Partial Vertical Integration, Ownership Structure and Foreclosure - Technical Appendix

Nadav Levy, Yossi Spiegel, and David Gilo*

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Abstract

In this Appendix we develop variants of two of the leading "raising your rivals costs" models of input foreclosure to show that the main implications of our basic setup are robust, and can be also derived from other models of vertical integration. We also show that the results in the main text of the paper generalize to the case where the input prices are determined by a more general bargaining process.

*Levy: School of Economics, Interdisciplinary Center (IDC), Herzliya. Email: nadavl@idc.ac.il. Spiegel: Recanati Graduate School of Business Adminstration, Tel Aviv University, CEPR, and ZEW. Email: spiegel@post.tau.ac.il, http://www.tau.ac.il/~spiegel; Gilo: The Buchman Faculty of Law, Tel-Aviv University, email: gilod@post.tau.ac.il.

1 Introduction

In this Appendix, we first develop variants of Ordover, Salop and Saloner (1990) and Salinger (1988), which are two of the leading "raising your rivals costs" models of input foreclosure. Although these models differ from the one we use in the main text in several respects (e.g., inputs are homogeneous rather than differentiated), they nonetheless give rise to a downstream gain, G, and an upstream loss, L, under foreclosure, just as our model does. The purpose of this analysis is to show that the main implications of our basic setup are robust, and can be also derived from other models of vertical integration.

In the last Section of this Appendix we show that the results in the main text of the paper, where we assume that the upstream suppliers make the two downstream firms take-it-or-leave-it offers, generalize to the case where the input prices are determined by a more general bargaining process.

2 A variant of Ordover, Salop and Saloner (1990)

We begin with a variant of the Ordover, Salop and Saloner (1990) model (henceforth OSS): two upstream suppliers, U_1 and U_2, produce a homogenous input and sell it to two symmetric downstream firms, D_1 and D_2, which produce substitute products and compete by setting prices. Since U_1 and U_2 sell homogenous inputs, they engage in Bertrand competition in the upstream market, so absent integration their profit is 0. By definition then, an upstream suppliers cannot lose from vertical integration. Clearly the OSS setting is extreme. To make it less extreme and ensure that U_1 and U_2 earn a profit before integration, we will modify the OSS setting slightly by assuming that the upstream costs are random.[1]

Specifically, we assume that the per unit cost of each upstream supplier i, c_i, is either high, $\overline{c}$ or low, $\underline{c}$, with equal probabilities, independently across the two suppliers (OSS assume that U_1 and U_2 have the same per unit cost, which is deterministic). Given their cost realizations, U_1 and U_2 set the prices of their respective inputs. Then downstream firms, D_1 and D_2, buy the inputs, convert each unit of input to one unit of the final product, at no additional cost, set their respective prices, and sell to final consumers.

[1] Another possibility is to assume that the inputs are imperfect substitutes. However, this modelling approach would require us to specify how the two inputs are combined into a final product, which would add another layer of complication. Our approah avoids this extra complication.

Let w_1 and w_2 be the prices that D_1 and D_2 pay for the input. Since inputs are converted to outputs on a 1:1 basis, w_1 and w_2 are also the marginal costs of D_1 and D_2. The profit of each downstream firm i is then given by

$$\pi_i = (p_i - w_i)\, q_i\,(p_i, p_j)\,,$$

where p_i and p_j are the downstream prices and $q_i\,(p_i, p_j)$ is firm i's quantity. Since the products of D_1 and D_2 are (imperfect) substitutes, $q_i\,(p_i, p_j)$ decreases with p_i and increases with p_j.

The equilibrium price of each downstream firm i is $p_i\,(w_i, w_j)$ and its corresponding quantity and profit are $q_i\,(w_i, w_j) \equiv q_i\,(p_i\,(w_i, w_j)\,, p_j\,(w_i, w_j))$ and $\pi_i\,(w_i, w_j)$.

Lemma A1: $\pi_i\,(w_i, w_j)$ *decreases with* w_i *and assuming that* p_i *increases with* w_j*,* $\pi_i\,(w_i, w_j)$ *also increases with* w_j*.*

Proof: Let $\widehat{w}_i > w_i$ and $\widehat{w}_j > w_j$. Then by revealed preference,

$$\begin{aligned}
\pi_i\,(w_i, w_j) &= (p_i\,(w_i, w_j) - w_i)\, q_i\,(w_i, w_j) \\
&\geq (p_i\,(\widehat{w}_i, w_j) - w_i)\, q_i\,(\widehat{w}_i, w_j) \\
&> (p_i\,(\widehat{w}_i, w_j) - \widehat{w}_i)\, q_i\,(\widehat{w}_i, w_j) \\
&= \pi_i\,(\widehat{w}_i, w_j)\,.
\end{aligned}$$

Moreover,

$$\begin{aligned}
\pi_i\,(w_i, \widehat{w}_j) &= (p_i\,(w_i, \widehat{w}_j) - w_i)\, q_i\,(w_i, \widehat{w}_j) \\
&> (p_i\,(w_i, \widehat{w}_j) - w_i)\, q_i\,(w_i, w_j) \\
&> (p_i\,(w_i, w_j) - w_i)\, q_i\,(w_i, w_j) \\
&= \pi_i\,(w_i, w_j)\,,
\end{aligned}$$

where the first inequality follows because $p_j\,(w_i, \widehat{w}_j) > p_j\,(w_i, w_j)$ and because the two final products are substitutes, so $p_j\,(w_i, \widehat{w}_j) > p_j\,(w_i, w_j)$ implies that $q_i\,(w_i, \widehat{w}_j) > q_i\,(w_i, w_j)$ and the second inequality follows by revealed preference. ■

2.1 Nonintegration

Since the input is homogenous, both input prices under nonintegration are equal to $\underline{c}$ if $c_1 = c_2 = \underline{c}$ and $\overline{c}$ if $c_1 = c_2 = \overline{c}$. When $c_i = \underline{c}$ and $c_j = \overline{c}$, U_i can always undercut U_j slightly and sell to both

D_1 and D_2, so in equilibrium, only U_i sells the input. We will assume that the difference between $\overline{c}$ and $\underline{c}$ is not too large in the sense that U_i will prefer to set the input price at $\overline{c}$.

Assuming that in case of a tie, D_1 and D_2 buy from the lowest cost supplier (and if costs are the same, they randomize their purchases), it follows that in equilibrium,

$$w_1 = w_2 = \begin{cases} \underline{c} & \text{if } c_1 = c_2 = \underline{c}, \\ \overline{c} & \text{otherwise.} \end{cases}$$

Let $\overline{q} \equiv q_1(\overline{c}, \overline{c}) = q_2(\overline{c}, \overline{c})$ be the equilibrium output levels when $w_1 = w_2 = \overline{c}$, and define $\underline{q}$ similarly. The associated downstream prices are $\overline{p} \equiv p_1(\overline{c}, \overline{c}) = p_2(\overline{c}, \overline{c})$ and $\underline{p} = p_1(\underline{c}, \underline{c}) = p_2(\underline{c}, \underline{c})$. Since the input is converted to output on a 1:1 basis, $\overline{q}$ and $\underline{q}$ are also the demands for the input. The expected profit of each supplier is then:

$$V_0^U = \frac{1}{4} \times 2(\overline{c} - \underline{c})\overline{q} = \frac{(\overline{c} - \underline{c})\overline{q}}{2}.$$

This expression reflects the fact that a nonintegrated supplier U_i earns a positive profit only when $c_i = \underline{c}$ and $c_j = \overline{c}$; the probability of this event is $\frac{1}{4}$. The supplier then sets a price of $\overline{c}$ and sells $\overline{q}$ units to each downstream firm. The associated expected profits of D_1 and D_2 is

$$V_0^D = \frac{3}{4}\pi_1(\overline{c}, \overline{c}) + \frac{1}{4}\pi_1(\underline{c}, \underline{c}),$$

where

$$\pi_1(\overline{c}, \overline{c}) = \overline{q}(\overline{p} - \overline{c}), \qquad \pi_1(\underline{c}, \underline{c}) = \underline{q}(\underline{p} - \underline{c}).$$

2.2 Integration

When U_1 and D_1 integrate, U_1 supplies D_1 at cost, unless $c_1 = \overline{c}$ and $c_2 = \underline{c}$, in which case U_2 sells the input to D_1 at a price equal to $\overline{c}$. Hence, D_1 buys the input from U_1 at $\underline{c}$ if $c_1 = \underline{c}$ and at $\overline{c}$ if $c_1 = c_2 = \overline{c}$, and buys it from U_2 at $\overline{c}$ if $c_1 = \overline{c}$ and $c_2 = \underline{c}$. Note that in all cases, $w_1 = c_1$.

As in OSS, we assume that when U_1 and D_1 integrate, U_1 commits not to sell to D_2.[2] Hence U_2 becomes the sole supplier to D_2 and sets the input price, w_2, to maximize its profit

$$(w_2 - c_2)\, q_2(c_1, w_2).$$

[2] As mentioned earlier, there is a debate about how realistic this assumption is. Yet, we choose to follow OSS because our purpose here is to show that (a variant of) their model also predicts that there are cases in which $G > L$ and there are cases in which $G < L$.

We will assume that this profit is concave in w_2. This assumption holds for example in the linear demand example shown below. The profit maximizing value of w_2 is defined implicitly by the following first-order condition:

$$q_2(c_1, w_2) + (w_2 - c_2)\frac{\partial q_2(c_1, w_2)}{\partial w_2} = 0. \tag{1}$$

The solution for the equation, $w_2(c_1, c_2)$, determines D_2's marginal cost. Clearly, $w_2(c_1, c_2) > c_2$ for all c_2. Moreover, $w_2(\overline{c}, \underline{c}) \geq \overline{c}$ provided that $q_2(\overline{c}, \overline{c}) + (\overline{c} - \underline{c})\frac{\partial q_2(\overline{c},\overline{c})}{\partial w_2} \geq 0$. Since $\frac{\partial q_2(\overline{c},\overline{c})}{\partial w_2}$ is bounded from above and $q_2(\overline{c}, \overline{c}) > 0$, this assumption 1 holds when $\overline{c} - \underline{c}$ is sufficiently small.

The expected profit of D_1 under integration is

$$V_1^D = \frac{1}{4}\pi_1(\underline{c}, w_2(\underline{c}, \underline{c})) + \frac{1}{4}\pi_1(\underline{c}, w_2(\underline{c}, \overline{c})) + \frac{1}{4}\pi_1(\overline{c}, w_2(\overline{c}, \underline{c})) + \frac{1}{4}\pi_1(\overline{c}, w_2(\overline{c}, \overline{c})). \tag{2}$$

Notice that since $\pi_i(w_i, \widehat{w}_j) > \pi_i(w_i, w_j)$ for $\widehat{w}_j > w_j$ and since $w_2(\overline{c}, \overline{c}) > w_2(\overline{c}, \underline{c}) \geq \overline{c}$ and $w_2(\underline{c}, \underline{c}) > \underline{c}$,

$$\begin{aligned}
V_1^D &= \frac{1}{4}\pi_1(\underline{c}, w_2(\underline{c}, \underline{c})) + \frac{1}{4} \times \pi_1(\underline{c}, w_2(\underline{c}, \overline{c})) + \frac{1}{4}\pi_1(\overline{c}, w_2(\overline{c}, \underline{c})) + \frac{1}{4}\pi_1(\overline{c}, w_2(\overline{c}, \overline{c})) \\
&> \frac{1}{4}\pi_1(\underline{c}, \underline{c}) + \frac{1}{4}\pi_1(\underline{c}, \overline{c}) + \frac{1}{4}\pi_1(\overline{c}, \overline{c}) + \frac{1}{4}\pi_1(\overline{c}, \overline{c}) \\
&> \frac{1}{4}\pi_1(\underline{c}, \underline{c}) + \frac{1}{4}\pi_1(\overline{c}, \overline{c}) + \frac{1}{4}\pi_1(\overline{c}, \overline{c}) + \frac{1}{4}\pi_1(\overline{c}, \overline{c}) \\
&= V_0^D.
\end{aligned}$$

That is, vertical integration and the foreclosure of D_2 boost the profit of D_1. Since U_1 commits not to sell to D_2 and since it transfers the input to D_1 at cost, its profit is $V_1^U = 0$. Given that its pre-merger profit is $V_0^U > 0$, it follows that integration and the foreclosure of D_2 involve a transfer of profits from U_1 to D_1.

Vertical integration is profitable if the downstream gain exceeds the upstream loss:

$$\underbrace{V_1^D - V_0^D}_{G} > \underbrace{V_0^U - V_1^U}_{L} = V_0^U, \tag{3}$$

where G is the downstream benefit from vertical integration and the foreclosure of D_2 and L is the associated upstream loss. The next example shows that $G > L$ for a broad range of parameters.

2.3 Example

Assume that $q_i = A - p_i + \gamma p_j$, where $\gamma \in [0, 1]$ is the degree of product differentiation. The profit of each downstream firm i is $\pi_i = q_i(p_i - w_i)$. The Nash equilibrium when both firms choose prices simultaneously is

$$p_1(w_1,w_2)=\frac{(2+\gamma)A+2w_1+\gamma w_2}{4-\gamma^2},\qquad p_2(w_1,w_2)=\frac{(2+\gamma)A+2w_2+\gamma w_1}{4-\gamma^2}.$$

The resulting quantities are

$$q_1(w_1,w_2)=\frac{(2+\gamma)A-\left(2-\gamma^2\right)w_1+\gamma w_2}{4-\gamma^2},\qquad q_2(w_1,w_2)=\frac{(2+\gamma)A-\left(2-\gamma^2\right)w_2+\gamma w_1}{4-\gamma^2}.$$

The equilibrium profits are $\pi_1(w_1,w_2)=q_1(w_1,w_2)^2$ and $\pi_2(w_1,w_2)=q_2(w_1,w_2)^2$. Notice that $\pi_i(w_i,w_j)$ decreases with w_i and increases with w_j as Lemma A1 above states.

Given these expressions, the expected pre-merger profits of D and U_1 are:

$$V_0^D=\frac{3}{4}\pi_1(\bar{c},\bar{c})+\frac{1}{4}\pi_1(\underline{c},\underline{c})=\frac{3\left(A-(1-\gamma)\bar{c}\right)^2+\left(A-(1-\gamma)\underline{c}\right)^2}{4\left(2-\gamma^2\right)},\tag{4}$$

and

$$V_0^U=\frac{(\bar{c}-\underline{c})}{2}\times\underbrace{\frac{(2+\gamma)A-\left(2-\gamma^2\right)\bar{c}+\gamma\bar{c}}{4-\gamma^2}}_{\bar{q}}=\frac{(\bar{c}-\underline{c})\left(A-(1-\gamma)\bar{c}\right)}{2(2-\gamma)}.\tag{5}$$

To calculate the price at which U_2 sells to D_2 after U_1 and D_1 integrate, recall that after integration, $w_1=c_1$. Substituting $q_2(c_1,w_2)$ into (1) and solving for w_2 yields

$$w_2(c_1,c_2)=\frac{(2+\gamma)A+\left(2-\gamma^2\right)c_2+\gamma c_1}{2\left(2-\gamma^2\right)}.$$

Hence, the profit of D_1, given c_1 and c_2, is

$$\pi_1(c_1,w_2(c_1,c_2))=q_1(c_1,w_2(c_1,c_2))^2=\left(\frac{\beta A-c_1\left(8-9\gamma^2+2\gamma^4\right)+\gamma c_2\left(2-\gamma^2\right)}{2\left(2-\gamma^2\right)\left(4-\gamma^2\right)}\right)^2,$$

where $\beta\equiv 8+6\gamma-3\gamma^2-2\gamma^3$. Substituting into (2) and rearranging,

$$\begin{aligned}V_1^D&=\frac{1}{4}\pi_1(\underline{c},w_2(\underline{c},\underline{c}))+\frac{1}{4}\pi_1(\underline{c},w_2(\underline{c},\bar{c}))+\frac{1}{4}\pi_1(\bar{c},w_2(\bar{c},\underline{c}))+\frac{1}{4}\pi_1(\bar{c},w_2(\bar{c},\bar{c}))\\&=\frac{2\beta^2A^2+\phi\left(\underline{c}^2+\bar{c}^2\right)-\gamma\left(32-52\gamma^2+26\gamma^4-4\gamma^6\right)\underline{c}\bar{c}-2(1-\gamma)A\beta^2(\underline{c}+\bar{c})}{8(2+\gamma)^2\left(4-2\gamma-2\gamma^2+\gamma^3\right)^2},\end{aligned}\tag{6}$$

where

$$\phi\equiv 64-16\gamma-140\gamma^2+26\gamma^3+109\gamma^4-13\gamma^5-35\gamma^6+2\gamma^7+4\gamma^8.$$

To simplify the computations, we will use the normalizations $A=1$ and $\underline{c}=0$. To ensure that $w_2(\bar{c},\underline{c})\geq\bar{c}$, we will also assume

$$\bar{c}\leq\frac{2+\gamma}{4-\gamma-2\gamma^2}.\tag{7}$$

Substituting from (4), (5), and (6) into (3) and using the normalizations, we get

$$\underbrace{V_1^D - V_0^D}_{G} - \underbrace{V_0^U}_{L} = \frac{2\beta^2 + \phi\bar{c}^2 - 2(1-\gamma)\beta^2\bar{c}}{8(2+\gamma)^2(4-2\gamma-2\gamma^2+\gamma^3)^2} - \frac{4-2(1-2\gamma)\bar{c} - (1-\gamma^2)\bar{c}^2}{4(2-\gamma)^2}. \tag{8}$$

This expression depends only on the degree of product differentiation, γ, and on $\bar{c}$. Figure 2 shows that the combinations of γ and $\bar{c}$ for which (8) holds. The relevant range of parameters which satisfy (7) are those below the $\frac{2+\gamma}{4-\gamma-2\gamma^2}$ curve. The figure shows that the downstream benefit from vertical integration and the foreclosure of D_2, G, exceeds the associated upstream loss, L, when γ is sufficiently large, i.e., the downstream products are sufficiently close substitutes. When γ is low, L exceeds G (note in particular that when $\gamma = 0$, D_1 and D_2 do not compete with each other, so $G = 0$, implying that $L > G$; by continuity this is also true when γ is positive but small).

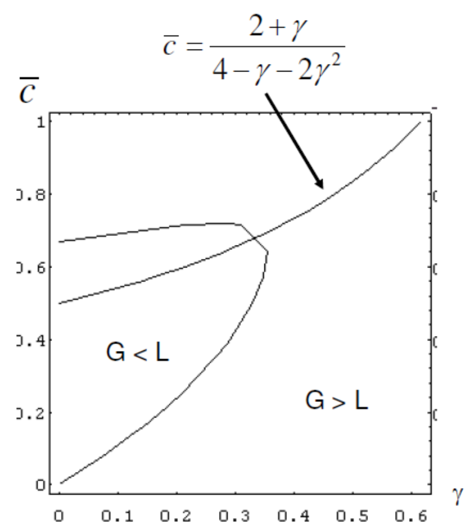

Figure 2: The profitability of vertical integration in a variant of the OSS model

3 A variant of Salinger (1988)

This example shows that our basic setup is also consistent with Salinger (1988). In his model, there are $N \geq 2$ symmetric upstream suppliers $U_1, ..., U_N$, which is produce a homogenous input at a cost c per unit (again, in the main text we consider differentiated inputs). The upstream firms compete by setting quantities and the input price, w, clears the input market. For simplicity, we will assume here only two downstream firms, D_1 and D_2, which convert the input to a final product on a 1:1 basis at no additional cost. D_1 and D_2 also compete by setting their respective quantities, q_1 and q_2. The demand for the final good is given by $p = A - Q$, where $Q = q_1 + q_2$.

3.1 Nonintegration

Since D_1 and D_2 convert the input to a final product on a 1:1 basis at no additional cost, their marginal costs are equal to the input price w. Noting that D_1 and D_2 engage in Cournot competition, the output of each firm is $\frac{A-w}{3}$. Hence, the total demand for the input is $Q = \frac{2(A-w)}{3}$, so the inverse demand for the input is $w = A - \frac{3Q}{2}$.

Each upstream supplier i chooses q_i to maximize its profit $q_i (w - c)$. The resulting Nash equilibrium output of each upstream firm is

$$q^* = \frac{2(A-c)}{3(N+1)},$$

and the equilibrium price of the input is

$$w^* = A - \frac{3Nq^*}{2} = \frac{A+Nc}{N+1}.$$

The equilibrium profit of each upstream firm then is

$$V_0^U = q^* (w^* - c) = \frac{2}{3}\left(\frac{A-c}{N+1}\right)^2, \tag{9}$$

and the equilibrium profit of each downstream firm is

$$V_0^D = \left(\frac{A-w^*}{3}\right)^2 = \left(\frac{N(A-c)}{3(N+1)}\right)^2. \tag{10}$$

3.2 Integration

As Salinger argues, when U_1 and D_1 integrate, U_1 finds it optimal to withdraw from the input market and supply only D_1, who buys the input at a cost c. Hence, $V_1^U = 0$, implying that the upstream loss from vertical integration is $L = V_0^U$.

Now, D_2 buys the input at w, while D_1 buys it at c. In a Nash equilibrium in the downstream market, the output of D_1 is $\frac{A-2c+w}{3}$ and the output of D_2 is $\frac{A-2w+c}{3}$. Since only D_2 buys the input in the upstream market (D_1 is supplied by U_1 at marginal cost), the inverse demand for the input is $w = \frac{A+c-3Q}{2}$.

The profit of each nonintegrated upstream supplier i is given by $q_i (w - c)$. Each upstream supplier i chooses q_i to maximize his profit. The resulting Nash equilibrium output of each upstream firm is

$$q^{**} = \frac{A-c}{3N},$$

and the equilibrium price of the input is

$$w^{**} = \frac{A + c - 3(N-1)\,q^{*}}{2} = \frac{A + (2N-1)\,c}{2N}.$$

Consequently, the equilibrium profit of D_1 is

$$V_1^D = \left(\frac{A - 2c + w^{**}}{3}\right)^2 = \left(\frac{(2N+1)(A-c)}{6N}\right)^2. \tag{11}$$

Using (9)-(11), we obtain

$$\begin{aligned}
\underbrace{V_1^D - V_0^D}_{G} - \underbrace{V_0^U}_{L} &= \left(\frac{(2N+1)(A-c)}{6N}\right)^2 - \left(\frac{N(A-c)}{3(N+1)}\right)^2 - \frac{2}{3}\left(\frac{A-c}{N+1}\right)^2 \\
&= \left(\frac{A-c}{6N(N+1)}\right)^2 \left(1 + 6N - 11N^2 + 12N^3\right),
\end{aligned}$$

which is positive for all N. Hence, vertical integration is always profitable in the Salinger model.

4 Relaxing the assumption that the upstream suppliers make the downstream firms take-it-or-leave-offers

Throughout the paper we assume that the upstream suppliers make the two downstream firms take-it-or-leave-it offers. We now show that our results generalize to the case where the input prices are determined by a more general bargaining process. To this end, suppose that each D_i pays each upstream supplier a price of $\mu\Delta_1(k,l)$ for the input, where $\mu \in \left[\frac{c}{\Delta_1(N,N)}, 1\right]$ measures the bargaining power of upstream suppliers,[3] then the post-acquisition values of D_1 and U_1 are

$$V_1^D = \pi(N, N-1) - N\mu\Delta_1(N, N-1), \qquad V_1^U = \mu\Delta_1(N, N-1) - c,$$

while their pre-acquisition values are

$$V_0^D = \pi(N,N) - N\mu\Delta_1(N,N), \qquad V_0^U = 2\left[\mu\Delta_1(N,N) - c\right].$$

As a result, the upstream loss from foreclosure becomes

$$L_\mu \equiv V_0^U - V_1^U = \mu\Delta_1(N,N) - c + \mu\Delta_{12}(N,N),$$

and the downstream gain from foreclosure becomes

$$G_\mu \equiv V_1^D - V_0^D = -\Delta_2(N,N) + N\mu\Delta_{12}(N,N).$$

By Assumptions A3 and A4, L_μ is increasing, while G_μ is decreasing with μ. Hence, an increase in the bargaining power of upstream suppliers, μ, expands the range of parameters for which D_2 is foreclosed. Intuitively, an increase in μ boosts upstream profits and lowers downstream profits and hence makes input foreclosure, which shifts profits from the upstream firm to the downstream firm, more attractive.

Hence, if input prices are determined by some bargaining process rather than by take-it-or-leave-it offers, G_μ and L_μ replace G and L. ■

[3] We assume that $\mu \geq \frac{c}{\Delta_1(N,N)}$ to ensure that the marginal willingness of D_i to pay for inputs exceeds their cost.

5 References

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