# Partial Vertical Integration, Ownership Structure, and Foreclosure<sup>†</sup>

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We study the incentive to acquire a partial stake in a vertically related firm and then foreclose rivals. We show that whether such partial acquisitions are profitable depends crucially on the initial ownership structure of the target firm and on corporate governance. (JEL D21, D43, G34, L13, L22)

One of the main antitrust concerns that vertical mergers raise is that the merger will result in the foreclosure of either upstream or downstream rivals.<sup>1</sup> Although most of the discussion has focused on full vertical mergers, in reality, many firms acquire partial stakes in suppliers (partial backward integration) or in buyers (partial forward integration). A case in point is the cable industry, where several operators acquired partial ownership stakes in cable or television networks (see Waterman and Weiss 1997, 24–32). This situation has raised the concern that non-integrated networks will be denied access to cable systems or will obtain access at unfavorable terms.<sup>2</sup> More broadly, policymakers seem to be increasingly concerned about the potential anticompetitive effects of partial vertical integration.<sup>3</sup>

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<sup>1</sup>For an overview of the competitive effects of vertical mergers and a summary of vertical enforcement actions in the US during the 1994–2013 period, see for instance Salop and Culley (2014).

<sup>2</sup>Recent prominent examples include News Corp.'s (a major owner of TV stations and programming networks) acquisition of a 34 percent stake in Hughes Electronics Corporation in 2003, which gave it a de facto control over DirecTV Holdings, LLC (a direct broadcast satellite service provider which is wholly-owned by Hughes), and the 2011 joint venture of Comcast, GE, and NBCU, which gave Comcast (the largest cable operator and Internet service provider in the US) a controlling 51 percent stake in the venture that owns broadcast TV networks and stations, and various cable programming. In the UK, BSkyB (a leading TV broadcaster) acquired in 2006 a 17.9 percent stake in ITV (the United Kingdom's largest TV content producer). The UK competition commission concluded that "BSkyB had acquired the ability materially to influence the policy of ITV, which gives rise to common control" and argued that BSkyB would use it to "reduce ITV's investment in content" and "influence investment by ITV in high-definition television (HDTV) or in other services requiring additional spectrum." For examples from other industries, see European Commission (2013b) and Gilo and Spiegel (2011).

<sup>3</sup>See, for example, European Commission (2013a), where the commission states that "... non-horizontal acquisitions of minority shareholdings that also provide material influence may raise competitive concerns of input foreclosure. For some minority shareholdings, foreclosure may even be more likely than when control is acquired ...". The industrial organization literature has mostly studied full vertical mergers and it implicitly assumes that vertical integration occurs if and only if the profit of the merged entity exceeds the sum of the merging firms' premerger profits. But when integration is partial and the target firm (the upstream supplier under partial backward integration or the downstream firm under partial forward integration) is initially held by more than one shareholder, then this is no longer true, because the merger also affects the target firm's remaining, passive, shareholders. If it lowers their payoff, the passive shareholders effectively subsidize the merger, while if it raises their payoff, the merger effectively subsidizes the passive shareholders. This suggests that the initial ownership structure of the target firm has important implications for the incentive to partially integrate and foreclose rival firms after integration. The goal of this paper is to explore these implications in the context of a model that explicitly takes into account the acquisition process.

To this end, we consider a model in which two downstream firms buy inputs from several upstream suppliers. In most of the paper, we study the conditions under which partial integration between one downstream firm,  $D_1$ , and one upstream supplier,  $U_1$ , leads to input foreclosure, i.e., after integration,  $U_1$  forecloses the nonintegrated downstream rival. Our model, however, can be also modified to consider customer foreclosure, under which  $D_1$  stops buying from nonintegrated rival suppliers after integration.

Input foreclosure weakens the downstream rival and hence boosts the profit of  $D_1$ . But then, it also lowers the profit of  $U_1$ , who now forgoes sales to the downstream rival. Under partial backward integration, part of the resulting upstream loss is borne by the passive shareholders of  $U_1$ . We show that partial backward integration, which leads to input foreclosure, is particularly profitable when  $U_1$  is initially held by dispersed shareholders. In that case,  $D_1$  can acquire the minimal stake that ensures control and hence minimize its share in the upstream loss from foreclosure. The rest of the loss is borne by the remaining shareholders of  $U_1$ , who effectively subsidize the input foreclosure. When  $U_1$  has initially a controlling shareholder,  $D_1$  needs to compensate him for the reduction in the value of his entire stake in order to induce him to relinquish control. Since this stake may well exceed the minimal stake needed for control, partial backward integration, which is followed by input foreclosure, is more costly in this case and therefore less likely to occur.

Under partial forward integration by contrast,  $U_1$  bears the entire upstream loss from foreclosure, but needs to share the associated downstream gain with the passive shareholders of  $D_1$ . We show that the resulting transfer of wealth to  $D_1$ 's passive shareholders renders foreclosure unprofitable when  $D_1$  is initially held by dispersed shareholders, but it may still be profitable when  $D_1$  has a controlling shareholder, whose controlling stake is sufficiently large (i.e.,  $D_1$  has relatively few passive shareholders who receive a subsidy).

We also use our model to study additional ownership structures. In particular, we consider the incentive to partially integrate and then foreclose rivals when the target has initially two controlling shareholders, the incentive to backward integrate when  $D_1$  initially holds a noncontrolling stake in  $U_1$  (i.e., a toehold), and the incentive of a controlling shareholder of  $D_1$  to acquire a stake in  $U_1$  either directly or through some other firm that he controls rather than through  $D_1$  itself.

A key driving force in our analysis is that the passive shareholders of the foreclosing firm ( $U_1$  under backward integration and input foreclosure and  $D_1$  under forward integration and costumer foreclosure) subsidize the foreclosure of rivals and hence foreclosure arises for a larger set of parameters when there are more passive shareholders. While this concern would vanish if the rights of the passive shareholders were effectively protected, in reality, policymakers appear to be skeptical about the ability of corporate governance to satisfactorily address this concern.<sup>4</sup>

Our paper merges ideas from corporate finance and from industrial organization. From a corporate finance perspective, our paper is related to the literature on takeovers (e.g., Grossman and Hart 1980; Bebchuk 1994; Burkart, Gromb, and Panunzi 1998, 2000; Burkart et al. 2014; Israel 1992; and Zingales 1995). This literature, however, abstracts from the interaction between the market for corporate control and competition in the product market and how this interaction depends on the initial ownership structure of the target. A main contribution of our paper is to develop a general framework that allows us to study this interaction and examine its potential antitrust implications. In particular, we explore how the ownership structure of the target and the acquirer affect the likelihood of partial vertical mergers and their competitive effects.<sup>5</sup>

The industrial organization literature on input foreclosure has three strands.<sup>6</sup> In Bolton and Whinston (1993), vertical integration strengthens the incentive of the integrated downstream firm to invest and weakens the incentive of the nonintegrated firm to invest. As a result, the latter is less likely to buy the input when its supply is limited. Bolton and Whinston interpret this situation as foreclosure. In our paper, by contrast, foreclosure is due to a deliberate refusal to deal with a nonintegrated rival rather than a by-product of the effect of integration on downstream investments.

The second strand of the literature, due to Hart and Tirole (1990), shows that an upstream supplier may prefer to deal exclusively with one downstream firm in order to alleviate an opportunism problem that prevents the supplier from extracting profits from the downstream firms. In our paper there is no opportunism problem and the role of foreclosure is to shift profits from one downstream firm to another.

Our model is closely related to the third strand, due to Ordover, Saloner, and Salop (1990) and Salinger (1988). In this strand, input foreclosure may be beneficial because it raises the costs of downstream rivals.<sup>7</sup> The idea in our model is similar,

<sup>&</sup>lt;sup>4</sup>For example, in its decision regarding News Corp.'s acquisition of a 34 percent stake in Hughes in 2003, the Federal Communications Commission (FCC) wrote: "We therefore discount the likelihood that corporate governance, corporate law or securities laws in general may be relied upon to adequately protect MVPD and video programming competitors from potential anti-competitive vertical foreclosure behavior on the part of Applicants." See FCC (2014, paragraph 100).

<sup>&</sup>lt;sup>5</sup>There are additional differences: backward integration in our paper lowers the target's value rather than increases it as in most of the corporate finance literature. Under forward integration, the acquisition increases the target's value, but this is because it affects the acquirer's strategy rather than the target's strategy. For this reason, the acquisition does not necessarily involve a controlling stake. Moreover, instead of enjoying private benefits as most of the corporate finance literature assumes, the acquirer, who under forward integration is an upstream supplier, sustains a loss due to the forgone sales to a downstream rival.

<sup>&</sup>lt;sup>6</sup>See Rey and Tirole (2007) and Riordan (2008) for literature surveys.

<sup>&</sup>lt;sup>7</sup>The assumption that the upstream supplier can commit to foreclose the downstream rival has been criticized as problematic, see Hart and Tirole (1990) and Reiffen (1992), and see Ordover, Salop, and Saloner (1990) for a reply. Several papers, including Ma (1997); Chen (2001); Choi and Yi (2000); Church and Gandal (2000); and Allain, Chambolle, and Rey (2016) have proposed models that are immune to this criticism. Moresi and Schwartz (2017)

although foreclosure lowers the value that a downstream rival can offer consumers rather than raises its cost. More importantly, we examine the incentive to acquire a partial stake in a vertically related firm and examine how this incentive depends on the initial ownership structure of the target firm and on corporate governance. Our model also generalizes to the case of customer foreclosure with minimal modifications and hence it differs from the models of Ordover, Saloner, and Salop (1990); Salinger (1988); or Hart and Tirole (1990), which cannot be naturally adapted to explain customer foreclosure.

There are only a few papers that consider the competitive effects of partial vertical integration. Riordan (1991), Greenlee and Raskovitch (2006), Choi et al. (2014), and Hunold and Stahl (2016) consider passive acquisitions, which affect the competitive strategy of the acquirer, but do not affect the target's strategy as in the case of backward integration in our paper.<sup>8</sup> We are aware of only four papers that consider the acquisition of a controlling stake in a vertically related firm. Baumol and Ordover (1994) show that a downstream firm, which controls a bottleneck owner with a partial ownership stake, has an incentive to divert business to itself, even if downstream rivals are more efficient.<sup>9</sup> Spiegel (2013) examines a model in the spirit of Bolton and Whinston (1993), in which (partial) vertical integration affects the incentives of downstream firms to invest in the quality of their products. He shows that relative to full integration, partial vertical integration may either alleviate or exacerbate the concern for input foreclosure and examines the resulting implications for consumers. Brito, Cabral, and Vasconcelos (2016) show that downstream firms will use their controlling stakes in an upstream supplier to lower the wholesale price; consequently, consumer surplus is higher and increasing in the size of the controlling stakes. Neither paper, however, studies the takeover game, nor examines how the incentive to partially integrate depends on the ownership structure of the target, which is the main focus of the current paper.

There is some empirical evidence on the foreclosure effect of vertical mergers. Waterman and Weiss (1996), Chipty (2001), Chen and Waterman (2007), Suzuki (2009), and Crawford et al. (2015) find evidence that vertically integrated cable systems tend to exclude nonaffiliated television networks, while favoring affiliated networks. Hastings and Gilbert (2005) show that a vertically integrated refiner charges higher wholesale prices in cities where it competes more with independent gas stations.

show that when a vertically integrated input monopolist supplies a differentiated downstream rival and sets a linear price for the input, it has an incentive to induce expansion by the rival irrespective of whether downstream competition involves prices or quantities.

<sup>&</sup>lt;sup>8</sup>Fiocco (2016) considers passive partial forward integration and shows that it allows the manufacturer to capture some of the information rents that accrue to a privately informed retailer and hence affects the contracts that the manufacturer offers the retailer and consequently the resulting competition in the downstream market. Höffler and Kranz (2011a,b) also study passive partial backward integration, but in their model, the upstream supplier, which is regulated, may internalize some of the downstream profits and consequently sabotage the access of rival downstream firms to its essential input.

<sup>&</sup>lt;sup>9</sup>Reiffen (1998) finds that the stock price of Chicago Northwestern (CNW) railroad reacted positively, rather than negatively, to events that made it more likely that Union Pacific (UP) Railroad will gain effective control over CNW with a partial ownership stake. This finding is inconsistent with the idea that UP would have used its control over CNW to foreclose competing railroads.

The rest of the paper is organized as follows. The basic model is presented in Section I. In Section II, we study two benchmarks: nonintegration and full integration. Our main results appear in Section III, where we examine how the incentives to engage in "input foreclosure" following partial backward or forward integration depend on the initial ownership structure of the target firm. In particular, we consider two polar cases: the target has a single controlling shareholder, and the target is owned by atomistic shareholders. In Section IV, we examine three additional ownership structures: the target has two or more large shareholders, the acquirer holds a noncontrolling stake (i.e., a toehold) in the target, and the acquisition is made by the controlling shareholder of the firm (or another firm he controls) rather than by the firm itself. In Section V, we show that our model also applies, with minimal modifications, to "customer foreclosure," and we also study two extensions of our basic setup. In Section VI, we conclude. The Appendix includes technical proofs and an example that shows how our reduced-form profits can be derived from an explicit model of downstream competition.

#### I. The Model

Consider two downstream firms,  $D_1$  and  $D_2$ , that provide a final good/service to consumers. The downstream firms use up to  $N \ge 1$  differentiated inputs, each of which is produced by a single upstream supplier  $U_i$ , i = 1, 2, ..., N. The cost that an upstream supplier incurs when it serves a downstream firm is c.<sup>10</sup> Let  $\pi(k, l)$  denote the (reduced-form) profit of a downstream firm when it uses k inputs and its rival uses l inputs, before any payments to upstream suppliers. In Appendix B, we show an example for how  $\pi(k, l)$  can be derived from an explicit model of downstream competition. The profit functions and upstream costs are common knowledge.

Throughout the analysis we will impose the following assumption.

# ASSUMPTION A1: $\pi(k, l)$ is increasing with k at a decreasing rate and decreasing with l.

Assumption A1 says that the upstream inputs are complementary in the sense that a downstream firm is more profitable when it uses more inputs, while its rival uses fewer inputs. The assumption is natural when downstream firms sell a variety of products and greater variety is valued by consumers. For example,  $D_1$  and  $D_2$  can be cable or satellite TV providers, which deliver TV channels to viewers. Assumption A1 then implies that viewers care for a variety of programming and hence, other things being equal, each TV provider faces a higher demand when it offers more channels, while its rival offers fewer channels. Likewise, if  $D_1$  and  $D_2$ are retailers, Assumption A1 says that each retailer faces a higher demand when it carries more brands, while its rival carries fewer brands.<sup>11</sup> Assumption A1 also fits

<sup>&</sup>lt;sup>10</sup>It is straightforward to modify this assumption and assume instead that the cost of serving two downstream firms is more or less than twice the cost of serving only one downstream firm.

<sup>&</sup>lt;sup>11</sup>Indeed, the Federal Trade Commission (FTC) opposed the 1999 merger between Barnes and Noble, Inc. (a major book retailer) with Ingram Book Group (a book wholesaler) on the grounds that Barnes and Noble could restrict access or raise prices of books to competing retailers. The merger was eventually abandoned. See Salop and Culley (2014).

well situations in which the downstream firms combine various components to offer products/services to final consumers. For example,  $D_1$  and  $D_2$  could be smartphone producers and the inputs could be various technologies that enhance the functionality of smartphones, or  $D_1$  and  $D_2$  could be providers of some service who rely on various components that enhance the value of the service.<sup>12</sup> While Assumption A1 does not fit situations where the upstream inputs are homogenous (say  $D_1$  and  $D_2$  produce concrete and buy cement from upstream suppliers, or are chains of gasoline stations and buy gasoline from upstream refineries), our analysis still goes through, because the main building blocks for our analysis can also be derived from models in which the upstream inputs are homogenous.<sup>13</sup>

The sequence of events is as follows. At the outset, all firms are independently owned. Then, either one downstream firm,  $D_1$ , acquires a stake  $\alpha$  in upstream supplier  $U_1$  (backward integration), or  $U_1$  acquires a stake  $\alpha$  in  $D_1$  (forward integration). We will say that integration is partial if  $\alpha < 1$ . In most of the paper we will assume that the acquisition gives the acquirer full control over the target if  $\alpha \geq \underline{\alpha}$ , where  $\underline{\alpha}$  is the minimal stake needed for de facto control.<sup>14</sup> In Section VB, we will relax this assumption, and show that our main results generalize to the case where a stake  $\alpha < 1$  gives the acquirer only partial control over the target.

Given the new ownership structure, each of N upstream suppliers decides whether to supply its input to both downstream firms or to only one. These decisions are publicly observable and irreversible.<sup>15</sup> Since each downstream firm either buys the input of upstream supplier *i* or does not buy, its payment for the input is a total payment rather than a per unit price. Then,  $U_1, \ldots, U_N$  make simultaneous take-itor-leave-it offers to  $D_1$  and  $D_2$ , and once the offers are accepted or rejected, the final product is produced and payoffs are realized.<sup>16</sup>

<sup>12</sup> A case in point is Google's acquisition in 2011 of ITA Software Inc., which develops and licenses software used by online travel intermediaries (OTIs) to provide customized flight searches. The Department of Justice alleged that Google, which planned to offer its own online travel search, would deny OTIs access to or raise their price for the software. Another example could be the 1999 merger of Provident Companies, Inc. and UNUM Corporation, both providers of insurance for individual disability policies. It was common practice for insures to share actuarial data through an industry association to facilitate the determination of risk for particular injuries. The FTC alleged that the merged firm would no longer have the incentive to share this information with rivals. See Salop and Culley (2014).

<sup>16</sup>The assumption that the upstream firms can make take-it-or-leave-it offers is not essential: in the online Appendix, available at https://www.tau.ac.il/~spiegel/papers/partialVI-tech-Appendix.pdf, we show that our results generalize to the case where input prices are determined by a more general bargaining process.

<sup>&</sup>lt;sup>13</sup>In a technical Appendix, available at https://www.tau.ac.il/~spiegel/papers/partialVI-tech-Appendix.pdf, we show that the main implications of our basic setup can also be derived from variants of Ordover, Salop, and Saloner (1990) and Salinger (1988), which are two of the leading "raising your rivals costs" models of input foreclosure, and where inputs are homogenous.

<sup>&</sup>lt;sup>14</sup>We do not restrict  $\underline{\alpha}$  to be above 50 percent, as is often assumed in the literature. Indeed, in the News Corp. and the BSkyB examples mentioned in footnote 2, stakes well below 50 percent were deemed sufficient to ensure effective control. One reason for this is that noncontrolling shareholders often do not vote. According to Broadridge Financial Solutions and PwC's Center for Board Governance (2015), during the first half of 2015, individual investors, who account for 32 percent of ownership, voted only 28 percent of the shares they owned, while institutional investors, who account for 68 percent of ownership, voted only 91 percent of their shares. Since combined, less than 71 percent of the shares are voted, on average, a 41.5 percent stake secures the majority of votes. In firms with capitalization of under \$300 million, voting participation is even lower, so on average, a stake of 29.1 percent is enough for control.

 $<sup>^{15}</sup>$  This assumption can be justified as in Church and Gandal (2000) and Choi and Yi (2000), where each upstream firm needs to adapt the input to the special needs of each downstream firm. As we discuss below, absent this assumption, we would have a Hart and Tirole (1990) type of commitment problem.

#### **II. Nonintegration and Input Foreclosure under Full Integration**

In this section, we study the nonintegration and full integration benchmarks. In later sections, we will consider the incentive to partially integrate, taking the merger process explicitly into account.

We begin by solving the last stage of the game in which the upstream suppliers make simultaneous take-it-or-leave-it offers to  $D_1$  and  $D_2$  for the sale of their inputs. To this end, suppose that  $D_i$  already buys k - 1 inputs and  $D_j$  buys l inputs. Then, the marginal willingness of  $D_i$  to pay for the *k*th input is

$$\Delta_1(k, l) \equiv \pi(k, l) - \pi(k - 1, l).$$

This expression represents the incremental profit from adding the *k*th input, given that the rival uses *l* inputs. Assumption A1 implies that  $\Delta_1(k, l)$  is positive but decreasing with *k*. For later use, we denote the negative externality that an increase in *l* imposes on  $D_i$ 's profit by

$$\Delta_2(k, l) \equiv \pi(k, l) - \pi(k, l-1).$$

By Assumption A1,  $\Delta_2(k, l) < 0$  for all k and l.

To ensure that selling N inputs is profitable, we will make the following assumption.

ASSUMPTION A2:  $\Delta_1(N, N) > c$ .

While Assumption A2 ensures that selling inputs is profitable if both downstream firms buy all N inputs, it is possible that an upstream supplier may prefer to sell its input to only one downstream firm. To see why, note that the change in  $D_i$ 's marginal willingness to pay for input k when  $D_j$  increases the number of its inputs from l - 1 to l, is given by

$$\Delta_{12}(k, l) \equiv \Delta_1(k, l) - \Delta_1(k, l-1).$$

Although in general  $\Delta_{12}(k, l)$  could be either positive or negative, the example in Appendix B shows that it is reasonable to assume that  $\Delta_{12}(k, l) < 0$ . That is,  $D_i$ 's marginal willingness to pay for inputs decreases when  $D_j$  uses one more input. For the sake of concreteness, we will assume that this is indeed the case.

ASSUMPTION A3: 
$$\Delta_{12}(k, l) \leq 0$$
 for all k, l.

Given Assumption A3, it is possible, at least in principle, that an upstream firm may be unwilling to supply both downstream firms. The following assumption rules out this possibility and ensures that under nonintegration, both downstream firms buy all *N* inputs.

ASSUMPTION A4: 
$$\Delta_1(k, l) - c > -\Delta_{12}(l, k)$$
 for all  $k, l$ .

Assumption A4 implies that the maximal profit that an upstream supplier can make by selling an extra input to  $D_i$ ,  $\Delta_1(k, l) - c$ , exceeds  $-\Delta_{12}(l, k)$ , which is the associated loss of profit from selling to  $D_i$ .

#### A. The Nonintegration Benchmark

With Assumption A4 in place, we now characterize the equilibrium behavior of non-integrated upstream suppliers.

LEMMA 1: In equilibrium, non-integrated upstream suppliers sell to both  $D_1$  and  $D_2$ , irrespective of whether  $D_1$  and  $U_1$  are partially or fully integrated and irrespective of whether  $U_1$  forecloses  $D_2$  or not. If  $D_1$  and  $U_1$  are integrated and  $U_1$  forecloses  $D_2$ , upstream suppliers 2, ..., N charge  $D_1$  an amount  $\Delta_1(N, N-1)$  for the input and charge  $D_2$  an amount  $\Delta_1(N-1, N)$ . If  $D_2$  is not foreclosed, all upstream suppliers charge  $D_2$  an amount  $\Delta_1(N, N)$  and all non-integrated upstream suppliers charge  $D_1$  an amount  $\Delta_1(N, N)$ .

Given that  $\Delta_1(N, N-1) > \Delta_1(N, N)$  by Assumption A3, Lemma 1 implies that when  $D_2$  is foreclosed,  $D_1$  ends up paying more for the *N* inputs. Notice that since the payments that downstream firms make for inputs are total payments rather than per unit prices, there is no double marginalization in our model.<sup>17</sup>

The following corollary follows immediately from Lemma 1.

COROLLARY 1: Under nonintegration, as well as absent foreclosure, both  $D_1$  and  $D_2$  buy all N inputs at a price of  $\Delta_1(N, N)$ . The resulting profit of each downstream firm is

$$V_0^D \equiv \pi(N, N) - N\Delta_1(N, N),$$

while the profit of each upstream supplier is

$$V_0^U \equiv 2(\Delta_1(N, N) - c),$$

where  $V_0^D > 0$  and  $V_0^U > 0$ .

#### B. The Full Integration Benchmark

Now suppose that  $D_1$  and  $U_1$  fully integrate. Absent foreclosure, the sum of  $D_1$  and  $U_1$ 's profits is  $V_0^D + V_0^U$ . When  $U_1$  forecloses  $D_2$ , the sum of the profits becomes  $V_1^D + V_1^U$ , where

$$V_1^D \equiv \pi(N, N-1) - N\Delta_1(N, N-1),$$

<sup>17</sup>With double marginalization, vertical integration can harm consumers both because  $D_2$  does not use one of the inputs and because  $D_1$  pays a higher price for the inputs it uses.

and

$$V_1^U \equiv \Delta_1(N, N-1) - c.$$

These profits reflect the fact that when  $D_2$  is foreclosed,  $D_1$ 's downstream profit is  $\pi(N, N-1)$  and it pays each upstream supplier (including  $U_1$ ) an amount  $\Delta_1(N, N-1)$  for inputs.<sup>18</sup>

Foreclosing  $D_2$  is profitable for the vertically integrated firm if and only if

$$V_1^D + V_1^U \ge V_0^D + V_0^U.$$

We will refer to an equilibrium in which integration between  $U_1$  and  $D_1$  leads to the foreclosure of  $D_2$  as a "foreclosure equilibrium."

**PROPOSITION 1:** Suppose that  $D_1$  and  $U_1$  are fully integrated. Then a foreclosure equilibrium exists and is unique if and only if

$$G \geq L$$
,

where

$$G \equiv V_1^D - V_0^D = -\underbrace{\left[\pi(N, N) - \pi(N, N-1)\right]}_{\Delta_2(N, N)} + N\underbrace{\left[\Delta_1(N, N) - \Delta_1(N, N-1)\right]}_{\Delta_{12}(N, N)}$$

is the downstream gain from foreclosure, and

$$L \equiv V_0^U - V_1^U = \Delta_1(N, N) - c + \underbrace{\left[\Delta_1(N, N) - \Delta_1(N, N-1)\right]}_{\Delta_{12}(N, N)}$$

is the associated upstream loss.

Proposition 1 simply says that foreclosure is profitable if the downstream gain from foreclosing  $D_2$  exceeds the associated upstream loss.<sup>19</sup> The latter consists of  $\Delta_1(N, N) - c$ , which is the forgone profit from not selling to  $D_2$ , plus  $\Delta_{12}(N, N) < 0$ , which is due to the fact that when  $D_2$  is foreclosed,  $U_1$  charges  $D_1$  for the input  $\Delta_1(N, N-1)$  instead of  $\Delta_1(N, N)$ . This increase moderates somewhat the upstream loss from foreclosure, though by Assumption A4, we still have L > 0. As for the downstream gain G, note that  $-\Delta_2(N, N) > 0$  is the extra downstream profit that  $D_1$  makes when  $D_2$  is foreclosed, while  $N\Delta_{12}(N, N) < 0$ , reflects the higher payments of  $D_1$  for the N inputs.

<sup>&</sup>lt;sup>18</sup>We assume for simplicity that  $D_1$  pays  $U_1$  the same amount it pays all other suppliers. This assumption is without loss of generality since under full integration,  $D_1$ 's payment to  $U_1$  is merely a transfer within the same organization, and hence is irrelevant.

<sup>&</sup>lt;sup>19</sup>Interestingly, Proposition 1 does not require more than one upstream supplier. Hence, input foreclosure can arise in our framework even if there is only one upstream supplier. This is in contrast to Ordover, Saloner, and Salop (1990), where the existence of two upstream suppliers is crucial for input foreclosure.

As far as we know, the adverse effect of foreclosure on the foreclosing firm's payment for inputs has not been identified earlier. While this effect is extreme in our model because we assume that upstream suppliers have all the bargaining power vis-à-vis  $D_1$  and  $D_2$ , the effect would not disappear completely unless  $D_1$  and  $D_2$  can make take-it-or-leave-it offers to the upstream suppliers.

Recall that we assume that the decisions of upstream firms to supply downstream firms are publicly observable and irreversible. To see why this assumption is needed, consider a foreclosure equilibrium, and recall that in such an equilibrium, suppliers  $U_2, \ldots, U_N$  charge  $D_2$  an amount of  $\Delta_1(N-1, N)$  for the input. If  $U_1$ 's decision to foreclose was not publicly observable and irreversible,  $U_1$  would have had an incentive to deviate from foreclosure by offering its input to  $D_2$  at slightly below  $\Delta_1(N-1, N)$ . In this case,  $D_2$  would have accepted the offer, and would have dropped one of the inputs of suppliers  $U_2, \ldots, U_N$  because the extra profit from using the Nth input,  $\Delta_1(N, N)$ , is lower than the associated payment,  $\Delta_1(N-1, N)$  (the reason why  $\Delta_1(N, N) < \Delta_1(N-1, N)$  is that  $\Delta_{11}(N, N) < 0$  by Assumption A1). Since  $D_2$  would continue to use N-1 inputs,  $D_1$  would still gain G downstream, while  $U_1$  would increase its upstream profit by  $\Delta_1(N-1, N) - c$ , where  $\Delta_1(N-1, N) - c > \Delta_1(N, N) - c > 0$  (the latter inequality follows from Assumption A2). Hence, a foreclosure equilibrium cannot exist when  $U_1$  cannot credibly commit to foreclose  $D_2$ .

#### **III. Input Foreclosure under Partial Integration**

Our analysis in the previous section shows that vertical integration leads to the foreclosure of  $D_2$  if and only if G > L. Although at first blush it would seem that vertical integration is profitable and would take place in this case, our analysis in this section and the next shows that this is not necessarily true when the target firm has passive shareholders.

To study the incentive for vertical integration, we assume that  $D_1$  and  $U_1$  are initially independent and then ask whether  $D_1$  would like to acquire a controlling stake,  $\alpha \geq \underline{\alpha}$ , in  $U_1$  (backward integration), or  $U_1$  would like to acquire a stake  $\alpha$ , not necessarily controlling, in  $D_1$  (forward integration). It turns out that the answer depends heavily on the initial ownership structure of the target firm ( $U_1$  in the case of backward integration and  $D_1$  in the case of forward integration).<sup>20</sup> In this section, we will consider two extreme cases:

(i) Initially, the target (U<sub>1</sub> in the case of partial forward integration and D<sub>1</sub> in the case of partial backward integration) has a single controlling shareholder whose stake is α<sub>C</sub> ∈ [α, 1]; the remaining 1 − α<sub>C</sub> stake in U<sub>1</sub> (if any) is held by passive shareholders.

<sup>&</sup>lt;sup>20</sup>The acquirer's ownership structure is irrelevant. The only assumption we need is that the acquiring firm has a controller who has a stake in the firm's profit (either due to shares, stock options, bonuses, or career concerns that depend on profits). Since this stake is constant, it does not affect the controller's decisions. In Section IVD, we study a case where the controller of the acquiring firm has personal benefits or costs from making decisions, due to holdings in other firms that are affected by his decisions. As we shall see, the controller's stake in the acquirer does matter in that case.

#### (ii) Initially, the target is owned by a mass 1 of atomistic shareholders.

In Section IV, we will consider additional possibilities.

Before we start, note that when  $D_1$  partially controls  $U_1$  (partial backward integration), it would like to pay  $U_1$  as little as possible (and thereby expropriate the wealth of  $U_1$ 's passive shareholders), while when  $U_1$  partially controls  $D_1$  (partial forward integration),  $U_1$  would like to charge  $D_1$  as much as possible (and thereby expropriate the wealth of  $D_1$ 's passive shareholders). The incentive to distort  $D_1$ 's payment for  $U_1$ 's input is often referred to as "tunneling" (see e.g., Johnson et al. 2000; and Bae, Kang, and Kim 2002).<sup>21</sup> To model tunneling, we will assume that under partial integration,  $D_1$  pays for  $U_1$ 's input the same amount it pays under nonintegration, but minus a discount *t* if  $D_1$  controls  $U_1$ , and plus a premium *t* if  $U_1$  controls  $D_1$ . The parameter *t* measures the extent of tunneling and is larger when the protection of minority shareholders is weaker. Notice that since the payments of downstream firms for inputs are total payments rather than per unit prices, *t* has no effect on the reduced-form profits of  $D_1$  or  $D_2$ .<sup>22</sup>

To simplify the analysis, we will assume that the effect of tunneling on the payoffs of  $D_1$  and  $U_1$  is smaller than the effect of foreclosure.

# ASSUMPTION A5: $t \leq \min\{G, L\}$ .

To reduce the number of cases we need to consider, we will also make the following assumption.

# ASSUMPTION A6: $G > \underline{\alpha}L$ .

Assumption A6 holds trivially when G > L; when G < L, Assumption A6 imposes an upper bound on  $\underline{\alpha}$ , which is the minimal stake that ensures control. As we shall see, without this assumption, foreclosure never arises in our model under backward integration.

#### A. Backward Integration when $U_1$ Has Initially a Single Controlling Shareholder

Suppose that  $U_1$  has initially a single controlling shareholder, whose controlling stake is  $\alpha_C$ . To acquire a controlling stake  $\alpha \in [\underline{\alpha}, \alpha_C]$  in  $U_1, D_1$  makes  $U_1$ 's controller a take-it-or-leave-it offer b. If the offer is accepted,  $D_1$  becomes the new controlling shareholder in  $U_1$ . If the offer is rejected, the two firms remain independent.

<sup>&</sup>lt;sup>21</sup> Although the foreclosure of  $D_2$  also tunnels wealth from  $U_1$  to  $D_1$  under partial backward integration, we will only refer to the manipulation of input prices as "tunneling" to distinguish it from vertical foreclosure. Note that tunneling can arise even if  $D_1$  is a monopoly in the downstream market.

<sup>&</sup>lt;sup>22</sup> Had payments been per unit prices, t would have lowered  $D_1$ 's marginal cost and would have induced  $D_1$  to expand output or lower prices, depending on the type of strategic interaction in the downstream market. This would have in turn affected  $D_2$ 's output or price, depending on whether the strategies of  $D_1$  and  $D_2$  are strategic substitutes or strategic complements.

As we shall see below, the assumption that  $D_1$  has all the bargaining power vis-à-vis  $U_1$ 's controller is not essential.<sup>23</sup>

Conditional on acquiring a controlling stake  $\alpha$  in  $U_1$ ,  $D_1$  would use its control to foreclose  $D_2$  if foreclosure increases its post-acquisition payoff:

$$V_1^D + t + \alpha \left( V_1^U - t \right) \geq V_0^D + t + \alpha \left( V_0^U - t \right) \quad \Rightarrow \quad \underbrace{V_1^D - V_0^D}_G \geq \alpha \underbrace{\left( V_0^U - V_1^U \right)}_L.$$

Hence, foreclosure arises if and only if the downstream gain from foreclosure, G, exceeds  $D_1$ 's share in the associated upstream loss,  $\alpha L$ . Recalling that under partial backward integration  $D_1$ 's payment for  $U_1$ 's input is discounted by t, and noting that Assumption A6 ensures that  $\underline{\alpha} < G/L$ , and that by definition,  $V_1^D \equiv V_0^D + G$  and  $V_1^U \equiv V_0^U - L$ , we can express the payoffs of  $D_1$  and  $U_1$  under backward integration as functions of  $\alpha$ :

(1) 
$$V_{BI}^{D}(\alpha) = \begin{cases} V_{0}^{D} & \text{if } \alpha < \underline{\alpha} \\ V_{1}^{D} + t \equiv V_{0}^{D} + G + t & \text{if } \underline{\alpha} \le \alpha \le \frac{G}{L} \\ V_{0}^{D} + t & \text{if } \alpha > \frac{G}{L} \end{cases}$$

and

(2) 
$$V_{BI}^{U}(\alpha) = \begin{cases} V_{0}^{U} & \text{if } \alpha < \underline{\alpha} \\ V_{1}^{U} - t \equiv V_{0}^{U} - L - t & \text{if } \underline{\alpha} \le \alpha \le \frac{G}{L} \\ V_{0}^{U} - t & \text{if } \alpha > \frac{G}{L} \end{cases}$$

Since  $\alpha \leq 1$ , the last lines in  $V_{Bl}^D(\alpha)$  and  $V_{Bl}^U(\alpha)$  are irrelevant when G > L. Given  $V_{Bl}^U(\alpha)$ , the minimal acceptable payment  $b^U$  that  $D_1$  must offer  $U_1$ 's controller should leave the controller indifferent between accepting and rejecting the offer:

(3) 
$$\underbrace{b^{U} + (\alpha_{C} - \alpha) V^{U}_{BI}(\alpha)}_{\text{Accepting the offer}} = \underbrace{\alpha_{C} V^{U}_{0}}_{\text{Rejecting the offer}} \Rightarrow b^{U} = \alpha V^{U}_{BI}(\alpha) + \alpha_{C} \left( V^{U}_{0} - V^{U}_{BI}(\alpha) \right).$$

That is,  $b^U$  is equal to the post-acquisition value of the acquired stake,  $\alpha V_{Bl}^U(\alpha)$ , plus a premium,  $\alpha_C (V_0^U - V_{Bl}^U(\alpha))$ , that compensates the initial controller of  $U_1$  for the change in the value of his entire initial stake.

 $<sup>^{23}</sup>$  The assumption that the acquirer (here  $D_1$ ) makes a take-it-or-leave-it offer is natural when the target's ownership is dispersed. We wish to make the same assumption when the target has an initial controller to ensure that the two scenarios differ only with respect to the target's ownership structure and not the relative bargaining powers of the acquirer and the target. In any event, this assumption is not essential.

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To find out if  $D_1$  will offer  $b^U$  for a controlling stake in  $U_1$ , note that  $D_1$ 's payoff is equal to its post-acquisition payoff,  $V_{Bl}^D(\alpha) + \alpha V_{Bl}^U(\alpha)$ , minus  $b^U$ . Using (1)–(3) and rearranging terms, we can rewrite  $D_1$ 's payoff as a function of the size of the acquired stake  $\alpha$ :

(4) 
$$Y^{D}(\alpha) = \begin{cases} V_{0}^{D} & \text{if } \alpha < \underline{\alpha} \\ V_{0}^{D} + G - \alpha_{C}L + (1 - \alpha_{C})t & \text{if } \underline{\alpha} \le \alpha \le \frac{G}{L} \\ V_{0}^{D} + (1 - \alpha_{C})t & \text{if } \alpha > \frac{G}{L} \end{cases}$$

As (4) shows,  $Y^{D}(\alpha)$  is maximized at  $\alpha \in [\underline{\alpha}, G/L]$  when  $G \geq \alpha_{C}L$ , and at  $\alpha > G/L$  when  $G < \alpha_{C}L$ .<sup>24</sup> Since the initial controller of  $U_{1}$  can sell at most a stake of  $\alpha_{C}$ ,  $D_{1}$  will acquire in the latter case a controlling stake  $\alpha \in [G/L, \alpha_{C}]$ .

**PROPOSITION 2:** Suppose that initially,  $U_1$  has a single controlling shareholder. Then, in equilibrium,  $D_1$  will acquire a controlling stake  $\alpha \in [\underline{\alpha}, \alpha_C]$  and will use it to foreclose  $D_2$  if

(5) 
$$G \ge \alpha_C L$$

When this condition fails,  $D_1$  will acquire a controlling stake  $\alpha \in [G/L, \alpha_C]$  in  $U_1$  but will not use it to foreclose  $D_2$  after the acquisition.

Proposition 2 implies that  $D_1$  always acquires a controlling stake in  $U_1$  because it allows it to buy  $U_1$ 's input at a discount, and thereby expropriate some of the wealth of  $U_1$ 's passive shareholders. If, in addition, condition (5) holds, i.e., the downstream gain from foreclosure exceeds the stake of  $U_1$ 's initial controller in the associated upstream loss, the acquisition leads to the foreclosure of  $D_2$ . Interestingly, the condition for foreclosure is independent of  $\alpha$ , which is the size of the acquired stake, because  $D_1$  needs to compensate the initial controller of  $U_1$  for the loss to his entire initial stake,  $\alpha_C$ , even if it does not fully acquire this stake.

The passive shareholders of  $U_1$  effectively subsidize foreclosure since they bear a fraction  $1 - \alpha_C$  of the loss from foreclosure. Not surprisingly then, condition (5) is more likely to hold when their stake in  $U_1$ ,  $1 - \alpha_C$ , gets larger. Recalling that under full integration foreclosure arises if and only if  $G \ge L$ , Proposition 2 suggests that antitrust authorities should be more concerned with partial backward integration than with full vertical integration, particularly when the controlling stake of the initial controller is relatively small. These concerns are alleviated to some extent when the protection of minority shareholders is effective, in which case  $D_1$  may find it harder to use it to control over  $U_1$  to foreclose downstream rivals, as well as engage in tunneling.

<sup>&</sup>lt;sup>24</sup>To the extent that tunneling and foreclosure are easier when the initial controller of  $U_1$  is out of the picture,  $D_1$  might wish to acquire the entire stake  $\alpha_C$  of  $U_1$ 's initial controller.

It should be emphasized that Proposition 2 continues to hold even if  $D_1$  does not have all the bargaining power vis-à-vis  $U_1$ 's initial controller. To see why, note from (1) and (2) that the joint payoff of the initial controllers of  $D_1$  and  $U_1$  under integration is

$$V_{BI}^{D}(\alpha) + \alpha_{C} V_{BI}^{U}(\alpha) = \begin{cases} V_{0}^{D} + \alpha_{C} V_{0}^{U} & \text{if } \alpha < \underline{\alpha} \\ V_{0}^{D} + \alpha_{C} V_{0}^{U} + G - \alpha_{C} L + (1 - \alpha_{C}) t & \text{if } \underline{\alpha} \le \alpha \le \frac{G}{L} \\ V_{0}^{D} + \alpha_{C} V_{0}^{U} + (1 - \alpha_{C}) t & \text{if } \alpha > \frac{G}{L} \end{cases}$$

Since the joint payoff absent integration is  $V_0^D + \alpha_C V_0^U$ , transferring control to  $D_1$  is jointly profitable, and it leads to foreclosure if and only if  $G \ge \alpha_C L$ , exactly as stated in Proposition 2.<sup>25</sup>

# B. Backward Integration when U<sub>1</sub>'s Ownership Is Initially Dispersed

We now turn to the case where  $U_1$  is initially held by a mass 1 of atomistic shareholders. To acquire a controlling stake  $\alpha \geq \underline{\alpha}$  in  $U_1$ ,  $D_1$  makes a restricted tender offer  $(\overline{\alpha}, V)$ , where  $\overline{\alpha} \leq 1$  is the maximal stake it offers to acquire, and V is the price for the entire firm. Given the tender offer, each of  $U_1$ 's shareholders decides whether to tender his shares. If more than  $\overline{\alpha}$  shares are tendered, the submitted shares are prorated. We will say that the tender offer succeeds if  $D_1$  manages to acquire at least a stake of  $\underline{\alpha}$  and gains control over  $U_1$ , and it fails otherwise. When the offer succeeds,  $D_1$  pays  $\overline{\alpha}V$  for the acquired shares.

To characterize the equilibrium of the tender offer game, note that the post-acquisition values of  $D_1$  and  $U_1$  are given by (1) and (2) and also note that  $V_{BI}^U(\alpha) \leq V_0^U$ . Whenever  $V_{BI}^U(\alpha) \leq V < V_0^U$ , it is optimal for each shareholder to tender his shares if the tender offer succeeds (and get V for the tendered shares instead of  $V_{BI}^U(\alpha)$ ), but hold on to his shares if the tender offer fails (in which case the shareholder gets V for the tendered shares, instead of  $V_0^U$ ).<sup>26</sup> Hence, the tendering subgame admits two equilibria: (i) all shareholders tender and the offer succeeds, and (ii) no shareholder tenders, so the offer fails. Since  $V_0^U \geq V_{BI}^U(\alpha)$ , equilibrium (ii) Pareto dominates equilibrium (i). We will therefore assume that whenever  $V_{BI}^U(\alpha) \leq V < V_0^U$ , equilibrium (ii) is played.<sup>27</sup> With this equilibrium selection criterion in place, we prove the following lemma.

LEMMA 2: Suppose that if  $V_{BI}^{U}(\alpha) \leq V < V_{0}^{U}$ , the initial shareholders of  $U_{1}$  do not tender their shares. Then, in equilibrium,  $V = V_{0}^{U}$ .

<sup>&</sup>lt;sup>25</sup> The relative bargaining power of  $D_1$  vis-à-vis  $U_1$ 's initial controller would matter, however, if  $D_1$  has some fixed cost associated with initiating a takeover. Then, the lower  $D_1$ 's bargaining power, the less likely the takeover is.

<sup>&</sup>lt;sup>26</sup> If the offer is conditional on success, the shareholders are indifferent between submitting and not submitting shares when the offer fails.

<sup>&</sup>lt;sup>27</sup>The same equilibrium selection criterion is also used in Grossman and Hart (1980); Burkart, Gromb, and Panunzi (1998); and Burkart et al. (2014). It rules out the implausible scenario where the target's shareholders tender at prices below the status quo value of the target.

Lemma 2 implies that in order to acquire shares,  $D_1$  needs to pay  $U_1$ 's dispersed shareholders the pre-acquisition value of the shares. This value exceeds the post-acquisition value of shares whenever  $D_1$  gains control. As we shall see shortly,  $D_1$  will therefore prefer to acquire the minimal stake,  $\underline{\alpha}$ , needed for control.

To examine  $D_1$ 's incentive to acquire a stake  $\alpha$  in  $U_1$ , note that given that  $D_1$  needs to pay  $\alpha V_0^U$  for the acquired shares, its post-acquisition payoff is  $V_{Bl}^D(\alpha) + \alpha V_{Bl}^U(\alpha) - \alpha V_0^U$ . Using (1) and (2) and rearranging terms, we can rewrite  $D_1$ 's payoff, as a function of  $\alpha$ , as follows:

(6) 
$$Y^{D}(\alpha) = \begin{cases} V_{0}^{D} & \text{if } \alpha < \underline{\alpha} \\ V_{0}^{D} + G - \alpha L + (1 - \alpha)t & \text{if } \underline{\alpha} \le \alpha \le \frac{G}{L} \\ V_{0}^{D} + (1 - \alpha)t & \text{if } \alpha > \frac{G}{L} \end{cases}$$

Since  $G > \underline{\alpha}L$  by Assumption A6,  $Y^{D}(\alpha)$  jumps upward at  $\alpha = \underline{\alpha}$ ; given that  $Y^{D}(\alpha)$  is continuous at  $\alpha > G/L$  and decreasing for all  $\alpha \geq \underline{\alpha}$ , we obtain the following result.

**PROPOSITION 3:** Suppose that initially,  $U_1$ 's ownership is dispersed. Then, in equilibrium,  $D_1$  will acquire the minimal stake,  $\underline{\alpha}$ , needed to control  $U_1$ , and given Assumption A6, will use it to foreclose  $D_2$ .

Since  $\underline{\alpha} \leq \alpha_C \leq 1$ , Propositions 1–3 imply that a foreclosure equilibrium exists for a wider range of parameters under partial backward integration than under full integration, especially when  $U_1$  is initially owned by dispersed shareholders, or if its initial controller has a small stake in the firm. In that sense, our results suggest that antitrust authorities should be particularly concerned about partial backward integration when the initial ownership of the upstream supplier is less concentrated. This is in stark contrast to the horizontal context, where greater ownership concentration is typically anticompetitive rather than pro-competitive as in our vertical setting.<sup>28</sup>

To see why greater ownership concentration is pro-competitive, note that foreclosing  $D_2$  diverts profits from  $U_1$  to  $D_1$ . When  $U_1$  has an initial controller,  $D_1$  must compensate him for his entire stake in the upstream loss,  $\alpha_C L$ , even if  $D_1$  buys only part of this stake. By contrast, when  $U_1$ 's ownership is dispersed,  $D_1$  can acquire a minimal stake,  $\underline{\alpha}$ , that ensures control and hence it internalizes only a fraction  $\underline{\alpha} \leq \alpha_C$  of the upstream loss, L. Put differently, when  $U_1$  has an initial controller,  $U_1$ 's passive shareholders bear a fraction  $1 - \alpha_C$  of the upstream loss from foreclosure, while under dispersed ownership this fraction increases to  $1 - \underline{\alpha}$ .

<sup>&</sup>lt;sup>28</sup> For the anticompetitive effects of cross ownership among horizontal competitors, as well as common ownership of the same shareholders in a set of horizontal competitors, see e.g., Dietzenbacher, Smid, and Volkerink (2000); Gilo, Moshe, and Spiegel (2006); Brito, Ribeiro, and Vasconcelos (2014); Azar, Schmalz, and Tecu (2016); and Azar, Raina, and Schmalz (2016).

Note that unlike Proposition 2, where  $D_1$  is indifferent about the size of the acquired controlling stake, here  $D_1$  wishes to acquire the minimal stake that ensures control. The reason is that in Proposition 2, the amount that  $D_1$  pays for the acquired stake depends on the controller's initial stake,  $\alpha_C$ , irrespective of how large  $\alpha$  is, whereas here, the amount paid is increasing with  $\alpha$  (note that  $D_1$  pays for each share its pre-acquisition value, which exceeds its post-acquisition value, and hence, would like to acquire as few shares as possible).

To conclude this subsection, five remarks are in order. First, t does not affect the incentive to foreclose. Hence, in our model, the incentive to foreclose rivals is independent of whether (partial) vertical integration leads to an upward or a downward distortion in  $D_1$ 's payment for  $U_1$ 's input. This feature allows us to separate the issue of tunneling, which can also arise when  $D_1$  and  $U_1$  do not have rivals, from the issue of foreclosure and its potential anticompetitive effects, which is our main focus.<sup>29</sup>

Second, when Assumption A6 is violated, the downstream gain from foreclosure is always lower than  $D_1$ 's share in the corresponding upstream loss; hence,  $D_1$  will never use its control over  $U_1$  to foreclose  $D_2$ .

Third, suppose that contrary to our equilibrium selection criterion, all of  $U_1$ 's shareholders tender shares when  $V_{BI}^U(\alpha) \leq V < V_0^U$ . While this allows  $D_1$  to pay less for a controlling stake  $\underline{\alpha}$  in  $U_1$ , the acquisition price is already sunk when  $D_1$ decides whether or not to use its control to foreclose  $D_2$ . Hence, foreclosure still arises if and only if Assumption A6 holds, i.e.,  $G \geq \alpha L$ .

Fourth, one may wonder whether an external investor may wish to acquire a sufficiently large stake from  $U_1$ 's dispersed shareholders and use it to oppose  $D_1$ 's decision to lower  $D_1$ 's payment for the input and to foreclose  $D_2$ . Such an acquisition raises  $U_1$ 's value from  $V_{BI}^U(\alpha)$  to  $V_0^U$ . But since the dispersed shareholders of  $U_1$  are atomistic, then as in Grossman and Hart (1980), the investor needs to pay them the post-acquisition value of their shares to induce them to submit their shares. As a result, such an acquisition is not profitable for the investor.

Finally, so far we implicitly assumed that when  $U_1$  has a controlling shareholder,  $D_1$  must acquire his stake,  $\alpha_C$ , to gain control over  $U_1$ . If we relax this assumption and assume in addition that  $\alpha_C < 1/2$ , then  $D_1$  can also gain control over  $U_1$  by acquiring a stake  $\alpha > \alpha_C$  from U<sub>1</sub>'s dispersed shareholders. However, this strategy gives  $D_1$  a payoff of  $G - \alpha L < G - \alpha_C L$ , where the latter is  $D_1$ 's payoff when it acquires the controlling stake of  $U_1$ 's controller.<sup>30</sup> Hence, bypassing  $U_1$ 's controller is not profitable for  $D_1$ .<sup>31</sup> Another possibility is that  $D_1$  threatens  $U_1$ 's controller that if he does not accept  $D_1$ 's offer,  $D_1$  would acquire a controlling stake from  $U_1$ 's dispersed shareholders. The controller may try to block  $D_1$  by increasing his stake in  $U_1$ 

<sup>&</sup>lt;sup>29</sup>This feature of our model depends on the assumption that the demand of downstream firms for inputs is inelastic. When the demand for inputs is elastic, the distortion in  $D_1$ 's payment for  $U_1$ 's input may affect the quantity of the input that  $D_1$  uses and hence the competition with  $D_2$  in the downstream market. <sup>30</sup>This conclusion is only strengthened if  $U_1$ 's controlling shareholder makes a counter offer to the dispersed

shareholders.

<sup>&</sup>lt;sup>31</sup>Burkart, Gromb, and Panunzi (2000) also shows that when the target has both a large shareholder and dispersed, atomistic, shareholders, the acquirer prefers to deal with the large shareholder. Their setting however differs from ours in several respects; in particular, they consider a value-increasing acquisition, while in our case the acquisition is value-decreasing.

from  $\alpha_C$  to 1/2, in which case he, rather than  $D_1$ , gains control over  $U_1$ . The highest amount,  $b_C^{\text{max}}$ , that the controller will agree to pay for the acquired stake is such that

$$\frac{1}{2}V_0^U - b_C^{\max} = \alpha_C (V_1^U - t) \implies$$

$$b_C^{\max} = \frac{1}{2}V_0^U - \alpha_C (V_1^U - t) = \left(\frac{1}{2} - \alpha_C\right)V_0^U + \alpha_C (L + t).$$

Since the associated price per share is  $b_C^{\max}/(1/2 - \alpha_C) = V_0^U + \alpha_C(L + t)/(1/2 - \alpha_C)$ ,  $D_1$  must offer  $U_1$ 's dispersed shareholders at least  $V_0^U + \alpha_C(L + t)/(1/2 - \alpha_C)$  per share in order to gain control over  $U_1$ . The offer is profitable for  $D_1$  if and only if

(7) 
$$\underbrace{V_1^D + t + \frac{1}{2}(V_1^U - t)}_{\text{Post-acquisition payoff}} - \underbrace{\frac{1}{2}\left(V_0^U + \frac{\alpha_C(L+t)}{\frac{1}{2} - \alpha_C}\right)}_{\text{Payment for the acquired stake}} \ge \underbrace{V_0^D}_{\text{Payoff absent an offer}} \Rightarrow$$

$$G+t \geq \frac{L+t}{2(1-2\alpha_C)}.$$

When (7) holds,  $D_1$ 's threat to acquire a controlling stake from  $U_1$ 's dispersed shareholders is credible, so  $U_1$ 's initial controller will accept an offer of  $\alpha_C(V_1^U - t)$  to sell his stake. Conditional on acquiring a stake,  $\alpha_C$ ,  $D_1$  will use its control to foreclose  $D_2$  if and only if  $G \ge \alpha_C L$ , exactly as we showed in Section IIIA. The acquisition price is lower though since  $D_1$  pays the controller only  $\alpha_C(V_1^U - t)$ , instead of  $\alpha_C V_0^U$ .

# C. Forward Integration

Next, suppose that  $U_1$  wishes to integrate forward. Unlike backward integration, control is no longer needed for foreclosure, since  $U_1$  can foreclose  $D_2$  regardless of whether it controls  $D_1$ . However, control allows  $U_1$  to tunnel wealth from  $D_1$  by inflating its payment for  $U_1$ 's input by t.

Conditional on acquiring a noncontrolling stake  $\alpha$  in  $D_1$ ,  $U_1$  will foreclose  $D_2$  if foreclosure increases its overall payoff:

$$V_1^U + \alpha V_1^D \ge V_0^U + \alpha V_0^D \quad \Rightarrow \quad \alpha \underbrace{\left(V_1^D - V_0^D\right)}_G \ge \underbrace{V_0^U - V_1^U}_L.$$

That is,  $U_1$  forecloses  $D_2$  if and only if its stake in the downstream gain from foreclosure exceeds its associated upstream loss. Put differently, foreclosure is profitable if and only if  $\alpha$  is sufficiently high in the sense that  $\alpha \ge L/G$ . When  $U_1$  acquires a controlling stake  $\alpha \ge \underline{\alpha}$  in  $D_1$ , it can also inflate  $D_1$ 's payment for the input, so  $U_1$ 's profit increases by t, while  $D_1$ 's profit decreases by t. Together with the fact that  $V_1^D \equiv V_0^D + G$  and  $V_1^U \equiv V_0^U - L$ , the payoffs of  $D_1$  and  $U_1$  under forward integration, as functions of  $\alpha$ , are given by

(8) 
$$V_{FI}^{D}(\alpha) = \begin{cases} V_{0}^{D} & \text{if } \alpha < \min\left\{\underline{\alpha}, \frac{L}{G}\right\} \\ V_{1}^{D} \equiv V_{0}^{D} + G & \text{if } \frac{L}{G} < \alpha < \underline{\alpha} \\ V_{0}^{D} - t & \text{if } \underline{\alpha} \le \alpha < \frac{L}{G} \\ V_{1}^{D} + t \equiv V_{0}^{D} + G - t & \text{if } \alpha \ge \max\left\{\underline{\alpha}, \frac{L}{G}\right\} \end{cases}$$

and

(9) 
$$V_{FI}^{U}(\alpha) = \begin{cases} V_{0}^{U} & \text{if } \alpha < \min\left\{\underline{\alpha}, \frac{L}{G}\right\} \\ V_{1}^{U} \equiv V_{0}^{U} - L & \text{if } \frac{L}{G} < \alpha < \underline{\alpha} \\ V_{0}^{U} + t & \text{if } \underline{\alpha} \le \alpha < \frac{L}{G} \\ V_{1}^{U} + t \equiv V_{0}^{U} - L + t & \text{if } \alpha \ge \max\left\{\underline{\alpha}, \frac{L}{G}\right\} \end{cases}$$

The second lines in (8) and (9) are relevant only if  $\underline{\alpha} > L/G$ , while the third lines are relevant only if  $\underline{\alpha} < L/G$ .

Having computed  $V_{FI}^{D}(\alpha)$  and  $V_{FI}^{U}(\alpha)$ , we now study  $U_1$ 's incentive to acquire a stake  $\alpha$  in  $D_1$  in the first place. We begin with the case where initially  $D_1$ 's shareholders are atomistic. Then, (8) implies that when  $\alpha < L/G$ , the acquisition either lowers  $D_1$ 's value or does not affect it. As in Lemma 2,  $U_1$  then needs to pay the atomistic shareholders of  $D_1$  the pre-acquisition value of their shares,  $V_0^D$ . By contrast, when  $\alpha \geq L/G$ ,  $U_1$  forecloses  $D_2$  after the acquisition, and by Assumption A5,  $D_1$ 's value increases even if  $U_1$  gains control and inflates  $D_1$ 's payment for the input. Consequently,  $U_1$  faces the well-known free-rider problem of Grossman and Hart (1980), and must set V equal to the post-acquisition value of  $D_1$ , which is either  $V_1^D$  absent tunneling or  $V_1^D - t$  with tunneling.<sup>32</sup> Using these prices, we prove the following result.

**PROPOSITION 4:** Suppose that initially,  $D_1$ 's ownership is dispersed. Then, in equilibrium,  $U_1$  will acquire the minimal stake,  $\underline{\alpha}$ , needed to control  $D_1$  if

(10) 
$$\underline{\alpha}G < L.$$

<sup>&</sup>lt;sup>32</sup>When the acquisition is value increasing, the atomistic shareholders of the target have a dominant strategy to hold on to their shares so long as  $V < V_1^D$ . It should be noted that this conclusion hinges on the assumptions that  $D_1$ 's shareholders are atomistic and the post-acquisition value of  $D_1$  is common knowledge. See Bagnoli and Lipman (1988) and Holmström and Nalebuff (1992) for analysis of value-increasing takeovers when the target is held by a finite number of shareholders and the acquirer has private information about the post-acquisition value of the target.

The acquisition, if it takes place, does not lead to the foreclosure of  $D_2$ . When the condition fails,  $U_1$  has no incentive to acquire a stake in  $D_1$ .

Intuitively,  $U_1$  has no incentive to foreclose  $D_2$  because foreclosure boosts the value of  $D_1$ , and this forces  $U_1$  to pay the atomistic shareholders of  $D_1$  a price equal to the post-acquisition value of  $D_1$ . But then,  $U_1$  breaks even on the acquisition, and since it bears an upstream loss, L, due to foreclosure, it has no incentive to pursue the acquisition.<sup>33</sup> The acquisition is profitable only when  $U_1$  acquires control over  $D_1$  and can use it to inflate  $D_1$ 's payment for the input, but the controlling stake is sufficiently low to ensure that  $U_1$  does not foreclose  $D_2$  after the acquisition (and hence there is no free-rider problem).<sup>34</sup> The important implication of Proposition 4 is that when  $D_1$ 's ownership is dispersed, forward integration does not lead to a foreclosure equilibrium.

Next, we turn to the case where  $D_1$  has a controlling shareholder whose stake is  $\alpha_C \in [\underline{\alpha}, 1]$ . Then, the minimal payment  $b^D$  that  $U_1$  needs to offer  $D_1$ 's controller to induce him to sell a stake  $\alpha \leq \alpha_C$  in  $D_1$  (this stake may or may not be controlling) is given by

(11) 
$$\underbrace{b^{D} + (\alpha_{C} - \alpha) V_{FI}^{D}(\alpha)}_{\text{Accepting the offer}} = \underbrace{\alpha_{C} V_{0}^{D}}_{\text{Rejecting the offer}} \Rightarrow$$

$$b^D = \alpha V_{FI}^D(\alpha) - \alpha_C (V_{FI}^D(\alpha) - V_0^D).$$

That is,  $b^D$  equals the post-acquisition value of the acquired shares,  $\alpha V_{FI}^D(\alpha)$ , minus  $\alpha_C (V_{FI}^D(\alpha) - V_0^D)$ , which is the change in the value of the initial stake of  $D_1$ 's initial controller due to forward integration. Using  $b^D$ , we prove the following result.

PROPOSITION 5: Suppose that initially,  $D_1$  has a single controlling shareholder. Then, in equilibrium,  $U_1$  will acquire a controlling stake,  $\alpha \in [\max\{\underline{\alpha}, L/G\}, \alpha_C]$ in  $D_1$  and will use it to foreclose  $D_2$  if

(12) 
$$\alpha_C G \ge L.$$

When this condition fails,  $U_1$  will acquire a controlling stake  $\alpha \in [\underline{\alpha}, \alpha_C]$  in  $D_1$ , but will not foreclose  $D_2$  after the acquisition.

 $<sup>^{33}</sup>$  As in Grossman and Hart (1980), the acquirer forgoes the acquisition because it needs to pay atomistic shareholders the entire increase in the target's value. However, unlike in Grossman and Hart, here the acquisition need not involve control and it boosts the target's value by affecting the acquirer's behavior (the foreclosure of  $D_2$ ) rather than the target's own behavior.

<sup>&</sup>lt;sup>34</sup>In a sense, tunneling serves the same role as dilution in Grossman and Hart (1980).

Proposition 5 shows that unlike the case where  $D_1$ 's ownership is dispersed, when  $D_1$  has initially a controlling shareholder, forward integration can lead to a foreclosure equilibrium, but only if the stake of the initial controller is sufficiently large to satisfy (12). The reason is that under forward integration,  $U_1$  bears the entire loss from foreclosure, so foreclosure can be profitable only if  $U_1$ 's share in the associated gain is sufficiently large. Put differently, under forward integration, input foreclosure subsidizes the passive shareholders of  $D_1$ . When the initial shareholders of  $D_1$  are atomistic, they demand the entire subsidy in order to sell their shares. The resulting free-rider problem renders forward integration unprofitable whenever the acquisition leads to input foreclosure. When  $D_1$  has an initial controller,  $U_1$ can negotiate with him a mutually beneficial price and hence the acquisition goes through, provided that there are not too many passive shareholders who continue to be subsidized by foreclosure.

Combined, Propositions 1–5 show that relative to full integration, partial backward integration facilitates foreclosure, while partial forward integration hinders it. In particular, input foreclosure occurs under partial backward integration when  $G \ge \alpha_C L$  or  $G \ge \underline{\alpha} L$ , depending on whether  $U_1$  has an initial controller or dispersed ownership, whereas under full integration it occurs only when  $G \ge L$ . By contrast, forward integration never leads to input foreclosure when  $D_1$ 's ownership is initially dispersed, and leads to foreclosure when  $D_1$  has an initial controller only when  $\alpha_C G \ge L$ .

As mentioned above, Proposition 5 continues to hold when  $U_1$  does not have all the bargaining power vis-à-vis  $D_1$ 's initial controller. To see why, note that given (8) and (9), the joint payoff of  $U_1$  and  $D_1$ 's initial controller if an acquisition goes through is

$$V_{FI}^{U}(\alpha) + \alpha_{C} V_{FI}^{D}(\alpha) = \begin{cases} V_{0}^{U} + \alpha_{C} V_{0}^{D} & \text{if } \alpha < \min\left\{\underline{\alpha}, \frac{L}{G}\right\} \\ V_{0}^{U} - L + \alpha_{C}(V_{0}^{D} + G) & \text{if } \frac{L}{G} < \alpha < \underline{\alpha} \\ V_{0}^{U} + t + \alpha_{C}(V_{0}^{D} - t) & \text{if } \underline{\alpha} \le \alpha < \frac{L}{G} \\ V_{0}^{U} - L + t + \alpha_{C}(V_{0}^{D} + G - t) & \text{if } \alpha \ge \max\left\{\underline{\alpha}, \frac{L}{G}\right\} \end{cases}$$

Since their joint payoff without an acquisition is  $V_0^U + \alpha_C V_0^D$ , transferring  $\alpha_C$  to  $U_1$  is always jointly profitable and foreclosing  $D_2$  is jointly profitable if and only if  $\alpha_C G \ge L$ , exactly as in Proposition 5. Hence, the relative bargaining powers of the two parties only determine how the joint surplus is divided, but not whether the acquisition would take place.

# D. Backward or Forward Integration?

Suppose that initially, both  $D_1$  and  $U_1$  have controlling shareholders whose respective stakes are  $\alpha_C^D$  and  $\alpha_C^U$ . Suppose that the two controllers can get together

and decide to integrate their firms. Will they agree that  $U_1$ 's controller sells his stake to  $D_1$  (partial backward integration), or that  $D_1$ 's controller sells his stake to  $U_1$  (partial forward integration)?

To address this question, suppose that  $U_1$ 's controller sells his stake to  $D_1$  for a price  $b^U$ . Then, the resulting joint post-acquisition payoff of the two controllers is

$$\alpha_C^D \left( V_{BI}^D (\alpha_C^U) + \alpha_C^U V_{BI}^U (\alpha_C^U) - b^U \right) + b^U,$$

where  $V_{Bl}^D(\alpha_C^U)$  and  $V_{Bl}^U(\alpha_C^U)$  are given by (1) and (2). Similarly, if  $D_1$ 's controller sells his stake to  $U_1$  for a price  $b^D$ , the joint post-acquisition payoff of the two controllers is

$$\alpha_C^U \left( V_{FI}^U (\alpha_C^D) + \alpha_C^D V_{FI}^D (\alpha_C^D) - b^D \right) + b^D,$$

where  $V_{FI}^D(\alpha_C^U)$  and  $V_{FI}^U(\alpha_C^U)$  are given by (8) and (9).

**PROPOSITION** 6: Suppose that initially, both  $D_1$  and  $U_1$  have single controlling shareholders and assume further that  $\alpha_C^U \leq G/L$  and  $\alpha_C^D \geq L/G$ , so that both partial backward integration and partial forward integration lead to the foreclosure of  $D_2$ . Then the two controllers will decide to pursue partial backward integration, regardless of the size of their initial controlling stakes.

To see the intuition, note that the controlling shareholders of  $D_1$  and  $U_1$  need to share the downstream gain, G, and the upstream loss, L, from foreclosure with the passive shareholders of  $D_1$  and  $U_1$ , and likewise they need to share the profit from tunneling with the passive shareholders of the acquiring firm. Backward integration is more profitable than forward integration because the price paid to  $U_1$ 's controlling shareholder under backward integration,  $b^U$ , exceeds the post-acquisition value of the acquired shares, so the passive shareholders of  $D_1$  subsidize part of the deal. Under forward integration, the price paid to  $D_1$ 's controlling shareholder,  $b^D$ , falls short of the post-acquisition value of the acquired shares, so the passive shareholders of  $U_1$  receive a subsidy.

#### **IV. Input Foreclosure under Additional Ownership Structures**

In the previous section, we considered the incentive to vertically integrate under two polar cases: the target has a single controlling shareholder or is owned by atomistic shareholders. In this section, we examine three more cases: initially the target has two or more controlling shareholders; backward integration when the acquirer,  $D_1$ , already holds a noncontrolling stake in  $U_1$  (i.e., a toehold); and backward integration when the acquisition is made by the controlling shareholder of  $D_1$  rather than by  $D_1$  itself.

# A. Backward Integration when U<sub>1</sub> Has Initially Two or More Large Shareholders

Suppose that initially,  $U_1$  has two large shareholders whose stakes  $\alpha_1$  and  $\alpha_2$  are such that  $\alpha_1 \leq \alpha_2 < \underline{\alpha} \leq \alpha_1 + \alpha_2 < 1$ . That is, neither shareholder alone has control, but together they do. To ensure that neither shareholder can block  $D_1$ 's attempt to acquire control over  $U_1$ , we will also assume that  $1 - \alpha_2 \geq \underline{\alpha}$ .<sup>35</sup> We now make three additional assumptions. First, we will assume that  $\underline{\alpha} \geq 1/2$ , so that  $\alpha \geq \underline{\alpha}$  is necessary and sufficient for control.<sup>36</sup> Second, to make the two large shareholders case comparable to the single controlling shareholder case, we will assume that  $\alpha_1 + \alpha_2 = \alpha_C$ . Third, we will assume that  $G \geq (\alpha_1 + \alpha_2)L \equiv \alpha_C L$ , so once  $D_1$  acquires the stakes of the two large shareholders, it uses its control over  $U_1$  to foreclose  $D_2$  and to buy  $U_1$ 's input at a discount.

We begin by observing that  $D_1$  can fully replicate the single controlling shareholder outcome by acquiring the stakes of the two large shareholders at their pre-acquisition values  $\alpha_1 V_0^U$  and  $\alpha_2 V_0^U$ . Given our assumption that  $G \ge (\alpha_1 + \alpha_2) L$ ,  $D_1$  will use its control to foreclose  $D_2$  and buy  $U_1$ 's input at a discount and will obtain a payoff of

$$\underbrace{V_1^D + t + (\alpha_1 + \alpha_2)(V_1^U - t)}_{\text{Post-acquisition payoff}} - \underbrace{(\alpha_1 + \alpha_2)V_0^U}_{\text{Payment for the acquired stake}}$$

$$\equiv V_0^D + G + t - \underbrace{(\alpha_1 + \alpha_2)}_{\alpha_C} (L + t),$$

exactly as in the single controlling shareholder (see the middle line in (4)).

However,  $D_1$  can acquire control over  $U_1$  at an even lower price by making sequential take-it-or-leave-it offers to the two shareholders. If both offers are accepted,  $D_1$ gains control over  $U_1$ . If both offers are rejected,  $D_1$  fails to gain control over  $U_1$ . If one offer is accepted and the other is rejected,  $D_1$  can make a restricted tender offer to  $U_1$ 's dispersed shareholders for the rest of the shares needed for control.<sup>37</sup> Thus,  $D_1$  gains control over  $U_1$  if its tender offer is accepted and if the shareholder who rejected the offer cannot make a counter tender offer, which is more profitable for  $U_1$ 's dispersed shareholders.

To characterize the resulting equilibrium, suppose that shareholder *j* accepts  $D_1$ 's offer, while shareholder *i* rejects it. To secure control,  $D_1$  needs to acquire a stake  $\underline{\alpha} - \alpha_j$  from  $U_1$ 's dispersed shareholders (since  $1 - \alpha_2 \ge \underline{\alpha}$ , there are enough dispersed shares for  $D_1$  to acquire). Assumption A6 ensures that if  $D_1$  succeeds, it will use its control to foreclose  $D_2$  and to buy  $U_1$ 's input at a discount. Shareholder *i* can block  $D_1$ 's tender offer by increasing his own stake from  $\alpha_i$  to  $\underline{\alpha}$ , in which case he,

<sup>&</sup>lt;sup>35</sup> If  $1 - \alpha_2 < \underline{\alpha}$ ,  $D_1$  cannot acquire enough shares from  $U_1$ 's dispersed shareholders to secure control over  $U_1$ , even if it acquires the stake of shareholder 1. Since  $\alpha_2 \ge \alpha_1$ ,  $1 - \alpha_2 < \underline{\alpha}$  implies  $1 - \alpha_1 < \underline{\alpha}$ .

<sup>&</sup>lt;sup>36</sup>Whenever  $\underline{\alpha} < 1/2$ , a stake  $\underline{\alpha}$  is necessary for control, but not sufficient. To secure control,  $D_1$  would have to acquire a stake of at least 1/2.

 $<sup>{}^{37}</sup>D_1$  does not acquire a larger stake in  $U_1$  because it is better off exploiting as many passive shareholders of  $U_1$  as possible after the acquisition.

rather than  $D_1$ , gains control over  $U_1$ .<sup>38</sup> The highest amount,  $b_i^{\text{max}}$ , that shareholder *i* agrees to pay for a stake  $\underline{\alpha} - \alpha_i$  in  $U_1$  is such that

$$\underline{\alpha} V_0^U - b_i^{\max} = \alpha_i (V_1^U - t) \implies$$

$$b_i^{\max} = \underline{\alpha} V_0^U - \alpha_i (V_1^U - t) = (\underline{\alpha} - \alpha_i) V_0^U + \alpha_i (L + t).$$

The implied price per share is  $b_i^{\max}/(\underline{\alpha} - \alpha_i) = V_0^U + \alpha_i(L+t)/(\underline{\alpha} - \alpha_i)$ . Therefore, to acquire a stake  $\underline{\alpha} - \alpha_j$  in  $U_1$ ,  $D_1$  must offer  $U_1$ 's dispersed share-

Therefore, to acquire a stake  $\underline{\alpha} - \alpha_j$  in  $U_1$ ,  $D_1$  must offer  $U_1$ 's dispersed shareholders at least  $V_0^U + \alpha_i(L+t)/(\underline{\alpha} - \alpha_i)$  per share. The offer is profitable for  $D_1$  if and only if

$$(13) \underbrace{V_1^D + t + \underline{\alpha}(V_1^U - t)}_{\text{Post-acquisition payoff}} - \underbrace{(\underline{\alpha} - \alpha_j)\left(V_0^U + \frac{\alpha_i(L+t)}{\underline{\alpha} - \alpha_i}\right)}_{\text{Payment for the acquired stake}} \ge \underbrace{V_0^D + \alpha_j V_0^U}_{\text{Payoff absent an offer}} \Rightarrow \frac{G+t}{L+t} \ge \frac{\underline{\alpha}^2 - \alpha_i \alpha_j}{\alpha - \alpha_i}.$$

When (13) holds, shareholder *i* cannot prevent  $D_1$  from gaining control over  $U_1$  once shareholder *j* accepted  $D_1$ 's offer, and will therefore agree to sell his stake to  $D_1$  at its post-acquisition value,  $\alpha_i(V_1^U - t)$ . When (13) fails, shareholder *i* is pivotal, in the sense that he can deny  $D_1$  control over  $U_1$  even if  $D_1$  has already acquired shareholder *j*'s stake. Notice that since  $\alpha_2 \ge \alpha_1$ ,  $(\underline{\alpha}^2 - \alpha_1 \alpha_2)/(\underline{\alpha} - \alpha_2) \ge (\underline{\alpha}^2 - \alpha_1 \alpha_2)/(\underline{\alpha} - \alpha_1)$ , shareholder 2, who is the larger shareholder between the two, is more likely to be pivotal.

We now prove the following proposition.

**PROPOSITION** 7: Suppose that initially,  $U_1$  has two large shareholders, whose stakes are such that  $\alpha_1 \leq \alpha_2 \leq \underline{\alpha} \leq \alpha_1 + \alpha_2 \leq 1$  and  $1 - \alpha_2 \geq \underline{\alpha}$ . Then, in equilibrium,  $D_1$  will acquire a controlling stake  $\alpha_1 + \alpha_2$ , and given our assumption that  $G \geq (\alpha_1 + \alpha_2) L$ , will use it to foreclose  $D_2$ . The total amount that  $D_1$  pays for the acquisition is

$$b = \begin{cases} \left(\alpha_1 + \alpha_2\right) \left(V_1^U - t\right) & \text{if } \frac{G+t}{L+t} \ge \frac{\underline{\alpha}^2 - \alpha_1 \alpha_2}{\underline{\alpha} - \alpha_2} \\ \\ \alpha_1 \left(V_1^U - t\right) + \alpha_2 V_0^U & \text{if } \frac{\underline{\alpha}^2 - \alpha_1 \alpha_2}{\underline{\alpha} - \alpha_2} > \frac{G+t}{L+t} > \frac{\underline{\alpha}^2 - \alpha_1 \alpha_2}{\underline{\alpha} - \alpha_1} \\ \\ \left(\alpha_1 + \alpha_2\right) V_0^U & \text{if } \frac{G+t}{L+t} < \frac{\underline{\alpha}^2 - \alpha_1 \alpha_2}{\underline{\alpha} - \alpha_1} \end{cases}$$

<sup>38</sup>Recall that we assume that  $\underline{\alpha}$  is necessary and sufficient for control. Without this assumption,  $D_1$  and shareholder *i* would have to compete for becoming the largest shareholder in  $U_1$  with a stake above  $\underline{\alpha}$ . Proposition 7 shows that when  $U_1$  has two large shareholders, partial backward integration leads to a foreclosure equilibrium when  $G \ge (\alpha_1 + \alpha_2) L$ . Although this is analogous to the case of a single controlling shareholder with a stake of  $\alpha_C = \alpha_1 + \alpha_2$ , the acquisition can be more profitable now because  $D_1$  may be able to acquire the stake of at least one large shareholder at below its pre-acquisition value. The reason for this is that unlike the case of a single controlling shareholder, who is pivotal and therefore rejects any offer below the pre-acquisition value of his stake, each of the two large shareholders alone is not necessarily pivotal. In particular,  $D_1$  can always acquire the stake of shareholder j at its pre-acquisition value, and threaten shareholder i that if he does not sell his own stake at its post-acquisition value,  $\alpha_i(V_1^U - t)$ ,  $D_1$  will gain control over  $U_1$  by acquiring a stake  $\underline{\alpha} - \alpha_j$  from  $U_1$ 's dispersed shareholders. When  $(G + t)/(L + t) \ge (\underline{\alpha}^2 - \alpha_1 \alpha_2)/(\underline{\alpha} - \alpha_i)$ , the acquisition is profitable for  $D_1$ , so the threat is credible. As a result, shareholder irealizes that if he rejects  $D_1$ 's offer, his stake will be worth  $\alpha_i(V_1^U - t)$  anyway, and hence he might as well accept  $D_1$ 's offer in the first place.

Proposition 7 is closely related to the "naked exclusion" result of Rasmusen, Ramseyer, and Wiley (1991) and Segal and Whinston (2000), where an incumbent supplier can costlessly exclude rival suppliers by signing downstream buyers to exclusive supply contracts. In our case, the foreclosure of  $D_2$  could also be costless for  $D_1$  if it acquires the stakes of the two large shareholders of  $U_1$  at their post-acquisition values. But as Proposition 7 shows, this is not always possible: whenever  $(G + t)/(L + t) < (\underline{\alpha}^2 - \alpha_1 \alpha_2)/(\underline{\alpha} - \alpha_i)$ ,  $D_1$  needs to pay shareholder *i* his entire pre-acquisition value, so although the foreclosure of  $D_2$  is still profitable, it is no longer costless for  $D_1$ .

Our analysis can now be readily extended to the case of  $n \ge 3$  large shareholders. To this end, assume that the stakes of the *n* large shareholders are such that  $\alpha_1 \le \alpha_2 \le \cdots \le \alpha_n < \sum_{i \ne 1} \alpha_i < \alpha \le \sum_i \alpha_i < 1$  and  $1 - \alpha_n \ge \alpha$ . The assumption that  $\sum_{i \ne 1} \alpha_i < \alpha \le \sum_i \alpha_i$  implies that, combined, the *n* large shareholders have control over  $U_1$ , but no subset of large shareholders has control. The assumption that  $1 - \alpha_n \ge \alpha$  ensures that there are enough dispersed shares that  $D_1$  can acquire in case a single shareholder rejects its offer. Otherwise, we maintain the same assumptions as in the two large shareholders case, and in particular, we assume that  $G \ge \sum_i \alpha_i L$ .

As in the two large shareholders case,  $D_1$  can fully replicate the outcome of the single controlling shareholder case by acquiring the stakes of the large shareholders at their pre-acquisition values. Hence,  $D_1$  will acquire a controlling stake  $\sum_i \alpha_i$  in  $U_1$  and since  $G \ge \sum_i \alpha_i L$ , it uses it to foreclose  $D_2$  and to buy  $U_1$ 's input at a discount.

In the next proposition, we show that as in the two large shareholders case,  $U_1$  can acquire the controlling stake at a lower price than it pays when  $D_1$  has a single controller. To see how, note that shareholder *i* is pivotal if (G + t)/(L + t)  $< (\underline{\alpha}^2 - \alpha_i \sum_{i \neq 1} \alpha_i)/(\underline{\alpha} - \alpha_i)$ . In the next proposition, we prove that  $\alpha_1 \leq \alpha_2$  $\leq \cdots \leq \alpha_n$  implies that

$$\frac{\underline{\alpha}^2 - \alpha_1 \sum_{i \neq 1} \alpha_i}{\underline{\alpha} - \alpha_1} \leq \frac{\underline{\alpha}^2 - \alpha_2 \sum_{i \neq 2} \alpha_i}{\underline{\alpha} - \alpha_2} \leq \cdots \leq \frac{\underline{\alpha}^2 - \alpha_n \sum_{i \neq n} \alpha_i}{\underline{\alpha} - \alpha_n}.$$

We then show that each pivotal shareholder *i* must be offered the pre-acquisition values of his stakes,  $\alpha_i V_0^U$ , while each non-pivotal shareholder *j* can be offered the post-acquisition value of his stake  $\alpha_i (V_1^U - t)$ .

**PROPOSITION 8:** Suppose that initially,  $U_1$  has n large shareholders whose stakes are  $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_n < \sum_{i \neq 1} \alpha_i < \alpha \leq \sum_i \alpha_i < 1$  and  $1 - \alpha_n \geq \alpha$ . Then, in equilibrium,  $D_1$  will acquire a controlling stake  $\sum_i \alpha_i$ , and given our assumption that  $G \geq \sum_i \alpha_i L$ , will use it to foreclose  $D_2$ .  $D_1$  will pay each shareholder j an amount

$$b = \begin{cases} \alpha_j (V_1^U - t) & \text{if } \frac{G+t}{L+t} \ge \frac{\underline{\alpha}^2 - \alpha_j \sum_{i \neq j} \alpha_i}{\underline{\alpha} - \alpha_j}, \\ \alpha_j V_0^U & \text{if } \frac{G+t}{L+t} < \frac{\underline{\alpha}^2 - \alpha_j \sum_{i \neq j} \alpha_i}{\underline{\alpha} - \alpha_j}. \end{cases}$$

Proposition 8 implies that if several upstream suppliers become potential targets,  $D_1$  will prefer to either acquire control in the one in which the combined stake of pivotal large shareholders is lowest, or in an upstream supplier whose ownership is initially dispersed. To see this, let  $\sum_i \alpha_i \equiv \alpha_C$  and assume that  $G > \alpha_C L$  (the acquisition leads to the foreclosure of  $D_2$ ). Using  $\alpha_p$  to denote the aggregate stake of pivotal large shareholders, Proposition 8 implies that the cost of the acquisition is  $\alpha_p V_0^U + (\alpha_C - \alpha_p) (V_1^U - t)$ . Since the post-acquisition value of the acquired shares is  $\alpha_C (V_1^U - t)$ , and since  $D_1$  enjoys a downstream gain of G + t, its resulting gain from partial backward integration is

$$G+t+\alpha_C(V_1^U-t)-\alpha_pV_0^U-(\alpha_C-\alpha_p)(V_1^U-t) = G+t-\alpha_p(L+t).$$

It is easy to see that  $D_1$ 's gain decreases with  $\alpha_p$  and is minimized when  $U_1$  has a single controller, in which case  $\alpha_p = \alpha_c$ . And, as Proposition 3 and (6) show,  $D_1$ 's gain from backward integration when  $U_1$ 's ownership is initially dispersed is  $G + t - \underline{\alpha}(L+t)$ . Clearly,  $G + t - \alpha_p(L+t) \ge G + t - \underline{\alpha}(L+t)$  as  $\underline{\alpha} \ge \alpha_p$ , meaning that  $D_1$  prefers to acquire control in an upstream supplier with large shareholders if  $\underline{\alpha} > \alpha_p$  and in an upstream supplier with dispersed ownership if  $\underline{\alpha} < \alpha_p$ .

#### B. Forward Integration when D<sub>1</sub> Has Initially Two Large Shareholders

We now briefly consider the case where initially,  $D_1$  has two large shareholders whose stakes are such that  $\alpha_1 \leq \alpha_2 < \alpha \leq \alpha_1 + \alpha_2 < 1$ . If  $(\alpha_1 + \alpha_2) G < L$ , acquiring  $\alpha_1$  and  $\alpha_2$  is insufficient to induce  $U_1$  to foreclose  $D_2$ . Foreclosure equilibrium arises in this case only if in addition to  $\alpha_1 + \alpha_2$ ,  $U_1$  also acquires shares from the dispersed shareholders of  $D_1$ , such that its final stake,  $\alpha$ , is such that  $\alpha G \geq L$ . Since foreclosure boosts  $D_1$ 's value,  $U_1$  needs to pay the dispersed shareholders the post-acquisition value of their shares,  $V_1^D - t$ , to induce them to sell their shares (since  $\alpha > \alpha_1 + \alpha_2 \geq \alpha$ ,  $U_1$  wins control over  $D_1$  and uses its control to sell the input to  $D_1$  at an inflated price). Hence,  $U_1$ 's gain from acquiring an additional stake  $\alpha - \alpha_1 - \alpha_2$  is

$$\underbrace{V_1^U + t + \alpha \left(V_1^D - t\right)}_{\text{Post-acquisition payoff}} - \underbrace{\left(\alpha - \alpha_1 - \alpha_2\right)\left(V_1^D - t\right)}_{\text{Payment for the acquired stake}} - \underbrace{\left[V_0^U + t + \left(\alpha_1 + \alpha_2\right)\left(V_0^D - t\right)\right]}_{\text{Payoff absent an acquisition}}$$

$$= L - (\alpha_1 + \alpha_2) G < 0.$$

As a result,  $U_1$  will not acquire the additional shares, and since its final stake in  $D_1$  is at most  $\alpha_1 + \alpha_2$ , it will not foreclose  $D_2$ .

Now suppose that  $(\alpha_1 + \alpha_2) G \ge L$ . Then,  $U_1$  can make the two large shareholders simultaneous public take-it-or-leave-it offers to sell their stakes at their pre-acquisition values,  $\alpha_1 V_0^D$  and  $\alpha_2 V_0^D$ , and make the offers conditional on the acceptance of both offers. There are two Nash equilibria in the resulting game: both shareholders accept or both reject their offers. However, in the Pareto-dominant equilibrium, both shareholders accept their respective offers. The acquisition is profitable for  $U_1$  because

$$\underbrace{V_1^U + t + (\alpha_1 + \alpha_2) (V_1^D - t)}_{\text{Post-acquisition payoff}} - \underbrace{(\alpha_1 + \alpha_2) V_0^D}_{\text{Payment to controlling shareholders}} - \underbrace{V_0^U}_{\text{Payoff absent an acquisition}}$$
$$= (\alpha_1 + \alpha_2) G - L + (1 - \alpha_1 - \alpha_2) t > 0.$$

In sum, we get a foreclosure equilibrium if and only if  $(\alpha_1 + \alpha_2) G \ge L$ . If we assume that  $\alpha_1 + \alpha_2 = \alpha_C$ , the situation is similar to that in the single controller's case.<sup>39</sup>

#### C. Toeholds

We now examine what happens when, at the outset,  $D_1$  already holds a noncontrolling stake,  $\alpha_1 < \underline{\alpha}$ , in  $U_1$  (i.e., a toehold). To gain control over  $U_1$ ,  $D_1$  must acquire an additional stake  $\alpha - \alpha_1$  in  $U_1$ , such that after the acquisition, its controlling stake in  $U_1$  is  $\alpha \geq \underline{\alpha}$ .

**PROPOSITION 9:** Suppose that initially,  $D_1$  holds a noncontrolling stake (toehold),  $\alpha_1$ , in  $U_1$ . The toehold has no effect on the equilibrium if  $U_1$  is initially held by dispersed shareholders. When  $U_1$  has initially a single controlling shareholder, a foreclosure equilibrium arises if and only if  $G \ge (\alpha_C + \alpha_1) L$ . An increase in  $\alpha_1$  shrinks the range of parameters for which  $D_2$  is foreclosed.

<sup>&</sup>lt;sup>39</sup>The difference is that  $U_1$  can actually make the two large shareholders sequential offers and threaten them that if they reject their offers,  $U_1$  will acquire shares from  $D_1$ 's dispersed shareholders. This may enable  $U_1$  to acquire the stakes of the two large shareholders at below their pre-acquisition values.

Intuitively, when  $D_1$  has a toehold in  $U_1$ , it internalizes part of the upstream loss from foreclosure, L. When  $U_1$  has an initial controller,  $D_1$  still needs to compensate him for the loss to his entire stake, so  $D_1$  internalizes a larger fraction of L. Under dispersed ownership by contrast, the toehold allows  $D_1$  to buy fewer shares from  $U_1$ 's dispersed shareholder and hence,  $D_1$  internalizes only a fraction  $\underline{\alpha}$  of L, exactly as in the no toehold case. Put differently, when  $U_1$  has an initial controller, there are  $1 - \alpha_C - \alpha_1$  passive shareholders in  $U_1$  who subsidize the foreclosure of  $D_2$ , while under dispersed ownership,  $D_1$  can gain control by acquiring only  $\underline{\alpha} - \alpha_1$  additional shares, so there are  $1 - \underline{\alpha} > 1 - \alpha_C - \alpha_1$  passive shareholders in  $U_1$  who can be exploited.

# D. Acquisition by a Controller

So far we have assumed that vertical integration arises when  $D_1$  buys a controlling stake in  $U_1$ , or  $U_1$  buys a controlling stake in  $D_1$ . However, cases exist in which the controlling shareholder of a firm, rather than the firm itself, buys a controlling stake in a vertically related firm, either directly or through other firms that he controls. For example, in 2000, Vivendi, which already held a controlling 49 percent stake in Canal+ (a major European producer of pay-TV channels, with a significant presence in the distribution of films and the licensing of broadcasting rights) acquired Seagram, which owned Universal Studios Inc.<sup>40</sup> Among other things, the acquisition raised a concern for the foreclosure of Canal+ rivals in the pay-TV market. Another example is the 2009 offer of International Petroleum Investment Company (IPIC), which was the controlling shareholder of Agrolinz Melamine International (AMI) (one of the leading melamine producers worldwide), to acquire a 70 percent stake in MAN Ferrostaal, which held a controlling 30 percent stake in Eurotecnica Melamine (the sole supplier and licensor of high pressure technology used in melamine production). The European Commission expressed the concern that after the acquisition, IPIC would foreclose AMI's competitors from Eurotecnica's technology.<sup>41</sup>

In order to study how acquisitions by controllers affect the concern for foreclosure, suppose that the controlling shareholder of  $D_1$  also controls m-1 other firms, which operate in other industries, and let  $\beta_1$  denote the controller's stake in  $D_1$  and  $\beta_2, \ldots, \beta_m$  denote his stakes in firms 2, ..., m. Now,  $D_1$ 's controller can acquire a controlling stake  $\alpha \ge \underline{\alpha}$  in  $U_1$  either directly, through  $D_1$ , or through firms 2, ..., m.

**PROPOSITION** 10: Suppose that  $D_1$  has a single controlling shareholder who also owns controlling stakes in m - 1 firms from other markets. Then, the controller acquires a controlling stake in  $U_1$  through firm i in which he holds the lowest stake among all firms under his control:

(i) If initially,  $U_1$  has a single controlling shareholder,  $D_1$ 's controller acquires in equilibrium a controlling stake  $\alpha \in [\underline{\alpha}, \alpha_C]$  in  $U_1$  through firm i and

<sup>&</sup>lt;sup>40</sup>See http://ec.europa.eu/competition/mergers/cases/decisions/m2050\_en.pdf.

<sup>&</sup>lt;sup>41</sup>See http://ec.europa.eu/competition/mergers/cases/decisions/m5406\_20090313\_20212\_en.pdf.

uses it to foreclose  $D_2$  if  $\beta_1 G \geq \beta_i \alpha_C L$ ; if  $\beta_1 G < \beta_i \alpha_C L$ , the controller acquires a controlling stake  $\alpha \in ((\beta_1/\beta_i)G/L, \alpha_C]$  through firm *i*, but does not use it to foreclose  $D_2$ . The payoff of the acquiring firm, firm *i*, is  $-\alpha_C(L+t) < 0$  if  $\alpha \in [\alpha, \alpha_C]$  and  $-\alpha_C t < 0$  if  $\alpha \in ((\beta_1/\beta_i)G/L, \alpha_C]$ .

(ii) If initially,  $U_1$ 's ownership is dispersed,  $D_1$ 's controller acquires in equilibrium a controlling stake  $\alpha = \underline{\alpha}$  through firm i and uses it to foreclose  $D_2$  if and only if  $\beta_1 G \ge \beta_i \underline{\alpha} L$ . The payoff of the acquiring firm, firm i, is  $-\underline{\alpha}(L+t) < 0$ .

Since  $\beta_i \leq \beta_1$ , the ability of  $D_1$ 's controller to choose whether to acquire a controlling stake in  $U_1$  through  $D_1$  or through another firm which he controls expands the range of parameters for which  $D_2$  is foreclosed (unless  $D_1$  happens to be the firm in which the controller has the lowest controlling stake). Moreover, so long as  $\beta_i < 1$ , the controller does not acquire a controlling stake in  $U_1$  directly, but rather through firm *i*. Intuitively, when the controller has a small stake in firm *i*, a large fraction of the upstream loss from foreclosing  $D_2$  is borne by the passive shareholders of *i*, who effectively subsidize the foreclosure of  $D_2$ . And, when  $\beta_1$  is large, a large fraction of the associated downstream gain accrues to the controller. Hence, a foreclosure equilibrium is more likely when  $\beta_i$  is small and  $\beta_1$  is large.

The result that firm i loses and firm 1 gains is consistent with Bae, Kang, and Kim (2002). They find that when a firm that belongs to a Korean business group (chaebol) makes an acquisition, on average, its stock price falls and its minority shareholders lose, but the firm's controlling shareholder benefits because the acquisition enhances the value of other firms in the same group.

#### V. Extensions

#### A. Customer Foreclosure

So far we have considered the effect of partial vertical integration on input foreclosure, but with minimal modifications, our model also applies to customer foreclosure. In that respect, our model differs from most existing theories of input foreclosure like Ordover, Saloner, and Salop (1990); Salinger (1988); or Hart and Tirole (1990), which cannot be naturally adapted to explain customer foreclosure.<sup>42</sup>

To consider "customer foreclosure," we will assume, without a serious loss of generality, that there are only two upstream suppliers (i.e., N = 2) and will also assume that while the cost of producing two units is once again 2c, the cost of the

<sup>&</sup>lt;sup>42</sup>In Ordover, Saloner, and Salop (1990), the foreclosure of  $D_2$  gives  $D_1$  a strategic advantage in the downstream market since  $D_2$  is forced to buy exclusively from  $U_2$ , and hence it pays more for the input. By contrast, the foreclosure of  $U_2$  by  $D_1$  does not give  $U_1$  any advantage in the upstream market since  $U_1$  and  $U_2$  still engage in Bertrand competition for the supply of the input to  $D_2$ . Likewise, in Hart and Tirole (1990), foreclosure solves an opportunism problem that arises when  $D_1$  and  $U_1$  renegotiate their supply contract in a way that diverts downstream sales from  $D_2$  to  $D_1$ . There is no equivalent diversion in the opposite case where  $D_1$  forecloses  $U_2$ .

first unit, denoted c(1), is above the cost of the second unit: c(1) > c (the cost of the second unit is 2c - c(1)).

We now consider the possibility that after  $D_1$  and  $U_1$  integrate (fully or partially),  $D_1$  stops buying the input from  $U_2$ . Then,  $U_2$  can only sell to  $D_2$ . When  $D_2$ buys the input from both  $U_1$  and  $U_2$ , its marginal willingness to pay for inputs is  $\Delta_1(2, 1) \equiv \pi(2, 1) - \pi(1, 1)$ . Since  $U_1$  and  $U_2$  make take-it-or-leave-it offers to  $D_1$  and  $D_2$ , the equilibrium profit of  $U_2$  is at most

$$\Delta_1(2, 1) - c(1).$$

If  $\Delta_1(2, 1) < c(1)$ ,  $U_2$  is better off exiting the market even if it can fully extract  $D_2$ 's profit from selling its input.

When  $U_2$  exits,  $D_1$  and  $D_2$  buy the input only from  $U_1$ , so their marginal willingness to pay is  $\Delta_1(1, 1) \equiv \pi(1, 1) - \pi(0, 1)$ . If  $U_1$  charges  $\Delta_1(1, 1)$  for the input, and assuming for simplicity that t = 0, its profit is

$$2\Delta_1(1, 1) - 2c = 2(\Delta_1(1, 1) - c).$$

This profit is positive by Assumption A2. To ensure that  $U_2$  cannot undercut  $U_1$ and supply  $D_2$  at  $\Delta_1(1, 1)$ , we will assume that  $\Delta_1(1, 1) < c(1)$ . Since  $\Delta_1(2, 1) < \Delta_1(1, 1)$  by Assumption A1, the assumption that  $\Delta_1(1, 1) < c(1)$  also ensures that  $\Delta_1(2, 1) < c(1)$ , so indeed  $U_2$  exits the market after being foreclosed by  $D_1$ .

The profit of  $U_1$  when  $U_2$  is not foreclosed is also  $2(\Delta_1(2, 2) - c)$ . Since  $\Delta_1(1, 1) > \Delta_1(2, 1)$ , and since by Assumption A3,  $\Delta_1(2, 1) > \Delta_1(2, 2)$ , then

$$2\big(\Delta_1(2,\,2)-c\big)\ <\ 2\big(\Delta_1(2,\,1)-c\big)\ <\ 2\big(\Delta_1(1,\,1)-c\big).$$

That is,  $U_1$  makes more money when  $U_2$  is foreclosed. The resulting gain of  $U_1$  from customer foreclosure is

$$G_c = 2(\Delta_1(1, 1) - \Delta_1(2, 2)).$$

This gain comes from the increase in the willingness of  $D_1$  and  $D_2$  to pay for  $U_1$ 's input.

As for  $D_1$ , recall that when  $U_2$  is foreclosed,  $D_1$  pays  $\Delta_1(1, 1)$  for the input, so its profit is

$$\pi(1, 1) - \Delta_1(1, 1) = \pi(1, 1) - (\pi(1, 1) - \pi(0, 1)) = \pi(0, 1).$$

Absent foreclosure,  $D_1$ 's profit is

$$\pi(2, 2) - 2\Delta_1(2, 2).$$

Thus,  $D_1$ 's resulting loss from customer foreclosure is

$$\begin{split} L_c &= \pi(2, 2) - 2\Delta_1(2, 2) - \pi(0, 1) \\ &= \underbrace{\pi(2, 2) - \pi(1, 2)}_{\Delta_1(2, 2)} + \underbrace{\pi(1, 2) - \pi(0, 2)}_{\Delta_1(1, 2)} - 2\Delta_1(2, 2) + \underbrace{\pi(0, 2) - \pi(0, 1)}_{\Delta_2(0, 2)} \\ &= \Delta_1(1, 2) - \Delta_1(2, 2) + \Delta_2(0, 2) \\ &= -\Delta_{11}(2, 2) + \Delta_2(0, 2). \end{split}$$

We will assume that  $L_c > 0$ .

Using  $G_c$  and  $L_c$ , we report the following proposition, which is analogous to the results from Section III.

**PROPOSITION 11:** Suppose that the cost of the first unit is higher than the cost of the second unit, i.e., c(1) > c, and suppose moreover, that  $c(1) > \Delta_1(1, 1)$  and  $-\Delta_{11}(2, 2) + \Delta_2(0, 2) > 0$ . Then:

- (i) Under forward integration, U<sub>1</sub> will acquire a controlling stake <u>α</u> in D<sub>1</sub> and will foreclose U<sub>2</sub> if and only if G<sub>c</sub> ≥ <u>α</u>L<sub>c</sub> when D<sub>1</sub>'s ownership is initially dispersed. When D<sub>1</sub> has initially a controlling shareholder, U<sub>1</sub> will acquire a controlling stake α<sub>c</sub> in D<sub>1</sub> and will foreclose U<sub>2</sub> if and only if G<sub>c</sub> ≥ α<sub>c</sub>L<sub>c</sub>.
- (ii) Under backward integration,  $D_1$  can foreclose  $U_2$  unilaterally, so when  $U_1$  has initially a single controlling shareholder, a foreclosure equilibrium arises if and only if  $\alpha_C G_c \ge L_c$ . Backward integration is not profitable when  $U_1$ 's ownership is initially dispersed.
  - B. Backward Integration under Partial Control

Up to now we have assumed that control is an all-or-nothing parameter: an ownership stake  $\alpha \geq \underline{\alpha}$  gives a shareholder full control over the target, while an ownership stake  $\alpha < \underline{\alpha}$  gives the shareholder no influence over the target. We now relax this assumption and assume instead that when  $D_1$  holds a stake  $\alpha$  in  $U_1$ , the management of  $U_1$  maximizes a weighted average of the payoffs of  $U_1$ 's passive shareholders and the payoff of  $D_1$ :

(14) 
$$(1 - \omega(\alpha))V^{U} + \omega(\alpha)(V^{D} + \alpha V^{U}),$$

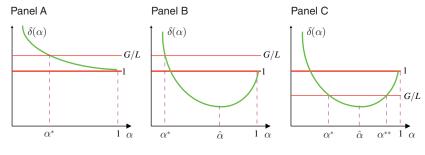


Figure 1. The  $\delta(\alpha)$  Function

where  $\omega'(\alpha) \ge 0$ , with  $\omega(0) = 0$  and  $\omega(1) = 1$ . Notice that our setup so far is a special case of (14) and arises when  $\omega(\alpha) = 0$  for  $\alpha < \underline{\alpha}$  and  $\omega(\alpha) = 1$  for  $\alpha \ge \underline{\alpha}$ .<sup>43</sup> The objective function (14) can also be expressed as

(15) 
$$V^{D} + \underbrace{\left(\frac{1}{\omega(\alpha)} + \alpha - 1\right)}_{\delta(\alpha)} V^{U}.$$

Note that  $\delta(0) = \infty$  and  $\delta(1) = 1$ , so when  $D_1$  does not acquire any stake in  $U_1$ , the management of  $U_1$  simply maximizes  $V^U$  (as in Section III when  $\alpha < \underline{\alpha}$ ), while under a full integration, it maximizes  $V^D + V^U$  (as in Section III when  $\alpha \ge \underline{\alpha}$ ). Moreover, note that

$$\delta'(\alpha) = 1 - \frac{\omega'(\alpha)}{\omega(\alpha)^2}.$$

Assuming that  $\omega'(0)$  is finite and recalling that  $\omega(0) = 0$ , it follows that  $\delta'(\alpha) < 0$ when  $\alpha$  is small. If we assume in addition that  $\omega(\alpha)$  is increasing with  $\alpha$  at a decreasing rate, i.e.,  $\omega'(\alpha) \ge 0 \ge \omega''(\alpha)$ , then  $\delta''(\alpha) > 0$ . Intuitively, an increase in  $\alpha$  raises the weight that  $U_1$ 's management assigns to  $D_1$ 's payoff and this lowers  $\delta(\alpha)$ , but when  $\alpha$  is large,  $U_1$ 's profit has a bigger effect on  $D_1$ 's payoff, which in turn raises  $\delta(\alpha)$ . If the first effect dominates the second,  $\delta(\alpha)$  is decreasing with  $\alpha$ ; otherwise,  $\delta(\alpha)$  is U-shaped.

In what follows, we will assume, again for simplicity, that t = 0. Let  $\hat{\alpha}$  denote the ownership stake of  $D_1$  in  $U_1$  at which  $\delta(\alpha)$  is minimized and let  $\hat{\delta}$  denote the minimum of  $\delta(\alpha)$ . Then, assuming that  $G > \hat{\delta}L$ , the equation  $G = \delta(\alpha)L$  defines either a unique value of  $\alpha$ , denoted  $\alpha^*$ , if G > L, or it generically defines two values of  $\alpha$ , denoted  $\alpha^*$  and  $\alpha^{**}$ , where  $\alpha^* < \hat{\alpha} < \alpha^{**}$ , if  $G \leq L$ . This is illustrated in Figure 1.

Given (15),  $U_1$ 's management decides to foreclose  $D_2$  if and only if

$$V_1^D + \delta(\alpha) V_1^U \ge V_0^D + \delta(\alpha) V_0^U \quad \Rightarrow \quad G \ge \delta(\alpha) L.$$

<sup>&</sup>lt;sup>43</sup>For alternative ways to capture partial control, see O'Brien and Salop (2000).

If  $G < \hat{\delta}L$ , the management of  $U_1$  will never foreclose  $D_2$ , so  $D_1$  will have no incentive to integrate backward, regardless of  $U_1$ 's initial ownership structure. The next proposition shows that whenever  $G \ge \hat{\delta}L$ , we always have a foreclosure equilibrium and it also characterizes the equilibrium.

**PROPOSITION 12:** Suppose that when  $D_1$  holds a stake  $\alpha$  in  $U_1$ , the objective function of  $U_1$ 's management is given by (15):

- (i) If  $U_1$  has initially a single controlling shareholder whose equity stake is  $\alpha_C$ and if G > L, a foreclosure equilibrium exists. In equilibrium,  $D_1$  acquires a stake  $\alpha \in [\alpha^*, \alpha_C]$  if  $\alpha^* < \alpha_C$  and  $\alpha^*$  otherwise. If  $G \leq L$ , a foreclosure equilibrium exists if only if  $G \geq \max\{\hat{\delta}, \alpha_C\}$  L and in equilibrium,  $D_1$ acquires a stake  $\alpha \in [\alpha^*, \min\{\alpha^{**}, \alpha_C\}]$ .
- (ii) If initially  $U_1$ 's ownership is dispersed, a foreclosure equilibrium exists if and only if  $G \ge \hat{\delta}L$ . In equilibrium,  $D_1$  acquires an ownership stake  $\alpha = \alpha^*$ .

Proposition 12 shows that our results in Section III carry over to the case where the acquirer obtains only partial control over the target, with some modifications. Still, the main point is that partial ownership affects the conditions under which foreclosure arises, and foreclosure is (weakly) easier when  $U_1$  is initially held by dispersed shareholders.

# C. A Mandatory Bid Rule (MBR)

A main insight of this paper is that partial backward integration promotes input foreclosure, while partial forward integration promotes customer foreclosure. The reason for this is that in both cases, foreclosure is effectively subsidized by the passive shareholders of the target. Therefore, it is quite obvious that a strong protection of minority shareholders alleviates, at least to some extent, the concern that partial integration will lead to foreclosure. In this section, we consider the effect of a mandatory bid rule (MBR), which applies in many countries, including most European countries (though not in the United States), and requires the acquirer of a sufficiently large controlling stake, typically 30–33 percent (see Clerc and Demarigny 2012), to extend the offer to the target's remaining shareholders.<sup>44</sup> We now briefly discuss how our theory might change under an MBR.

Consider first backward integration and suppose that  $\underline{\alpha}$  is above the MBR threshold (otherwise the MBR has no bite). If  $U_1$  has an initial controller and  $D_1$  acquires his stake,  $D_1$  must extend the offer to  $U_1$ 's passive shareholders. If the passive shareholders accept the offer,  $D_1$  becomes the sole owner of  $U_1$ , so foreclosure

<sup>&</sup>lt;sup>44</sup>The rule is also known as the Equal Opportunity Rule (EOR). EU Directive 2004/25/EC on takeover bids requires all EU member states to adopt the rule, although it allows states to maintain exceptions from the rule; see Annex 3 in Commission of the European Communities (2007). For analysis of the effect of the MBR or EOR on takeovers, see Bebchuk (1994) and Burkart and Panunzi (2004).

arises if and only if  $G \ge L$ . To find out if the acquisition is worthwhile, note that as in Section IIIA,  $D_1$  needs to pay  $U_1$ 's initial controller  $b^U = \alpha V_{BI}^U(\alpha) + \alpha_C (V_0^U - V_{BI}^U(\alpha))$  for his stake  $\alpha_C$ . This payment implies a value of  $b^U / \alpha_C$  for the entire firm, so  $D_1$ 's post-acquisition payoff is

$$\begin{split} Y^{D} &= V^{D}_{BI} + V^{U}_{BI} - b^{U} - (1 - \alpha_{C}) \frac{b^{U}}{\alpha_{C}} \\ &= V^{D}_{BI} + V^{U}_{BI} - \left(1 + \frac{1 - \alpha_{C}}{\alpha_{C}}\right) \underbrace{\left(\alpha_{C} V^{U}_{BI} + \alpha_{C} (V^{U}_{0} - V^{U}_{BI})\right)}_{b^{U}} \\ &= V^{D}_{BI} + V^{U}_{BI} - V^{U}_{0}. \end{split}$$

Since foreclosure arises if and only if  $G \ge L$  and since there is no tunneling under full integration, it follows that when G < L,  $V_{BI}^D + V_{BI}^U = V_0^D + V_0^U$ , so  $Y^D = V_0^D$ . Hence,  $D_1$  has no incentive to pursue the acquisition. When  $G \ge L$ ,  $V_{BI}^D + V_{BI}^U$  $= V_0^D + G + V_0^U - L$ , so  $Y^D = V_0^D + G - L \ge V_0^D$ , implying that the acquisition is profitable.

When  $U_1$ 's ownership is dispersed, Section IIIA shows that  $D_1$  acquires the lowest stake needed for control,  $\underline{\alpha}$ , and pays  $V_0^U$  for the entire  $U_1$ . If  $D_1$  needs to acquire all shares, its resulting payoff is again  $V_{BI}^D + V_{BI}^U - V_0^U$ , so the acquisition is once again profitable if and only if  $G \ge L$ . Hence, the situation under an MBR is exactly as in the full integration case.

Under forward integration,  $U_1$  does not need to acquire control in  $D_1$ , so the MBR may not apply. To see what happens if it does, recall from Section IIIC that if  $D_1$  has an initial controller,  $U_1$  needs to pay him  $b^D = \alpha V_{FI}^D(\alpha) - \alpha_C (V_{FI}^D(\alpha) - V_0^D)$  for a stake  $\alpha$ . Offering the same price to  $D_1$ 's passive shareholders, implies a price per share of  $b^D/\alpha$ . Now suppose by way of negation that the passive shareholders accept the offer. Then,  $U_1$  becomes the sole owner of  $D_1$ , so there is no tunneling. By (8),  $V_{FI}^D(\alpha) \ge V_0^D$ , so  $b^D/\alpha = V_{FI}^D(\alpha) - (\alpha_C/\alpha) (V_{FI}^D(\alpha) - V_0^D) \le V_{FI}^D(\alpha)$ . Implying that,  $D_1$ 's passive shareholders are better off rejecting the offer, a contradiction. Hence, an MBR is irrelevant. When  $D_1$ 's ownership is dispersed,  $U_1$  needs to offer them the post-acquisition value of the shares, otherwise they will reject the offer. But then  $U_1$  breaks even on the acquisition, and since its upstream profit weakly falls, it has no incentive to purse an acquisition.<sup>45</sup>

In sum, an MBR has no effect on foreclosure under forward integration, but under backward integration, it implies that a foreclosure equilibrium arises if and only if it also arises under full integration. Since the latter occurs for a more limited set of parameters than under backward integration without an MBR, it follows that an MBR reduces the set of parameters for which backward integration leads to foreclosure. Hence, while the corporate finance literature has already pointed out that an

<sup>&</sup>lt;sup>45</sup> In Section IIIC,  $U_1$  may still wish to buy a controlling stake in  $D_1$  by making a restricted offer for a stake  $\underline{\alpha}$  and then exploit the remaining passive shareholders by engaging in tunneling. Here, by contrast,  $U_1$  cannot make a restricted offer and hence it cannot gain from tunneling.

MBR can protect minority shareholders against inefficient transfers of control (see e.g., Bebchuk 1994), our results show that an MBR can also alleviate the concern for input foreclosure under backward integration.

#### VI. Conclusion

We have merged ideas from IO and from corporate finance in order to develop a general framework that allows us to study the interaction between the takeover process and the antitrust implications of partial vertical integration. In particular, we studied the incentive to acquire partial stakes in vertically related firms and then foreclose a downstream rival. Such foreclosure generates a downstream gain by weakening the downstream rival, but entails an upstream loss due to the forgone sales to the downstream rival. Although we focused on the foreclosure of a downstream rival (input foreclosure), our theory can also apply, with some modifications, to the foreclosure of upstream rivals (customer foreclosure).

A main insight from our analysis is that under backward integration, the passive shareholders of  $U_1$  bear part of the upstream loss from input foreclosure, so partial backward integration is more profitable when their post-acquisition stake is large. We showed that when  $U_1$  is initially held by dispersed shareholders,  $D_1$  can acquire just the minimal stake that ensures control, and hence there are relatively many passive shareholders who subsidize the foreclosure. When control is acquired from an initial controller, whose controlling stake is above the minimum needed for control, the stake of passive shareholders is smaller, so partial backward integration is less profitable. Acquisition of control from an initial controller is even less profitable when  $D_1$  holds a toehold in  $U_1$ , because the toehold reduces further the stake of passive shareholders in  $U_1$  who can be exploited. By contrast, a toehold is irrelevant when  $U_1$ 's ownership is initially dispersed, because the toehold allows  $D_1$  to acquire a smaller stake in order to gain control, and hence it does not affect the ultimate stake of passive shareholders in  $U_1$ . We also showed that input foreclosure is particularly profitable when  $D_1$  has an initial controller who can acquire a controlling stake in  $U_1$  through some other firm which he controls. In that case, some of the upstream loss from foreclosure is borne by the passive shareholders of the acquiring firm.

Under forward integration, input foreclosure can arise regardless of whether  $U_1$  gains control over  $D_1$  because the decision to foreclose is taken by  $U_1$  rather than by  $D_1$ . Nevertheless, the foreclosure of a downstream rival boosts the value of  $D_1$ , so when  $D_1$  is initially held by dispersed shareholders, the shareholders agree to sell their shares only if  $U_1$  offers them the post-acquisition value of their shares. This renders the acquisition unprofitable because  $U_1$  breaks even on the acquired shares and cannot cover the associated upstream loss. When  $D_1$  has an initial controller, the stake of passive shareholders, who capture part of the downstream gain from foreclosure, is smaller, so now the acquisition might be profitable, provided that the downstream gain is sufficiently larger than the upstream loss.

From an antitrust perspective, our analysis suggests that antitrust authorities should pay special attention to the post-acquisition stake of passive shareholders in the target firm and to whether these shareholders stand to gain or lose from foreclosure. Our theory also implies that strong corporate governance is another important factor that should be taken into account, because it affects the acquirer's ability to exploit the passive shareholders of the target.

Although we considered several scenarios and extensions, there are still many open questions which are left for future research. We now briefly mention three questions. First, we only considered the possibility that one upstream firm and one downstream firm integrate. But as Ordover, Saloner, and Salop (1990) show, foreclosure may induce another pair of upstream and downstream firms to merge; this possibility constrains the ability of the merged entity to foreclose the downstream rival. It would be interesting to consider how the possibility of a countermerger affects the incentive to integrate in our model in the first place. Second, we did not consider competition for acquiring the target. If the two downstream firms, say, compete to acquire a stake in  $U_1$ , each will have a stronger incentive to control  $U_1$ because it can both foreclose the rival and prevent the rival from foreclosing it. The question then is how this extra incentive affects matters. Third, firms in our model are symmetric. The question is whether asymmetry in either the downstream profits or upstream costs makes vertical foreclosure more or less likely, relative to the symmetric case, and whether the more or less efficient firms are more likely to be the first to integrate.

#### **APPENDIX A: PROOFS**

Following are the proofs of Lemmas 1–2, Corollary 1, and Propositions 4–10, and 12.

# PROOF OF LEMMA 1:

By Assumption A2, in equilibrium each supplier sells to at least one downstream firm. Now suppose by way of negation that there exists an equilibrium in which  $k_1$ suppliers sell exclusively to  $D_1$ ,  $k_2$  suppliers sell exclusively to  $D_2$ , and  $N - k_1 - k_2 \ge 0$  suppliers sell to both downstream firms. In this equilibrium,  $D_1$  buys  $N - k_2$ inputs and  $D_2$  buys  $N - k_1$  inputs. Hence, the marginal willingness of  $D_1$  to pay for inputs is  $\Delta_1(N - k_2, N - k_1)$ , while the marginal willingness of  $D_2$  to pay for inputs is  $\Delta_1(N - k_1, N - k_2)$ . Since the upstream suppliers make take-it-or-leave-it offers to the two downstream firms, in equilibrium, each downstream firm pays a price equal to its marginal willingness to pay. Consequently, the profit of each supplier that sells exclusively to  $D_1$  is

$$\Delta_1(N-k_2, N-k_1)-c.$$

If the supplier also sells to  $D_2$ , its profit becomes

$$\underbrace{\Delta_1(N-k_2, N-k_1+1)}_{\text{The price that } D_1 \text{ pays}} + \underbrace{\Delta_1(N-k_1+1, N-k_2)}_{\text{The price that } D_2 \text{ pays}} - 2c.$$

Selling to both  $D_1$  and  $D_2$  is more profitable since

$$\begin{split} & \left[\Delta_1(N-k_2, N-k_1+1) + \Delta_1(N-k_1+1, N-k_2) - 2c\right] - \left[\Delta_1(N-k_2, N-k_1) - c\right] \\ & = \Delta_1(N-k_1+1, N-k_2) - c - \underbrace{\left[\Delta_1(N-k_2, N-k_1) - \Delta_1(N-k_2, N-k_1+1)\right]}_{-\Delta_{12}(N-k_2, N-k_1+1)} > 0, \end{split}$$

where the inequality follows from Assumption A4. A similar argument applies when suppliers sell exclusively to  $D_2$ . Hence, in equilibrium, suppliers 2, ..., N sell to both  $D_1$  and  $D_2$ .

The last part of the lemma follows because  $D_1$  and  $D_2$  pay input prices that reflect their marginal willingness to pay.

#### PROOF OF COROLLARY 1:

Assumption A1 implies that  $\Delta_{11}(\cdot, \cdot) < 0$ , so  $\Delta_1(k, N) > \Delta_1(N, N)$  for all k < N. Hence,

$$V_0^D = \pi(0, N) + \sum_{k=1}^N \Delta_1(k, N) - N \Delta_1(N, N)$$

$$= \pi(0, N) + \sum_{k=1}^{N} (\Delta_1(k, N) - \Delta_1(N, N)) > 0.$$

By Assumption A2,  $V_0^U \equiv 2(\Delta_1(N, N) - c) > 0.$ 

#### PROOF OF LEMMA 2:

First, notice that if  $V \leq V_{BI}^U(\alpha)$  (the price that  $D_1$  offers is below the postacquisition value of  $U_1$ ), it is a dominant strategy for each shareholder not to tender. And, given the assumption in the lemma, shareholders also do not tender if  $V_{BI}^U(\alpha) \leq V < V_0^U$ . Hence, the tender offer fails for sure if  $V < V_0^U$ . By contrast, if  $V \geq V_0^U$ , it is a weakly dominant strategy for each shareholder to fully tender his shares: if the offer succeeds, the shareholder gets  $V_0^U$  on the sold shares, but gets only  $V_{BI}^U(\alpha) \leq V_0^U$  on retained shares; if the offer fails, the value of the shares is  $V_0^U$  regardless of whether they are tendered. Since the tender offer surely succeeds, it is optimal for  $D_1$  to set  $V = V_0^U$ , which is the lowest price that ensures success.

#### **PROOF OF PROPOSITION 4:**

Recall that when  $\alpha < L/G$ ,  $U_1$  needs to pay the atomistic shareholders of  $D_1$  the pre-acquisition value of their shares,  $V_0^D$ , and when  $\alpha \ge L/G$ ,  $U_1$  needs to set V equal to the post-acquisition value of  $D_1$ , which is either  $V_1^D$  absent tunneling, or  $V_1^D - t$  with tunneling. Recalling in addition that  $V_1^U \equiv V_0^U - L$ , and rearranging

terms, the post-acquisition payoff of  $U_1$ , as a function of the size of the acquired stake,  $\alpha$ , is given by

$$Y^{U}(\alpha) = \begin{cases} V_{0}^{U} + \alpha V_{0}^{D} - \alpha V_{0}^{D} = V_{0}^{U} & \text{if } \alpha < \min\left\{\underline{\alpha}, \frac{L}{G}\right\} \\ V_{1}^{U} + \alpha V_{1}^{D} - \alpha V_{1}^{D} = V_{0}^{U} - L & \text{if } \frac{L}{G} < \alpha < \underline{\alpha} \\ V_{0}^{U} + t + \alpha \left(V_{0}^{D} - t\right) - \alpha V_{0}^{D} = V_{0}^{U} + (1 - \alpha)t & \text{if } \underline{\alpha} \le \alpha < \frac{L}{G} \\ V_{1}^{U} + t + \alpha \left(V_{1}^{D} - t\right) - \alpha \left(V_{1}^{D} - t\right) = V_{0}^{U} - L + t & \text{if } \alpha \ge \max\left\{\underline{\alpha}, \frac{L}{G}\right\} \end{cases}$$

Given Assumption A5,  $V_0^U + (1 - \alpha)t \ge V_0^U > V_0^U - L + t \ge V_0^U - L$ , where the first and third inequalities are strict when t > 0. Since  $V_0^U + (1 - \alpha)t$  decreases with  $\alpha$ ,  $Y^U(\alpha)$  is maximized at  $\alpha = \alpha$  if  $\alpha < L/G$ . If  $\alpha > L/G$ , the third line of  $Y^U(\alpha)$  is irrelevant, so now  $Y^U(\alpha)$  is maximized at  $\alpha < L/G$  (the first line of (A1)). Since at the optimum  $Y^U(\alpha) = V_0^U$ ,  $U_1$  has no incentive to pursue an acquisition.

# **PROOF OF PROPOSITION 5:**

Given  $b^D$ ,  $U_1$ 's payoff if it acquires a stake  $\alpha$  is  $V_{FI}^U(\alpha) + \alpha V_{FI}^D(\alpha) - b^D$ . Using (8), (9), and (11) and rearranging terms,  $U_1$ 's payoff as a function of  $\alpha$  is

$$(A2) Y^{U}(\alpha) = \begin{cases} V_{0}^{U} & \text{if } \alpha < \min\left\{\underline{\alpha}, \frac{L}{G}\right\} \\ V_{0}^{U} + \alpha_{C}G - L & \text{if } \frac{L}{G} < \alpha < \underline{\alpha} \\ V_{0}^{U} + (1 - \alpha_{C})t & \text{if } \underline{\alpha} \le \alpha < \frac{L}{G} \\ V_{0}^{U} + \alpha_{C}G - L + (1 - \alpha_{C})t & \text{if } \alpha \ge \max\left\{\underline{\alpha}, \frac{L}{G}\right\} \end{cases}$$

There are now two possibilities: (i) If  $\alpha_C G \ge L$ ,  $Y^U(\alpha)$  is maximized at  $\alpha \ge \max{\{\underline{\alpha}, L/G\}}$  (the last line in (A2)). In equilibrium,  $U_1$  will foreclose  $D_2$ . (ii) If  $\alpha_C G < L$ ,  $Y^U(\alpha)$  is maximized at  $\alpha \in [\underline{\alpha}, \alpha_C]$  (the third line of (A2)). In equilibrium,  $U_1$  will not foreclose  $D_2$ .

### **PROOF OF PROPOSITION 6:**

Suppose that  $\alpha_C^D \ge L/G$  and  $\alpha_C^U \le G/L$ , so we get a foreclosure equilibrium regardless of whom controls the partially integrated firm. Given (1), (2), (8), and (9), the difference between the joint payoffs of the two controllers under backward and forward integration is given by

$$\begin{split} \Delta &= \left[ \alpha_C^D \left( V_1^D + t + \alpha_C^U (V_1^U - t) - b^U \right) + b^U \right] \\ &- \left[ \alpha_C^U \left( V_1^U + t + \alpha_C^D (V_1^D - t) - b^D \right) + b^D \right] \\ &= \left( 1 - \alpha_C^U \right) \left( \alpha_C^D (V_1^D + t) - b^D \right) - \left( 1 - \alpha_C^D \right) \left( \alpha_C^U (V_1^U + t) - b^U \right). \end{split}$$

To determine the sign of  $\Delta$ , notice that  $\Delta = 0$  when  $\alpha_C^D = \alpha_C^U = 1$  (but since  $\alpha_C^D \ge L/G$  and  $\alpha_C^U \le G/L$ , this can occur only when G = L). Otherwise,  $U_1$ 's controller will agree that  $U_1$  acquires the stake of  $D_1$ 's controller only if this forward integration boosts the value of his controlling stake in  $U_1$ , i.e., only if  $\alpha_C^U(V_1^U + t + \alpha_C^D(V_1^D - t) - b^D) \ge \alpha_C^U V_0^U$  or  $b^D \le \alpha_C^D(V_1^D - t) + V_1^U - V_0^U + t$ . Moreover, to induce  $U_1$ 's controller to sell his stake to  $D_1$ ,  $b^U$  must be at least equal to the controller's payoff absent a deal, i.e.,  $b^U \ge \alpha_C^U V_0^U$ . Hence,

$$\begin{split} \Delta &\geq \left(1 - \alpha_{C}^{U}\right) \left(\alpha_{C}^{D}(V_{1}^{D} + t) - \alpha_{C}^{D}(V_{1}^{D} - t) - V_{1}^{U} + V_{0}^{U} - t\right) \\ &- \left(1 - \alpha_{C}^{D}\right) \left(\alpha_{C}^{U}(V_{1}^{U} + t) - \alpha_{C}^{U}V_{0}^{U}\right) \\ &\equiv \left(1 - \alpha_{C}^{U}\right) \left(2\alpha_{C}^{D}t + L - t\right) + \left(1 - \alpha_{C}^{D}\right)\alpha_{C}^{U}(L - t) > 0, \end{split}$$

where the last inequality follows from Assumption A5. Moreover, the joint payoff of the two controllers under backward integration exceeds their joint payoff absent integration:

$$\begin{aligned} \alpha_C^D \left( V_1^D + t + \alpha_C^U (V_1^U - t) - b^U \right) + b^U \\ &\geq \alpha_C^D \left( V_1^D + t + \alpha_C^U (V_1^U - t) \right) + \alpha_C^U (1 - \alpha_C^D) V_0^U \\ &\equiv \alpha_C^D \left( V_0^D + G + t + \alpha_C^U (V_0^U - L - t) \right) + \alpha_C^U (1 - \alpha_C^D) V_0^U \\ &= \alpha_C^D V_0^D + \alpha_C^U V_0^U + \alpha_C^D \left( G - \alpha_C^U L + (1 - \alpha_C^U) t \right) \\ &> \alpha_C^D V_0^D + \alpha_C^U V_0^U, \end{aligned}$$

where the inequality follows since by assumption,  $G \ge \alpha_C^U L$ , and since t > 0.

### **PROOF OF PROPOSITION 7:**

Since  $D_1$  can fully replicate the outcome of the single controlling shareholder case and since  $G \ge (\alpha_1 + \alpha_2)L$ , it is clear that  $D_1$  always gains control over  $U_1$  and will use it to foreclose  $D_2$  and to buy  $U_1$ 's input at a discount. The only remaining question is at what cost? Since  $\alpha_2 \ge \alpha_1$ ,  $(\underline{\alpha}^2 - \alpha_1 \alpha_2)/(\underline{\alpha} - \alpha_2)$  $\ge (\underline{\alpha}^2 - \alpha_1 \alpha_2)/(\underline{\alpha} - \alpha_1)$ , there are three cases to consider:

**Case 1:**  $(G + t)/(L + t) \ge (\underline{\alpha}^2 - \alpha_1 \alpha_2)/(\underline{\alpha} - \alpha_2)$ . In this case neither large shareholder is pivotal, so  $D_1$  can acquire their stakes at their post-acquisitions values,  $\alpha_1(V_1^U - t)$  and  $\alpha_2(V_1^U - t)$ . The acquisition is profitable since our assumption that  $G \ge (\alpha_1 + \alpha_2) L$  implies that it is profitable to acquire the two stakes even at their pre-acquisition values, which are higher. To prove that shareholders 1 and 2 will sell their stakes at  $\alpha_1(V_1^U - t)$  and  $\alpha_2(V_1^U - t)$ , suppose that  $D_1$  approaches shareholder *j* before it approaches shareholder *i*. If shareholder *j* accepts  $D_1$ 's offer, shareholder *i* cannot prevent  $D_1$  from gaining control over  $U_1$ , and will therefore agree to sell his stake to  $D_1$  at  $\alpha_i(V_1^U - t)$ . As for shareholder *j*, assume by way of

negation that his payoff exceeds  $\alpha_j (V_1^U - t)$  if he rejects  $D_1$ 's offer. In that case, shareholder *i* is pivotal, so  $D_1$  must offer him the pre-acquisition value of his stake,  $\alpha_i V_0^U$ . After acquiring a stake of  $\alpha_i$ ,  $D_1$  can acquire an additional stake  $\underline{\alpha} - \alpha_i$  from  $U_1$ 's dispersed shareholders and gain control over  $U_1$ . Since  $(G + t)/(L + t) \geq (\underline{\alpha}^2 - \alpha_1 \alpha_2)/(\underline{\alpha} - \alpha_j)$ , shareholder *j* cannot prevent  $D_1$ 's control, so his payoff is  $\alpha_j (V_1^U - t)$ , contrary to our assumption that shareholder *j* gains by rejecting  $D_1$ 's offer to sell his stake at  $\alpha_j (V_1^U - t)$ .

**Case 2:**  $(\underline{\alpha}^2 - \alpha_1 \alpha_2)/(\underline{\alpha} - \alpha_2) > (G + t)/(L + t) > (\underline{\alpha}^2 - \alpha_1 \alpha_2)/(\underline{\alpha} - \alpha_1)$ . Now shareholder 2 is pivotal, while shareholder 1 is not. Since shareholder 2 is pivotal,  $D_1$  must offer him the pre-acquisition value of his stake,  $\alpha_2 V_0^U$ , regardless of whether shareholder 1 accepts or rejects  $D_1$ 's offer. However, since shareholder 1 is not pivotal,  $D_1$  only needs to offer him the post-acquisition value of his stake,  $\alpha_1(V_1^U - t)$ . The reason is that if shareholder 1 rejects  $D_1$ 's offer (whether this offer is made before or after the offer to shareholder 2),  $D_1$  can profitably acquire a stake  $\underline{\alpha} - \alpha_2$  from  $U_1$ 's dispersed shareholders in which case shareholder 1's payoff is also  $\alpha_1(V_1^U - t)$ . Hence, the total cost of the acquisition in this case is  $\alpha_1(V_1^U - t) + \alpha_2 V_0^{U.46}$ 

**Case 3:**  $(G + t)/(L + t) < (\underline{\alpha}^2 - \alpha_1 \alpha_2)/(\underline{\alpha} - \alpha_1)$ . Now both large shareholders are pivotal, so  $D_1$  must offer both of them the pre-acquisition values of their stakes,  $\alpha_1 V_0^U$  and  $\alpha_2 V_0^U$ .

**PROOF OF PROPOSITION 8:** 

Note that if k > j,

$$\begin{split} \frac{\underline{\alpha}^2 - \alpha_k \sum_{i \neq k} \alpha_i}{\underline{\alpha} - \alpha_k} &- \frac{\underline{\alpha}^2 - \alpha_j \sum_{i \neq j} \alpha_i}{\underline{\alpha} - \alpha_j} \\ &= \frac{\alpha_k \alpha_j \left( \sum_{i \neq k} \alpha_i - \sum_{i \neq j} \alpha_i \right) - \underline{\alpha} \left( \alpha_k \sum_{i \neq k} \alpha_i - \alpha_j \sum_{i \neq j} \alpha_i \right) + \underline{\alpha}^2 (\alpha_k - \alpha_j)}{(\underline{\alpha} - \alpha_j) (\underline{\alpha} - \alpha_j) (\underline{\alpha} - \alpha_k)} \\ &= \frac{-\alpha_k \alpha_j (\alpha_k - \alpha_j) - \underline{\alpha} (\alpha_k - \alpha_j) \sum_{i \neq k, j} \alpha_i + \underline{\alpha}^2 (\alpha_k - \alpha_j)}{(\underline{\alpha} - \alpha_j) (\underline{\alpha} - \alpha_k)} \\ &= \frac{(\alpha_k - \alpha_j) \left( \underline{\alpha} \left( \underline{\alpha} - \sum_{i \neq k, j} \alpha_i \right) - \alpha_k \alpha_j \right)}{(\underline{\alpha} - \alpha_j) (\underline{\alpha} - \alpha_k)} \\ &> \frac{\underline{\alpha} (\alpha_k - \alpha_j) \left( \underline{\alpha} - \sum_{i \neq k, j} \alpha_i - \alpha_j \right)}{(\underline{\alpha} - \alpha_j) (\underline{\alpha} - \alpha_k)} > 0, \end{split}$$

<sup>46</sup>Notice that  $D_1$  can acquire shareholder 1's stake at  $\alpha_1 V_0^U$  and then threaten shareholder 2 that if he does not sell his stake at below its pre-acquisition value  $\alpha_2 V_0^U$ ,  $D_1$  will make a tender offer to  $U_1$ 's dispersed shareholder for a stake of  $\underline{\alpha} - \alpha_1$  that ensures  $D_1$  control. Although  $(\underline{\alpha}^2 - \alpha_1 \alpha_2)/(\underline{\alpha} - \alpha_2) > (G+t)/(L+t)$  implies that shareholder 2 can defeat this offer, doing so requires him to acquire a stake of  $\underline{\alpha} - \alpha_2$  at a premium and hence he may accept  $D_1$ 's offer. However, if we assume, realistically, that making a tender offer to  $U_1$ 's dispersed shareholders entails some arbitrarily small cost to  $D_1$ , the threat is not credible.

where one inequality before last follows because  $\alpha_k < \underline{\alpha}$  and the last inequality follows because  $\sum_{i \neq k,j} \alpha_i + \alpha_j = \sum_{i \neq k} \alpha_i \leq \sum_{i \neq 1} \alpha_i < \underline{\alpha}$ . Hence,  $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_n$  implies

$$\frac{\left(\underline{\alpha}^2 - \alpha_1 \sum_{i \neq 1} \alpha_i\right)(L+t)}{\underline{\alpha} - \alpha_1} \leq \frac{\left(\underline{\alpha}^2 - \alpha_2 \sum_{i \neq 2} \alpha_i\right)(L+t)}{\underline{\alpha} - \alpha_2}$$
$$\leq \cdots \leq \frac{\left(\underline{\alpha}^2 - \alpha_n \sum_{i \neq n} \alpha_i\right)(L+t)}{\underline{\alpha} - \alpha_n}$$

If  $(G + t)/(L + t) < (\underline{\alpha}^2 - \alpha_1 \sum_{i \neq 1} \alpha_i)/(\underline{\alpha} - \alpha_1)$ , all shareholders are pivotal, so  $D_1$  must offer each shareholder *i* the pre-acquisition value of his stake,  $\alpha_i V_0^U$ . But if  $(\underline{\alpha}^2 - \alpha_1 \sum_{i \neq k} \alpha_i)/(\underline{\alpha} - \alpha_k) \leq (G + t)/(L + t)$  $\leq (\underline{\alpha}^2 - \alpha_1 \sum_{i \neq k+1} \alpha_i)/(\underline{\alpha} - \alpha_{k+1})$  for  $1 \leq k \leq n$ , only shareholders  $k + 1, \ldots, n$  are pivotal and must be offered the pre-acquisition values of their stake. By contrast, each shareholder  $j = 1, \ldots, k$  is not pivotal, so  $D_1$  can threaten him that unless he accepts the post-acquisition value of his stake  $\alpha_j(V_1^U - t)$ ,  $D_1$  would acquire the stakes of all remaining shareholders in the sequence at their pre-acquisition values (the shareholders will agree to sell at this price), and would acquire the rest of the shares needed for control from  $U_1$ 's dispersed shareholders. Since shareholder j is not pivotal, the threat is credible and shareholder j will accept the offer.

### **PROOF OF PROPOSITION 9:**

First, suppose that  $U_1$  is initially held by a continuum of atomistic shareholders. As in Section IIIB, given  $D_1$ 's eventual stake  $\alpha$ , foreclosure will arise if and only if  $G \ge \alpha L$ . By Lemma 2,  $D_1$  will offer the dispersed shareholders a price that reflects a value of  $V_0^U$  for the entire firm and will therefore pay a total of  $(\alpha - \alpha_1) V_0^U$  for the acquired stake. Hence, the post-acquisition value of  $D_1$  is

$$Y^{D}(\alpha) = \begin{cases} V_{0}^{D} + \alpha V_{0}^{U} - (\alpha - \alpha_{1})V_{0}^{U} & \text{if } \alpha < \underline{\alpha} \\ V_{1}^{D} + t + \alpha (V_{1}^{U} - t) - (\alpha - \alpha_{1})V_{0}^{U} & \text{if } \underline{\alpha} \le \alpha \le \frac{G}{L} \\ V_{0}^{D} + t + \alpha (V_{0}^{U} - t) - (\alpha - \alpha_{1})V_{0}^{U} & \text{if } \alpha > \frac{G}{L} \end{cases}$$

Using the definitions of G and L, and rearranging, we get

$$Y^{D}(\alpha) = \begin{cases} V_{0}^{D} + \alpha_{1} V_{0}^{U} & \text{if } \alpha < \underline{\alpha} \\ V_{0}^{D} + \alpha_{1} V_{0}^{U} + G - \alpha L + (1 - \alpha)t & \text{if } \underline{\alpha} \le \alpha \le \frac{G}{L} \\ V_{0}^{D} + \alpha_{1} V_{0}^{U} + (1 - \alpha)t & \text{if } \alpha > \frac{G}{L} \end{cases}$$

By Assumption A6,  $Y^D(\alpha)$  is maximized at  $\alpha = \underline{\alpha}$ . Hence,  $D_1$  will acquire  $\underline{\alpha} - \alpha_1$  shares in  $U_1$  and will foreclose  $D_2$  after the acquisition. Since the condition that ensures foreclosure,  $G \ge \underline{\alpha}L$ , is identical to that in Proposition 3, the toehold does not affect the equilibrium.

Next, suppose that  $U_1$  is initially controlled by a single shareholder, whose initial stake is  $\alpha_C$ . The minimal offer that  $D_1$  needs to make to induce  $U_1$ 's initial controller to sell is given by (3), except that now,  $\alpha - \alpha_1$  replaces  $\alpha$ . Hence, the post-acquisition payoff of  $D_1$  is given by

$$V_{BI}^{D}(\alpha) + \alpha V_{BI}^{U}(\alpha) - \underbrace{\left(\left(\alpha - \alpha_{1}\right)V_{BI}^{U}(\alpha) + \alpha_{C}\left(V_{0}^{U} - V_{BI}^{U}(\alpha)\right)\right)}_{b^{U}}$$
$$= V_{BI}^{D}(\alpha) + \alpha_{1}V_{BI}^{U}(\alpha) - \alpha_{C}\left(V_{0}^{U} - V_{BI}^{U}(\alpha)\right).$$

Using (1) and (2) and rearranging terms,  $D_1$ 's post-acquisition payoff becomes

$$Y^{D}(\alpha) = \begin{cases} V_{0}^{D} + \alpha_{1} V_{0}^{U} & \text{if } \alpha < \underline{\alpha} \\ V_{0}^{D} + \alpha_{1} V_{0}^{U} + G - (\alpha_{C} + \alpha_{1}) L + (1 - \alpha_{C} - \alpha_{1}) t & \text{if } \underline{\alpha} \le \alpha \le \frac{G}{L} \\ V_{0}^{D} + \alpha_{1} V_{0}^{U} + (1 - \alpha_{C} - \alpha_{1}) t & \text{if } \alpha > \frac{G}{L} \end{cases}$$

Note that  $Y^{D}(\alpha)$  is maximized at  $\alpha \in [\underline{\alpha}, G/L]$  when  $G \geq (\alpha_{C} + \alpha_{1})L$ , and at  $\alpha > G/L$  when  $G < (\alpha_{C} + \alpha_{1})L$ . In the former case, we get foreclosure.

#### **PROOF OF PROPOSITION 10:**

Conditional on acquiring a controlling stake  $\alpha \geq \underline{\alpha}$  in  $U_1$  through firm  $i = 0, 1, \ldots, m$  ("firm 0" means that the controller acquires the stake directly, so naturally,  $\beta_0 = 1$ ), the controller will use his control over  $U_1$  to lower  $D_1$ 's payment for  $U_1$ 's input by t if his stake in  $D_1$  is at least as large as his stake in  $U_1$ , i.e., if  $\beta_1 \geq \beta_i \alpha$ , or  $\underline{\alpha} \leq \alpha \leq \beta_1/\beta_i$ . If  $\alpha > \beta_1/\beta_i$ , the controller will raise  $D_1$ 's payment for  $U_1$ 's input by t (the controller can do it even if he has no control over  $U_1$ ). In this case, t < 0 (i.e., funds flow from  $D_1$  to  $U_1$ ).

Moreover, the controller will use his control over  $U_1$  to foreclose  $D_2$  if this increases his post-acquisition payoff:

$$\beta_1 (V_1^D + t) + \beta_i \alpha (V_1^U - t) \geq \beta_1 (V_0^D + t) + \beta_i \alpha (V_0^U - t) \Rightarrow$$
$$\beta_1 \underbrace{(V_1^D - V_0^D)}_G \geq \beta_i \alpha \underbrace{(V_0^U - V_1^U)}_L.$$

That is, foreclosure arises if and only if  $\alpha \leq (\beta_1/\beta_i)G/L$ . Note that the decision to foreclose  $D_2$  is independent of *t*. The payoffs of  $D_1$  and  $U_1$  under (partial) backward integration are given by

(A3) 
$$V_{BI}^{D}(\alpha) = \begin{cases} V_{0}^{D} & \text{if } \alpha < \underline{\alpha} \\ V_{1}^{D} + t \equiv V_{0}^{D} + G + t & \text{if } \underline{\alpha} \le \alpha \le \frac{\beta_{1}}{\beta_{i}} \frac{G}{L}, \\ V_{0}^{D} + t & \text{if } \alpha > \frac{\beta_{1}}{\beta_{i}} \frac{G}{L} \end{cases}$$

and

(A4) 
$$V_{Bl}^{U}(\alpha) = \begin{cases} V_{0}^{U} & \text{if } \alpha < \underline{\alpha} \\ V_{1}^{U} - t \equiv V_{0}^{U} - L - t & \text{if } \underline{\alpha} \le \alpha \le \frac{\beta_{1}}{\beta_{i}} \frac{G}{L} \\ V_{0}^{U} - t & \text{if } \alpha > \frac{\beta_{1}}{\beta_{i}} \frac{G}{L} \end{cases}$$

where t > 0 if  $\alpha \leq \beta_1/\beta_i$  and t < 0 if  $\alpha > \beta_1/\beta_i$ .

Given that firm *i* pays *b* for a stake  $\alpha$  in  $U_1$ , the controller's payoff is given by

(A5) 
$$\beta_1 V_{BI}^D(\alpha) + \beta_i \left( \alpha V_{BI}^U(\alpha) - b \right) + \sum_{j=2}^m \beta_j V_j,$$

where  $V_j$  is the value of firm  $j = 2, \ldots, m$ .

Now, suppose that  $U_1$  has an initial controller, whose stake is  $\alpha_C$ . The minimal offer that firm *i* needs to make to induce  $U_1$ 's initial controller to sell his shares is given by (3). Substituting for  $b = b^U$  from (3) in (A5), the post-acquisition payoff of  $D_1$ 's controller becomes

$$\beta_1 V_{BI}^D(\alpha) + \beta_i \left[ \alpha V_{BI}^U(\alpha) - \underbrace{\left( \alpha V_{BI}^U(\alpha) + \alpha_C \left( V_0^U - V_{BI}^U(\alpha) \right) \right)}_{b^U} \right] + \sum_{j=2}^m \beta_j V_j$$
$$= \beta_1 V_{BI}^D(\alpha) - \beta_i \alpha_C \left( V_0^U - V_{BI}^U(\alpha) \right) + \sum_{j=2}^m \beta_j V_j.$$

Using equations (A3) and (A4) and rearranging, the post-acquisition payoff of  $D_1$ 's controller becomes

$$Y^{C}(\alpha) = \sum_{j=2}^{m} \beta_{j} V_{j} + \begin{cases} \beta_{1} V_{0}^{D} & \text{if } \alpha < \underline{\alpha} \\ \beta_{1} V_{0}^{D} + \beta_{1} G - \beta_{i} \alpha_{C} L + (\beta_{1} - \beta_{i} \alpha_{C}) t & \text{if } \underline{\alpha} \le \alpha \le \frac{\beta_{1}}{\beta_{i}} \frac{G}{L} \\ \beta_{1} V_{0}^{D} + (\beta_{1} - \beta_{i} \alpha_{C}) t & \text{if } \alpha > \frac{\beta_{1}}{\beta_{i}} \frac{G}{L} \end{cases}$$

Notice that  $Y^{C}(\alpha)$  is decreasing with  $\beta_{i}$ , which is obvious when t > 0, but is also true when t < 0 since by Assumption A5, L > t. Hence, if  $D_{1}$ 's controller acquires a controlling stake in  $U_{1}$ , he does it via the firm in which he holds the lowest controlling stake. As a result,  $\beta_{i} \leq \beta_{1}$ , which implies in turn that  $\alpha \leq \beta_{1}/\beta_{i}$ , so t > 0: after the acquisition,  $D_{1}$ 's controller uses his control over  $U_{1}$  to lower  $D_{1}$ 's payment for  $U_{1}$ 's input.

Since t > 0, the maximum of  $Y^{C}(\alpha)$  is attained at the second line of (A6) if  $\beta_{1}G \geq \beta_{i}\alpha_{C}L$ , and since  $\alpha \leq \alpha_{C}$  (the initial controller's stake is at most  $\alpha_{C}$ ), the size of the acquired controlling stake is  $\alpha \in [\underline{\alpha}, \alpha_{C}]$ . If  $\beta_{1}G < \beta_{i}\alpha_{C}L$ ,  $Y^{C}(\alpha)$  attains a maximum at the third line of (A6), and since  $\alpha \leq \alpha_{C}$ , the size of the acquired controlling stake now is  $\alpha \in ((\beta_{1}/\beta_{i})G/L, \alpha_{C}]$ . Foreclosure arises only in the former case. Note from (A6) that the payoff of the acquiring firm, firm *i* (the terms multiplied by  $\beta_{i}$ ) is  $-\alpha_{C}(L+t) < 0$  if  $\alpha \in [\underline{\alpha}, \alpha_{C}]$  and  $-\alpha_{C}t < 0$  if  $\alpha \in ((\beta_{1}/\beta_{i})G/L, \alpha_{C}]$ .

Next, suppose that  $U_1$  is initially held by a continuum of atomistic shareholders. By Lemma 2, the acquirer *i* will offer the dispersed shareholders a price that reflects a value of  $V_0^U$  for the entire firm and will therefore pay  $b = \alpha V_0^U$  for the acquired stake. Substituting in (A5), using equations (A4) and (A3) and rearranging terms, the controller's post-acquisition payoff becomes

(A7)

$$Y^{C}(\alpha) = \sum_{j=2}^{m} \beta_{j} V_{j} + \begin{cases} \beta_{1} V_{0}^{D} & \text{if } \alpha < \underline{\alpha} \\ \beta_{1} V_{0}^{D} + \beta_{1} G - \beta_{i} \alpha L + (\beta_{1} - \beta_{i} \alpha) t & \text{if } \underline{\alpha} \le \alpha \le \frac{\beta_{1}}{\beta_{i}} \frac{G}{L} \\ \beta_{1} V_{0}^{D} + (\beta_{1} - \beta_{i} \alpha) t & \text{if } \alpha > \frac{\beta_{1}}{\beta_{i}} \frac{G}{L} \end{cases}$$

Equation (A7) differs from (A6) only in that  $\alpha$  replaces  $\alpha_C$ . Hence,  $D_1$ 's controller will acquire a controlling stake in  $U_1$  through the firm in which he holds the lowest controlling stake. Moreover,  $Y^C(\alpha)$  decreases with  $\alpha$  and hence is maximized at  $\alpha = \underline{\alpha}$ . In equilibrium,  $D_2$  will be foreclosed if and only if  $\beta_1 G \geq \beta_i \underline{\alpha} L$ . The payoff of the acquiring firm, firm *i*, is now  $-\underline{\alpha}(L+t) < 0$ .

## **PROOF OF PROPOSITION 12:**

Suppose that  $G \ge \hat{\delta}L$ , otherwise foreclosure never arises, and suppose that initially,  $U_1$  has an initial controller whose ownership stake is  $\alpha_C$ . Suppose first that G > L; as Figure 1 shows, in this case,  $\delta(\alpha^*) > \delta(\alpha)$  for all  $\alpha \in (\alpha^*, 1]$ .

If  $\alpha_C \geq \alpha^*$ , an acquisition of  $\alpha \in [\alpha^*, \alpha_C]$  leads to foreclosure since  $G = \delta(\alpha^*)L \geq \delta(\alpha)L$ . Now note that  $\omega(\alpha) \leq 1$  implies that  $\delta(\alpha) \geq \alpha$  and recall that  $G \equiv V_1^D - V_0^D$ . Then, the acquisition is profitable for  $D_1$  since

after paying the initial controller  $b^U = \alpha V_1^U + \alpha_C L$  (see (3) and recall that we assume that t = 0),  $D_1$ 's payoff exceeds its pre-acquisition payoff,  $V_0^D$ :

$$V_1^D + \alpha V_1^U - \underbrace{\left(\alpha V_1^U + \alpha_C L\right)}_{b^U} = V_0^D + G - \alpha_C L$$
$$\geq V_0^D + G - \delta(\alpha_C) L$$
$$\geq V_0^D + G - \delta(\alpha^*) L$$
$$= V_0^D,$$

where the second inequality follows because  $\alpha_C \ge \alpha^*$  implies  $\delta(\alpha_C) \le \delta(\alpha^*)$ . Since  $D_1$ 's payoff is independent of  $\alpha$ ,  $D_1$  will acquire in this case any  $\alpha \in [\alpha^*, \alpha_C]$ .

If  $\alpha_C < \alpha^*$ , an acquisition of  $\alpha \le \alpha_C$  is insufficient to induce foreclosure because  $G = \delta(\alpha^*)L < \delta(\alpha_C)L \le \delta(\alpha)L$ . However,  $D_1$  can buy an additional stake  $\alpha - \alpha_C$  from  $U_1$ 's passive shareholders, such that after the acquisition, its stake is  $\alpha \ge \alpha^*$ , in which case  $U_1$  will foreclose  $D_2$ . Since  $D_1$  needs to pay both the initial controller and the passive shareholders of  $U_1$  a price that reflects the pre-acquisition value of  $U_1$ , the resulting payoff of  $D_1$  exceeds its pre-acquisition payoff:

$$V_{1}^{D} + \alpha V_{1}^{U} - \alpha_{C} V_{0}^{U} - (\alpha - \alpha_{C}) V_{0}^{U} = V_{0}^{D} + G - \alpha \underbrace{(V_{0}^{U} - V_{1}^{U})}_{L}.$$

Since this expression decreases with  $\alpha$ ,  $D_1$  will only acquire from the passive shareholders a stake of  $\alpha^* - \alpha_C$ , such that its final stake in  $U_1$  is  $\alpha^*$ . Given that  $\delta(\alpha) \geq \alpha$ , the acquisition is profitable since  $D_1$ 's resulting payoff is

(A8) 
$$V_0^D + G - \alpha^* \underbrace{\left(V_0^U - V_1^U\right)}_L \ge V_0^D + G - \delta(\alpha^*)L = V_0^D.$$

In sum, when  $U_1$  has an initial controller and G > L, we get a foreclosure equilibrium, and  $D_1$ 's stake in  $U_1$  is any  $\alpha \in [\alpha^*, \alpha_C]$  if  $\alpha^* < \alpha_C$  and  $\alpha^*$  otherwise.

Next, suppose that  $G \leq L$ . Then as Figure 1 shows,  $G > \delta(\alpha)L$  for all  $\alpha \in (\alpha^*, \alpha^{**})$ . The analysis is as before when  $\alpha_C \leq \alpha^{**}$ . Things are different, however, when  $\alpha_C > \alpha^{**}$ . To induce foreclosure,  $D_1$  must acquire a stake  $\alpha \leq \min\{\alpha^{**}, \alpha_C\}$ . After paying the initial controller  $b^U = \alpha V_1^U + \alpha_C L$  for this stake,  $D_1$ 's post-acquisition payoff is

$$V_1^D + \alpha V_1^U - \underbrace{\left(\alpha V_1^U + \alpha_C L\right)}_{b^U} = V_0^D + G - \alpha_C L.$$

This payoff is (weakly) higher than the pre-acquisition payoff if and only if  $G \ge \alpha_C L$ . Again,  $D_1$  is indifferent as to the actual stake it acquires provided that it induces foreclosure, i.e., is such that  $\alpha \in [\alpha^*, \alpha^{**}]$ .

Finally, assume that initially  $U_1$ 's ownership is dispersed and assume that  $D_1$  acquires a stake  $\alpha$ , which leads to the foreclosure of  $D_2$  (otherwise the acquisition is not profitable). Since  $D_1$  needs to pay the passive shareholders of  $U_1$  a price that reflects the pre-acquisition value of  $U_1$ , the post-acquisition payoff of  $D_1$  is

$$V_{1}^{D} + \alpha V_{1}^{U} - \alpha V_{0}^{U} = V_{0}^{D} + G - \alpha \underbrace{\left(V_{0}^{U} - V_{1}^{U}\right)}_{L}.$$

Since this payoff decreases with  $\alpha$ ,  $D_1$  will acquire the minimal stake that ensures foreclosure, i.e.,  $\alpha^*$ . Hence,  $D_1$ 's resulting payoff is as in (A8), so the acquisition is profitable.

### APPENDIX B: THE PROPERTIES OF THE REDUCED-FORM PROFITS

Following is an explicit model of downstream competition that is intended to motivate the assumptions made in Sections I and II on the downstream profit functions. Suppose that  $D_1$  and  $D_2$  are located at the two ends of a unit line and compete by setting prices. Consumers are uniformly distributed on the line and the utility of a consumer located at point *x* is

$$U_1(x) = v \log(n_1 + 1) - \tau x - p_1,$$

if he buys from  $D_1$  and

$$U_2(x) = v \log(n_2 + 1) - \tau(1 - x) - p_2,$$

if he buys from  $D_2$ , where  $v \log(n_1 + 1)$  and  $v \log(n_2 + 1)$  are the "qualities" of  $D_1$  and  $D_2$ , which increases with the number of inputs that  $D_1$  and  $D_2$  use;  $\tau > 0$  is the transportation cost per unit of distance, and  $p_1$  and  $p_2$  are the prices that  $D_1$  and  $D_2$  charge. If the consumer does not buy at all, his utility is 0.47

Assuming that the market is fully covered, the location of the indifferent consumer between  $D_1$  and  $D_2$  is

(B1) 
$$x^*(p_1, p_2, n_1, n_2) = \frac{1}{2} + \frac{p_2 - p_1 + v \log\left(\frac{n_1 + 1}{n_2 + 1}\right)}{2\tau}$$

Assuming in addition that  $D_1$  and  $D_2$  pay a fixed price for the inputs (the input prices are independent of actual sales) and normalizing their additional costs to 0, the gross profits of  $D_1$  and  $D_2$  are given by

$$\pi_1 = p_1 x^* (p_1, p_2, n_1, n_2), \qquad \pi_2 = p_2 (1 - x^* (p_1, p_2, n_1, n_2)).$$

<sup>47</sup>This formulation is similar to the simple model in Gavazza (2011), where the willingness of consumers to pay increases with the number of different products that each firm offers. Gavazza referes to this effect as "demand spillover." Unlike here, Gavazza considers a discrete choice model and  $\log(n_i)$  is the intercept of the consumer's utility when he buys from firm *i*.

Solving for the Nash equilibrium prices, we obtain

$$p_1^*(n_1, n_2) = \tau + \frac{v}{3} \log\left(\frac{n_1+1}{n_2+1}\right), \qquad p_2^*(n_1, n_2) = \tau - \frac{v}{3} \log\left(\frac{n_1+1}{n_2+1}\right).$$

To avoid uninteresting complications, we shall assume that  $\tau$  is large but not too large:

$$\frac{v}{3} \left| \log \left( \frac{n_1 + 1}{n_2 + 1} \right) \right| < t < \frac{v}{3} \log (n_1 + 1) (n_2 + 1).$$

This assumption ensures that  $p_1^*(n_1, n_2)$  and  $p_2^*(n_1, n_2)$  are both nonnegative and the market is covered as we assumed.<sup>48</sup>

Substituting  $p_1^*(n_1, n_2)$ ,  $p_2^*(n_1, n_2)$  in the profit functions and using (B1), the profit of a downstream firm when it uses k inputs and its rival uses l inputs (e.g., the profit of  $D_1$  when  $n_1 = k$  and  $n_2 = l$ ) is

$$\pi(k, l) = \frac{\left[t + \frac{v}{3}\log\left(\frac{k+1}{l+1}\right)\right]^2}{2t}.$$

Our assumption that  $\tau > \frac{v}{3} \left| \log((n_1 + 1)/(n_2 + 1)) \right|$  ensures that  $\pi(k, l)$  is increasing with *k* and decreasing with *l* as Assumption A1 states.

Now,

$$\Delta_1(k, l) \equiv \pi(k, l) - (k - 1, l) = \frac{v}{6\tau} \log\left(\frac{k + 1}{k}\right) \left[2\tau + \frac{v}{3}\log\left(\frac{k(k + 1)}{(l + 1)^2}\right)\right],$$

and

$$\Delta_2(k, l) \equiv \pi(k, l) - (k, l-1) = \frac{\nu}{6\tau} \log\left(\frac{l}{l+1}\right) \left[2\tau + \frac{\nu}{3} \log\left(\frac{(k+1)^2}{l(l+1)}\right)\right].$$

Assumption A3 holds since

$$\begin{split} \Delta_{12}(l,\,k) \, &\equiv \, \underbrace{\left(\pi(l,\,k) - (l-1,\,k)\right)}_{\Delta_1(l,k)} - \underbrace{\left(\pi(l,\,k-1) - (l-1,\,k-1)\right)}_{\Delta_1(l,k-1)} \\ &= \, \frac{v^2}{9\tau} \log\left(\frac{k+1}{k}\right) \log\left(\frac{l}{l+1}\right) \, < \, 0. \end{split}$$

Assumption A2 holds if

$$\frac{\nu}{6\tau} \log\left(\frac{N+1}{N}\right) \left[2\tau + \frac{\nu}{3} \log\left(\frac{N}{N+1}\right)\right] > c,$$

<sup>48</sup> Similar restrictions are also needed in the textbook version of the Hotelling model since when  $\tau$  is too high, the two firms become local monopolies, and when  $\tau$  is too small, the equilibrium is unstable (starting from a candidate interior equilibrium, firms may wish to cut prices drastically and corner the market).

and Assumption A4 holds if

$$\frac{\nu}{6\tau} \log\left(\frac{k+1}{k}\right) \left[ 2\tau + \frac{\nu}{3} \log\left(\frac{k(k+1)l^2}{(l+1)^4}\right) \right] > c.$$

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