The incentive to participate in open source projects: a signaling approach*

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Abstract

This paper examines the incentive of unpaid programmers to contribute to open source software (OSS) projects in order to signal their talents. The analysis shows that if programmers contribute to OSS projects at all, then generically there are multiple equiliria. In these equilibria, an increase in the visibility of performence, an increase in the sensitivity of performence to effort, and an increase in the informativeness of performence about talent may or may not boost the signaling incentive of programers.

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1 Introduction

Open source software (OSS) is a computer program whose source code - the instructions for the program, written in a human readable format - is distributed free of charge and can be modified, extended, adapted, and incorporated into other programs with relatively few restrictions. OSS is a rapidly expanding phenomenon: some OSS such as the Apache web server, dominate their product categories. In the personal computer market, some OSS such as the operating system Linux, the web browser Firefox, and the office suites OpenOffice.org gain rapid popularity.[1]

Apart from having millions of OSS users, there are also tens of thousands of participating programmers who contribute to various OSS projects, and there is also a growing number of firms who sell services, support, and documentation for OSS. The majority of the programmers who participate in OSS projects are unpaid volunteers. For example, Hars and Ou (2002) have surveyed 81 individuals involved in open source projects and found that only 16% received any direct monetary compensation for their contribution. This raises obvious questions about the incentives and motivations of the participating programmers. There are three main, mostly complimentary, explanations for the willingness of programmers to contribute to OSS projects. The first two involve intrinsic motivations while the third involves extrinsic motivations.

The first explanation is that programmers simply like to be involved in open source projects, either because they enjoy being creative, or due to a sense of obligation or community related reasons, or simply due to sheer altruism.[2] Indeed, a web-based survey conducted by Lakhani and Wolf (2003) reveals that the responding programmers were mainly driven by enjoyment-based intrinsic motivations.

The second explanation involves another type of intrinsic motivation. According to this explanation, individual users such as system managers (e.g., users of Apache), who make all sorts of software improvements for their own benefit, are willing to share these improvements with other users in their community. A model along these lines is offered by Johnson (2002), who views participation in OSS projects as a private provision of a public good (see Bessen, 2006, Bitzer and Schröder, 2005, and Bitzer, Schrettl, and Schröder, 2007, for related models).

[1] It is estimated that as of May 2009 there are 29 million users of Linux worldwide (see http://counter.li.org/estimates.php), and that by the end of April 2009, there were 894 million downloads of Firefox (see http://twitter.com/FirefoxTweets), and about 59 million downloads of OpenOffice.org from the OpenOffice.org site (see http://marketing.openoffice.org/marketing_bouncer.html)

[2] Athey and Ellison (2006) consider a dynamic model of the evolution of open source software projects, in which altruistic programmers who have used the software in the past are motivated to publish their own improvements for the benefit of other users.

The third explanation, suggested by Lerner and Tirole (2002), is that programmers are willing to contribute to OSS projects in order to signal their ability to potential employers, venture capitalists, or to peers. This enables programmers to boost their human capital or enhance their social status within the programmers' community. Fershtman and Gandal (2007) examine a large data set on programmers' participation in OSS projects and find that the output per contributor is much higher when the OSS is distributed under a less restrictive license and is more commercially oriented. They argue that this result is consistent with the hypothesis that contributing programmers are driven by signaling incentives. Another piece of evidence for this hypothesis is due to Hann et al (2004) who examine a panel data on contributions to three major OSS projects under the control of the Apache Software Foundation for the period 1998 to 2002. They find that credentials earned through the merit-based ranking system within the Apache open source community are associated with a $13\% - 27\%$ increase in wages, depending on the rank attained.

Drawing on the "career concerns" literature (e.g., Holmström, 1999), Lerner and Tirole (2002) conjecture that the signaling incentive to participate in OSS projects will become stronger as (i) performance becomes more visible to the relevant audience, (ii) effort has a stronger impact on performance, and (iii) performance becomes more informative about talent. While these conjectures are intuitively appealing, it is also possible to think about the opposite conjectures. For instance, if effort has a greater impact on performance and/or if performance becomes more visible, then even a small amount of effort might enable talented programmers to produce a visible positive signal about their talent.

In this paper I study the signaling incentive to participate in OSS projects in the context of a formal model and then use it to examine the Lerner and Tirole conjectures. In this model, programmers are privately informed about their types: some are "talented" and have high productivity, while others are "untalented" and have low productivity.[3] To signal their talent to prospective employers, programmers participate in OSS projects and each programmer either "succeeds" (i.e., "solves a problem" or "advances within the community's ranks") or "fails." Talented agents can boost their chances to succeed by exerting effort. Prospective employers then imperfectly observe whether specific programmers have succeeded or not and this observation, together with their beliefs on the effort of talented agents determine the wages that they offer programmers.

I show that the model always admits a no-effort equilibrium in which firms do not expect

[3] My model therefore differs from Holmström (1999) where agents do not have private information about their talent.

programmers to exert effort in OSS projects, and programmers indeed do not exert such effort. However, the model may also admit interior equilibria in which talented programmers exert effort and observed success translates into higher wages. When these equilibria exist, then generically their number is even. Interestingly, this multiplicity of equilibria is not driven by out-of-equilibrium beliefs as in Spence style signaling models, because the mapping from effort to success/failure in my model is stochastic, so there are no out-of-equilibrium signals. It is well-known that comparative statics results can go the "wrong" way when equilibria are unstable. The analysis shows however that even when we restrict attention to stable equilibria, conjectures (i)-(iii) may or may not hold. This suggests in turn that it is hard to say a-priori which factors will boost the signaling incentive of talented agents and which factors will weaken it.

There are three closely related papers that also study the signaling incentive of programmers who participate in OSS projects. These papers differ from mine both in terms of their set up and in terms of their main focus. In Lee, Moisa, and Weiss (2003) programmers need to choose between joining closed-source software firms and OSS projects. If they join software firms, their wage reflects the expected productivity of all programmers who join closed-source software firms (more and less talented ones). On the other hand, if they join OSS projects, they forgo current wages, but can signal their productivity to software firms and thereby boost their future wages. The main focus of their analysis is on the relative sizes of the closed-source and the open-source systems. They show that an open-source system can exist only if there are sufficiently many talented programmers.

Blatter and Niedermayer (2008) also consider the programmers' choice between closed-source software firms and OSS projects, but assume that working for OSS projects generates a public signal about performance whereas working for a software firm generates a signal which is observed only by the current employer. They show that talented programmers may prefer OSS projects even if they are not paid because a public signal on their performance improves their bargaining position vis-a-vis future employers.

Leppämäki and Mustonen (2004) consider a model in which programmers signal their talent to software firms by choosing how many lines of code to contribute to an OSS project. Talented programmers have a lower cost of writing code and hence they separate themselves from untalented programmers by writing sufficiently many lines of code. Their model departs from the traditional Spence signaling model in that the freely available OSS project imposes either a positive or a negative externality on commercial software, depending on whether the two are substitutes or complements. This externality in turn affects the wages that software firms are willing to offer and hence the marginal benefit to signaling. As a result, talented programmers contribute to the OSS project less (more) if the OSS and commercial software are substitutes (complements) than if they are independent of each other.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 shows that the model can give rise to multiple equilibria and characterizes them. Section 4 studies the comparative static properties of the model and examines how the incentive to contribute to OSS projects is affected by the visibility of the programmers' performance to prospective employers, the sensitivity of performance to effort, and the informativeness of performance about talent. Concluding remarks are in Section 5.

2 The model

In order to examine the signaling incentive of agents, I present a model in which agents contribute to OSS projects in order to generate positive signals about their talent. The likelihood that agents succeed to generate positive signals is increasing with both their effort and talent. Prospective employers (firms) use the signals they observe to update their beliefs about the talent of individual agents.

Specifically, I consider a competitive job market with a large number of agents, each of whom is either "talented" or "untalented." If hired by a firm, the marginal productivity of a talented agent is w, while the marginal productivity of an untalented agent is normalized to 0. Under full information, the wage of each agent is simply equal to his marginal productivity.

Under asymmetric information, it is common knowledge that the fraction of talented agents in the population is α, but firms cannot tell the agents' types before hiring them (agents however know their own types). Before the labor market opens up, agents participate in OSS projects in the hope of convincing prospective employers that they are talented.[4] I assume that each agent either succeeds (i.e., "solves a problem" or "advances within the community's ranks") or fails (i.e., "fails to come up with satisfactory results" or "does not advance within the community's ranks"). The probability that an untalented agent succeeds is exogenous and equal to p_0. Talented agents by contrast can boost their probability of success by exerting effort: if a talented agent exerts effort e in OSS projects, his probability of success increases from p_0 to $p(e)$, where $p(e)$ is increasing and

[4] To simplify matters, I assume that the cost of participation is sufficiently low to ensure that all agents participate.

strictly concave, with $p(0) = p_0$ and $\lim_{e\to\infty} p(e) = 1$.[5]

In and of themselves, OSS projects do not benefit the firms nor the agents directly.[6] The only advantage of participation is that it generates a signal on the agents' talent. Firms cannot directly observe if and how much effort each agent exerts; rather they can only observe successful performance.[7] Since one of the goals of this paper is to examine the effect of the visibility of performance on the signaling incentive of the agents, I will assume that the probability that firms observe a successful performance is $Q(p(e), \beta) < p(e)$, where $\beta \in [0, 1]$ is a shift parameter that reflects the visibility of the agents' performance to prospective employers. For instance, a higher value of β could be associated with more popular or more prominent projects that attract the attention of more firms. With probability $1 - Q(p(e), \beta)$, firms observe nothing and cannot tell whether the agent failed or whether he succeeded but his success was not observed. I will assume that $Q(p(e), \beta)$ is increasing and concave in $p(e)$, which implies in turn that $Q(e, \beta) \equiv Q(p(e), \beta)$ is increasing and strictly concave in e. When an agent is untalented, his probability of success is p_0 and the probability that firms will observe a successful performance is $Q_0(\beta) \equiv Q(p_0, \beta)$. Moreover, I will assume that $Q(e, \beta)$ is increasing with β.

The payoff of each agent is increasing with his expected wage, Ew, and decreasing with his effort level, e:

$$U = Ew - e.$$

3 Equilibrium characterization

I now look for a perfect Bayesian equilibrium in which talented agents exert effort, untalented agents do not exert effort, and the beliefs of firms are consistent with the agents' strategies.[8] To characterize this equilibrium, suppose that firms believe that the effort of talented agents is $\widehat{e}$ and hence expect that talented agents will produce positive signals about their performance (i.e., will succeed and their success will be observed) with probability $Q(\widehat{e}, \beta)$. Recalling that the fraction of talented agents is α and that untalented agents produce positive signals about their performance

[5] The assumption that untalented agents cannot increase their probability of success is not essential: the main results go through even if this assumption is relaxed (albeit the analysis becomes more involved).

[6] In order to focus on the signaling incentive of agents I ignore intrinsic motivations to participate in OSS projects. Incorporating such motivations in my model is starightforward and does not yield any new insights.

[7] My model therefore involves "noisy" signaling since the mapping from efforts to observed success is stochastic. This approch differ from the approach in Leppämäki and Mustonen (2004) where prospective employers can perfectly observe the action of each agent which is how many lines of code to write.

[8] Untalented agents do not exert effort because their success probability is independent of their effort and equal to p_0.

with probability $Q_0(\beta) \equiv Q(p_0, \beta)$, it follows that conditional on observing a success, firms believe that an agent is talented with probability

$$q(\widehat{e} \mid s) = \frac{\alpha Q(\widehat{e}, \beta)}{\alpha Q(\widehat{e}, \beta) + (1-\alpha) Q_0(\beta)}. \tag{1}$$

If firms do not observe a success, they cannot tell whether (i) the agent is talented, exerted effort, and either failed or his success was not observed, or (ii) the agent is untalented and either failed or his success was not observed. The probability of (i) is $\alpha(1 - Q(\widehat{e}, \beta))$, while the probability of (ii) is $(1-\alpha)(1 - Q_0(\beta))$. Hence, conditional on not observing a positive signal, firms believe that an agent is talented with probability

$$q(\widehat{e} \mid n) = \frac{\alpha(1 - Q(\widehat{e}, \beta))}{\alpha(1 - Q(\widehat{e}, \beta)) + (1-\alpha)(1 - Q_0(\beta))}. \tag{2}$$

Note that $q(0 \mid s) = q(0 \mid n) = \alpha$: if firms expect talented agents to exert no effort, then observed success is not informative about talent. Moreover, note that $q(\widehat{e} \mid s)$ approaches 1 as $Q_0(\beta)$ approaches 0: if untalented agents never generate positive signals about their performance, then a positive signal is a sure sign that the agent is talented.

3.1 The effort choice of talented agents

To characterize the effort that talented agents will exert, note that since the labor market is competitive, the wage of agents is $q(\widehat{e} \mid s)w$ following an observed success and $q(\widehat{e} \mid n)w$ otherwise. Hence, the expected payoff of talented agents given their effort level, e, and given the belief of firms, $\widehat{e}$, is

$$U(e, \widehat{e}) = Q(e, \beta)\, q(\widehat{e} \mid s)w + (1 - Q(e, \beta))\, q(\widehat{e} \mid n)w - e. \tag{3}$$

The first term on the right-hand side of (3) reflects the idea that with probability $Q(e, \beta)$, a talented agent manages to produce a positive signal about his talent. The second term reflects the idea that with probability $1 - Q(e, \beta)$, a talented agent fails to produce a positive signal about his talent either because his success was not observed by firms or because the agent simply failed. In both cases, firms cannot tell whether the agent is talented or not and hence they pay him $q(\widehat{e} \mid n)w$. The last term on the right-hand side of (3) is the agent's cost of effort.

Since $Q(e, \beta)$ is strictly concave in e, and using subscripts to denote partial derivatives, the effort that each talented agent will choose given the firms' beliefs, $\widehat{e}$, is defined implicitly by the following first order condition:

$$\frac{\partial U(e,\widehat{e})}{\partial e} = Q_e\,(e,\beta)\,\Delta(\widehat{e},\beta)w - 1 \leq 0, \qquad e\frac{\partial U(e,\widehat{e})}{\partial e} = 0, \tag{4}$$

where

$$\Delta(\widehat{e},\beta) \equiv q(\widehat{e} \mid s) - q(\widehat{e} \mid n), \tag{5}$$

is the increase in the probability that firms assign to an agent being talented following a positive signal. The expression $Q_e\,(e,\beta)\,\Delta(\widehat{e},\beta)w$ represents the marginal benefit from effort; it is equal to the marginal impact of effort on the probability of producing a positive signal, $Q_e\,(e,\beta)$, times the "wage premium" in this event, $\Delta(\widehat{e},\beta)w$. At an interior optimum, the marginal benefit of effort must be equal to the marginal cost, which is 1.

Since w is simply a constant in my model, I will refer to $\Delta(\widehat{e},\beta)$ in what follows as the "signaling wage premium." The following lemma reports two important properties of $\Delta(\widehat{e},\beta)$:

Lemma 1: *The signaling wage premium, $\Delta(\widehat{e},\beta)$, is increasing with $\widehat{e}$ and $\Delta(0,\beta) = 0$.*

Proof: It is easy to see that $q(\widehat{e} \mid s)$ increases and $q(\widehat{e} \mid n)$ decreases with $Q\,(\widehat{e},\beta)$ which in turn increases with $\widehat{e}$; hence $\Delta_e(\widehat{e},\beta) > 0$. $\Delta(0,\beta) = 0$ because $q(0 \mid s) = q(0 \mid n) = \alpha$. ■

Lemma 1 implies that when firms believe that talented agents exert more effort, they are willing to pay higher wages to agents who were observed to be successful.

3.2 Stable and unstable equilibria

Let $BR(\widehat{e})$ denote the solution of (4). This function defines the best response of each talented agent against the firms' beliefs about his effort level. In equilibrium, the firms' beliefs must be consistent with the true efforts of talented agents. Hence, the equilibrium effort level, e^*, is defined implicitly by the equation

$$e^* = BR(e^*). \tag{6}$$

Given its central role in what follows, I will now study the properties of $BR(\widehat{e})$ in the next lemma. The following assumption is needed to ensure that $BR(\widehat{e})$ is positive for sufficiently large values of $\widehat{e}$:

Assumption (*): *The marginal impact of effort on the probability of success is large for* $e = 0$*:*

$$Q_e(0,\beta) > \frac{(\alpha Q_1(\beta) + (1-\alpha)Q_0(\beta))\,(1 - \alpha Q_1(\beta) - (1-\alpha)\,Q_0(\beta))}{\alpha(1-\alpha)\,(Q_1(\beta) - Q_0(\beta))\,w}, \tag{7}$$

where $Q_0(\beta) \equiv Q(p_0,\beta)$ *and* $Q_1(\beta) \equiv \lim_{e\to\infty} Q(p(e),\beta)$.

When Assumption (*) fails, it never pays talented agents to exert effort, no matter how high $\widehat{e}$ is, so $BR(\widehat{e}) = 0$ for all $\widehat{e}$.

Lemma 2: *Suppose that Assumption (*) holds. Then, the best response of talented agents against the firms' beliefs about their effort level,* $BR(\widehat{e})$*, has the following properties:*

(i) $BR(\widehat{e}) = 0$ *for all* $0 < \widehat{e} \leq \widehat{e}_1$;

(ii) $BR(\widehat{e}) > 0$ *for all* $\widehat{e} > \widehat{e}_1$ *(talented agents exert effort only if firms expect them to exert a sufficiently large level of effort), where* $BR(\widehat{e})$ *is implicitly defined by*

$$Q_e(e,\beta)\,\Delta(\widehat{e},\beta)w = 1, \tag{8}$$

and $\widehat{e}_1$ *is implicitly defined by* $\frac{\partial U(0,\widehat{e})}{\partial e} = 0$.

(iii) *For all* $\widehat{e} > \widehat{e}_1$,

$$BR'(\widehat{e}) = -\frac{Q_e(e,\beta)\,\Delta_e(\widehat{e},\beta)}{Q_{ee}(e,\beta)\,\Delta(\widehat{e},\beta)} > 0, \tag{9}$$

and $\lim_{\widehat{e}\to\infty} BR'(\widehat{e}) = 0$.

Proof: See the Appendix.

To characterize the equilibrium effort level, e^*, note from (6) that e^* is attained at the intersection of the best-response function, $BR(\widehat{e})$, with the 45^0 line in the $(\widehat{e}, e)$ space (the 45^0 line reflects the requirement that in equilibrium, firms must hold correct beliefs about the effort of talented agents). Since $BR(\widehat{e})$ passes through the origin, $e^* = 0$ is an equilibrium. Hence, there always exists a no-effort equilibrium in which talented agents are not expected and indeed do not exert effort.[9] The question is whether there also exist interior equilibria with $e^* > 0$.

[9] Intrestingly, the Athey and Ellison (2006) model also admits a "no-effort" equilibrium in which programmers do not contribute to open source projects. In their dynamic model, this equilibrium is driven by the fact that potential contributors to open source projects are former users. An open source software with 0 quality attracts no users, and hence has no future contributors.As a result, its quality can never improve.

To address this question, I present $BR(\widehat{e})$ in Figure 1, using Lemma 2. As the figure shows, $BR(\widehat{e})$ coincides with the horizontal axis for sufficiently small values of $\widehat{e}$. Assumption (*) ensures that there exists a critical value of $\widehat{e}$, denoted $\widehat{e}_1$, above which $BR(\widehat{e})$ becomes positive. Part (iii) of Lemma 2 shows that $BR(\widehat{e})$ increases for all $e > \widehat{e}$, although eventually it becomes flat. Recalling that the equilibrium effort of talented agents, e^*, is determined by the intersection of $BR(\widehat{e})$ with the 45^0 line, it is clear from Figure 1 that in general, there are two possibilities.

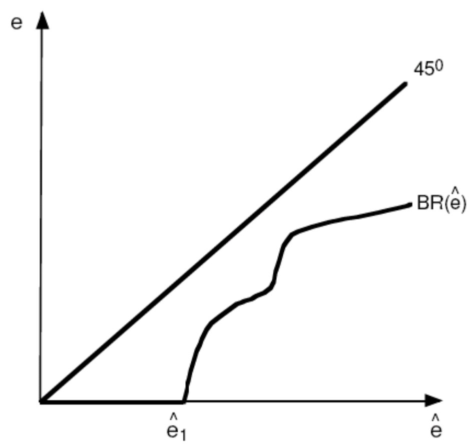

Figure 1a: No interior equilibria

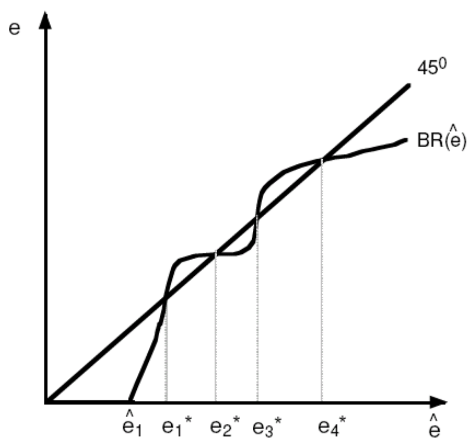

Figure 1b: Four interior equilibria

The first possibility, illustrated in Figure 1a, arises when $BR(\widehat{e})$ lies below the 45^0 line for all $\widehat{e} > 0$. In this case, the model does not admit interior equilibria with $e^* > 0$. A sufficient (though not necessary) condition for this case is that $BR'(\widehat{e}) < 1$ for all $\widehat{e} > \widehat{e}_1$. This condition is likely to hold if $Q_e(e, \beta)$ is small relative to $-Q_{ee}(e, \beta)$.

The second possibility, illustrated in Figure 1b, arises when $BR(\widehat{e})$ intersects the 45^0 line at least once from below at some $\widehat{e} > \widehat{e}_1$.[10] Since $\lim_{\widehat{e}\to\infty} BR'(\widehat{e}) = 0$, $BR(\widehat{e})$ must also intersect the 45^0 line from above at least once. Hence, if there are interior equilibria with $e^* > 0$, then generically, their number must be even.

Notice that whenever $BR'(e^*) < 1$, the best response of talented agents, evaluated at the equilibrium point, is flatter than the 45^0 line and hence must cut it from above. The resulting interior equilibria (e_2^* and e_4^* in Figure 1b) are then stable in the sense that, starting from any close

[10] It is also possible that $BR(\widehat{e})$ is just tangent to the 45^0 line. Such tangency point is also an equilibrium, but this equilibrium is non-generic in the sense that it will vanish following small perturbations that shift $BR(\widehat{e})$ either upward or downward. In the rest of the paper, I will restrict attention to generic equilibria.

neighborhood of e^*, a Cournot tatônnement process will converge to e^*. Notice that the no-effort equilibrium is also stable since $BR'(0) = 0$. On the other hand, whenever $BR'(e^*) > 1$, $BR(e^*)$ is steeper than the 45^0 line and hence must cut it from below. Consequently, the resulting equilibria (e_1^* and e_3^* in Figure 1b) are unstable. The following proposition summarizes this discussion:

Proposition 1: *The model always admits a (stable) no-effort equilibrium in which $e^* = \widehat{e}^* = 0$. A sufficient (but not necessary) condition for this equilibrium to be unique is that $BR'(\widehat{e}) < 1$ for all $\widehat{e} > \widehat{e}_1$. However, if the model admits interior equilibria with $e^* > 0$, then generically, their number is even, with half being stable and half being unstable.*

4 Comparative statics

Having characterized the equilibrium, I can now examine its comparative statics properties. In particular, I will examine the following three conjectures that are due to Lerner and Tirole (2002) and state that the signaling incentive of agents is stronger when:

(i) performance becomes more visible to the relevant audience,

(ii) effort has a stronger impact on performance, and

(iii) performance becomes more informative about talent.

In all three cases, I will use the following result which is clear from Figure 1b:

Lemma 3: *An upward shift of $BR(\widehat{e})$ increases the equilibrium effort level, e^*, in stable equilibria and decreases it in unstable equilibria, and conversely when $BR(\widehat{e})$ shifts downward.*

Lemma 3 implies that the comparative statics of interior equilibria depend crucially on their stability. In what follows, I will restrict my attention to the comparative statics properties of stable interior equilibria.[11]

4.1 The effect of increased visibility of performance

To examine conjecture (i), I will examine how an increase in β, which implies that the agents's performance becomes more visible to firms, affects e^*. As it turns out, the result depends on

[11] Lemma 3 implies that the comparative statics results are reversed in the case of unstable interior equilibria.

whether effort and visibility are complements or substitutes in the production of positive signals of performance: I will say that effort and visibility are complements in the production of positive signals of performance if $Q_{e\beta}(e,\beta) > 0$ (i.e., the marginal impact of effort on the probability of a positive signal increases when there is a greater visibility) and substitutes if $Q_{e\beta}(e,\beta) < 0$ (i.e., the marginal impact of effort on the probability of a positive signal decreases when there is a greater visibility). In principle, either case is plausible. For example, if successful performance is observed with probability β independent of the agent's effort, then $Q(e,\beta) = \beta p(e)$. Clearly then, $Q_{e\beta}(e,\beta) > 0$: effort and visibility are complements. On the other hand, if effort contributes not only to the agent's performance but is also required to attract attention to the agent's performance, then an exogenous increase in visibility may allow the agent to attract the same amount of attention with less effort. For instance, imagine that $Q(p(e),\beta) = 1 - (1 - p(e))\, m^{e+\beta}$, where $m \in (0,1)$. Then, holding $p(e)$ fixed, $Q(\cdot,\beta)$ increases with e, implying that effort contributes to the visibility of the agent's success. It is straightforward to verify that $Q(e,\beta)$ is increasing and concave in e and is increasing with β. Now, $Q_{e\beta}(e,\beta) = -m^{e+\beta}\log(m)\left(\log(m)(1-p(e)) - p_e(e)\right) < 0$, where the inequality follows because $m < 1$ and $p_e(e) > 0$. Hence, effort and visibility are substitutes in the production of positive signals on performance.

To examine the effect of β on e^*, note that by Lemma 3, it is sufficient to examine whether an increase in β shifts the best response of talented agents against the firms' beliefs about their effort, $BR(\widehat{e})$, upward or downward. Using equation (8),

$$\frac{\partial BR(\widehat{e})}{\partial \beta} = -\frac{Q_{e\beta}(\cdot,\beta)\Delta(\widehat{e},\beta) + Q_e(\cdot,\beta)\Delta_\beta(\widehat{e},\beta)}{Q_{ee}(\cdot,\beta)}, \tag{10}$$

where $Q_{ee}(\cdot,\beta) < 0$ by the strict concavity of $Q(\cdot,\beta)$ in e. Hence, the equilibrium level of effort, e^*, is increasing with β if the numerator of $\frac{\partial BR(\widehat{e})}{\partial \beta}$ is positive and decreasing with β if the numerator of $\frac{\partial BR(\widehat{e})}{\partial \beta}$ is negative. The sign of the numerator depends in turn on the signs of $Q_{e\beta}(\cdot,\beta)$ and $\Delta_\beta(\widehat{e},\beta)$. As mentioned above, $Q_{e\beta}(\cdot,\beta)$ is positive if effort and visibility are complements and negative if they are substitutes in the production of positive signals on performance. As for $\Delta_\beta(\widehat{e},\beta)$, then it is positive if greater visibility raises the signaling wage premium that agents enjoy following an observed success and it is negative if greater visibility lowers the signaling wage premium.

The following proposition establishes sufficient conditions for $Q_{e\beta}(\cdot,\beta)$ and $\Delta_\beta(\widehat{e},\beta)$ to have the same sign, in which case, an increase in β has an unambiguous effect on e^*.

Proposition 2: *Consider an increase in β (the agents' performance becomes more visible to firms).*

Then,

(i) *If* $Q_{e\beta}(\cdot,\beta) > 0$ *(effort and visibility are complements in the production of positive signals on performance), then* $\frac{\partial}{\partial\beta}\left(\frac{Q(\cdot,\beta)}{Q_0(\beta)}\right) \geq 0$ *is sufficient (but not necessary) for an increase in the effort level of talented agents in stable interior equilibria.*

(ii) *If* $Q_{e\beta}(\cdot,\beta) < 0$ *(effort and visibility are substitutes in the production of positive signals on performance), then* $\frac{\partial}{\partial\beta}\left(\frac{1-Q(\cdot,\beta)}{1-Q_0(\beta)}\right) \geq 0$ *is sufficient (but not necessary) for a decrease in the effort level of talented agents in stable interior equilibria.*

Proof: See the Appendix.

Proposition 2 shows that the signaling incentive of agents may or may not become stronger as their performance becomes more visible to the relevant audience. To understand the intuition, note an increase in β creates two effects: first, it affects the marginal impact of effort on the probability of generating a positive signal on performance. This effect is positive if effort and visibility are complements in generating positive signals but it is negative if they are substitutes. Second, visibility affects the signaling wage premium that firms offer agents following an observed success. This effect can also be either positive or negative because higher visibility makes it more likely that firms will observe not only the success of talented agents who exerted effort but also the success of untalented agents who simply "got lucky." Proposition 2 establishes sufficient conditions for the two effects of visibility to have the same signs.

To illustrate Proposition 2 consider first the case where $Q(e,\beta) = \beta p(e)$. In this case, $Q_{e\beta}(e,\beta) > 0$ and moreover, $\Delta(\widehat{e},\beta)$ increases with β because by equations (1) and (2), $q(\widehat{e} \mid s)$ is independent of β while $q(\widehat{e} \mid n)$ is decreasing with β (greater visibility does not affect the probability that firms assign to an agent being talented following an observed success but it lowers this probability otherwise). Since both $Q_{e\beta}(e,\beta)$ and $\Delta_\beta(\widehat{e},\beta)$ are positive, talented agents exert more effort when their success is more likely to observable to prospective employers.

Now suppose that $Q(e,\beta) = 1 - (1 - p(e))\, m^{e+\beta}$, where $m \in (0,1)$. Then, as shown above, $Q_{e\beta}(e,\beta) < 0$, so effort and visibility are substitutes in the production of positive signals on performance. Moreover, in this example,

$$\Delta_\beta(\widehat{e},\beta) = \frac{\alpha(1-\alpha)(1-p(\widehat{e}))\, m^\beta \left(1-m^{\widehat{e}}\right)\log(m)}{\left(1-(1-p(\widehat{e}))\, m^\beta\left(1-\alpha\left(1-m^{\widehat{e}}\right)\right)\right)^2} < 0.$$

Consequently, increased visibility lowers the effort of talented agents in stable interior equilibria, implying that, contrary to conjecture (i), the signaling incentive of agents is weakened when their performance becomes more visible to the relevant audience.[12]

4.2 The effect of increased sensitivity of performance to effort

Next, I examine the conjecture that the signaling incentive of agents will become stronger when effort has a stronger impact on performance. To this end, I introduce a new shift parameter, γ, which increases the probability of talented agents to succeed at each effort level. That is, I assume that the probability that a talented agent will succeed in OSS projects is given by $p(e,\gamma)$, where $p_\gamma(e,\gamma) > 0$. The probability that an untalented agent will succeed remains p_0. Given $p(e,\gamma)$ and suppressing the parameter β in order to simplify the notation, the probability that a talented agent will produce a positive signal on his talent is $Q(e,\gamma) \equiv Q(p(e,\gamma))$. The probability that an untalented agent will produce a positive signal on his talent is given by $Q_0 \equiv Q(p_0)$, where once again, the parameter β is suppressed.

Notice that since $p_\gamma(e,\gamma) > 0$ and $Q(p(e,\gamma))$ is increasing with $p(e,\gamma)$, then $Q_\gamma(e,\gamma) > 0$: talented agents are more likely to produce positive signals when γ increases (effort has a stronger impact on performance). Although the parameter β also raises the probability that talented agents will produce a positive signal, it differs from γ in that β (which reflects the visibility of performance) also raises the probability that untalented agents will produce a positive signal ($Q_0'(\beta) > 0$), whereas γ (which reflects the impact of effort on performance) has no effect on Q_0.

As in the case of β, Lemma 3 implies that in order to examine the effect of γ on e^*, it is sufficient to examine whether an increase in γ shifts the best response of talented agents, $BR(\widehat{e})$, upward or downward. Using equation (8),

$$\frac{\partial BR(\widehat{e})}{\partial \gamma} = -\frac{Q_{e\gamma}(\cdot)\Delta(\widehat{e},\gamma) + Q_e(\cdot)\Delta_\gamma(\widehat{e},\gamma)}{Q_{ee}(\cdot)}, \tag{11}$$

where $Q_{ee}(\cdot) < 0$ by the strict concavity of $Q(\cdot)$ in e. The sign of the numerator of $\frac{\partial BR(\widehat{e})}{\partial \gamma}$ depends

[12] Blatter and Niedermayer (2008) also show that increased visibility may lead to lower effort. In their model, low visibility is associated with working for a closed-source software firm which privately observes the agent's performance, while high visibility is associated with joining an OSS project in which case performence becomes common knowledge. They show that agents may exert more effort when working for closed-source firms even though performance is less visible. This is especially so when software firms are willing to pay untalented agents a relatively high wage since then, untalented agents may prefer to work for software firms. Hence, the decision to join an OSS project already sends a strong signal that the agent is talented, implying that talented agents do not need to exert that much effort to signal their talent.

on the signs of $Q_{e\gamma}(\cdot)$ and $\Delta_\gamma(\widehat{e},\gamma)$. Using equations (1) and (2), we can write $\Delta(\widehat{e},\gamma)$ as

$$\Delta(\widehat{e},\gamma)=\frac{\alpha}{\alpha+(1-\alpha)\frac{Q_0}{Q(\widehat{e},\gamma)}}-\frac{\alpha}{\alpha+(1-\alpha)\frac{1-Q_0}{1-Q(\widehat{e},\gamma)}}.$$

It is easy to see from this expression that $\Delta(\widehat{e},\gamma)$ is increasing with $Q(\widehat{e},\gamma)$, which is in turn increasing with γ. Hence, $\Delta_\gamma(\widehat{e},\gamma)>0$.

By contrast, the sign of $Q_{e\gamma}(p\,(\widehat{e},\gamma))$ is in general ambiguous. To see why, note that

$$Q_{e\gamma}(p\,(\widehat{e},\gamma))=Q_{pp}\,(\cdot)\,p_e\,(e,\gamma)\,p_\gamma\,(e,\gamma)+Q_p\,(\cdot)\,p_{e\gamma}\,(e,\gamma)\,.$$

The first term in $Q_{e\gamma}(p\,(\widehat{e},\gamma))$ is nonpositive since $Q_{pp}\,(\cdot)\leq 0$. The second term depends on the sign of $p_{e\gamma}\,(e,\gamma)$ which in general can be either positive or negative.[13] When $p_{e\gamma}\,(e,\gamma)>0$, an increase in γ boosts the marginal impact of effort on the probability of success and conversely when $p_{e\gamma}\,(e,\gamma)<0$. Hence, the sign of $Q_{e\gamma}(p\,(e,\gamma))$ is in general ambiguous. Nonetheless, it is easy to see that a sufficient condition for $Q_{e\gamma}(p\,(e,\gamma))>0$ is that $p_{e\gamma}\,(e,\gamma)>0$ and that $Q\,(\cdot)$ is not too concave.

Proposition 3: *Consider an increase in γ (effort has a stronger impact on performance). Then $Q_{e\gamma}(p\,(\widehat{e},\gamma))>0$ is sufficient (but not necessary) for an increase in the effort level of talented agents in stable interior equilibria. The sufficient condition holds so long as $Q\,(p\,(e,\gamma))$ is not too concave in $p\,(e,\gamma)$* and $p_{e\gamma}\,(e,\gamma)>0$.

Proposition 3 provides a sufficient condition for conjecture (ii) to hold. However, the next example shows that for some parameter values, an increase in γ can actually lower e^*, contrary to the conjecture.

Suppose that $p(e,\gamma)=\frac{\exp\left(1-\left(\frac{1}{2}\right)^{t+\gamma e}\right)-1}{\exp(1)-1}$, where $\gamma,t>0$, and $Q(p)=\beta\log\left(1+(\exp\left(1\right)-1)\,p\right)$. It is easy to verify that $Q(p)$ is increasing and concave and $p(e,\gamma)$ is increasing in e and γ, strictly concave in e, and $\lim_{e\to\infty}p(e,\gamma)=1$. Using these expressions, $Q\,(e,\gamma)\equiv Q\,(p\,(e,\gamma))=\beta\left(1-\left(\frac{1}{2}\right)^{t+\gamma e}\right)$. Note that $Q_e(e,\gamma)=\beta\gamma\ln(2)\left(\frac{1}{2}\right)^{t+\gamma e}$ is first increasing and then decreasing with γ. To ensure that $BR(\widehat{e})>0$ for sufficiently large values of $\widehat{e}$, I will now impose Assumption (*),

[13] Although an increase in γ raises the probability of success, $p\,(e,\gamma)$, it need not neccesarily also raise the marginal effect of effort on this probability, $p_e\,(e,\gamma)$. Hence, $p_{e\gamma}(e,\gamma)$ can be either positive or negative.

which requires in the present context that:

$$w > \frac{\left(1-(1-\alpha)\left(\frac{1}{2}\right)^{t}\right)\left(1-\beta\left(1-(1-\alpha)\left(\frac{1}{2}\right)^{t}\right)\right)}{\alpha(1-\alpha)\beta\gamma\left(\frac{1}{2}\right)^{2t}\ln(2)}.$$

Substituting for $p(e,\gamma)$ and $p_e(e,\gamma)$ in equation (8) and rearranging terms, the best-response function of talented agents for sufficiently large values of $\widehat{e}$, is given by

$$BR(\widehat{e}) = \frac{\ln(\beta\gamma\ln(2)\Delta(\widehat{e})w) - t\ln(2)}{\gamma\ln(2)},$$

where

$$\Delta(\widehat{e}) \equiv \frac{\alpha\left(1-\left(\frac{1}{2}\right)^{t+\gamma\widehat{e}}\right)}{1-\alpha\left(\frac{1}{2}\right)^{t+\gamma\widehat{e}}-(1-\alpha)\left(\frac{1}{2}\right)^{t}} - \frac{\alpha\left(1-\beta\left(1-\left(\frac{1}{2}\right)^{t+\gamma\widehat{e}}\right)\right)}{1-\beta\left(1-\alpha\left(\frac{1}{2}\right)^{t+\gamma\widehat{e}}-(1-\alpha)\left(\frac{1}{2}\right)^{t}\right)}.$$

Now, let $\alpha = \beta = 0.5$, $t = 0.01$, and $w = 20$. Figure 2a shows $BR(\widehat{e})$ for two values of γ: 0.5 and 1. In both cases, $BR(\widehat{e})$ intersects the 45^0 line twice. The stable interior equilibrium is attained at the large intersection point where $BR(\widehat{e})$ crosses the 45^0 line from above. When γ increases from 0.5 to 1, $BR(\widehat{e})$ shifts upward and the effort of talented agents in the stable interior equilibrium increases from 1.856 to 2.066. However, Figure 2b shows that when γ increases from 0.5 to 2, $BR(\widehat{e})$ rotates clockwise, and the effort of talented agents in the stable interior equilibrium decreases from 1.856 to 1.563. Consequently, the relationship between γ and e^* is non-monotonic.

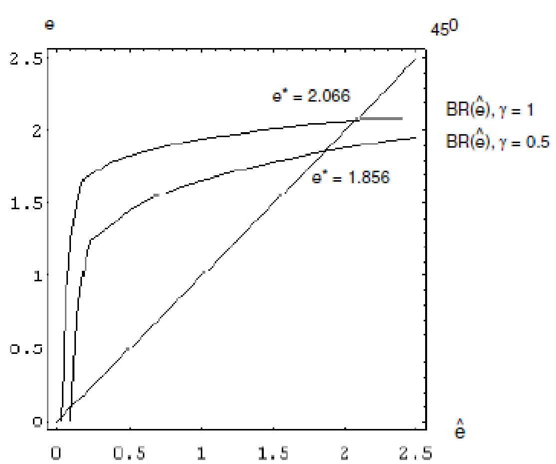

Figure 2a: The effect of γ on e^* when γ inccreases from 0.5 to 1.
Parameter values: $\alpha = \beta = 0.5$, $t = 0.01$, $w = 20$

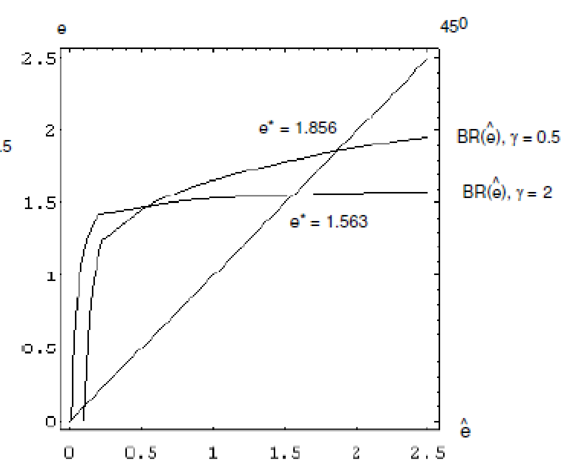

Figure 2b: The effect of γ on e^* when γ increases from 0.5 to 2.
Parameter values: $\alpha = \beta = 0.5$, $t = 0.01$, $w = 20$

To explore this nonmonotonicity further, I present e^* as a function of γ in Figure 3 for

$\alpha = \beta = 0.5$, $t = 0.01$, and $w = 20$. When γ is small, there do not exist interior equilibria. When $\gamma > 0.356$, there exist, for each value of γ, two interior equilibria: a stable equilibrium with a high e^* and an unstable equilibrium with a low e^*. Focusing on stable equilibria (the upper contour in Figure 3), one can see that e^* increases as γ increases from 0.356 to 0.754, but once $\gamma > 0.754$, a further increase in γ leads to a decrease in e^*. Hence, the effect of γ on e^* can be either positive or negative, depending on whether γ is initially high or low.

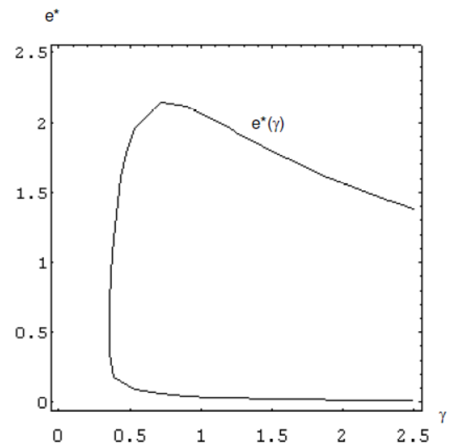

Figure 3: the effect of γ on e*
Parameter values: $\alpha = \beta = 0.5$, t = 0.01, w = 20

4.3 The effect of the informativeness of performance about talent

Conjecture (iii) states that the signaling incentive of agents will become stronger as performance becomes more informative about talent. This occurs in my model when the probability that firms assign to an agent being talented following an observed success, $q(\widehat{e} \mid s)$, increases. Since $q(\widehat{e} \mid s)$ in turn increases with α (the fraction of talented agents in the population) and decreases with p_0 (the likelihood that an untalented agent succeeds), the conjecture can be examined by either studying the effect of an increases in α or a decreases in p_0 on the equilibrium effort of talented agents, e^*. Since the parameters β and γ play no role in this analysis, I will suppress them and write the probability that firms observe a successful performance as $Q(e) \equiv Q(p(e))$ if the agent is talented and $Q_0 \equiv Q(p_0)$ if the agent is untalented.

Proposition 4: *Consider a decrease in p_0 (performance is more informative about talent). Then the effort level of talented agents increases in stable interior equilibria.*

Proof: Differentiating equation (8) with respect to e^* and p_0 and recalling that $\Delta(\widehat{e}) \equiv q(\widehat{e} \mid$

$s) - q(\widehat{e} \mid n)$ yields,

$$\begin{aligned}\frac{\partial BR(\widehat{e})}{\partial p_0} &= -\frac{Q_e(e)\frac{\partial \Delta(\widehat{e})}{\partial p_0}}{Q_{ee}(e)\Delta(\widehat{e})} \\ &= -\frac{Q_e(e)}{Q_{ee}(e)\Delta(\widehat{e})}\left[\frac{\partial q(\widehat{e} \mid s)}{\partial p_0} - \frac{\partial q(\widehat{e} \mid n)}{\partial p_0}\right] < 0,\end{aligned}$$

where the inequality follows because by assumption, $Q_e(e) > 0 > Q_{ee}(e)$ and because (1) and (2) imply that $q(\widehat{e} \mid s)$ decreases and $q(\widehat{e} \mid n)$ increases with Q_0 which is in turn increasing with p_0. Hence, a decrease in p_0 shifts $BR(\widehat{e})$ upward. Lemma 3 now implies the result. ■

Propositions 4 confirms the conjecture that agents have a stronger incentive to signal their talent when performance becomes more informative about talent. However, as the next result shows, this is not necessarily the case when performance becomes more informative about talent due to an increase in the fraction of talented agents in the population.

Proposition 5: *Consider an increase in α (the pool of agents is on average more talented). Then, the effort level of talented agents in stable interior equilibria increases if α is relatively small but decreases if α is relatively large.*

Proof: Differentiating equation (8) with respect to e^* and α, yields,

$$\begin{aligned}\frac{\partial BR(\widehat{e})}{\partial \alpha} &= -\frac{Q_e(e)\frac{\partial \Delta(\widehat{e})}{\partial \alpha}}{Q_{ee}(e)\Delta(\widehat{e})} \\ &= -\frac{Q_e(e)}{Q_{ee}(e)\Delta(\widehat{e})}\left[\frac{(Q(\widehat{e}) - Q_0)\, T(\alpha)}{(\alpha Q(\widehat{e}) + (1-\alpha)Q_0)^2\,(1 - \alpha Q(\widehat{e}) - (1-\alpha)Q_0)^2}\right],\end{aligned}$$

where

$$T(\alpha) \equiv (1-\alpha)^2 Q_0 (1 - Q_0) - \alpha^2 Q(\widehat{e})(1 - Q(\widehat{e}))\,.$$

Noting that $Q_e(e) > 0 > Q_{ee}(e)$ and $Q(\widehat{e}) > Q_0$ for all $\widehat{e} > 0$, it follows that the sign of $\frac{\partial BR(\widehat{e})}{\partial \alpha}$ depends on the sign of $T(\alpha)$. Clearly, $T'(\alpha) < 0$ and $T(0) > 0 > T(1)$. Hence, for each $\widehat{e}$, there exists a unique value of α, denoted $\overline{\alpha}(\widehat{e})$, where $\overline{\alpha}(\widehat{e}) \in (0,1)$, such that $\frac{\partial BR(\widehat{e})}{\partial \alpha} > 0$ for $\alpha \in [0, \overline{\alpha}(\widehat{e}))$ and $\frac{\partial BR(\widehat{e})}{\partial \alpha} < 0$ for $\alpha \in (\overline{\alpha}(\widehat{e}), 1]$. Lemma 3 now implies the result. ■

Intuitively, as increase in α creates two opposing effects. First, it makes it more likely that an observed success is associated with a talented agent. As a result, agents who were observed to be successful receive a higher wage. This effect boosts the signaling incentive of agents. Second, an increase in α makes it more likely that agents who were not observed to be successful are in fact talented. Hence, the wage of agents who were not observed to be successful also increases with α. This effect weakens the signaling incentive of agents. Proposition 5 shows that the first positive effect dominates when α is relatively small and the second negative effect dominates when α is relatively large.

An interesting implication of Propositions 5 is that agents may have a stronger incentive to exert effort if the OSS project attracts fewer talented agents. Therefore, if an OSS project wants to provide participants with a strong signaling incentive, then it may be better off not attracting too many high talented participants.[14] Moreover, Proposition 5 suggests the following interesting dynamics: suppose that an OSS project starts with a relatively talented pool of programmers. Over time, some will succeed and will be hired away by commercial software companies. Since talented programmers are more likely to produce positive signals and be hired away, the remaining pool of programmers will have on average a lower fraction of talented programmers. Proposition 5 implies that the faster attrition rate of talented programmers will first induce the remaining talented programmers to exert more effort and this will increase their probability of success and therefore accelerate their rate of attrition. Once the fraction of talented programmers drops below a critical level, the process will be reversed since the faster attrition of talented agents will now induce the remaining talented programmers to induce less effort and hence will lower their probability of producing positive signals and hence their rate of attrition.

5 Conclusion

The main finding in this paper is that the signaling incentive of programmers to contribute to OSS projects is more complex than it might seem at first glance. First, there always exists a no-effort stable equilibrium, and moreover this equilibrium may be unique if, for example, the marginal impact of effort on the probability of success is relatively small. This implies in turn that OSS projects may never take off. Second, when interior equilibria exist, there are generically an even number of them. This multiplicity of equilibria suggests that a given OSS project may induce a small level of effort or even no effort at all even though a seemingly identical project induces a

[14] Of course, if programmers enjoy interacting with talented programmers and if there are complimentarities among programmers (talented programmers create positive externalities), then the more talented the pool of programmers is, the more productive other participants are going to be. These considerations however are outside my model as I focus on the signaling incentive of programmers.

large level of effort. Third, the comparative static properties of interior equilibria may in general go either way. In particular, shifts in exogenous parameters, like an increase in the visibility of performance and an increase in the marginal productivity of effort, may either boost or weaken the signaling incentive of talented agents depending on whether the equilibrium is stable or unstable and depending on the properties of the probability that successful performance will be observed by prospective employers. Therefore, a-priori it is in general impossible to tell whether increased visibility of performance and increased sensitivity of performance to effort will induce talented agents to exert more or less effort.

6 Appendix

Following are the proofs of Lemma 2 and Proposition 2.

Proof of Lemma 2: (i)-(ii) Since $Q(e,\beta)$ is strictly concave in e, it follows from equation (4) that $\frac{\partial U(e,\widehat{e})}{\partial e}$ is a strictly decreasing function of e for all $\widehat{e}>0$. Moreover, since $Q(e,\beta)$ is concave in e and bounded from above by 1, $\lim_{e\to\infty} Q_e(e,\beta)=0$, so $\lim_{e\to\infty}\frac{\partial U(e,\widehat{e})}{\partial e}=-1$ for all $\widehat{e}>0$. Since $\frac{\partial U(e,\widehat{e})}{\partial e}$ is continuous in e, this implies that there exists a unique value of e at which $\frac{\partial U(e,\widehat{e})}{\partial e}=0$ if and only if

$$\frac{\partial U(0,\widehat{e})}{\partial e}=Q_e(0,\beta)\,\Delta(\widehat{e},\beta)w-1>0. \tag{12}$$

Since Lemma 1 implies that $\Delta(0,\beta)=0$, condition (12) clearly fails when $\widehat{e}=0$, and by continuity, it also fails for sufficiently small values of $\widehat{e}$. Hence, $BR(0)=0$ for small values of $\widehat{e}$. On the other hand, since $\Delta_e(\widehat{e},\beta)>0$, it follows that $\frac{\partial U(0,\widehat{e})}{\partial e}$ is increasing with $\widehat{e}$. Moreover, noting that

$$\begin{aligned}\Delta(\widehat{e},\beta) &= q(\widehat{e}\mid s)-q(\widehat{e}\mid n)\\ &= \frac{\alpha Q(\widehat{e},\beta)}{\alpha Q(\widehat{e},\beta)+(1-\alpha)Q_0(\beta)}-\frac{\alpha\,(1-Q(\widehat{e},\beta))}{\alpha\,(1-Q(\widehat{e},\beta))+(1-\alpha)\,(1-Q_0(\beta))},\end{aligned} \tag{13}$$

it follows that

$$\begin{aligned}\lim_{\widehat{e}\to\infty}\frac{\partial U(0,\widehat{e})}{\partial e} &= Q_e(0,\beta)\left(\lim_{\widehat{e}\to\infty}\Delta(\widehat{e},\beta)\right)w-1\\ &= \frac{Q_e(0,\beta)\,w\alpha(1-\alpha)\,(Q_1(\beta)-Q_0(\beta))}{(\alpha Q_1(\beta)+(1-\alpha)Q_0(\beta))\,(1-\alpha Q_1(\beta)-(1-\alpha)\,Q_0(\beta))}-1>0,\end{aligned}$$

where the inequality follows by Assumption (*). Therefore, there exists a unique value of $\widehat{e}$, denoted $\widehat{e}_1$, such that $\frac{\partial U(0,\widehat{e})}{\partial e}>0$ for all $\widehat{e}>\widehat{e}_1$ and $\frac{\partial U(0,\widehat{e})}{\partial e}<0$ otherwise, where $\widehat{e}_1$ is implicitly defined by the equation

$$\frac{\partial U(0,\widehat{e})}{\partial e}=Q_e\left(0,\beta\right)\Delta(\widehat{e},\beta)w-1=0.$$

In sum, whenever $\widehat{e}\leq\widehat{e}_1$, $\frac{\partial U(e,\widehat{e})}{\partial e}<0$ for all e, implying that $BR(\widehat{e})=0$. On the other hand, whenever $\widehat{e}>\widehat{e}_1$, there exists a unique value of e that solves the equation $\frac{\partial U(e,\widehat{e})}{\partial e}=0$. Hence, $BR(\widehat{e})>0$ for all $\widehat{e}>\widehat{e}_1$.

(iii) As parts (i)-(ii) show, $BR(\widehat{e})$ is defined implicitly by the equation $\frac{\partial U(BR(\widehat{e}),\widehat{e})}{\partial e}=0$ for all $\widehat{e}>\widehat{e}_1$. Fully differentiating this equation with respect to $\widehat{e}$ and using Lemma 1 and the fact that $Q(e,\beta)$ is increasing and strictly concave in e, reveals that $BR'(\widehat{e})$, defined by equation (9), is positive.

Using equations (9) and (13),

$$\begin{aligned}\lim_{\widehat{e}\to\infty}BR'(\widehat{e}) &= -\frac{Q_e(e,\beta)}{Q_{ee}(e,\beta)}\lim_{\widehat{e}\to\infty}\frac{\Delta_e(\widehat{e},\beta)}{\Delta(\widehat{e},\beta)}\\ &= -\frac{Q_e(e,\beta)}{Q_{ee}(e,\beta)}\lim_{\widehat{e}\to\infty}\left[\frac{\frac{\alpha(1-\alpha)Q_e(\widehat{e},\beta)Q_0(\beta)}{(\alpha Q(\widehat{e},\beta)+(1-\alpha)Q_0(\beta))^2}+\frac{\alpha(1-\alpha)Q_e(\widehat{e},\beta)(1-Q_0(\beta))}{(\alpha(1-Q(\widehat{e},\beta))+(1-\alpha)(1-Q_0(\beta)))^2}}{\Delta(\widehat{e},\beta)}\right].\end{aligned}$$

The strict concavity of $Q(e,\beta)$ and the assumption that $\lim_{e\to\infty}Q_e(e,\beta)=Q\left(1,\beta\right)\leq 1$ imply that $\lim_{\widehat{e}\to\infty}Q_e(\widehat{e},\beta)=0$. Hence, $\lim_{\widehat{e}\to\infty}BR'(\widehat{e})=0$. ■

Proof of Proposition 2: To prove the result, I will establish sufficient conditions for $Q_{e\beta}(\cdot,\beta)$ and $\Delta_\beta(\widehat{e},\beta)$ to have the same sign. To this end, note that

$$\Delta(\widehat{e},\beta)\equiv q(\widehat{e}\mid s)-q(\widehat{e}\mid n)=\frac{\alpha}{\alpha+(1-\alpha)\frac{Q_0(\beta)}{Q(\widehat{e},\beta)}}-\frac{\alpha}{\alpha+(1-\alpha)\frac{1-Q_0(\beta)}{1-Q(\widehat{e},\beta)}}.$$

It is easy to see that $\frac{\partial}{\partial\beta}\left(\frac{Q_0(\beta)}{Q(\widehat{e},\beta)}\right)\leq 0\leq\frac{\partial}{\partial\beta}\left(\frac{1-Q_0(\beta)}{1-Q(\widehat{e},\beta)}\right)$ implies that $\Delta_\beta\left(\widehat{e},\beta\right)\geq 0$ and $\frac{\partial}{\partial\beta}\left(\frac{Q_0(\beta)}{Q(\widehat{e},\beta)}\right)\geq 0\geq\frac{\partial}{\partial\beta}\left(\frac{1-Q_0(\beta)}{1-Q(\widehat{e},\beta)}\right)$ implies that $\Delta_\beta\left(\widehat{e},\beta\right)\leq 0$.

Now, consider the case where $Q_{e\beta}(\cdot,\beta)>0$. Then $\frac{\partial}{\partial\beta}\left(\frac{1-Q_0(\beta)}{1-Q(\cdot,\beta)}\right)=\frac{-Q_0'(\beta)(1-Q(\cdot,\beta))+Q_e(\cdot,\beta)(1-Q_0(\beta))}{(1-Q_0(\beta))^2}>\frac{-Q_0'(\beta)+Q_e(\cdot,\beta)}{1-Q_0(\beta)}>0$, where the first inequality follows since $Q(e,\beta)$ is increasing with e, so $Q(\widehat{e},\beta)>Q_0(\beta)$. Hence, $\frac{\partial}{\partial\beta}\left(\frac{Q(\cdot,\beta)}{Q_0(\beta)}\right)\geq 0$ (which implies that $\frac{\partial}{\partial\beta}\left(\frac{Q_0(\beta)}{Q(\cdot,\beta)}\right)\leq 0$) is sufficient for $\Delta_\beta(\widehat{e},\beta)\geq 0$.

Next, consider the case where $Q_{e\beta}(\cdot,\beta)<0$. Then $\frac{\partial}{\partial\beta}\left(\frac{Q_0(\beta)}{Q(\cdot,\beta)}\right)=\frac{Q_0'(\beta)Q(\cdot,\beta)-Q_e(\cdot,\beta)Q_0(\beta)}{(Q(\cdot,\beta))^2}>\frac{Q_0'(\beta)-Q_e(\cdot,\beta)}{Q(\cdot,\beta)}>0$, where the first inequality follows since $Q(\widehat{e},\beta)>Q_0(\beta)$. Hence, $\frac{\partial}{\partial\beta}\left(\frac{1-Q(\cdot,\beta)}{1-Q_0(\beta)}\right)\geq 0$ (which implies that $\frac{\partial}{\partial\beta}\left(\frac{1-Q_0(\beta)}{1-Q(\cdot,\beta)}\right)\leq 0$) is sufficient for $\Delta_\beta(\widehat{e},\beta)\leq 0$. ■

7 References

Athey S. and G. Ellison (2006), "Dynamics of Open Source Movements," Mimeo.

Bessen J. "Open Source Software: Free Provision of Complex Public Goods," in Bitzer J. and P.Schröder , eds., *The Economics of Open Source Software Development*, Elsevier B. V.

Bitzer J. and P. Schröder (2005), "Bug-Fixing and Code-Writing: The Private Provision of Open Source Software," *Information Economics and Policy*, **17(3)**, 389-406.

Bitzer J., W. Schrettl, and P. Schröder (2007), "Intrinsic Motivation in Open Source Software Development," *Journal of Comparative Economics*, **35**, 160–169.

Blatter M. and A. Niedermayer (2008), "Informational Hold-Up, Disclosure Policy, and Career Concerns on the Example of Open Source Software Development," NET Institute Working Paper #08-06.

Fershtman C. and N. Gandal (2007), "Open Source Software: Motivation and Restrictive Licensing," *Journal of International Economics and Economic Policy*, **4**, 209-225.

Hars A. and S. Ou (2002), "Working for Free? Motivations for Participating in Open-Source Projects," *International Journal of Electronic Commerce*, **6(3)**, 25-39.

Holmström, B. (1999) "Managerial Incentive Problems: A Dynamic Perspective," *Review of Economic Studies*, **66**, 169-182.

Hann I.-H., J. Roberts, S. Slaughter, and R. Fielding (2004), "An Empirical Analysis of Economic Returns to Open Source Participation," Mimeo, Carnegie-Mellon University.

Johnson J. (2002), "Open Source Software: Private Provision of a Public Good," *Journal of Economics and Management Strategy*, **11(4)**, 121-139.

Lakhani K. and R. Wolf (2002), "Why Hackers Do What They Do: Understanding Motivation and Effort in Free/Open Source Software Projects," MIT Solan Working Paper No. 4425-03.

Lee S., N. Moisa, and M. Weiss (2003), "Open Source as a Signaling Device - An Economic Analysis," WP Series Finance & Accounting, Johann Wolfgang Goethe-University, Frankfurt/Main.

Leppämäki M. and M. Mustonen (2004), "Signaling and Screening with Open Source Programming," Helsinki School of Economics, Working Papers W-377.

Lerner J. and J. Tirole (2002), "Some Simple Economics of Open Source," *The Journal of Industrial Economics*, **50(2)**, 197-234.