(p, k, \theta) = [V(k, \theta) - p]^{1-\gamma} (p - c - sk)^{\gamma}.$$  

(1)

The resulting regulated price, denoted by $p^*(k, \theta)$, allocates the surplus generated by the firm according to the Nash bargaining solution for the regulatory process. This approach follows Spulber’s (1989) and Besanko and Spulber’s (1992) models of the rate-setting process as a bargaining problem between consumers and the firm, with the regulator playing the role of an arbitrator. It is also consistent with Peltz-

9. Alternatively, one can view $s$ as representing the “hard” portion of investment, which regulators can credibly commit to reimburse the firm for, while $1 - s$ is “soft” investment, which cannot be verifiable in a court and hence regulators can expropriate if they wish to.
man's (1976) political-economy model of rate regulation, in which case \( W(p, k, \theta) \) can be viewed as the regulator's (Cobb-Douglas) utility function. Our approach is very general. In particular, it can accommodate as special cases both regulatory capture by the firm (the case where \( \gamma = 1 \)) and regulatory capture by consumers (the case where \( \gamma = 0 \)). The parameter \( \gamma \) is therefore a measure of the regulatory climate: A low value of \( \gamma \) indicates a hostile regulatory climate, while \( \gamma \) close to one indicates a lenient regulatory climate. The parameter \( \gamma \), together with the parameter \( s \) that measures the degree to which the firm's investment is sunk, captures the notion of regulatory opportunism. Specifically, regulatory opportunism becomes more problematic as \( \gamma \) and \( s \) decrease.

The capital market is assumed to be perfectly competitive. This assumption implies that the firm's securities are priced fairly in the sense that the market value of the firm, i.e., the combined value of the firm's equity and debt, accurately reflects the capital market's assessment of the future profits of the firm. Let \( b^0 \) be the capital market's prior belief that \( \theta = \theta^0 \). In period 2, after observing the firm's investment level \( k \), the capital market updates its prior belief. Let \( b(k) \) be the posterior belief of the capital market as a function of the firm's investment level. Define

\[
\hat{\pi}(k, b(k)) = [1 - b(k)]\pi(p^0(k, \theta^0), k) + b(k)\pi(p^*(k, \theta^k), k)
\]

\[
= [1 - b(k)]p^0(k, \theta^0) + b(k)p^*(k, \theta^k) - c - sk
\]

as the expected profit of the firm according to the capital market's posterior belief in period 2. Normalizing the intertemporal discount factor to 1, the market value of the firm in period 2 is equal to \( \hat{\pi}(k, b(k)) \). In period 3, the capital market learns the true value of \( \theta \), so the market value of the firm becomes equal to its actual profit, given by \( \pi(k, \theta) = \pi(p^*(k, \theta), k) \).

An investment strategy for the firm is a mapping from its private information about \( \theta \) to an investment level \( k(\theta) \). We assume that \( k(\theta) \) is financed by retained earnings or riskless debt. This assumption allows us to ignore the capital structure of the firm and focus on the role of investment in signaling the firm's private information to the capital market.\(^{10}\) Since in our model there are two distinct periods in which the firm's securities are priced in the capital market (period 2

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10. The effects of capital structure on investment in a full-information setting are examined by Spiegel and Spulber (1994) and Spiegel (1994). The use of capital structure to signal private information to the capital market and to a regulator is studied in Spiegel and Spulber (1993).
and period 3), it is natural to assume that the firm's management is concerned with the firm's value in both periods.\textsuperscript{11} This may reflect either the fact that management owns some of the firm's stock and may wish to sell it on the capital market in both periods, or the fact that managerial compensation is tied to the firm's performance in both periods (through bonuses and stock options), or the fact that managers have career concerns that are linked to the firm's performance in both periods through the probability that the management team would be replaced by the board of directors, as well as through the human capital of managers in the market for corporate control. Thus we may write down an induced utility function for the manager defined over the value of the firm in each period. For the sake of analytical tractability, we assume that the utility function is linear.\textsuperscript{12} As a positive transformation of utilities will not affect the analysis, we assume that the firm's management chooses $k(\theta)$ with the objective of maximizing the expression

$$U(k, b(k), \theta) = \alpha \pi(k, b(k)) + (1 - \alpha)\pi(k, \theta) - (1 - s)k,$$  \hspace{1cm} (3)

where $\alpha (0 \leq \alpha < 1)$ is an exogenously specified parameter that reflects the management's relative degree of concern about the period 2 and period 3 market value of the firm, and $(1 - s)k$ is the sunk cost of investment. Note that when $\alpha = 1$ we have the special case that managerial compensation depends only on the firm's current stock price and career concerns are absent. In this case, investment cannot serve as a credible signal, since the firm's management cares only about the current market value of the firm. Consequently the management's utility is independent of the true demand parameter, implying that a low-demand firm can mimic the investment level of a high-demand firm at no cost to the management. Hence it is clear that $\alpha < 1$ is essential for investment to serve as a credible signal.

3. INVESTMENT AND FIRM VALUATION UNDER FULL INFORMATION

In this section, we solve the model under the assumption that the demand parameter (firm's type), $\theta$, is common knowledge. This establishes the full-information benchmark, and helps in developing useful intuition for the more complex asymmetric-information case. Our so-

\textsuperscript{11} This assumption is by now standard in the finance literature; see, e.g., Bebchuk and Stole (1993), Bernheim (1991), Miller (1987), and Stein (1989).

\textsuperscript{12} Including nonlinear terms in the management's utility function complicates the analysis without yielding any new insights.
olution concept here is subgame perfect equilibrium, and we therefore solve the model backwards. We begin by considering period 3 of the model, in which the regulator chooses the regulated price with the objective of maximizing the function $W(p, k, \theta)$. The resulting regulated price as a function of the firm’s investment level $k$ and the demand parameter $\theta$ is

$$p^*(k, \theta) = \gamma V(k, \theta) + (1 - \gamma)(c + sk). \quad (4)$$

Note that $p^*(k, \theta)$ is independent of the sunk cost of investment, $(1 - s)k$, thus reflecting regulatory opportunism. Also note that at the extreme where $\gamma = 0$, the regulator is captured by consumers, so $p^*(k, \theta)$ will only cover the firm’s avoidable costs (i.e., will leave the firm just indifferent to operating in the market). At the other extreme, where $\theta = 1$, the regulator is captured by the firm, and hence $p^*(k, \theta)$ will be set so as to extract from consumers their entire surplus. Given $p^*(k, \theta)$, the firm’s profit becomes

$$\pi(k, \theta) = p^*(k, \theta) - c - sk = \gamma[V(k, \theta) - c - sk]. \quad (5)$$

In equilibrium, the capital market correctly anticipates the firm’s profit. Moreover, since $\theta$ is common knowledge, the firm’s market value in periods 2 and 3 equals $\pi(k, \theta)$. Consequently, the objective function of the firm’s management becomes $\pi(k, \theta) - (1 - s)k$. In equilibrium, the management chooses an investment level, $k^{**}(\theta)$, to maximize this expression. The first-order condition for $k^{**}(\theta)$ is

$$V_k(k, \theta) = (1 - s) + s = 0. \quad (6)$$

The existence of a unique interior solution is ensured by the assumptions that $\lim_{k \to 0} V_k(k, \theta) = \infty$, $\lim_{k \to \infty} V_k(k, \theta) = 0$, and $V_{kk}(k, \theta) < 0, \theta \in \Theta$. Equation (6) states that the firm chooses its level of investment by equating its marginal benefits from investment, given by the left side of the equation, with its marginal cost of investment, given by $\phi$. Since $V_k(k, \theta)$ increases with $\theta$, it follows immediately that $k^{**}(\theta') < k^{**}(\theta'')$. Moreover, since $V_k(k, \theta)$ decreases with increasing $k$, $k^{**}(\theta)$ decreases with increasing $\phi$, which in turn decreases with increasing $s$. Therefore, an increase in assets redeployability, i.e., an increase in $s$, leads to an increase in $k^{**}(\theta)$. This result is intuitive, since an increase in $s$ means that a smaller portion of investment has to be sunk prior to the regulatory process, thus alleviating the extent of regulatory opportunism and encouraging the firm to invest more than it would otherwise. Finally, an increase in $\gamma$ leads to a decrease in $\phi/\gamma$, thereby leading to an increase in $k^{**}(\theta)$. Again, this result is intuitive, because an increase in $\gamma$ means that the firm receives a larger share
of the surplus it generates, and will therefore have a stronger incentive to invest. The properties of \( k^* (\theta) \) are now recorded in the following proposition:

**PROPOSITION 1:** The equilibrium level of investment under full information, \( k^* (\theta) \), is defined implicitly by eq. (6), and it has the following properties:

(i) \( k^* (\theta) \) increases with the demand parameter \( \theta \), i.e., \( k^* (\theta^l) < k^* (\theta^h) \).

(ii) \( k^* (\theta) \) increases both with the degree to which assets are redeployable \( s \) and with the weight that the regulator assigns to firm profits in his objective function \( \gamma \).

The socially optimal level of investment, \( k^{fb} \), is obtained by maximizing the surplus generated by the new service, \( V(k, \theta) - k \). Thus, \( k^{fb} \) is defined implicitly by the first-order condition \( V_k (k, \theta) = 1 \). Comparing this condition with eq. (6) and noting that \( \gamma, s \leq 1 \) imply \( \phi / \gamma \geq 1 \) (with equality holding for \( \gamma = 1 \) or \( s = 1 \)), it follows that \( k^* (\theta) = k^{fb} \) only when \( \gamma = 1 \) or \( s = 1 \). Otherwise, \( k^* (\theta) < k^{fb} \), so the regulated firm underinvests relative to the social optimum.

4. The Case of Asymmetric Information

Having established the full-information benchmark, we now solve the model under the assumption that the demand parameter \( \theta \) is private information for the firm in the first two periods of the game. As before, \( \theta \) becomes common knowledge at the beginning of period 3, before the regulated price is set.

4.1 Equilibrium Concept

We restrict attention to pure strategies and employ perfect Bayesian equilibrium (PBE) as our solution concept. In the current model, a PBE in pure strategies is a pair of strategies \( (p^*(k, \theta), k^*(\theta)) \), a fair pricing of the firm’s securities in the capital market, and a belief function \( b^*(k) \), satisfying the following four conditions:

(E1) Given the demand parameter \( \theta \) and the firm’s investment level \( k \), the regulator’s pricing strategy \( p^*(k, \theta) \) maximizes the social welfare function \( W(p, k, \theta) \).

(E2) Given the firm’s investment level \( k \), the capital market’s (correct) expectation \( p^*(k, \theta) \) about the regulated price that the regulator would select for a given \( \theta \), and the capital market’s posterior
belief $b(k)$, the market values of the firm in periods 2 and 3, respectively, are $\pi(k, b(k))$ and $\pi(k, \theta)$.$^{13}$

(E3) Given its correct expectations $p^*(k, \theta)$ about the regulated price, and the equilibrium market value of the firm, the management of a regulated firm facing a demand parameter $\theta$ chooses its level of investment $k^*(\theta)$ to maximize $U(k, b(k), \theta)$.

(E4) The capital market's posterior belief is derived from Bayes's rule whenever it is applicable. In particular, on the equilibrium path this belief is correct, that is,

$$
b(k^*(\theta')) = 0 \text{ and } b(k^*(\theta^h)) = 1 \quad \text{if } k^*(\theta') \neq k^*(\theta^h),
$$

$$
b(k^*(\theta')) = b(k^*(\theta^h)) = b^0 \quad \text{if } k^*(\theta') = k^*(\theta^h).$$

(7)

If the top line holds, the equilibrium is separating, and if the bottom line holds, it is pooling.

As usual with signaling models, the current model admits multiple equilibria.$^{14}$ This multiplicity arises because condition (E4) does not place any restrictions on the capital market's beliefs off the equilibrium path. Thus, equilibria exist that are supported by "unreasonable" beliefs. To eliminate such equilibria, we apply the refinement of undefeated equilibrium due to Mailath et al. (1993). This refinement is appealing because it ensures that any adjustment of out-of-equilibrium beliefs is consistent with beliefs at other information sets, including some information sets along the equilibrium path.$^{15}$ Another advantage of this refinement is that it is not biased against pooling equilibria like other belief-based refinements [e.g., the intuitive criterion (Cho and Kreps, 1987), or the D2 criterion (Banks and Sobel, 1987)]. In the present model, the refinement of undefeated equilib-

13. Technically, a complete description of the game should specify a strategy for each one of the (potentially many) outside investors. In a perfect Bayesian equilibrium, these strategies should be optimal not only given the equilibrium strategies of the firm and the regulator, but also against one another. However, this would complicate the analysis without affecting any of the results: in equilibrium each investor would still have to earn zero net expected return, for otherwise there would be profitable deviations for some investors.

14. More specifically, the model admits a continuum of separating PBE, and if $\alpha$ is sufficiently large, it also admits a continuum of pooling PBE. A complete characterization of the PBE in pure strategies of the model is available from the authors upon request.

15. Thus, undefeated equilibrium is immune to the so-called Stiglitz critique. This property distinguishes this refinement from other belief-based refinements such as the intuitive criterion (Cho and Kreps, 1987) and perfect sequential equilibrium (Grossman and Perry, 1986). For a detailed discussion on the properties of undefeated equilibria and a comparison between this refinement concept and other refinements, see Mailath et al. (1993).
rium places the following consistency restriction on the capital market's beliefs off the equilibrium path:

(E5) Consider a proposed equilibrium $\sigma$, and an investment level $k'$ that is not chosen in $\sigma$, but is chosen by at least one type of firm in an alternative equilibrium, $\sigma'$. Let $T$ be the set of firm's types that choose $k'$ in $\sigma'$. If each member of $T$ prefers $\sigma'$ to the $\sigma$, with strict preference for at least one type, then upon observing $k'$, the capital market's belief must be consistent with the set $T$. The capital market's belief is said to be inconsistent with the set $T$ if

$$b(k') \neq \frac{b^0m(\theta^h)}{b^0m(\theta^h) + (1 - b^0)m(\theta^l)} \quad (8a)$$

for any $m: \Theta \rightarrow [0, 1]$ satisfying

$$m(t) = 1 \ \forall t \in T_1, \quad m(t) = 0 \ \forall t \notin T, \quad (8b)$$

where $T_1$ is the set of types that strictly prefer the alternative equilibrium to the proposed one.\(^{16}\)

If the capital market's posterior belief satisfies (E5), then the proposed equilibrium is said to be undefeated. In other words, an equilibrium is undefeated if there is no alternative equilibrium that defeats it. Intuitively, consider a putative equilibrium, and suppose that there is an alternative equilibrium with the property that the investment level $k'$ used by some type in the alternative equilibrium is never an equilibrium play in the putative equilibrium, and moreover, this type's payoff is higher in the alternative equilibrium than its payoff in the putative equilibrium. Then upon observing this investment level $k'$, it seems natural to ask, "Could this be an equilibrium signal?" If the answer is yes, then the putative equilibrium seems internally inconsistent if it is supported only by beliefs inconsistent with those

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16. In our two-type model, condition (E5) implies that $b(k') = 1$ if only type $h$ prefers $\sigma'$ to $\sigma$, $b(k') = 0$ if only type $l$ prefers $\sigma'$ to $\sigma$, $b(k') = b^0$ if both types prefer $\sigma'$ to $\sigma$, $0 \leq b(k') \leq b^0$ if type $l$ strongly prefers $\sigma'$ to $\sigma$ while type $h$ weakly prefers it, and $b^0 \leq b(k') \leq 1$ if type $h$ strongly prefers $\sigma'$ to $\sigma$ while type $l$ weakly prefers it. That is, upon observing $k'$, the capital market interprets the deviation to $k'$ as a message sent by the firm. When one type prefers $\sigma'$ to $\sigma$, but not the other type, the market believes that the deviation has been played by the former type. When both types prefer $\sigma'$ to $\sigma$, the capital market finds the message of playing $k'$ uninformative (both types are equally likely to have sent it), so the prior cannot be revised. The situation is a bit more complex when one type strongly prefers $\sigma'$ to $\sigma$, but the other type only weakly prefers it. In this case, the capital market believes that the former type surely have played $k'$, while the latter type may or may not have played it. As a result, the prior is revised by increasing the weight assigned to the type that surely have played $k'$. 

that support \( k' \) as an equilibrium investment level in the alternative equilibrium. That is, the putative equilibrium is defeated if there is a deviation from this equilibrium that is played in the alternative equilibrium by some (or all) types of the firm, all of whom prefer the alternative equilibrium to the putative one.

### 4.2 Preliminary Results and Notation

Before characterizing the set of undefeated PBEs of the model, we first establish some preliminary results and introduce notation that will be useful for what follows. We begin by considering the regulatory process that takes place in period 3. Since at this point \( \theta \) is common knowledge, the regulator’s pricing strategy is the same as under full information, so \( p^*(k, \theta) \) is given by eq. (4). Since the capital market observes the demand parameter \( \theta \) at the beginning of period 3, the market value of the firm in this period is \( \pi(k, \theta) \).

Next, consider period 2. The expected profit of the firm, and hence its market value in this period according to the capital market’s belief, is given by eq. (2). Substituting for \( p^*(k, \theta) \) from eq. (4) into eq. (2), the period 2 market value of the firm becomes

\[
\tilde{\pi}(k, b(k)) = \gamma \tilde{V}(k, b(k)) - c - sk,
\]

where

\[
\tilde{V}(k, b(k)) = [1 - b(k)] V(k, \theta') + b(k) V(k, \theta^h).
\]

Note that \( \tilde{V}(k, b(k)) \) increases with \( b(k) \), which is the probability that the capital market assigns to the demand parameter being \( \theta^h \). As a result, whenever \( \gamma > 0 \), the firm will attempt to convince the capital market that the future demand for its service is high even when in fact it is low.

Substituting for \( \tilde{\pi}(k, b(k)) \) from eq. (9) and \( \pi(k, \theta) \) from eq. (5) into eq. (3), the objective function of the management of a firm with a demand parameter \( \theta \) is

\[
U(k, b(k), \theta) = \alpha \tilde{\pi}(k, b(k)) + (1 - \alpha) \pi(k, \theta) - (1 - s)k
\]

\[
= \gamma[\alpha \tilde{V}(k, b(k)) + (1 - \alpha) V(k, \theta) - c] - \phi k,
\]

where \( \phi = (1 - s) + \gamma s \). The first term on the second line of eq. (11) is the net benefit of investment, due to its effect on the capital market’s assessment of the willingness of consumers to pay in period 3. The cost of investment is captured by \( \phi k \). Note that the cost of investment is independent of the firm’s private information. Nevertheless, the
single-crossing property is still satisfied in this model because $V_k(k, \theta^h) > V_k(k, \theta^l)$, so the benefits from signaling are increasing in the firm's private information, i.e., $U_k(k, b, \theta^h) > U_k(k, b, \theta^l)$.

Now, consider period 1, in which the regulated firm chooses its level of investment in the new technology. If $k^*(\theta^l) \neq k^*(\theta^h)$, the equilibrium is separating, so the firm's demand parameter $\theta$ is fully revealed. Then, by Bayes's rule, $b^*(k^*(\theta^l)) = 0$ and $b^*(k^*(\theta^h)) = 1$. Since a low-demand firm cannot fool the capital market into believing that its demand parameter is high, it would choose in any separating equilibrium its full-information level of investment. Hence,

**Lemma 1:** In any separating PBE, a low-demand-parameter firm chooses its full-information level of investment, i.e., $k^*(\theta^l) = k^*(\theta^h)$.

Since by definition $\tilde{\pi}(k, 0) = \pi(k, \theta^l)$, it follows from eq. (11) and from Lemma 1 that the payoff of a low-demand firm in a separating equilibrium is

$$U^*(\theta^l) = U[k^*(\theta^l), 0, \theta^l] = \pi(k^{**}(\theta^l), \theta^l) - (1 - s)k^{**}(\theta^l)$$

$$= \gamma[V(k^{**}(\theta^l), \theta^l) - c] - \phi k^{**}(\theta^l) \quad (12)$$

Note that $U^*(\theta^l)$ is the minimum payoff that a low-demand firm can guarantee itself. To support a separating equilibrium, the investment level of the high-demand firm, $k^*(\theta^h)$, must be sufficiently large to deter the low-demand firm from mimicking it. Define $k^l$ as the highest level of investment that a low-demand firm will be willing to take in order to pretend that its demand parameter is high. Formally,

$$k^l = \max\{k : U(k, 1, \theta^l) = U^*(\theta^l)\}. \quad (13)$$

Since $U(k, b, \theta)$ is increasing in $b$ and concave in $k$, eq. (13) implies that $k^{**}(\theta^l) < k^l$. Note that by definition, investment levels above $k^l$ are dominated strategies for a low-demand firm and hence will never be chosen by such a firm in equilibrium.

Pooling equilibria arise when $k^*(\theta^l) = k^*(\theta^h) = k^*$. In this case, the firm's investment level does not reveal any information, so by Bayes's rule, $b^*(k^*) = b^0$. Hence, given $k^*$, the period 2 market value of the firm is $\hat{\pi}(k^*, b^0)$. To support a pooling equilibrium, $k^*$ cannot be too small or too large; otherwise the low-demand firm will prefer to choose its full-information level of investment, $k^{**}(\theta^l)$. Define $k^p$ and $\overline{k^p}$, respectively, as the lowest and highest levels of investment that a low-demand firm is willing to take in order to pool with a high-demand firm. That is, in a pooling equilibrium, $k_p^l \leq k^* \leq \overline{k^p}$. Formally, since $U(k, b, \theta^l)$ is strictly increasing in $b$ and strictly concave in $k$, we have that $k^p_l$ and $\overline{k^p}$, respectively, are the smallest and largest solutions
to the equation
\[ U(k, b^0, \theta') = U^*(\theta'), \]  
where \( U^*(\theta') \) is defined by eq. (12). The strict concavity of \( U(k, b^0, \theta') \) in \( k \) ensures that eq. (14) has exactly two solutions. Moreover it implies that \( k_p^l < k^{**}(\theta') < k_p^l \), and that \( U(k, b^0, \theta') > U^*(\theta') \) for all \( k \in [k_p^l, k_p^l] \). This last property means that a low-demand firm is always better off in a pooling equilibrium than it is in a separating equilibrium.

The investment levels \( k^{**}(\theta') \), \( k_p^l \), and \( k_p^l \) are shown in Figure 2 in the \((k, \pi'(\cdot, \cdot))\) space. The three curves labeled \( \pi(k, 1) \), \( \pi(k, b^0) \), and \( \pi(k, 0) \) show the market value of the firm as a function of the level of investment, \( k \), for different beliefs of the capital market. These curves provide a constraint on the firm's payoff. The U-shaped curve, labeled \( U^*(\theta') \), describes the combinations of investment and period 2 market value that give a low-demand firm the same payoff it receives in a separating equilibrium. In the full-information case, the market
knows the demand parameter, so the investment level \( k^{**}(\theta') \) must lie on the curve \( \bar{\pi}(k, 0) \). Then \( k^{**}(\theta') \) maximizes the payoff of the management of a low-demand firm, \( U(k, b, \theta') \), along the curve \( \bar{\pi}(k, 0) \) [that is, \( k^{**}(\theta') \) is characterized in Fig. 2 by the point at which the curve \( U(k, b, \theta') \) is tangent to the curve \( \bar{\pi}(k, 0) \)]. To find the investment levels \( k^*_l \) and \( k^*_h \), note that in a pooling equilibrium, the capital market's belief is such that \( b = b^0 \). Moreover, recall that the low-demand firm can ensure itself at least a payoff of \( U_l(0') \). Thus, \( k^*_l \) and \( k^*_h \) are characterized in Fig. 2 by the intersection of the curves \( U_l(0') \) and \( \bar{\pi}(k, b^0) \). Similarly, by fooling the capital market into believing that its demand parameter is high, a low-demand firm can obtain a payoff along the curve \( \bar{\pi}(k, 1) \). Thus, \( \bar{k} \) is the intersection point of the curves \( U_l(0') \) and \( \bar{\pi}(k, 1) \).17

We are now ready to characterize the set of undefeated PBEs of the model.

4.3 Undefeated PBE

Consider first the case where \( \bar{k} \leq k^{**}(\theta') \), i.e., the highest investment level that a low-demand firm is willing to take is less than the full-information investment level for the high-demand firm. Then we can prove the following (the proof appears in the Appendix):

**Proposition 2:** Consider the case where \( \bar{k} \leq k^{**}(\theta') \). Then the model admits an essentially unique undefeated PBE. In this equilibrium, \( k^*(\theta') = k^{**}(\theta') \) and \( k^*(\theta^h) = k^{**}(\theta^h) \). That is, both types choose their full information levels of investment. Since \( k^{**}(\theta') < k^{**}(\theta^h) \), the equilibrium is separating.

Proposition 2 is quite intuitive: since a low-demand firm will never invest \( k^{**}(\theta^h) \), it is "reasonable" to assume that the capital market will believe that only a high-demand firm will select this level of investment. Given this belief, the high-demand firm chooses its full-information level of investment (which gives the firm the highest payoff it can hope for), knowing that the capital market will correctly infer its type. An interesting property of the equilibrium is that neither type of firm distorts its choice of investment relative to the full-information case. In other words, asymmetric information has no effect on the equilibrium levels of investment.18

17. Note that the condition \( U_l(\theta') = \bar{\pi}(k, 1) \) is equivalent to the condition stated in eq. (12), because \( U(k, 1, \theta') \) is the payoff of a low-demand firm subject to the capital market's belief being \( b(k) = 1 \), i.e., \( U(k, 1, \theta') = a\bar{\pi}(k, 1) + (1 - a)\pi(k, \theta') - (1 - s)k \).

18. It is important to note that while we use a relatively strong refinement concept to obtain (essentially) uniqueness, the result of Proposition 2 can also be obtained by
Next, consider the case where $\bar{k}' > k^{**}(\theta')$. In this case, the low-demand firm will find it profitable to mimic the behavior of the high-demand firm if the latter were to select its full-information level of investment. As a result, the equilibrium levels of investment are distorted away from their full-information levels. Before we characterize the set of undefeated PBEs in this case, note that the separating equilibrium in which $k^*(\theta') = \bar{k}'$ is the Riley equilibrium in our model, i.e., it is the (unique) Pareto-undominated separating equilibrium. This is because the low-demand firm receives the same payoff in every separating equilibrium, while the high-demand firm receives the highest payoff among all separating equilibria when it invests $\bar{k}'$, which is the lowest investment level that induces separation. We now prove the following proposition (the proof appears in the Appendix):

**Proposition 3:** Consider the case where $\bar{k}' > k^{**}(\theta')$. Then the Riley equilibrium, i.e., the Pareto-undominated separating PBE, is the unique undefeated PBE of the model if it gives the high-demand firm the highest payoff among all PBEs. Otherwise, the set of undefeated PBEs coincides with the set of Pareto-undominated pooling equilibria that give the high-demand firm a higher payoff than it receives in the Riley equilibrium.

Proposition 3 implies that the model admits either an undefeated separating equilibrium, or (possibly a continuum of) undefeated pooling equilibria. The first case arises when the Riley equilibrium gives the high-demand firm the highest payoff among all PBEs. Then the low-demand firm invests the same amount as in the full-information case [$k^*(\theta') = k^{**}(\theta')$], while a high-demand firm invests more than it does in the full-information case [$k^*(\theta') = \bar{k}' > k^{**}(\theta')$]. Thus, asymmetric information leads to a distortion in the investment decision of the high-demand firm upward relative to the full-information case. Since the latter is less than the socially optimal level of investment on account of regulatory opportunism, it follows that asymmetric information alleviates the underinvestment problem, and may even lead to overinvestment if $\bar{k}'$ is sufficiently large. When the undefeated equilibrium is pooling, asymmetric information distorts the investment decisions of both types of firm. This is because the set of Pareto-
undominated pooling-equilibrium investment levels (which contains the set of undefeated equilibrium investment levels) is bounded below by \( \bar{k}_p = \text{argmax}_k U(k, b^0, \theta^*) \) and bounded above by \( \hat{k}_p = \text{argmax}_k U(k, b^0, \theta^*) \). But, since \( U(k, b, \theta) \) is increasing in \( b \) for \( \theta \in \Theta \), then \( \bar{k}_p > k^{**}(\theta^*) \) and \( \hat{k}_p < k^{**}(\theta^*) \). Hence, the low-demand firm invests more and the high-demand firm invests less than they respectively invest in the full-information case \([k^{**}(\theta^*) < k^* < k^{**}(\theta^*)]\). This means that when the undefeated equilibrium is pooling, asymmetric information alleviates the underinvestment problem only when the firm has a low demand, but exacerbates it otherwise.

According to Proposition 3, the model admits undefeated pooling PBE when \( \bar{k}^l > k^{**}(\theta^*) \) if and only if there exists an investment level \( k^* \) such that \( U(k^*, b^0, \theta^*) > U(\bar{k}^l, 1, \theta^*) \); otherwise, the equilibrium is separating. That is, the high-demand firm would rather pool with the low-demand firm by investing \( k^* \) than separate itself by investing \( \bar{k}^l \). But, as the next lemma shows, when \( \bar{k}_p \geq \hat{k}_p \), there does not exist such a \( k^* \). The proof of the lemma appears in the Appendix.

**Lemma 2:** Suppose that \( \bar{k}_p \geq \hat{k}_p \). Then the model does not admit undefeated pooling PBE. Consequently, the unique undefeated PBE of the model is the Riley equilibrium.

An implication of Lemma 2 is that undefeated pooling equilibria may exist only if \( \bar{k}^l > \hat{k}_p \). Now, recall from Proposition 2 that whenever \( \bar{k}^l \geq k^{**}(\theta^*) \) the model does not admit undefeated pooling equilibria. Together with Proposition 3, we therefore have the following:

**Proposition 4:** Necessary and sufficient conditions for the existence of undefeated pooling PBE are: (i) \( \bar{k}^l > k^{**}(\theta^*) \), (ii) \( \bar{k}^l > \hat{k}_p \), and (iii) \( U(\bar{k}_p, b^0, \theta^*) > U(\hat{k}_p, 1, \theta^*) \). When condition (i) fails, the model admits an essentially unique PBE that is separating and in which both types choose their full-information levels of investment. When either condition (ii) or (iii) fails, the Riley equilibrium is the unique undefeated PBE of the model.

We now use Proposition 4 to show that the existence of undefeated pooling equilibria depends critically on the parameter \( \alpha \). That is, we are able to prove that, given the regulatory environment, the characteristics of the set of undefeated equilibria depend on the degree to which the firm’s management cares about future profits. The proof of the proposition appears in the Appendix.

**Proposition 5:** There exists a critical value of \( \alpha \), denoted \( \alpha^* \), such that the set of undefeated pooling PBEs is nonempty if and only if \( \alpha > \alpha^* \). Otherwise, the unique undefeated PBE of the model is the Riley equilibrium.

Intuitively, when \( \alpha = 0 \), the firm’s management cares only about period 3 profits, which are independent of the capital market’s beliefs.
Hence, the management chooses the full-information investment level of the firm, which is higher for a high-demand firm. On the other hand, when $\alpha$ approaches 1, the management becomes completely myopic, so the firm's payoff becomes independent of its true demand parameter. As a result, the high-demand firm cannot separate itself, so all equilibria are pooling. By continuity, then, there exists a critical value $\alpha$ of $\alpha$, such that the set of undefeated pooling PBE is nonempty if and only if $\alpha > \alpha$.

5. **Empirical Implications**

In this section we derive the empirical implications of our model. We begin by examining the firm's value in the capital market. In our model, we have to distinguish between the initial market value of the firm, which is based only on the capital market's prior belief on $\theta$, $b^0$; the period 2 market value of the firm, which is based on the capital market's posterior belief on $\theta$, $b(k)$; and the period 3 market value of the firm which is based on the true value of $\theta$.

Earlier we saw that whether the equilibrium is separating or pooling has important implications for the underinvestment problem. We now show that it also has important implications for the firm's valuation. Consider first the case where the equilibrium turns out to be pooling. Then, the period 2 and 3 market values of the firm, respectively, are $\pi(k^*, b^0)$ and $\pi(k^*, \theta)$. Since in equilibrium the capital market correctly anticipates the investment level that the firm would choose in period 2, the initial market value of the firm is also equal to $\pi(k^*, b^0)$. Thus, the firm's market value does not change in period 2 after the firm invests, because the capital market anticipates $k^*$ and because no information is revealed in period 2. On the other hand, since $\pi(k^*, \theta') < \pi(k^*, \theta^0)$, the market value of a high-demand firm increases in period 3 once the new service is introduced and the firm's type is revealed, while the market value of a low-demand firm decreases. Thus, when the equilibrium is pooling, the market value of the firm is positively correlated with its demand parameter, but is independent of the level of investment.

Next, consider the case where the equilibrium turns out to be separating. Then, the capital market learns the firm's type once it invests, so the market value of the firm in periods 2 and 3 is $\pi(\theta) = \pi(k^*(\theta), \theta)$. That is, the market value of the firm does not change between periods 2 and 3, because the market learns all the relevant information on the firm in period 2. The initial market value of the firm is $b^0\pi(\theta^0) + (1 - b^0)\pi(\theta')$. In the Appendix we show that in every separating equilibrium, $\pi(\theta^0) > \pi(\theta')$. Thus, the market value
of a high-demand firm increases in period 2 after it invests, while the market value of a low-demand firm decreases. Hence, in contrast with the case where the equilibrium is pooling, now the market value of the firm is positively correlated with its level of investment, while the demand realization itself does not add to the firm’s value beyond what is added by investment.

The above discussion suggests that in estimating valuation models of regulated firms it is important to separate firms into two distinct groups: one to include all firms for which a pooling equilibrium obtains, and the other to include all firms for which the equilibrium is separating. To shed more light on this issue, we study in the rest of this section the effects of regulation (captured by the parameters $\gamma$ and $s$) on the type of equilibria that arise in the model. The parameter $\gamma$ reflects the regulatory climate, while the parameter $s$ reflects the degree to which the firm is exposed to the risk of regulatory opportunism. In particular, we are interested in finding out whether changes in $\gamma$ and $s$ can move the equilibrium from being a separating one to pooling, and in the former case, whether or not the high-demand firm has to distort its investment level upward relative to the full-information case to induce separation. To this end, consider the special case where $V(k, \theta) = \theta + 2\theta k^{1/2}$ and $\theta^i = 1$. In addition, let $\Delta = \theta^h - 1$ be the difference between the demand parameters of the two types of firms. That is, $\Delta$ is a measure of the range of possible demand realizations for the new service. A tedious but straightforward calculation using eqs. (6) and (11–14) shows that

$$k^{**}(\theta^h) = \left(\frac{\gamma \theta^h}{\phi}\right)^2, \quad k^{*}_p = \left(\frac{\gamma \theta^h}{\phi} - \frac{\alpha \gamma(1 - b^0)\Delta}{\phi}\right)^2,$$

(15)

$$k^i = \frac{\gamma}{\phi^2} \left[\sqrt{\gamma (1 + \alpha \Delta)} + \sqrt{\alpha \Delta} \sqrt{\phi + \gamma(2 + \alpha \Delta)}\right]^2,$$

(16)

and

$$\bar{k}^i_p = \frac{\gamma}{\phi^2} \left[\sqrt{\gamma (1 + ab^0 \Delta)} + \sqrt{ab^0 \Delta} \sqrt{\phi + \gamma(2 + ab^0 \Delta)}\right]^2.$$

(17)

According to Proposition 4, when $k^i \leq k^{**}(\theta^h)$, the unique undefeated PBE of the model is separating and both types of firm choose their full information levels of investment. Using eqs. (15) and (16), it follows that $\bar{k}^i \leq k^{**}(\theta^h)$ if and only if

$$\alpha(\phi + 2\gamma) \leq \gamma \Delta (1 - 2\alpha).$$

(18)

When $\alpha \geq \frac{1}{2}$, the right side of eq. (18) is less than or equal to zero, so the condition can never be satisfied. That is, when the firm’s manage-
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...ment cares more about the present than about the future, the full-information levels of investment cannot be supported as the outcome of an undefeated PBE. On the other hand, when \( \alpha < \frac{1}{2} \), the condition (18) is satisfied whenever

\[
\Delta \geq \Delta = \frac{\alpha(\phi + 2\gamma)}{\gamma(1 - 2\alpha)}. 
\]

That is, when the firm's management cares more about the future than about the present, the full-information levels of investment can be supported as the unique outcome of an undefeated PBE, provided that the difference between the demand parameters of the two types of firms is sufficiently large. Now, an increase in \( \gamma \) and \( s \) lowers \( \Delta \), so the condition (19) is satisfied for a wider set of \( \Delta \)'s. Hence, a decrease in regulatory opportunism (an increase in \( \gamma \)) or a decrease in the degree to which the firm's investment is sunk (an increase in \( s \)) makes the full-information levels of investment more likely to be supported as the unique outcome of an undefeated PBE.

Proposition 4 states that an undefeated pooling equilibrium exists if and only if (i) \( \tilde{k}^l > k^{**}(\theta^p) \), (ii) \( \tilde{k}^b > k^{b}_p \), and (iii) \( U(k^{b}_p, b^0, \theta^h) > U(\tilde{k}^l, 1, \theta^h) \). From eqs. (15-17) it follows that condition (i) is satisfied if and only if

\[
\alpha[\phi + 2 + \alpha\Delta] > \gamma\Delta(1 - \alpha)^2, 
\]

while condition (ii) is satisfied if and only if

\[
\alpha b^0[\phi + 2 + \alpha b^0\Delta] > \gamma\Delta(1 - \alpha)^2. 
\]

Substituting from eqs. (15) and (16) into eq. (11) and rearranging terms, it follows that condition (iii) is satisfied if and only if

\[
\alpha b^0[\phi + 2 + \alpha b^0\Delta] > \gamma\Delta(1 - \alpha)^2 + 2(1 - \alpha) \sqrt{\gamma\Delta} M, 
\]

where

\[
M = \sqrt{\alpha[\phi + 2 + \alpha\Delta]} - \sqrt{\gamma\Delta (1 - 2\alpha + \alpha b^0)}. 
\]

Since \( b^0 \leq 1 \), eq. (21) implies eq. (20). Moreover, if \( M \geq 0 \), then eq. (22) implies eq. (21). Otherwise, however, the condition (22) does not imply, nor is it implied by the condition (21). In general, the condition (22) depends on \( \gamma \) and \( s \) in a complex manner. Therefore, we simplify matters by assuming that \( \alpha = b^0 = \frac{1}{2} \).\(^{19}\) In this case, \( M \geq 0 \), so eq.

19. Note that when \( \alpha = 0 \), the condition (21) [and hence the condition (20)] fails, so the model does not admit undefeated pooling PBE. On the other hand, when \( \alpha = 1 \), all three conditions are satisfied, so the model admits (a continuum of) undefeated pooling PBE.
(22) is a necessary and sufficient condition for the undefeated equilibrium to be pooling. Substituting for $\alpha = b^0 = \frac{1}{2}$ in eq. (22) and rearranging terms, the condition becomes

$$\Delta < \frac{4(\sqrt{2} - 5)(\phi + 2\gamma)}{21\gamma}.$$  

Thus, the model admits undefeated pooling equilibria provided that the difference between the demand parameters of the two types of firms is not too large. Now, an increase in $\gamma$ and $s$ lowers $\Delta$, so the condition (24) is satisfied for a smaller set of $\Delta$'s. The implication of this result is that a decrease in regulatory opportunism (an increase in $\gamma$) or a decrease in the degree to which the firm's investment is sunk (an increase in $s$) makes it less likely that the equilibrium will be pooling.

In light of our earlier discussion on market values, one can conclude that investment in a new technology is likely to affect the market value of a regulated firm if the range of possible demand realizations is large, if regulators are not too hostile to the firm, and if investment does not require too much sunk cost. Otherwise, the equilibrium is pooling, so the capital market cannot infer the firm's private information about the future demand for its service from its investment. As a result, the market value of the firm will not be correlated with the magnitude of its investment.

6. Conclusion

This paper is motivated by the observation that regulated firms often make large long-term investments in infrastructure. Our objective is to examine the capital market's reaction to such investments. To this end, we have considered a three-period asymmetric information model, in which the firm invests in the first period, the capital market values the firm in the second period, and the regulator sets the price for the firm's output in the third period. Our model, therefore, emphasizes the interaction between rate regulation and capital markets and their effects on a firm's incentive to invest.

We have focused attention on equilibria that satisfy the belief-based refinement of undefeated equilibrium. We have shown that the existence and properties of such equilibria in our model depend critically on the degree to which the firm's management cares about future profits, the range of possible demand realizations, the degree of regulatory opportunism, and the degree to which new investments require sunk costs. More specifically, we have shown that under cer-
tain conditions, which include a low degree of managerial myopia, a large range of possible demand realizations, a lenient regulatory climate, and low sunk costs, investment can signal the firm's private information regarding the future demand for its output to the capital market and hence affect the market value of the firm. When these conditions fail, the level of investment in a new technology conveys no private information, so the current market value of the firm will merely reflect the capital market's beliefs about the average future returns and will not be correlated with the magnitude of investment.

**APPENDIX**

**Proof of Proposition 2:** First, note that the full-information levels of investment can be supported as the outcome of a PBE. One belief function that supports this outcome is \( b(k) = 1 \) if \( k = k^{**}(\theta^h) \) and \( b(k) = 0 \) otherwise (this outcome can also be supported by other belief functions that differ only with respect to their specification of out-of-equilibrium beliefs, so the equilibrium is only essentially unique). Given this belief, \( k^*(\theta^h) = k^{**}(\theta^h) \), which is the most preferred level of investment for a high-demand firm. A low-demand firm, on the other hand, will never invest \( k^{**}(\theta^h) \), since \( k' < k^{**}(\theta^h) \) implies that investing \( k^{**}(\theta^h) \) is a dominated strategy for the low-demand firm. Thus, the equilibrium is separating, so by Lemma 1, \( k^*(\theta^l) = k^{**}(\theta^l) \).

Given this belief, the high-demand firm is indeed better off deviating to \( k^{**}(\theta^h) \), so the putative equilibrium is defeated. □

**Proof of Proposition 3:** There are two cases to consider. The first arises when the Riley equilibrium (RE) gives the high-demand firm the highest payoff among all PBEs. Then, this equilibrium is the unique undefeated PBE of the model.²⁰ To see why, let \( k^* \) be an investment level that can be supported as an equilibrium outcome of some pooling equilibrium. Now, the RE cannot be defeated by any pooling PBE,

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²⁰ This claim, in fact, is implied by Theorem 2 in Mailath et al. (1993).
because, by assumption, the high-demand firm becomes worse off in such equilibria than it is in the RE. Consequently, condition (E5) does not restrict \( b(k^*) \), which can therefore be set equal to 0, implying that deviations to \( k^* \) cannot upset the RE. Second, since \( k^*(\theta') = k^{**}(\theta') \) in any separating equilibrium, the only deviations that can possibly upset the RE are those by a high-demand firm to investment levels that it chooses in some other separating equilibria (recall that in order to check whether a PBE is undefeated, we only need to consider deviations to investment levels that are played in alternative PBEs). Such deviations, however, are not profitable for the high-demand firm, since RE dominates all other separating equilibria. Third, using the same arguments, it is easy to see that any separating PBE in which \( k^*(\theta') \neq \tilde{k}' \) can be upset by a deviation of the high-demand firm to \( \tilde{k}' \). By condition (E5), \( b(\tilde{k}') = 1 \), so the high-demand firm would indeed deviate. Thus, the RE defeats all other PBEs.

The second case that has to be considered arises when some pooling PBE gives the high-demand firm a higher payoff than it receives in RE. Then we claim that all such pooling equilibria, which are also Pareto-undominated, can be supported as undefeated PBE. To see why, note first that any Pareto-dominated pooling equilibrium can be defeated by a Pareto-undominated one. This is because both types are better off in the latter equilibrium than they are in the former, so condition (E5) requires that \( b(k^*) = b^0 \), where \( k^* \) is an investment level that can be supported as an equilibrium outcome of some undominated pooling equilibrium. Given this belief, both types are indeed better off deviating, so the putative Pareto-dominated pooling equilibrium is defeated. Second, recall that the low-demand firm is always better off under pooling than under separation. Thus, when the high-demand firm is also better off in some pooling PBE than it is in the RE, condition (E5) requires that \( b(k^*) = b^0 \) following a deviation to \( k^* \). Given this belief, both types will deviate, so RE is defeated. Hence, the only candidates for undefeated equilibria are the undominated pooling equilibria that give the high-demand firm a higher payoff than it receives in RE. Using the same arguments, it is easy to see that none of these equilibria can be defeated by other PBEs. Hence, such equilibria are the only undefeated PBEs in the model. 

Proof of Lemma 2: First, recall that in any pooling equilibrium, \( k^* < \tilde{k}' \); otherwise the low-demand firm would rather invest \( k^{**}(\theta') \) and separate itself. Moreover, recall that \( \tilde{k}' \) is the optimal investment level for the high-demand firm given the capital market’s prior belief, \( b^0 \). Thus, when \( \tilde{k}' \geq \tilde{k}'_p \), the pooling equilibrium that the high-demand firm prefers the most is the one in which \( k^* = \tilde{k}' \). Now, recall that
by definition, \( U(\bar{k}_p', b^0, \theta') = U(\bar{k}', 1, \theta') = U^*(\theta') \). But, since \( \bar{k}_p' \leq \bar{k}' \) and \( V_k(k, \theta') < V_k(k, \theta^p) \), the first equality implies that \( U(\bar{k}_p', b^0, \theta^p) < U(\bar{k}', 1, \theta^p) \). Thus, the high-demand firm is better off in the RE than it is in any pooling equilibrium. By Proposition 3, the unique undefeated PBE in this case is the RE. □

**Proof of Proposition 5:** To prove the proposition, we first show that when \( \alpha = 0 \), condition (ii) in Proposition 4 fails, so the model does not admit an undefeated pooling PBE. Then we show that as \( \alpha \) approaches 1, all three conditions in Proposition 4 are satisfied, so the set of undefeated pooling PBEs is nonempty. By continuity, then, there exists a critical value \( \alpha^* \) of \( \alpha \) such that the set of undefeated pooling PBEs is nonempty if and only if \( \alpha > \alpha^* \).

Suppose that \( \alpha = 0 \). Then it follows from eq. (10) that \( U(k, b, \theta') = \pi(k, \theta) - (1-s)k, \theta \in \Theta \). Since the firm’s payoff is independent of \( b \), it follows from eq. (13) that \( \bar{k}_p = k^*(\theta') \). Similarly, \( \bar{k}_p^b = \argmax_k U(k, b, \theta^p) = \argmax_k U(k, b, \theta^p) = k^*(\theta^p) \). But, from Proposition 1, \( k^*(\theta') < k^*(\theta^p) \). Consequently, \( \bar{k}_p < \bar{k}_p^b \).

Now let \( \alpha \to 1 \). Using eq. (11), \( \lim_{\alpha \to 1} U(k, b, \theta) = \pi(k, b) - (1-s)k \), so the firm’s payoff is independent of \( \theta \). Consequently, \( \lim_{\alpha \to 1} k^*(\theta^p) = \lim_{\alpha \to 1} \argmax_k U(k, b, \theta^p) = \lim_{\alpha \to 1} \argmax_k U(k, b, \theta^p) \).

But, since \( U(k, b, \theta) \) increases in \( b \) and is strictly concave in \( k \), we have \( \bar{k}_p > \argmax_k U(k, 1, \theta') \). Hence, \( \bar{k}_p > \lim_{\alpha \to 1} k^*(\theta^p) \), implying that condition (i) in Proposition 4 is satisfied. Similarly, since the firm’s payoff is independent of \( b \), it follows that \( \lim_{\alpha \to 1} \bar{k}_p = \lim_{\alpha \to 1} \argmax_k U(k, b, \theta) \).

Now, \( \lim_{\alpha \to 1} U(k, b, \theta) = \lim_{\alpha \to 1} U(k, b, 0) = \lim_{\alpha \to 1} U(k, b, \theta) \).

Hence, \( \bar{k}_p > \lim_{\alpha \to 1} \bar{k}_p \), implying that condition (ii) in Proposition 4 is also satisfied. Finally, by definition, \( U(\bar{k}_p, b^0, \theta^p) = \lim_{\alpha \to 1} (k^*(\theta^p)) \).

Hence, \( \lim_{\alpha \to 1} U(\bar{k}_p, b^0, \theta^p) = \lim_{\alpha \to 1} U(k^*(\theta^p), b^0, \theta^p) = \lim_{\alpha \to 1} U(k^*(\theta^p), b^0, \theta') = \lim_{\alpha \to 1} U^*(\theta') \), where the last equality follows because \( U(k, b, \theta) \) increases in \( b \). But from eq. (14) it follows that \( U^*(\theta') = U(\bar{k}', 1, \theta') \).

Moreover, eq. (11) implies that \( \lim_{\alpha \to 1} U(\bar{k}_p, 1, \theta') = \lim_{\alpha \to 1} U(\bar{k}', 1, \theta^p) \). Consequently, \( \lim_{\alpha \to 1} U(\bar{k}_p, b^0, \theta^p) > \lim_{\alpha \to 1} U(\bar{k}', 1, \theta^p) \).

**Proof that in every separating equilibrium, \( \pi(\theta') < \pi(\theta^p) \):** By revealed preferences it follows that \( U^*(\theta') = \pi(\theta') - (1-s)k^*(\theta') < \pi(k^*(\theta'), \theta^p) - (1-s)k^*(\theta') = U^*(\theta^p) \), where the first inequality follows because \( V(k, \theta') < V(k, \theta^p) \forall k \). Hence, \( \pi(\theta') - (1-s)k^*(\theta') < \pi(\theta^p) - (1-s)k^*(\theta^p) \), or \( \pi(\theta') - \pi(\theta^p) > (1-s)[k^*(\theta^p) - k^*(\theta')] \). The proof is completed by observing that in a separating equilibrium, \( k^*(\theta^p) = \max \{ k^*(\theta^p), \bar{k}' \} > k^*(\theta') = k^*(\theta') \).

□


References


