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The antitrust prohibition of excessive pricing☆

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A B S T R A C T

Excessive pricing by a dominant firm is unlawful in many countries. To assess whether it is excessive, the dominant firm's price is often compared with price benchmarks. We examine the competitive implications of two such benchmarks: a retrospective benchmark where the price that prevails after a rival enters the market is used to assess whether the dominant firm's pre-entry price was excessive, and a contemporaneous benchmark, where the dominant firm's price is compared with the price that the firm charges contemporaneously in another market. We show that the two benchmarks restrain the dominant firm's behavior when it acts as a monopoly, but soften competition when the dominant firm competes with a rival. Moreover, a retrospective benchmark promotes entry, but may lead to inefficient entry.

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1. Introduction

Excessive pricing by a dominant firm is considered an unlawful abuse of dominant position in many countries. For example, Article 102 of the Treaty of the Functioning of the European Union prohibits a dominant firm from "directly or indirectly imposing unfair purchase or selling prices." Courts have interpreted this prohibition as including a prohibition of "excessive pricing."[1] A similar prohibition exists in many other countries, including all OECD countries, except the U.S., Canada, Australia, New Zealand, and Mexico.[2]

The antitrust prohibition of excessive pricing is highly controversial.[3] Opponents claim that the prohibition may have a chilling effect on the incentive of firms to invest and that it creates considerable legal uncertainty due to the difficulty in determining whether prices are excessive. They also claim that there is no need for antitrust intervention because excessive pricing may attract competition and hence the problem is self-correcting, and that in any event, the task of preventing dominant firms from charging high prices should be left to regulators who have the expertise and resources to set prices. They point out that ex ante price regulation avoids the legal uncertainty associated with antitrust enforcement, which is backward looking and condemns excessive prices that were set in the past.[4] By contrast, proponents of the policy argue that many antitrust rules create legal uncertainty, that excessive pricing is not a self-correcting problem, and that since price regulation is itself inefficient, the antitrust prohibition of excessive pricing should complement price regulation rather than substitute it.[5]

Part of the controversy surrounding the antitrust prohibition of excessive pricing stems from the fact that we still know very little about its competitive effects both in terms of theory, as well in terms of empirical research. In fact, as far as we know, existing literature on the topic is all informal. The purpose of this paper is to contribute to the discussion by studying the competitive effects of the prohibition of excessive pricing in the context of a formal model. Although the model abstracts from many important real-life considerations, it does highlight two new effects, that to the best of our knowledge, were not mentioned earlier and may have important implications.

[1] Traditionally, courts have interpreted unfair prices as being low predatory prices. However, in the landmark General Motors case in 1975, the European Court of Justice held that a dominant firm's price is unfair if it is "excessive in relation to the economic value of the service provided." See Case 26/75, General Motors Continental v. Commission [1975] ECR 1367, at para. 12. The court did not clarify, however, what the "economic value of the service provided" is, or indeed, how to measure it. The court reiterated this position in the United Brands case in 1978 and held that "charging a price which is excessive because it has no reasonable relation to the economic value of the product supplied would be... an abuse." The court further held that "It is advisable therefore to ascertain whether the dominant undertaking has made use of the opportunities arising out of its dominant position in such a way as to reap trading benefits which it would not have reaped if there had been normal and sufficiently effective competition." Case 27/76, United Brands v. Commission [1978] ECR 207, at para. 249–250.

[2] See http://www.oecd.org/competition/abuse/49604207.pdf.

[3] For a recent overview of the debate, see Jenny (2016).

[4] See e.g., Evans and Padilla (2005), Motta and de Streel (2006, 2007), O'Donoghue and Padilla (2006), OECD (2011), and Gal and Nevo (2015), Gal and Nevo (2016).

[5] See e.g., Ezrachi and Gilo (2009) and Gilo (2018).

A main obstacle to effective implementation of the prohibition of excessive pricing is the lack of a commonly agreed upon definition of what constitutes an "excessive price" or how to measure it. In practice, antitrust authorities and plaintiffs in excessive pricing cases often base their claims on a comparison of the dominant firm's price with some competitive benchmark, such as the firm's cost of production, or the firm's own price in other time periods, in different geographical markets, or in different market segments.[6]

In this paper, we consider two such price benchmarks. The first is a retrospective benchmark where the price that prevails after a rival enters the market is used to assess whether the dominant firm's pre-entry price was excessive. Retrospective benchmarks were used for example in class actions in Israel.[7] In an early class action filed in 1998, the plaintiff alleged that the incumbent acquirer of Visa cards charged excessive merchant fees based on the fact that the fees dropped significantly following the entry of a new credit card company.[8] In another case filed in 1997, the plaintiff alleged that prices for international phone calls charged by the former telecom monopoly Bezeq were excessive based on the fact that Bezeq lowered its prices by approximately 80% after the market was liberalized and two new rivals entered.[9,10]

The second price benchmark we study is a contemporaneous benchmark. Here, the dominant firm's price is compared with the price the firm charges contemporaneously in another market. This type of benchmark was used for example by the European Court to determine that British Leyland charged an excessive price for certificates that left-hand drive vehicles conform to an approved type by comparing it to the price for certificates for right-hand drive vehicles.[11] Similarly, the British OFT has determined that the price that Napp charged to community pharmacies in the UK for sustained release morphine was excessive by comparing it to the price charged to hospitals.[12]

[6] Motta and de Streel (2006) document various benchmarks used by the European Commission, including substantial differences between the dominant firm's prices across different geographic markets, or relative to the prices of smaller rivals. The OFT (2004) suggests similar benchmarks, including prices in other time periods, the prices of the same products in different markets, or the underlying costs when it is possible to measure them in an economically meaningful way.

[7] See Spiegel (2018) for an overview of these cases and for a broad overview of the antitrust treatment of excessive pricing in Israel.

[8] The class action was initially certified by the Tel Aviv District court, but was ultimately dismissed by the Israeli Supreme Court on appeal, mainly because the entrant went out of business shortly after entering the market, implying that the post-entry merchant fees were not a valid benchmark. See D.C.M. (T.A) 106462/98 Howard Rice v. Cartisei Ashrai Leisrael Ltd., P.M 2003(1) and Permission for Civil Appeal 2616/03 Isracard Ltd. v. Howard Rice, P.D. 59 (5) 701 [14.3.2005].

[9] Similarly to the Visa cards case, this class action was also dismissed on appeal by the Israeli Supreme Court after being initially certified by the Tel Aviv District Court. The ground for dismissal was that before liberalization, prices were set by regulators, meaning that Bezeq did not abuse its dominant position. See, D.C.A (T.A) 2298/01 Kav Machshava v. Bezeq Beinleumi Ltd. (Nevo, 25.12.2003) and See Permission for Civil Appeal 729/04 State of Israel et al., v. Kav Machshava et al., (Nevo, 26.4.2010).

[10] In another class action in Israel which is currently pending in court, the plaintiff alleges that Osem (one of the largest food suppliers in Israel and a subsidiary of Nestlé) charged an excessive price for Israeli couscous (toasted pasta shaped like rice grains or little balls), based on the fact that following the entry of Sugat (the leading supplier of sugar in Israel) into the market in August 2013, Osem, which is the dominant firm in the market, lowered its price from 6.30 NIS to 4.99 in July 2015.

[11] See British Leyland Public Ltd. Co. v. Commission [1986].

[12] See "Napp Pharmaceutical Holdings Limited And Subsidiaries (Napp)," Decision of the Director General of Fair Trading, No Ca98/2/2001, 30 March 2001.

A third type of price benchmark which is often used (but one that we do not consider in the current paper) is based on price hikes, typically after price controls are lifted. In a recent example, the British CMA imposed in December 2016 a £84.2 million fine on Pfizer and a £5.2 million fine on its distributor, Flynn Pharma, for charging an excessive price for phenytoin sodium capsules, which are used to treat epilepsy. The claim was based on a price hike of 2,300% – 2,600% following the de-branding of the drug, which meant that it was no longer subject to price regulation.[13] Similarly, the Italian Market Competition Authority fined Aspen over € 5 million in September 2016 for charging excessive prices for four anti-cancer drugs; Aspen raised their prices by 300% to 1500% after acquiring the rights to commercialize them from GlaxoSmithKline.[14]

To study the competitive effects of retrospective benchmarks, we consider a two-period model, in which firm 1 is a monopoly in period 1, but may face competition in period 2 from an entrant, firm 2. Under a retrospective benchmark, firm 1 anticipates that if entry occurs in period 2 and its post-entry price drops, its pre-entry price may be deemed excessive. Then firm 1 pays a fine proportional to its excess revenue in period 1, which depends in turn on the difference between its post-entry and its pre-entry prices. We then modify the model to consider a contemporaneous benchmark: instead of two periods, there are now two markets. Firm 1 is a monopoly in market 1, but competes with firm 2 in market 2. Firm 1's price in market 1 may be deemed excessive if it exceeds firm 1's price in market 2.

Our analysis has a number of interesting implications. First, the prohibition of excessive pricing lowers firm 1's monopoly price (the pre-entry price under a retrospective benchmark and the price in market 1 under a contemporaneous benchmark), but raises its benchmark price when it competes with firm 2. Firm 1 lowers the gap between the two prices in order to lower its excess revenue when it acts as a monopoly and hence the resulting expected fine when its price is deemed excessive. Firm 1's softer behavior under competition induces firm 2 to be more aggressive if the two firms compete by setting quantities, but softer if they compete by setting prices. Either way, the benchmark price is higher, implying that from the point of view of consumers, using retrospective or contemporaneous benchmarks to enforce a prohibition of excessive pricing involves a trade off between lower prices when firm 1 is a monopoly and higher prices when firms 1 and 2 compete.

[13] See https://www.gov.uk/government/news/cma-fines-pfizer-and-flynn-90-million-for-drug-price-hike-to-nhs. After patents expired in September 2012, Pfizer sold the rights for distributing the drug in the UK to Flynn Pharma, which in turn de-branded it (to avoid price regulation), and raised its price to the British National Health Services from £2.83 to £67.50, before reducing it to £54 in May 2014.

[14] See https://www.natlawreview.com/article/italy-s-agcm-market-competition-authority-fines-aspen-eur-5-million-excessive The European Commission recently announced that it is opening an investigation against Aspen for excessive pricing of the drugs outside of Italy. See http://europa.eu/rapid/press-release_IP-17-1323_en.htm. Another recent example is a class action in Israel alleging that Dead Sea Works Ltd, which is a monopoly in the supply of potash, charged farmers an excessive price for potash. The Central District Court approved a settlement partly on the grounds that Dead Sea Works raised its price from $200 per ton in 2007 to $1, 000 per ton in 2008-9, after it allegedly joined an international potash cartel. See Class Action (Central District Court) 41838-09-14 Weinstein v. Dead Sea Works, Inc. (Nevo, 29.1.2017).

Second, in a quantity-setting model with homogenous products and a linear demand function, the decrease in the pre-entry price due to a retrospective benchmark benefits consumers more than the increase in the post-entry price hurts them. By contrast, in a price-setting model with symmetric costs, a retrospective benchmark to assess whether the pre-entry price was excessive allows firm 2, if it enters, to monopolize the market at a price that exceeds the price that would have prevailed without a prohibition of excessive pricing. Firm 1 can therefore safely charge the monopoly price before entry occurs: absent entry it continues to charge the monopoly price, so its past price cannot be deemed excessive; if entry occurs, firm 1 makes no sales, so there is no benchmark to determine that its pre-entry price was excessive. Either way, a retrospective benchmark harms consumers when entry occurs, without benefitting them prior to entry. Things are different when firm 2's cost is randomly drawn from some distribution. Then, firm 1 can undercut firm 2's cost when firm 2's cost is relatively high and still maintain its monopoly position. Since by assumption, firm 2's cost is below the monopoly price (otherwise entry is blockaded), now firm 1 has an incentive to lower its pre-entry price below the monopoly price to reduce its pre-entry excess revenue and therefore the fine it may have to pay. As a result, even if it is short lived, a retrospective benchmark benefits consumers prior to entry.

Third, under a retrospective benchmark, firm 1 has a stronger incentive to ensure that its pre-entry price is low when the discounted probability of entry high. Consequently, consumers benefit more when the probability of entry is high. This result is interesting because it is often argued that there is no need to intervene in excessive pricing cases when the probability of entry is high, since then the "market will correct itself."[15] This argument, however, merely points out that the harm to consumers prior to entry is going to be short lived and ignores the fact that a retrospective benchmark can reduce this harm, especially when the probability of entry is high.

Fourth, a retrospective benchmark makes firm 1 softer once entry occurs and hence promotes entry. This result stands in sharp contrast to the often made claims that the prohibition of excessive pricing discourages entry by inducing the incumbent to lower its price.[16] Although in our model the incumbent charges a lower price prior to entry, what matters for the entrant is not the pre-entry behavior of the incumbent but rather its post-entry behavior. And, as we show, a retrospective benchmark induces the incumbent

[15] For instance, the OECD competition committee (OECD, 2011) emphasizes that "The existence of high and non-transitory structural entry barriers are probably considered the most important single requirement for conducting an excessive price case." It also adds that "This requirement is based on the fundamental proposition that competition authorities should not intervene in markets where it is likely that normal competitive forces over time eliminate the possibilities of a dominant company to charge high prices." Likewise, O'Donoghue and Padilla (2006) write that "The key consideration is to limit intervention to cases in which entry barriers are very high and, therefore, where there is a reasonable prospect that consumers could be exploited" (pp. 635–636). Similarly, Motta and de Streel (2006) write that "exploitative practices are self-correcting because excessive prices will attract new entrants" (p. 15).

[16] For instance, Areeda and Hovenkamp (2001) write: "While permitting the monopolist to charge its profit-maximizing price encourages new competition, forcing it to price at a judicially administered 'competitive' level would discourage entry and thus prolong the period of such pricing" (para. 720b). Similar arguments appear in Whish (2003, p. 688–689), and in Economic Advisory Group on Competition Policy (2005, p. 11).

to be softer following entry in order to ensure that its post-entry price does not fall by too much below its pre-entry price.

As far as we know, our paper is the first to examine the competitive implications of the prohibition of excessive prices in the context of a formal economic model. Existing literature on excessive prices is all based on informal legal policy discussion. Evans and Padilla (2005), Motta and de Streel (2006 and 2007), O'Donoghue and Padilla (2006), and Green (2006) critically examine the case law and policy issues, consider different possible benchmarks that can be used to assess if prices are excessive, and discuss their potential drawbacks. Gal (2004) compares the EU and U.S. antitrust laws that apply to the prohibition of excessive pricing and explains the difference between the two systems. Ezrachi and Gilo (2010a,b) critically discuss the main grounds for the reluctance of some antitrust agencies and courts to intervene in excessive pricing cases, while Ezrachi and Gilo (2009) discuss the retrospective benchmark that we also consider in this paper, but do not use a formal model.

Our analysis is related to the literature on most-favoured-customer (MFN) clauses, which guarantee past consumers a rebate if the price falls. Cooper (1986), Neilson and Winter (1993), and Schnitzer (1994) show that competing firms have an incentive to adopt retroactive MFN's in order to facilitate collusion (MFN's make firms reluctant to cut future prices in order to avoid paying rebates to past consumers). Although the fine that the dominant firm may have to pay when the post-entry price falls is akin to a rebate to past consumers, the dominant firm is better off without it, since then it is free to exploit its monopoly power prior to entry, and can respond optimally to entry if it occurs. Moreover, while MFN's facilitate collusion, the prohibition of excessive pricing may be pro-competitive.

A retrospective benchmark for assessing whether the incumbent's pre-entry price is excessive is reminiscent of the legal rules proposed by Williamson (1977), Baumol (1979), and Edlin (2002), to deter predatory pricing. These rules are also based on the response of a dominant firm to entry.[17] Unlike these papers, we do not propose a new legal rule, but rather examine the competitive implications of existing price benchmarks that are used in practice and are likely to become even more popular, especially in private antitrust enforcement.

The rest of the paper is organized as follows: Section 2 studies the competitive implications of a retrospective benchmark under quantity competition, and Section 3 studies its competitive implications under price competition. Section 4 studies the competitive implications of a contemporaneous benchmark under quantity and price competition. We conclude in Section 5. The Appendix contains technical proofs.

[17] Williamson (1977) proposes that following entry, the dominant firm will not be able to raise output above the pre-entry level for $12-18$ months. Edlin (2002) proposes to block a dominant firm from significantly cutting its price for a period of $12-18$ months following entry. Both rules prevent predation. Baumol (1979) proposes that the incumbent will not be allowed to raise its price if and when the entrant exits the market, unless this is justified by cost or demand changes. This rule prevents recoupement. Edlin et al. (2016) provide experimental evidence that both the Edlin and Baumol rules significantly improve consumer welfare when subjects are experienced.

2. A retrospective benchmark under quantity competition

We begin by studying the competitive effects of retrospective benchmarks for assessing whether the price of a dominant firm is excessive under the assumption that firms produce homogenous products and compete by setting quantities. The assumption that products are homogenous is a reasonable approximation for the two Israeli class actions mentioned in the Introduction, as well as several other class actions that are currently pending in court.[18]

There are two time periods. In period 1, firm 1 operates as a monopoly. In period 2, firm 1 continues to operate as a monopoly with probability $1-\alpha$, but faces competition from firm 2 with probability α. For simplicity, we assume that both firms have the same constant marginal cost c and denote the (downward sloping) inverse demand function by $p(Q)$, where Q is the aggregate output level. To ensure that the market is viable, we assume that $p(0)>c$. The intertemporal discount factor is δ.

The prohibition of excessive pricing is enforced in period 2 as follows: if entry occurs and the period 2 price, p_2, falls below the period 1 price, p_1, a court rules that p_1 was excessive with probability γ. The parameter γ reflects various legal factors, including the stringency of antitrust enforcement against excessive pricing, the availability of data on prices and quantities needed to support the case, and potential defenses that the dominant firm may have for its high prices, such as the need to recoup large investments. When p_1 is deemed excessive, firm 1 has to pay a fine in proportion to its excess revenue in period 1; the fine is given by $\tau(p_1-p_2)Q_1$, where $\tau>0$, Q_1 is firm 1's output in period 1, and p_1-p_2 is the per-unit excess revenue. To ensure interior solutions, we will make the following assumptions:

A1 $p'(Q)+p''(Q)(1+\gamma\tau)Q<0$
A2 $\gamma\tau<1$

Assumption A1 is a modified version of the standard assumption that $p'(Q)+p''(Q)Q<0$. It is stronger because $\gamma\tau>0$, but like the standard assumption, it also holds when the demand function is concave or not too convex. The assumption ensures that the marginal revenue functions are downward sloping. Assumption A2 ensures that the expected fine that firm 1 pays is not so large that firm 1 wishes to exit in period 2 when firm 2 enters.[19]

In the next two sections we characterize the equilibrium in our model. We begin in Section 2.1 by considering the equilibrium in period 2, and then we turn to period 1 in Section 2.2.

[18] Currently, there are 22 pending class action law suits in Israel alleging excessive pricing (see Spiegel, 2018 for details). Among the products involved in these cases are natural gas, white cheese, yellow cheese, heavy cream, cocoa powder, margarine, and green tea. Arguably, these products are fairly homogenuous.

[19] For example, in the Israeli cases mentioned in the Introduction, τ was equal to 1 as plaintiffs were suing for the actual damages. Since $\gamma<1$, $\gamma\tau$ was indeed below 1.

2.1. The equilibrium in period 2

Absent entry in period 2, there is no competitive benchmark which the court can use to evaluate whether p_1 was excessive. Hence, firm 1 maximizes its period 2 profit by producing the monopoly output, Q^M, defined implicitly by $MR(Q) \equiv p(Q) + p'(Q)Q = c$ ("M" stands for "Monopoly").[20]

Now suppose that firm 2 enters in period 2 and let q_1 and q_2 be the resulting output levels and $p_2 = p(q_1 + q_2)$ be the resulting price. Given firm 1's output in period 1, Q_1, the period 1 price, $p_1 = p(Q_1)$, can be deemed excessive if and only if $p_2 < p_1$, which is equivalent to $q_1 + q_2 > Q_1$. Recalling that p_1 is deemed excessive with probability γ and the fine that firm 1 pays in this event is $\tau(p_1 - p_2)Q_1$, the period 2 profits of firms 1 and 2 are given by

$$\pi_1(q_1, q_2) = \begin{cases} (p(q_1 + q_2) - c)q_1, & q_1 + q_2 \le Q_1, \\ (p(q_1 + q_2) - c)q_1 - \gamma\tau[p(Q_1) - p(q_1 + q_2)]Q_1, & q_1 + q_2 > Q_1, \end{cases}$$

and

$$\pi_2(q_1, q_2) = (p(q_1 + q_2) - c)q_2.$$

The next result characterizes the best-response functions of the two firms in period 2.

Lemma 1 (The best-response functions under entry)**.** *Suppose that firm 2 enters in period 2. Then, firm 2's best-response function is given by* $BR_2(q_1) = r_2^C(q_1)$. *Given Assumption A1, firm 1's best-response function is given by*

$$BR_1(q_2) = \begin{cases} r_1^C(q_2), & p(Q_1) + p'(Q_1)(Q_1 - q_2) < c, \\ Q_1 - q_2, & p(Q_1) + p'(Q_1)(Q_1 - q_2) > c > p(Q_1) + p'(Q_1) \\ & \quad ((1 + \gamma\tau)Q_1 - q_2), \\ r_1^E(q_2), & p(Q_1) + p'(Q_1)((1 + \gamma\tau)Q_1 - q_2) > c, \end{cases}$$

where $r_i^C(q_j)$ *is the "Cournot" best-response function ("C" stands for "Cournot"), defined implicitly by*

$$p(q_i + q_j) + p'(q_i + q_j)q_i = c, \tag{1}$$

and $r_1^E(q_2)$ *is firm 1's best-response function against* q_2 *when* p_1 *is excessive ("E" stands for "Excessive"), defined implicitly by*

$$p(q_1 + q_2) + p'(q_1 + q_2)(q_1 + \gamma\tau Q_1) = c. \tag{2}$$

[20] Note that if $Q_1 > Q^M$, the price in period 2, p_2, exceeds the price in period 1, p_1, and in principle may be deemed excessive. However, without a competitive benchmark in either period, it is hard to make the case that p_2 is excessive. Although in the Introduction we mentioned a few cases where prices were deemed excessive due to price hikes, these hikes followed the removal of price controls (Pfizer), a sale of the distribution rights to a new distributor (Pfizer and Aspen), or alleged cartelization (Dead Sea Works).

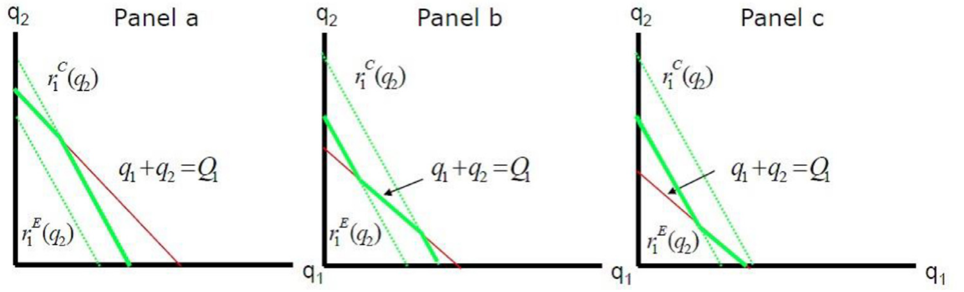

Fig. 1. The best-response function of firm 1 in period 2.

Assumption A1 ensures that both best-response functions are downward sloping in the (q_1, q_2) *space and* $BR_1'(\cdot) \le -1 \le BR_2'(\cdot) < 0$, *with* $BR_1'(\cdot) = -1$ *only when* $q_1 + q_2 = Q_1$.

Proof. See the Appendix. □

The Cournot best-response function of firm 1, $r_1^C(q_2)$, as well as its best-response function when p_1 is excessive, $r_1^E(q_2)$, are presented in Fig. 1.[21] The latter lies everywhere below $r_1^C(q_2)$, because firm 1 incurs an extra marginal cost when p_1 is excessive. This cost is due to the fact that an increase in q_1 lowers $p_2 = p(q_1, q_2)$ and therefore increases the excessive revenue, $(p_1 - p_2)Q_1$, on which firm 1 pays a fine if p_1 is deemed excessive. As Fig. 1 shows, there are three different cases, depending on how large Q_1 is. When $q_1 + q_2 = Q_1$ lies above $r_1^C(q_2)$, the aggregate output in period 2, $r_1^C(q_2) + q_2$, falls short of the output in period 1, Q_1, so $p_2 > p_1$, meaning that p_1 is not excessive. Hence, the best-response of firm 1 is indeed given by $r_1^C(q_2)$. By contrast, when $q_1 + q_2 = Q_1$ lies below $r_1^E(q_2)$, the aggregate output in period 2, $r_1^E(q_2) + q_2$, exceeds Q_1, so now p_1 is excessive and the best-response function of firm 1 is given by $r_1^E(q_2)$. And, when $q_1 + q_2 = Q_1$ lies below $r_1^C(q_2)$ but above $r_1^E(q_2)$, firm 1 sets q_1 such that $q_1 + q_2 = Q_1$ to ensure that $p_1 = p_2$. Note that while p_1 is not deemed excessive in this case, firm 1 cannot play its Cournot best response against q_2 either, because if it did, p_1 would be deemed excessive. In other words, firm 1 is constrained in this case to keep q_1 below the $q_1 + q_2 = Q_1$ line to ensure that p_1 is not retrospectively deemed excessive. Overall then, the best-response function of firm 1 is given by the thick downward sloping line in Fig. 1.

Let (q_1^*, q_2^*) be the Nash equilibrium in period 2 following entry. Then,

Lemma 2. *(Both firms are active in the market when firm 2 enters)* $q_1^* > 0$ *and* $q_2^* > 0$.

Proof. See the Appendix. □

[21] The best-response functions in Figs. 1 and 2 are drawn as linear only for convenience; in general they need not be linear. It the following analysis, however, we do not rely on the linearity of the best-response functions.

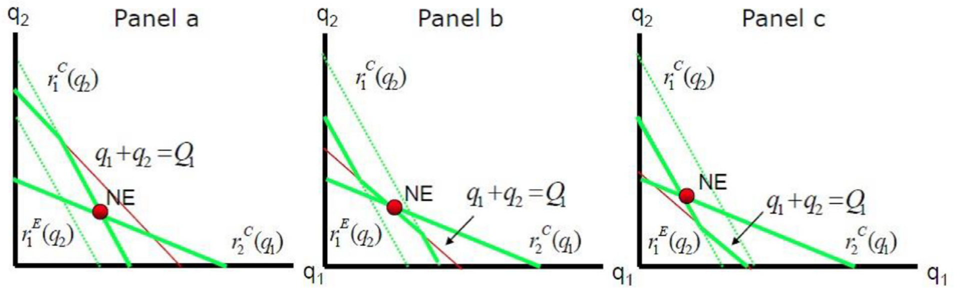

Fig. 2. The Nash equilibrium in period 2.

Given Lemma 2, three types of equilibria can emerge, depending on how high Q_1 is. We illustrate the three equilibria in Fig. 2.

The first type of equilibrium, illustrated in Fig. 2a, is the Cournot equilibrium, (q_1^C, q_2^C). It arises when $Q_1 > q_1^C + q_2^C \equiv Q^C$, so p_1 is not excessive. In Lemma 6 below, however, we prove that this situation cannot arise in equilibrium, since in period 1 firm 1 sets Q_1 such that $Q_1 \leq Q^C$.

The second type of equilibrium emerges when $Q_1 < Q^C$, but exceeds the aggregate output produced at the intersection of $r_1^E(q_2)$ and $r_2^C(q_1)$. As Fig. 2b illustrates, firm 1 sets q_1 such that $q_1 + q_2 = Q_1$, to ensure that $p_1 = p_2$. The equilibrium then, (q_1^*, q_2^*), is defined by the intersection of $q_1 + q_2 = Q_1$ with $r_2^C(q_1)$. Since $q_1 + q_2 = Q_1$ passes below the Cournot equilibrium point, (q_1^C, q_2^C), the equilibrium point (q_1^*, q_2^*) lies above the diagonal in the $(q_1,\ q_2)$ space, meaning that $q_2^* > q_1^*$.

The third equilibrium, illustrated in Fig. 2c, arises when Q_1 is even lower than the aggregate output produced when $r_1^E(q_2)$ and $r_2^C(q_1)$ intersect. Now firm 1 plays a best response against q_2, despite the fact that the resulting price renders p_1 excessive. The equilibrium is then defined by the intersection of $r_1^E(q_2)$ and $r_1^C(q_1)$. Since $r_1^E(q_2) < r_1^C(q_1)$, the equilibrium point again lies above the diagonal in the $(q_1,\ q_2)$ space, so once again, $q_2^* > q_1^*$.

Noting that in Fig. 2b and 2c, firm 1's best-response function lies below its Cournot best-response function, the Nash equilibrium in period 2 is attained in the $(q_1,\ q_2)$ space below a 45^0 line that passes through Q^c. Consequently, $q_1^* + q_2^* \leq Q^C$, with equality holding only when $\gamma\tau = 0$, in which case $r_1^E(q_2) = r_1^C(q_1)$.

We summarize the discussion in the next Lemma.

Lemma 3. *(The equilibrium in period 2 under entry) The equilibrium in period 2 when firm 2 enters, (q_1^*, q_2^*), is defined implicitly by the intersection of $r_1^E(q_2)$ and $r_2^C(q_1)$ if p_1 is excessive, and by the intersection of $q_1 + q_2 = Q_1$ and $r_2^C(q_1)$ if p_1 is not excessive. Either way, $q_1^* \leq q_1^C = q_2^C \leq q_2^*$ and $q_1^* + q_2^* \leq Q^C$, with equalities holding only when $\gamma\tau = 0$.*

Lemma 3 implies that when firm 2 enters in period 2, the period 1 output level, Q_1, matters: either p_1 is excessive and firm 1 pays in expectation a fine that depends on Q_1, or firm 1 chooses its output in period 2 such that $q_1 + q_2 = Q_1$ to ensure that p_1 is not excessive. Either way, in equilibrium, q_1 and q_2 depend on Q_1.

An important implication of Lemma 3 is that whenever $\gamma\tau > 0$, $\pi_2(q_1^*, q_2^*) > \pi_2\left(q_1^*, q_2^C\right) > \pi_2\left(q_1^C, q_2^C\right)$, where the first inequality follows by revealed preferences and the second follows because $q_1^* < q_1^C$. Hence, the prohibition of excessive pricing softens the behavior of firm 1 and thereby boosts the profit of firm 2, which encourages entry, contrary to what many scholars claim.

Let $\overline{Q}_1$ be the critical value of Q_1 such that p_1 is excessive if $Q_1 < \overline{Q}_1$ (as in Fig. 2c) and is not excessive if $Q_1 \geq \overline{Q}_1$ (as in Fig. 2b). Note that $\overline{Q}_1$ is attained when $q_1 + q_2 = Q_1$ passes through the intersection of $r_1^E(q_2)$ and $r_2^C(q_1)$. Hence, $\overline{Q}_1$ satisfies (1) when $q_i = q_2$, (2), and $q_1 + q_2 = \overline{Q}_1$. Substituting for q_2 from the last equation into (1) and (2), adding the two equations and simplifying, $\overline{Q}_1$ is implicitly defined by the equation

$$p(\overline{Q}_1) + \left(\frac{1+\gamma\tau}{2}\right)p'(\overline{Q}_1)\overline{Q}_1 = c. \tag{3}$$

Lemma 4. *(The properties of $\overline{Q}_1$) $\overline{Q}_1$ is such that $Q^M < \overline{Q}_1 < Q^C$ is decreasing with the size of the expected fine, $\gamma\tau$.*

Proof. See the Appendix. □

Lemma 4 provides a lower and upper bound on $\overline{Q}_1$, which is the critical value of Q_1 that delineates equilibria in which p_1 is excessive from equilibria in which p_1 is not excessive. Since p_1 is excessive only when $Q_1 < \overline{Q}_1$, the second part of Lemma 4 implies that as the expected fine, $\gamma\tau$, increases, firm 1 is more likely to set Q_1 such that p_1 will not end up being excessive. In the limit, as $\gamma\tau$ approaches 1, (3) coincides with the first-order condition for Q^M, implying that $\overline{Q}_1 = Q^M$. Since in Lemma 7 below we prove that $Q_1 > Q^M$, it follows that as $\gamma\tau$ approaches 1, $Q_1 > Q^M = \overline{Q}_1$, implying that p_1 is never deemed excessive in period 2.

We conclude this section by studying the effect of Q_1 on the equilibrium in period 2.

Lemma 5. *(The effect of Q_1 on the equilibrium in period 2) Suppose that $Q_1 < \overline{Q}_1$ (p_1 is excessive); then $-\gamma\tau < \frac{\partial q_1^*}{\partial Q_1} < 0 < \frac{\partial q_2^*}{\partial Q_1} < \gamma\tau$ and $-\gamma\tau < \frac{\partial(q_1^*+q_2^*)}{\partial Q_1} < 0$. If $Q_1 \geq \overline{Q}_1$ (p_1 is not excessive), then $\frac{\partial q_1^*}{\partial Q_1} > 1$, $\frac{\partial q_2^*}{\partial Q_1} < 0$, and $\frac{\partial(q_1^*+q_2^*)}{\partial Q_1} = 1$.*

Proof. See the Appendix. □

Lemma 5 shows that Q_1 has a non-monotonic effect on the equilibrium output levels in period 2: q_1^* is decreasing with Q_1 when $Q_1 < \overline{Q}_1$, and increasing with Q_1 when $Q_1 \geq \overline{Q}_1$, and conversely for q_2^*. Intuitively, whenever $Q_1 < \overline{Q}_1$, p_1 is excessive, so firm 1 has an incentive to cut q_1^* in order to keep p_2 high, and thereby lower the expected fine it pays. This incentive becomes stronger as Q_1 increases because the fine is proportional to Q_1. However, once $Q_1 \geq \overline{Q}_1$, firm 1 chooses q_1^* such that $q_1^* + q_2^* = Q_1$, so now an increase in Q_1 allows firm 1 to expand q_1^*. Since q_2^* and q_1^* are strategic substitutes, Q_1 has the opposite effect on q_2^*.

2.2. The equilibrium in period 1

Firm 1 chooses Q_1 in period 1 to maximize the discounted sum of its period 1 and period 2 profits:

$$\Pi_1(Q_1) = (p(Q_1) - c)Q_1 + \delta\left[(1-\alpha)\pi^M + \alpha\pi_1(q_1^*, q_2^*)\right],$$

where $(p(Q_1) - c)Q_1$ is firm 1's profit in period 1, $\pi^M \equiv \pi_1\left(q_1^M, 0\right)$ is firm 1's monopoly profit in period 2 absent entry, $\pi_1(q_1^*, q_2^*)$ is firm 1's profit in period 2 when entry occurs, and δ is the intertemporal discount factor.

Before proceeding, it is worth noting that in equilibrium it must be that $Q_1 \geq Q^M$, otherwise firm 1 can raise Q_1 slightly towards Q^M and make more money in period 1, while lowering p_1 and hence making it less likely to be deemed excessive in period 2. Moreover, it must be that $Q_1 \leq Q^C$, otherwise firm 1 can raise its profit in period 1 by lowering Q_1 slightly towards Q^C, without rendering p_1 excessive (recall that the aggregate output in period 2 can be at most equal to the aggregate Cournot level, Q^C). Hence,

Lemma 6. *(A bound on Q_1) The period 1 output of firm 1, Q_1, is between the monopoly output and the aggregate Cournot output: $Q^M \leq Q_1 \leq Q^C$.*

Since $Q_1 \leq Q^C$, the equilibrium is attained in the $(q_1,\, q_2)$ space either on the $q_1 + q_2 = Q_1$ line (as in Fig. 1b) or below it (as in Fig. 1c). Therefore the discounted expected profit of firm 1 can be rewritten as:

$$\Pi_1(Q_1) = \begin{cases} \begin{gathered}(p(Q_1) - c)Q_1 + \delta(1-\alpha)\pi^M \\ + \delta\alpha[(p(q_1^* + q_2^*) - c)q_1^* - \gamma\tau(p(Q_1) - p(q_1^* + q_2^*))Q_1],\end{gathered} & Q^M \leq Q_1 < \overline{Q}_1, \\ (p(Q_1) - c)Q_1 + \delta(1-\alpha)\pi^M + \delta\alpha(p(Q_1) - c)q_1^*, & Q_1 \geq \overline{Q}_1, \end{cases} \tag{4}$$

where (q_1^*, q_2^*) is defined by the intersection of $r_1^E(q_2)$ and $r_2^C(q_1)$ if $Q_1 < \overline{Q}_1$ and by the intersection of $q_1 + q_2 = \overline{Q}_1$ and $r_2^C(q_1)$ if $\overline{Q}_1 \leq Q_1 \leq Q^C$. Note that $\Pi_1(Q_1)$ is continuous at $Q_1 = \overline{Q}_1$ because $q_1^* + q_2^* = \overline{Q}_1$ and since q_1^* is equal in the first and second lines of (4) when $Q_1 = \overline{Q}_1$.

Let Q_1^* denote the optimal choice of Q_1. In order to characterize Q_1^*, we shall make the following assumption:

A3 $\Pi_1(Q_1)$ is piecewise concave (i.e., concave in each of its two relevant segments)

In the next subsection, we show that when the demand function is linear, Assumption A3 holds provided that the discounted probability of entry in period 2, $\delta\alpha$, is below 0.9. Indeed, it is easy to see that when $\alpha = 0$, $\Pi_1(Q_1)$ is concave by Assumption A1; by continuity this is also true so long as α is not too large. Given Assumption A3, we now characterize the equilibrium choice of Q_1.

Lemma 7. *(The choice of Q_1^*) $Q_1^* > Q^M$. Let $\frac{\partial q_1^*}{\partial Q_1^+}$ be the derivative of q_1^* with respect to Q_1 when $Q_1 \geq \overline{Q}_1$ (p_1 is not excessive) and $\frac{\partial q_2^*}{\partial Q_1^-}$ the derivative of q_2^* with respect to Q_1 when $Q_1 < \overline{Q}_1$ (p_1 is excessive). Then,*

(i) *$Q_1^* < \overline{Q}_1$, implying that p_1 ends up being excessive if $\frac{\partial q_1^*}{\partial Q_1^+} < \frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)}$ and $\frac{\partial q_2^*}{\partial Q_1^-} > \frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)}$. Both inequalities hold when $\delta\alpha$ is sufficiently small. Moreover, $\delta\alpha < \frac{1-\gamma\tau}{2\gamma\tau}$ is necessary for the first inequality and sufficient for the second.*

(ii) *$Q_1^* > \overline{Q}_1$, implying that p_1 does not end up being excessive if $\frac{\partial q_1^*}{\partial Q_1^+} \geq \frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)}$, and $\frac{\partial q_2^*}{\partial Q_1^-} > \frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)}$. $\delta\alpha > \frac{1-\gamma\tau}{2\gamma\tau}$ is sufficient for the first inequality and necessary for the second inequality.*

(iii) *Firm 1's problem has two local optima, one below and one above $\overline{Q}_1$ if $\frac{\partial q_1^*}{\partial Q_1^+} > \frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)}$, and $\frac{\partial q_2^*}{\partial Q_1^-} < \frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)}$, where $\delta\alpha > \frac{1-\gamma\tau}{2\gamma\tau}$ is sufficient for both inequalities.*

Proof. See the Appendix. □

To understand Lemma 7, note that when $Q_1 = Q^M$, firm 1 maximizes its profit in period 1, but if entry takes place in period 2, the aggregate output in period 2 exceeds Q^M. Since by Lemma 4 $Q^M < \overline{Q}_1$, where $\overline{Q}_1$ is the critical value of Q_1 below which p_1 is excessive, $Q_1 = Q^M$ implies that p_1 is excessive so firm 1 may have to pay a fine. Raising Q_1 slightly above Q^M entails a second-order loss of profits in period 1, but leads to a first-order reduction of the expected fine that firm 1 pays in period 2. Hence, firm 1 sets $Q_1 > Q^M$, implying that the prohibition of excessive pricing has a pro-competitive effect on the pre-entry behavior of firm 1, even if eventually firm 1 is not found liable in period 2.

A further increase in Q_1 involves a trade-off: firm 1's profit in period 1 falls as Q_1 increases further above Q^M, but then firm 1 can expand its period 2 output without increasing the fine it pays when p_1 is deemed excessive. Lemma 7 shows that when the discounted probability of entry, $\delta\alpha$, is large relative to $\frac{1-\gamma\tau}{2\gamma\tau}$, firm 1 may raise Q_1 beyond $\overline{Q}_1$ to ensure that p_1 is no longer excessive.[22] By contrast, when $\delta\alpha$, is not too large, firm 1 sets $Q_1 < \overline{Q}_1$, in which case p_1 is excessive.

Using Lemma 5, that shows that $\frac{\partial(q_1^*+q_2^*)}{\partial Q_1} < 0$ if $Q_1 < \overline{Q}_1$ and $\frac{\partial(q_1^*+q_2^*)}{\partial Q_1} = 1$ if $Q_1 \geq \overline{Q}_1$, we have the following result.

Proposition 1. *(The effect of a retrospective benchmark on output) Using a retrospective benchmark for excessive pricing raises Q_1 above the monopoly level Q^M, but lowers the*

[22] Note though that when $\gamma\tau < \frac{1}{3}$, $\frac{1-\gamma\tau}{2\gamma\tau} > 1$, so $\delta\alpha$ can never be large enough to ensure that $Q_1^* > \overline{Q}_1$.

period 2 aggregate output below the Cournot level, Q^C. When $Q_1^* < \overline{Q}_1$ (p_1 is excessive), the expansion of Q_1 exceeds the contraction of aggregate output in period 2. When $Q_1^* > \overline{Q}_1$ (p_1 is not excessive), the expansion of Q_1 is equal to the contraction of aggregate output in period 2.

Proposition 1 shows that from the perspective of consumers, using a retrospective benchmark to assess whether the pre-entry price of the incumbent is excessive involves a trade-off between restraining the monopoly power of firm 1 in period 1, and softening competition in period 2 after entry takes place. The expansion of output in period 1 exceeds the reduction of output in period 2 when p_1 is excessive, but the two are equal when p_1 is not excessive. Now recall that by Assumption A1, demand is either concave or not too convex. If demand is concave or linear, the result that the expansion of output in period 1 is larger or equal to the contraction of output in period 2 implies that p_1 decreases more than p_2 increases. By continuity, p_1 also decreases more than p_2 increases, so long as demand is not too convex.

Proposition 2 (Comparative statics of Q_1^*)**.** *Q_1^* increases with the discounted probability of entry in period 2, $\delta\alpha$, but is independent of $\gamma\tau$ when $Q_1^* > \overline{Q}_1$ (p_1 is not excessive).*

Proof. See the Appendix. □

Intuitively, when $Q_1^* < \overline{Q}_1$ (p_1 is excessive), an increase in $\delta\alpha$ implies that entry is more likely, in which case firm 1 may have to pay a fine. Hence firm 1 has a stronger incentive to expand Q_1 and thereby relax the constraint on its period 2 output. When $Q_1^* > \overline{Q}_1$, p_1 is no longer excessive because firm 1 keeps its period 2 output below its Cournot best-response function to ensure that $q_1^* + q_2^* = Q_1$. Nonetheless, an increase in $\delta\alpha$, which makes it more likely that q_1^* will be constrained in period 2, induces firm 1 to expand Q_1; the expansion of Q_1 relaxes the constraint on q_1^* and allows firm 1 to move closer to its Cournot best-response function when entry occurs.

The fact that firm 1 expands Q_1 as $\delta\alpha$ increases means that consumers benefit from the prohibition of excessive pricing before entry takes place, especially when the probability of entry is high. This result is interesting because, as mentioned in the Introduction, it is often argued that when the probability of entry is high, there is no reason to intervene in excessive pricing cases, since “the market will correct itself.” This argument, however, ignores the harm to consumers before entry occurs and simply says that this harm is not going to last for a long time. While this is true, our analysis shows that nonetheless, the retrospective benchmark that we consider restrains the dominant firm’s pre-entry behavior and is therefore pro-competitive, particularly when the probability of entry is high.

Proposition 2 also shows that, as might be expected, the expected fine, $\gamma\tau$, has no effect on Q_1^* when p_1 is not excessive. When p_1 is excessive, i.e., $Q_1^* < \overline{Q}_1$, an increase in $\gamma\tau$ affects Q_1^* both directly through the expected fine that firm 1 may have to pay in period 2, as well as indirectly through its effect on the equilibrium in period 2. In the next subsection, we show that when demand is linear, an increase in $\gamma\tau$ induces firm 1 to expand Q_1^* when p_1 is excessive.

2.3. The linear demand case

In this section we assume that the inverse demand function is linear and given by $p = a - Q$. This assumption allows us to obtain a closed-form solution and therefore assess the overall effect of a retrospective benchmark on consumers. To ensure that firm 1's problem in period 1 is piecewise concave, we will assume that $\delta\alpha < 0.9$, i.e., the discounted probability of entry is not too high. The analysis, which appears in the Appendix, shows that in equilibrium, firm 1 sets its period 1 output, Q_1, such that p_1 ends up being excessive when $\delta\alpha < Z(\gamma\tau)$, but not excessive when $\delta\alpha \geq Z(\gamma\tau)$, where $Z(\gamma\tau)$ is given by Eq. (21) in the Appendix. Since $Z'(\gamma\tau) < 0$ and $Z(1) = 0$, it follows that either an increase in $\delta\alpha$ (the discounted probability of entry) or an increase in $\gamma\tau$ (the expected per-unit fine) induce firm 1 to expand Q_1 to ensure that p_1 is not excessive.

Given an aggregate output Q, the per-period consumers' surplus in the linear demand case is given by

$$CS(Q) = \int_0^Q (a - x)dx - (a - Q)Q = \frac{Q^2}{2}.$$

In period 1, firm 1's equilibrium output, Q_1^*, is given by Eq. (22) in the Appendix. The aggregate output in period 2 depends on whether there is entry. With probability $1 - \alpha$, there is no entry, so firm 1 produces the monopoly output $\frac{A}{2}$. With probability α, firm 2 enters, and the resulting aggregate output is $\frac{2A - \gamma\tau Q_1^*}{3}$ if $\delta\alpha < Z(\gamma\tau)$ (p_1 is excessive), and Q_1^* if $\delta\alpha \geq Z(\gamma\tau)$ (p_1 is not excessive). In the latter case, firm 1 sets q_1^* such that $q_1^* + q_2^* = Q_1^*$ to ensure that p_1 is not excessive. The next proposition characterizes the overall consumers' surplus over the two periods.

Proposition 3. *(Consumers' surplus in the linear demand case) Suppose that $p = a - Q$ and assume that $\delta\alpha < 0.9$ (the discounted probability of entry is not too high). Then, the overall expected discounted consumers' surplus over the two periods is given by*

$$CS(Q_1^*) = \begin{cases} \frac{(Q_1^*)^2}{2} + \frac{\delta(1-\alpha)}{2}\left(\frac{A}{2}\right)^2 + \frac{\delta\alpha}{2}\left(\frac{2A - \gamma\tau Q_1^*}{3}\right)^2, & \delta\alpha < Z(\gamma\tau), \\ \frac{(Q_1^*)^2}{2} + \frac{\delta(1-\alpha)}{2}\left(\frac{A}{2}\right)^2 + \frac{\delta\alpha}{2}(Q_1^*)^2, & \delta\alpha \geq Z(\gamma\tau). \end{cases} \tag{5}$$

$CS(Q_1^*)$ increases with the discounted probability of entry, $\delta\alpha$, increases with the expected per-unit fine, $\gamma\tau$, when $\delta\alpha < Z(\gamma\tau)$ (p_1 is excessive), and is independent of $\gamma\tau$ when $\delta\alpha \geq Z(\gamma\tau)$ (p_1 is not excessive).

Proof. See the Appendix. □

We have already shown earlier that using a retrospective benchmark for assessing whether prices are excessive involves a trade-off between a higher pre-entry output and a lower post-entry output. Proposition 4 allows us to examine the overall effect on consumers' surplus. Noting that $\gamma\tau = 0$ is equivalent to having no prohibition of excessive pricing, the proposition shows that the prohibition of excessive pricing benefits consumers, and more so as $\gamma\tau$ increases towards $Z(\gamma\tau)$.

It is important to stress that Proposition 4 should be interpreted cautiously, because our model abstracts from a number of factors that are likely to be relevant in reality, like the cost of detecting excessive prices, the legal costs involved with court cases, and the potential effect of the prohibition of excessive prices on the firm's incentive to invest. Still, Proposition 4 shows that, at least in the context of quantity competition and linear demand, the effects that we identify - a decrease in p_1 and an increase in p_2 - benefit consumers on balance.

3. A retrospective benchmark under price competition

In this section we consider the same model as in Section 2, except that now we assume that firms set prices rather than quantities. In the absence of a prohibition of excessive pricing, firm 1 sets the monopoly price in period 1 and also in period 2 if there is no entry, but if entry occurs, both firms charge a price equal to marginal cost c.

When excessive pricing is prohibited, the above strategies are no longer an equilibrium because following entry, the equilibrium price drops, so p_1 may be deemed excessive. This possibility affects firm 1's behavior. To characterize the resulting equilibrium, it is important to note that unlike quantity competition, which features a single market clearing price, now the two firms may charge different prices. We will assume in what follows that only firm 1's price in period 2 serves as a benchmark to evaluate whether p_1 was excessive, but only if firm 1 actually makes sales in period 2 (otherwise firm 1's price is merely theoretic).

Let $Q(p)$ denote the demand function (previously we worked with the inverse demand function, $p(Q)$). Absent entry, firm 1 simply sets in period 2 the monopoly price $p^M \equiv \arg\max Q(p)(p-c)$ and earns the monopoly profit, $\pi^M \equiv Q\left(p^M\right)\left(p^M-c\right)$. If entry occurs, firms 1 and 2 set prices p_{12} and p_{22} and consumers buy from the lowest price firm. If $p_{12} = p_{22}$, consumers buy from the more efficient firm.[23] In our case, the more efficient firm in period 2 is firm 2, since firm 1 may have to pay a fine if p_1 is deemed excessive. The profit functions in period 2 following entry are given by

$$\pi_1(p_{12}, p_{22}) = \begin{cases} 0, & p_{12} \geq p_{22}, \\ Q(p_{12})(p_{12}-c) - \gamma\tau[p_1 - p_{12}]^+ Q(p_1), & p_{12} < p_{22}, \end{cases} \tag{6}$$

[23] This tie-breaking rule is standard (see e.g., Deneckere and Kovenock (1996)).

and

$$\pi_2(p_{12}, p_{22}) = \begin{cases} Q(p_{22})(p_{22} - c), & p_{12} \geq p_{22}, \\ 0, & p_{12} < p_{22}, \end{cases} \tag{7}$$

where $[p_1 - p_{12}]^+ \equiv \max\{p_1 - p_{12}, 0\}$ because firm 1 pays a fine in period 2 only if it makes sales and $p_1 > p_{12}$. We shall assume that the bottom line in (6) is quasi-concave in p_{12} and the top line in (7) is quasi-concave in p_{22}. Let $\underline{\mathrm{p}}(p_1)$ be the value of p_{12} at which the bottom line in (6) vanishes when $p_1 > p_{12}$. That is, $\underline{\mathrm{p}}(p_1)$ is the value of p_{12} at which $Q(p_{12})(p_{12} - c) = \gamma\tau(p_1 - p_{12})Q(p_1)$.[24] Note that $\underline{\mathrm{p}}(p_1)$ is increasing with $\gamma\tau$ and equal to c when $\gamma\tau = 0$. The intuition is that an increase in $\gamma\tau$ raises the expected fine that firm 1 has to pay, and hence, the price it must charge in period 2 to break even.

Since firm 1 can guarantee itself a profit of 0 in period 2 by setting a price above p^M (setting a price above p^M is a dominated strategy for firm 2, so any price above p^M ensures firm 1 a profit of 0), firm 1 will never set a price below $\underline{\mathrm{p}}(p_1)$. Moreover, $\underline{\mathrm{p}}(p_1) > c$, since at $p_{12} = c$, the bottom line of (6) is equal to $-\gamma\tau Q(c)(p_1 - p_{12})Q(p_1) < 0$. Hence, firm 2 is indeed more efficient than firm 1 since it can make a profit at prices between $\underline{\mathrm{p}}(p_1)$ and c, while firm 1 cannot.

Proposition 4. *(The equilibrium under price competition with symmetric marginal cost) Suppose that firms 1 and 2 have the same marginal cost c and compete by setting prices. Moreover, assume that p_1 is deemed excessive if firm 1 makes sales in period 2 at a price below p_1. Then, when entry occurs, both firms charge $\underline{\mathrm{p}}(p_1)$ and all consumers buy from firm 2. Since firm 1 makes no sales in period 2, p_1 cannot be deemed excessive, and hence firm 1 sets $p_1 = p^M$.*

Proposition 4 implies that the prohibition of excessive pricing harms consumers in period 2 by raising the equilibrium price, without lowering the price in period 1. The reason is that due to the fine it may have to pay, firm 1 never lowers p_{12} below $\underline{\mathrm{p}}(p_1)$. This allows firm 2 to monopolize the market in period 2 by charging $\underline{\mathrm{p}}(p_1)$. Since $\underline{\mathrm{p}}(p_1) > c$, the period 2 price is higher than it would be absent a prohibition of excessive pricing. While the prohibition of excessive pricing also raises the equilibrium price in period 2 under quantity competition, here it does not help consumers in period 1 because firm 1 makes no sales in period 2 when firm 2 enters, and hence it can safely charge p^M in period 1.[25] Under quantity competition, firm 1 makes sales in period 2 even when firm 2

[24] The bottom line of (6) has two roots since $Q(p_{12})(p_{12} - c)$ is an inverse U-shape function of p_{12}, while $\gamma\tau(p_1 - p_{12})Q(p_1)$ is linearly decreasing with p_{12}. The relevant root is the smaller between the two since firm 1 does not set $p_1 > p^M$, where $p^M = \arg\max_{p_{12}} Q(p_{12})(p_{12} - c)$.

[25] This result depends on our assumption that p_1 can be deemed excessive only if firm 1 makes sales in period 2. Otherwise, firm 1 has an incentive to raise p_{12} to p^M to avoid paying a fine. But then firm 2 also wishes to raise its price to p^M, in which case firm 1 has an incentive to undercut firm 2. The resulting equilibrium in period 2 is then in mixed strategies. In such an equilibrium, firm 1 makes sales in period 2 with a positive probability and hence also pays a fine with a positive probability. As a result, firm 1 also has an incentive to set p_1 below p^M. Also note that if firm 2's price in period 2 can also be used as a benchmark, firm 1 will have an incentive to lower p_1 below p^M even if it makes no sales in period 2.

enters, and therefore has an incentive to expand Q_1 in order to lower p_1 and hence relax the constraint on its behavior in period 2.

Proposition 4 depends on our assumption that firm 2's cost is equal to firm 1's cost. Next, we relax this assumption and assume that firm 2's cost, c_2, is randomly drawn from the interval $[0,\ p^M]$, according to a distribution function $f(c_2)$ with a cumulative distribution $F(c_2)$. With probability $1-\alpha$, firm 2 is not born, so firm 1 is a monopoly in period 2 and charges the monopoly price, p^M. With probability α, firm 2 is born and enters the market. If $c_2 \le \underline{\mathrm{p}}(p_1)$, firm 2 monopolizes the market by charging $\underline{\mathrm{p}}(p_1)$. If $c_2 \in (\underline{\mathrm{p}}(p_1),\ p^M]$, firm 1 remains a monopoly in period 2 and charges c_2, exactly as in the absence of an antitrust prohibition of excessive pricing. Note that firm 2's entry is inefficient when $c_2 \in [c,\ \underline{\mathrm{p}}(p_1))$, because then, the market is served by the less efficient firm 2. The reason that firm 2 can monopolize the market in this case despite being less efficient is that firm 1 does not price below $\underline{\mathrm{p}}(p_1)$ due to the fine it may have to pay when p_1 is deemed excessive.

In sum, when firm 2 is born, it monopolizes the market when $c_2 \le \underline{\mathrm{p}}(p_1)$, in which case the prohibition of excessive pricing harms consumers in period 2 because it raises the price from $\max\{c,\ c_2\}$ to $\underline{\mathrm{p}}(p_1)$. When $c_2 > \underline{\mathrm{p}}(p_1)$, firm 1 remains a monopoly and the prohibition of excessive pricing has no effect on consumers in period 2. Interestingly, the prohibition of excessive pricing harms consumers in period 2 precisely when firm 2 enters and monopolizes the market. The reason is that absent a prohibition of excessive pricing, entry would have led to a lower price.

We now turn to p_1, which is the price that firm 1 sets in period 1. This price is chosen to maximize firm 1's expected discounted profit, given by

$$\begin{aligned}\Pi(p_1) = {} & Q(p_1)(p_1 - c) + \delta(1-\alpha)\pi^M \\ & + \delta\alpha \int_{\underline{p}(p_1)}^{p^M} \Big[Q(c_2)(c_2 - c) - \gamma\tau[p_1 - c_2]^+ Q(p_1) \Big] dF(c_2).\end{aligned}$$

In the next proposition, we characterize the equilibrium value of p_1 and examine how the retrospective benchmark affects consumers.

Proposition 5. *(The effect of a retrospective benchmark under price competition when firms have asymmetric costs) Suppose that firms 2's cost is randomly drawn from the interval* $[0,\ p^M]$, *and* p_1 *is deemed excessive if firm 1 makes sales in period 2 at* $p_{12} < p_1$. *Then, the prohibition of excessive pricing raises the equilibrium price in period 2 from* $\max\{c,\ c_2\}$ *to* $\underline{\mathrm{p}}(p_1)$ *when firm 2 enters the market and monopolizes it, but has no effect on the equilibrium price in period 2 when firm 1 remains a monopoly. In period* 1, *the prohibition of excessive pricing lowers* p_1. *Moreover,* p_1 *is decreasing with* $\delta\alpha$, *which is the discounted probability that firm 2 is born.*

Proof. See the Appendix. □

Proposition 5 shows that, as in the case of quantity competition, a retrospective benchmark leads to a lower p_1. Intuitively, when firm 2 is relatively inefficient, i.e., $c_2 \in (\underline{\mathrm{p}}(p_1), p^M]$, firm 1 continues to monopolize the market in period 2, but its equilibrium price is $c_2 < p^M$, meaning that p_1 may be deemed excessive. As a result, firm 1 has an incentive to lower p_1 in order to lower the expected fine it may have to pay. Consumers then benefit unambiguously since p_1 drops, while the price in period 2 continues to be c_2. By contrast, when firm 2 is relatively efficient, i.e., $c_2 \leq \underline{\mathrm{p}}(p_1)$, firm 2 monopolizes the market in period 2 and charges $\underline{\mathrm{p}}(p_1)$. Now, from the perspective of consumers, the lower p_1 comes at the expense of a higher price in period 2.

4. A contemporaneous benchmark

Another price benchmark which is often used in practice to assess whether the price of a dominant firm is excessive is the price that the same firm is contemporaneously charging in other markets. This benchmark was used for example in the British Leyland and the Napp cases that were mentioned in the Introduction.[26]

We will now study the competitive effects of a contemporaneous benchmark for excessive pricing under both quantity and price competition. To this end, we will assume that firm 1 is a monopoly in market 1, but faces competition from firm 2 in market 2. Both firms have a common marginal cost c.[27] In both markets, the demand system is derived from the preferences of a representative agent whose utility function is given by

$$U(q_1, q_1, m) = a(q_1 + q_2) - \frac{q_1^2 + q_2^2 + 2bq_1q_2}{2} + m, \tag{8}$$

where $b \in [0, 1]$ is a measure of product differentiation and m is the monetary expenditure on all other goods. The demand system in market 2 is derived by maximizing $U(q_1, q_1, m)$ subject to the representative consumer's budget constraint, $p_1q_1 + p_2q_2 + m = I$, where I is the agent's income. The demand in market 1, where firm 1 is a monopoly, is derived similarly, but subject to the constraint that $q_2 = 0$.

4.1. Quantity competition

Let Q_1 and p_1 be the output and price of firm 1 in market 1, and let q_1 and q_2 be the outputs of firms 1 and 2 in market 2, and p_{12} and p_{22} be their corresponding prices. The

[26] Another example is the Israeli potash case mentioned in the Introduction. The Central District Court held that Dead Sea Works not only raised its price substantially in 2008–2009 relative to 2007, but also charged a much higher price in Israel than overseas. See Class Action (Central District Court) 41838-09-14 Weinstein v. Dead Sea Works, Inc. (Nevo, 29.1.2017).

[27] If firm 1 has a different cost in markets 1 and 2, one would have to compare its price-cost margins across the two markets instead of simply comparing its prices. It should be noted that in real-life cases, establishing the relevant cost in each market is typically complicated and often highly contentious, say due to the need to allocate common fixed costs to individual products.

inverse demand functions are then given by:

$$p_1 = a - Q_1, \qquad p_{12} = a - q_1 - bq_2, \qquad p_{22} = a - q_2 - bq_1. \tag{9}$$

Notice that when $b = 1$, the model is identical to the model in Section 2, except that instead of having two time periods, we now have two separate markets. The main implication in terms of the analysis is that now, Q_1, q_1, and q_2, are set simultaneously, instead of Q_1 being set before q_1 and q_2. Recalling that $A \equiv a - c$, the profit functions of firms 1 and 2 are given by

$$\Pi_1(Q_1, q_1, q_2) = \begin{cases} (A - Q_1)Q_1 + (A - q_1 - bq_2)q_1, & q_1 + bq_2 \leq Q_1, \\ (A - Q_1)Q_1 + (A - q_1 - bq_2)q_1 & \\ -\gamma\tau[(a - Q_1) - (a - q_1 - bq_2)]Q_1, & q_1 + bq_2 > Q_1, \end{cases}$$

and

$$\Pi_2(q_1, q_2) = (A - q_2 - bq_1)q_2.$$

To ensure that $\Pi_1(Q_1,\ q_1,\ q_2)$ is concave we will assume that $\gamma\tau \leq 0.8$. We now characterize the Nash equilibrium.

Proposition 6. *(The equilibrium in the contemporaneous benchmark case) Suppose that firm 1 is a monopoly in market 1, but competes with firm 2 in market 2 and assume that the inverse demand functions are given by (9).*

(i) *If $\gamma\tau < \frac{b(2-b)}{4+b-2b^2}$, the equilibrium is given by*

$$Q_1^* = \frac{A(2-b)(2-\gamma\tau+b(1-\gamma\tau))}{(2-b^2)H+4(1-\gamma\tau)}, \tag{10}$$

$$q_1^* = \frac{A(4-6\gamma\tau-H)}{(2-b^2)H+4(1-\gamma\tau)}, \qquad q_2^* = \frac{A(H+2(1-\gamma\tau)-b(2-3\gamma\tau))}{(2-b^2)H+4(1-\gamma\tau)}, \tag{11}$$

where $H \equiv 2 - 2\gamma\tau - (\gamma\tau)^2$. In equilibrium, $p_1 > p_{12}$, so p_1 is excessive. Moreover, p_1 decreases with $\gamma\tau$, while p_{12} and p_{22} increase with $\gamma\tau$.

(ii) *If $\gamma\tau \geq \frac{b(2-b)}{4+b-2b^2}$, the equilibrium is given by*

$$Q_1^* = \frac{A\left(4+b-2b^2\right)}{8-3b^2}, \qquad q_1^* = \frac{A(4-3b)}{8-3b^2}, \qquad q_2^* = \frac{2A(2-b)}{8-3b^2}. \tag{12}$$

Now, $p_1 = p_{12}$, so p_1 is not excessive.

Proof. See the Appendix. □

Proposition 6 shows that p_1 is excessive only when $\gamma\tau < \frac{b(2-b)}{4+b-2b^2}$. When $\gamma\tau \geq \frac{b(2-b)}{4+b-2b^2}$, a contemporaneous benchmark induces firm 1 to set its output levels such that $p_1 = p_{12}$.

The critical value below which p_1 is excessive, $\frac{b(2-b)}{4+b-2b^2}$, is increasing with b and equal to 0 when $b = 0$. In the latter case, firm 1 does not compete with firm 2 in market 2 and hence it sets the monopoly price in both markets 1 and 2, so p_1 is not excessive. As b increases, competition between firms 1 and 2 intensifies, so firm 1 has an incentive to expand its output in market 2. Consequently, p_{12} drops below p_1 and hence p_1 may be deemed excessive. Firm 1 responds by expanding Q_1 to lower p_1 and limit the gap between p_1 and p_{12}, but by doing so, it sacrifices some of its profit in market 1. The willingness of firm 1 to expand Q_1 increases with $\gamma\tau$. When $\gamma\tau = \frac{b(2-b)}{4+b-2b^2}$, firm 1 expands Q_1 and contracts q_1 to ensure that $p_1 = p_{12}$, in which case it avoids paying fines.

Proposition 6 also shows that an increase in $\gamma\tau$ induces firm 1 to further expand Q_1 and contract q_1 in order to lower the gap between p_1 and p_{12} and thereby lower its excessive revenue in market 1 on which it may pay a fine. This benefits consumers in market 1, but harms firm 1's consumers in market 2. Under quantity competition, the strategies of firms 1 and 2 in market 2 are strategic substitutes, so firm 2 expands its own output. The contraction of firm 1's output has a stronger effect on p_{22} than the expansion of firm 2's own output, so overall firm 2's price increases, making firm 2's consumers worse off.

Noting that $\gamma\tau = 0$ implies that excessive pricing is not prohibited, it follows that a contemporaneous benchmark involves a tradeoff between the welfare of consumers in markets 1 and 2. To examine the overall effect on consumers, note that if we substitute for m from the budget constraint $p_1q_1 + p_2q_2 + m = I$ into (8) and use the inverse demand functions (9), overall consumers' surplus is given by

$$CS(Q_1, q_1, q_2) = \frac{Q_1^2}{2} + \frac{q_1^2 + q_2^2 + 2bq_1q_2}{2}.$$

Substituting the equilibrium quantitates from Proposition 6 into $CS(Q_1,\ q_1,\ q_2)$, yields the overall consumers' surplus in equilibrium, $CS^* \equiv CS(Q_1^*, q_1^*, q_2^*)$, where CS^* is a ratio of two 4th degree polynomials in $\gamma\tau$ and b. Absent a prohibition of excessive pricing, consumers' surplus is given by $CS_0^* \equiv CS^*|_{\gamma\tau=0}$. Hence, we can study the effect of a contemporaneous benchmark on consumers by looking at $CS^* - CS_0^*$. The next figure shows $CS^* - CS_0^*$ as a function of $\gamma\tau$ and b.

Recall from Proposition 6 that p_1 is excessive only when $\gamma\tau < \frac{b(2-b)}{4+b-2b^2}$. Fig. 3 shows that in this range, using a contemporaneous benchmark to determine whether p_1 is excessive benefits consumers when $\gamma\tau$ is relatively high, but harms them when $\gamma\tau$ is relatively small. The boundary between the two regions increases with b. To see the intuition, suppose that $b = \gamma\tau = 0$. Then $p_1 = p_{12}$. As b increases, competition in market 2 intensifies, so p_{12} falls and consumers in market 2 become better off. But then $p_{12} < p_1$, so p_1 may be deemed excessive. If $\gamma\tau$ increases from 0, firm 1 has an incentive to expand output in market 1 and contract output in market 2 in order to lower p_1 and raise p_{12} and thereby lower the expected fine it pays when p_1 is deemed excessive. These changes benefit consumers in market 1 and harm consumers in market 2. When $\gamma\tau$ is low, the

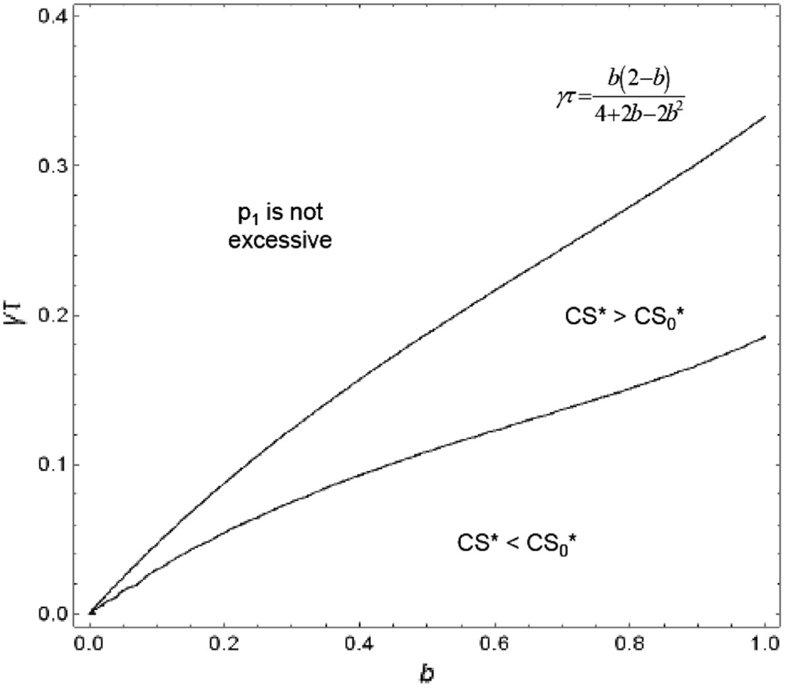

Fig. 3. The effect of a contemporeneous bechmark for excessive pricing on consumers under quantity competition.

harm to consumers in market 2 exceeds the benefit to consumers in market 1. But as $\gamma\tau$ increases towards $\frac{b(2-b)}{4+b-2b^2}$, firm 1 narrows the gap between p_1 and p_{12}. Since strategies are strategic substitutes, firm 2 responds to the contraction of firm 1's output in market 2 by expanding its own output, so its consumers are better off. As a result, overall consumers' surplus increases.

A contemporaneous benchmark for excessive pricing is reminiscent of contemporaneous MFN's, which prevent firms from offering selective discounts to some consumers. Besanko and Lyon (1993) show that contemporaneous MFN's relax price competition, though in equilibrium firms may not wish to adopt them unilaterally. Their model differs from ours in that they assume that firms compete in both markets (the market for shoppers and the market for non-shoppers in their model), while in our model, firm 1 is a monopoly in market 1. Moreover, they assume that an MFN renders price discrimination impossible, while in our model, firm 1 can still charge different prices in the two markets, but then may have to pay a fine with probability γ. In terms of results, Besanko and Lyon (1993) show that MFN's harm consumers because they raise the average price across the two markets, while in our framework a prohibition of excessive pricing benefits consumers in the monopoly market and harms consumers in the benchmark market.

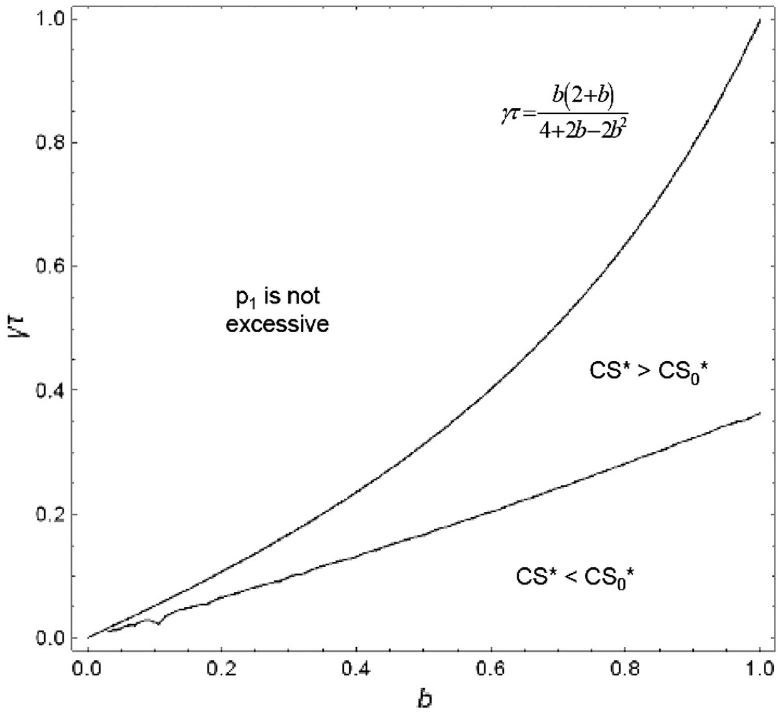

Fig. 4. The effect of a contemporaneous benchmark for excessive pricing on consumers under price competition.

4.2. Price competition

To study the case where firms 1 and 2 compete in market 2 by setting prices, we first invert the inverse demand system (9) to obtain the following demand system:

$$Q_1 = a - p_1, \qquad q_1 = a(1+b) - \frac{p_{12} - bp_{22}}{1-b^2}, \qquad q_2 = a(1+b) - \frac{p_{22} - bp_{12}}{1-b^2}.$$

Using this demand system, we repeat the same steps as in the case of quantity competition. The results are qualitatively similar and are reported in the Appendix. The next figure shows $CS^* - CS_0^*$ as a function of b and $\gamma\tau$: p_1 is excessive when $\gamma\tau < \frac{b(2+b)}{4+b-2b^2}$, and as in the case of quantity competition, consumers are better off under a contemporaneous benchmark when $\gamma\tau$ is relatively high, but worse off when $\gamma\tau$ is relatively small.

5. Conclusion

We have examined the competitive effects of the prohibition of excessive pricing by a dominant firm. A main problem when implementing this prohibition is to establish an appropriate competitive benchmark to assess whether the dominant firm's price is indeed excessive. In this paper we study two benchmarks, which are used in practice: a retrospective benchmark, where the price that the dominant firm charges following entry into the market is used to determine whether its pre-entry price was excessive, and a contemporaneous benchmark, where the price that the dominant firm is charging in a more competitive market is used to determine whether its price in a market where it is dominant is excessive. If the dominant firm's price is deemed excessive, the firm pays a fine proportional to its excess revenue in the dominant market. The latter is equal to the difference between the actual price and the benchmark price, times the firm's output in the dominant market.

We find that the two benchmarks lead to a tradeoff: they restrain the dominant firm's behavior when it acts as a monopoly, but soften its behavior in the benchmark market (the post-entry market in the retrospective benchmark case and the more competitive market in the contemporaneous benchmark case). We show that when the dominant firm and the rival compete in the benchmark market by setting quantities and products are homogenous, the pro-competitive effect of a retrospective benchmark in the monopoly market outweighs the corresponding anticompetitive effect in the benchmark market. Hence, a retrospective benchmark for excessive pricing benefits consumers overall. By contrast, under price competition with homogenous products and symmetric costs, a retrospective benchmark softens competition post-entry without lowering the pre-entry price, implying that consumers are overall worse off. We also show that a contemporaneous benchmark benefits consumers when the expected fine that the firm pays when its price in the monopoly market is deemed excessive is relatively high, but harms consumers when the expected fine is relatively low. These results hold under both quantity and price competition and indicate that the overall competitive effect of the prohibition of excessive pricing depends on the precise nature of competition, as well as the magnitude of the expected fines that the dominant firm pays when its price is deemed excessive.

We also show that using a retrospective benchmark to enforce the prohibition of excessive pricing may actually promote entry into the market, as it induces the incumbent firm to behave more softly once entry takes place. This soft behavior may enable an inefficient entrant to successfully enter the market.

While our analysis highlights two new effects that were not discussed earlier, we abstract from many considerations which are likely to be important in real-life cases. For example, our model does not take into account the incentive of firms to invest in R&D, advertising, or product quality, hold inventories, offer a variety of products, choose locations and other non-price decisions that affect competition and consumer welfare. Our model also abstracts from the cost of litigation and other legal costs, as well as from demand and cost fluctuations which make it harder to compare prices across different time periods or markets. Hence, more research is needed before we fully understand the competitive implications of the prohibition of excessive pricing.

Our analysis can be extended in a number of ways. We mention here three possible extensions. First, it is possible to endogenize the probability of entry, α, by assuming that the entrant incurs an entry cost, drawn from some known interval. In such a setting, the dominant firm will have to take into account the effect of its pre-entry output on the post-entry equilibrium and hence on the probability of entry. Second, the probability that the dominant firm is found liable, γ, may depend on the gap between the pre- and post-entry prices. For instance, if there is a safe harbor, $\gamma = 0$ if the gap is below some threshold Δ; otherwise, $\gamma > 0$ and increasing with the gap. It should be interesting to examine how consumers' surplus changes with Δ. Third, when the dominant firm's marginal cost is private information, its period 1 output will signal its cost and will therefore affect its pricing strategy. We leave these extensions and others for future research.

Appendix

Following are the proofs of Lemmas 1, 2, 4, 5, and 7, and Propositions 2, 3, 5 and 6, and the characterization of the equilibrium in the retrospective benchmark under quantity competition when demand is linear and in the contemporaneous benchmark case under price competition.

Proof of Lemma 1. We begin by proving that $\pi_1(q_1, q_2)$ is concave in q_1 and $\pi_2(q_1, q_2)$ is concave in q_2. To this end, note that differentiating the first line in $\pi_1(q_1, q_2)$ yields:

$$\frac{\partial^2 \pi_1(q_1, q_2)}{\partial q_1^2} = 2p'(q_1 + q_2) + p''(q_1 + q_2)q_1.$$

If $p''(\cdot) \leq 0$, we are done. If $p''(q_1 + q_2) > 0$,

$$\frac{\partial^2 \pi_1(q_1, q_2)}{\partial q_1^2} < 2p'(q_1 + q_2) + p''(q_1 + q_2)(q_1 + q_2) < 0,$$

where the last inequality follows from Assumption A1. Hence, $\pi_1(q_1, q_2)$ is concave in q_1 when $q_1 + q_2 \leq Q_1$. The proof that $\pi_2(q_1, q_2)$ is concave in q_2 is identical.

Differentiating the second line in $\pi_1(q_1, q_2)$ yields:

$$\frac{\partial^2 \pi_1(q_1, q_2)}{\partial q_1^2} < 2p'(q_1 + q_2) + p''(q_1 + q_2)(q_1 + \gamma\tau Q_1).$$

Again, if $p''(\cdot) \leq 0$, we are done. If $p''(q_1 + q_2) > 0$,

$$\begin{aligned}\frac{\partial^2 \pi_1(q_1, q_2)}{\partial q_1^2} &< 2p'(q_1 + q_2) + p''(q_1 + q_2)(q_1 + \gamma\tau(q_1 + q_2)) \\ &< 2p'(q_1 + q_2) + p''(q_1 + q_2)(1 + \gamma\tau)(q_1 + q_2) < 0,\end{aligned}$$

where the first inequality follows because $q_1 + q_2 > Q_1$, and the last inequality follows from Assumption A1. Hence, $\pi_1(q_1, q_2)$ is also concave in q_1 when $q_1 + q_2 > Q_1$.

Since firm 2's profit is concave in q_2, $BR_2(q_1) = r_2^C(q_1)$, where $r_2^C(q_1)$ is defined by (1). To characterize $BR_1(q_2)$, note first that $\pi_1(q_1, q_2)$ is continuous at $q_1 + q_2 = Q_1$ and is piecewise concave in q_1 (i.e., both when $q_1 + q_2 \le Q_1$, as well as when $q_1 + q_2 > Q_1$). Now,

$$\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} = \begin{cases} p(q_1+q_2) + p'(q_1+q_2)q_1 - c, & q_1 + q_2 \le Q_1, \\ p(q_1+q_2) + p'(q_1+q_2)(q_1 + \gamma\tau Q_1) - c, & q_1 + q_2 > Q_1. \end{cases} \tag{13}$$

Note that since $p'(q_1+q_2) < 0$, $\frac{\partial \pi_1(q_1,q_2)}{\partial q_1} < 0$ as $q_1 + q_2$ approaches Q_1 from below also implies that $\frac{\partial \pi_1(q_1,q_2)}{\partial q_1} < 0$ as $q_1 + q_2$ approaches Q_1 from above. Together with the fact that $\pi_1(q_1,\, q_2)$ is continuous at $q_1 + q_2 = Q_1$ and piecewise concave, it follows that

(i) $\pi_1(q_1,\, q_2)$ attains a maximum at $q_1 < Q_1 - q_2$ if $\frac{\partial \pi_1(q_1,q_2)}{\partial q_1} < 0$ as $q_1 + q_2$ approaches Q_1 from below, i.e., when $p(Q_1) + p'(Q_1)(Q_1 - q_2) < c$,

(ii) $\pi_1(q_1,\, q_2)$ attains a maximum at $q_1 > Q_1 - q_2$ if $\frac{\partial \pi_1(q_1,q_2)}{\partial q_1} > 0$ as $q_1 + q_2$ approaches Q_1 from above, i.e., when $p(Q_1) + p'(Q_1)((1+\gamma\tau)Q_1 - q_2) > c$,

(iii) $\pi_1(q_1,\, q_2)$ attains a maximum at $q_1 = Q_1 - q_2$ if $\frac{\partial \pi_1(q_1,q_2)}{\partial q_1} > 0$ as $q_1 + q_2$ approaches Q_1 from below, and $\frac{\partial \pi_1(q_1,q_2)}{\partial q_1} < 0$ as $q_1 + q_2$ approaches Q_1 from above, i.e., when $p(Q_1) + p'(Q_1)(Q_1 - q_2) > c > p(Q_1) + p'(Q_1)((1+\gamma\tau)Q_1 - q_2)$.

In case (i), p_1 is not excessive, and firm 1's best-response function is defined by $r_1^C(q_2)$. In case (ii), p_1 is excessive and firm 1's best-response function is defined by $r_1^E(q_2)$. And in case (iii), firm 1 sets q_1 to ensure that $q_1 + q_2 = Q_1$; this ensures that p_1 is not deemed excessive.

To study the slopes of the best-response functions in the $(q_1,\, q_2)$ space, notice first that

$$BR_2'(\cdot) = -\frac{\dfrac{\partial \pi_2^2(q_1^*, q_2^*)}{\partial q_1 \partial q_2}}{\dfrac{\partial \pi_2^2(q_1^*, q_2^*)}{\partial q_2^2}} = -\frac{p' + p'' q_2}{2p' + p'' q_2},$$

where the arguments of p' and p'' are suppressed to ease notation. Assumption A1 is sufficient to ensure that $-1 \le BR_2'(\cdot) < 0$. The proof that $BR_1'(\cdot) < -1$ when $BR_1(q_2) = r_1^C(q_2)$ is similar. When $BR_1(q_2) = Q_1 - q_2$, it is obvious that $BR_1'(\cdot) = -1$. Finally, when $BR_1(q_2) = r_1^E(q_2)$, then

$$BR_1'(\cdot) = -\frac{\dfrac{\partial \pi_1^2(q_1^*, q_2^*)}{\partial q_1^2}}{\dfrac{\partial \pi_1^2(q_1^*, q_2^*)}{\partial q_1 \partial q_2}} = -\frac{2p' + p''(q_1 + \gamma\tau Q_1)}{p' + p''(q_1 + \gamma\tau Q_1)} < -1,$$

where the inequality is implied by Assumption A1. □

Proof of Lemma 2. Suppose that p_1 is excessive and assume by way of negation that $q_1^* = 0$ when firm 2 enters. Then firm 2 produces the monopoly output in period 2, since $r_2^C(0) = Q^M$. For p_1 to be excessive, it must be that $Q_1 < Q^M$. Moreover, given

Assumption A2, $\gamma\tau Q_1 < Q_1 \leq Q^M$. Since p_1 is excessive, firm 1's best-response function against q_2 is $r_1^E(q_2)$. Evaluating the second line in (13) at (0, Q^M), and noting that Q^M is defined implicitly by the first-order condition $p(Q^M) + p'(Q^M)Q^M = c$,

$$\frac{\partial \pi_1(0, Q^M)}{\partial q_1} = p(Q^M) - c + \gamma\tau p'(Q^M)Q_1 = -p'(Q^M)(Q^M - \gamma\tau Q_1) > 0,$$

where the inequality follows since $p'(\cdot) < 0$ and $Q^M > \gamma\tau Q_1$. Hence, $r_1^E(Q^M) > 0$, contradicting the assumption that $q_1^* = 0$. $\square$

Proof of Lemma 4. The monopoly output, Q^M is defined by

$$\pi'(Q_1) = p(Q_1) + p'(Q_1)Q_1 - c = 0.$$

Evaluating $\pi'(Q_1)$ at $\overline{Q}_1$ and using (3),

$$\pi'(\overline{Q}) = p(\overline{Q}_1) + p'(\overline{Q}_1)\overline{Q}_1 - c = p'(\overline{Q}_1)\left(\frac{1-\gamma\tau}{2}\right)\overline{Q}_1 < 0,$$

where the inequality follows by Assumption A2. Hence, $\overline{Q} > Q^M$.

The Cournot equilibrium is defined by $\pi_1'(q_1, q_2) = 0$ and $\pi_2'(q_1, q_2) = 0$, where

$$\pi_i'(q_1, q_2) = p(q_1 + q_2) + p'(q_1 + q_2)q_i - c.$$

Since the equilibrium is symmetric, $q_1^C = q_2^C = \frac{Q^C}{2}$:

$$\pi_i'\left(\frac{Q^C}{2}, \frac{Q^C}{2}\right) = p(Q^C) + p'(Q^C)\frac{Q^C}{2} - c = 0.$$

Evaluating this equation at $\overline{Q}_1$ and using (3),

$$\pi_i'\left(\frac{\overline{Q}_1}{2}, \frac{\overline{Q}_1}{2}\right) = p(\overline{Q}_1) + p'(\overline{Q}_1)\frac{\overline{Q}_1}{2} - c = -\gamma\tau p'(\overline{Q}_1)\frac{\overline{Q}_1}{2} > 0.$$

Hence, $\overline{Q} < Q^C$.

Next, differentiating equation (3) with respect to $\overline{Q}_1$ and $\gamma\tau$, and rearranging terms,

$$\frac{\partial \overline{Q}_1}{\partial(\gamma\tau)} = \frac{-\frac{p'(\overline{Q}_1)\overline{Q}_1}{2}}{p'(\overline{Q}_1)(1 + \frac{1+\gamma\tau}{2}) + p''(\overline{Q}_1)(\frac{1+\gamma\tau}{2})\overline{Q}_1}.$$

Assumption A1 ensures that the denominator is negative and hence, $\frac{\partial \overline{Q}_1}{\partial(\gamma\tau)} < 0$. $\square$

Proof of Lemma 5. First, suppose that $Q_1 < \overline{Q}_1$. Then the Nash equilibrium is defined by the intersection of $r_1^E(q_2)$ and $r_2^C(q_1)$. Fully differentiating this system yields the following comparative statics matrix

$$\begin{vmatrix} 2p' + p''(q_1^* + \gamma\tau Q_1) & p' + p''(q_1^* + \gamma\tau Q_1) \\ p' + p''q_2^* & 2p' + p''q_2^* \end{vmatrix} \times \begin{vmatrix} \frac{\partial q_1^*}{\partial Q_1} \\ \frac{\partial q_2^*}{\partial Q_1} \end{vmatrix} = \begin{vmatrix} -\gamma\tau p' \\ 0 \end{vmatrix},$$

where the arguments of p' and p'' are suppressed to ease notation. Hence,

$$\begin{aligned}\frac{\partial q_1^*}{\partial Q_1} &= \frac{-\gamma\tau p'(2p' + p''q_2^*)}{(2p' + p''(q_1^* + \gamma\tau Q_1))(2p' + p''q_2^*) - (p' + p''(q_1^* + \gamma\tau Q_1))(p' + p''q_2^*)} \\ &= -\frac{\gamma\tau(2p' + p''q_2^*)}{3p' + p''(q_1^* + q_2^* + \gamma\tau Q_1)},\end{aligned}$$

and

$$\begin{aligned}\frac{\partial q_2^*}{\partial Q_1} &= \frac{\gamma\tau p'(p' + p''q_2^*)}{(2p' + p''(q_1^* + \gamma\tau Q_1))(2p' + p''q_2^*) - (p' + p''(q_1^* + \gamma\tau Q_1))(p' + p''q_2^*)} \\ &= \frac{\gamma\tau(p' + p''q_2^*)}{3p' + p''(q_1^* + q_2^* + \gamma\tau Q_1)}.\end{aligned}$$

Since $\frac{p'+p''q_2^*}{3p'+p''(q_1^*+q_2^*+\gamma\tau Q_1)} = \frac{1}{1+\frac{2p'+p''(q_1^*+\gamma\tau Q_1)}{p'+p''q_2^*}} < 1$ by Assumption A1, $-\gamma\tau < \frac{\partial q_1^*}{\partial Q_1} < 0 < \frac{\partial q_2^*}{\partial Q_1} < \gamma\tau$. Moreover, using Assumption A1, it follows that whenever $Q_1 < \overline{Q}_1$,

$$\begin{aligned}\frac{\partial(q_1^* + q_2^*)}{\partial Q_1} &= -\frac{\gamma\tau(2p' + p''q_2^*)}{3p' + p''(q_1^* + q_2^* + \gamma\tau Q_1)} + \frac{\gamma\tau(p' + p''q_2^*)}{3p' + p''(q_1^* + q_2^* + \gamma\tau Q_1)} \\ &= \frac{-p'\gamma\tau}{3p' + p''(q_1^* + q_2^* + \gamma\tau Q_1)} \in (-\gamma\tau, 0).\end{aligned}$$

Now suppose that $Q_1 \geq \overline{Q}_1$. Then the Nash equilibrium is defined by the intersection of $q_1 = Q_1 - q_2$ and $r_2^C(q_1)$. Fully differentiating this system, yields the following comparative statics matrix

$$\begin{vmatrix} 1 & 1 \\ p' + p''q_2^* & 2p' + p''q_2^* \end{vmatrix} \times \begin{vmatrix} \frac{\partial q_1^*}{\partial Q_1} \\ \frac{\partial q_2^*}{\partial Q_1} \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix},$$

Hence, by Assumption A1,

$$\frac{\partial q_1^*}{\partial Q_1} = \frac{2p' + p''q_2^*}{p'} > 1, \qquad \frac{\partial q_2^*}{\partial Q_1} = -\frac{p' + p''q_2^*}{p'} < 0.$$

It is now easy to see that $\frac{\partial(q_1^*+q_2^*)}{\partial Q_1} = 1$. □

Proof of Lemma 7. First, we prove that $Q_1^* > Q^M$. To this end, note that since $\overline{Q}_1 > Q^M$ by Lemma 4, p_1 is excessive when $Q_1 = Q^M$, so $\Pi_1(Q_1)$ is given by the first line of (4).

Differentiating the expression and using the envelope theorem (the derivative of the square bracketed term with respect to q_1^* vanishes), yields

$$\begin{aligned}\Pi_1'(Q_1) &= MR(Q_1) - c \\ &\quad -\delta\alpha\left[\gamma\tau(MR(Q_1) - p(q_1^* + q_2^*)) - p'(q_1^* + q_2^*)(q_1^* + \gamma\tau Q_1)\frac{\partial q_2^*}{\partial Q_1^-}\right], \end{aligned} \tag{14}$$

where $\frac{\partial q_2^*}{\partial Q_1^-}$ is the derivative of q_2^* with respect to Q_1 when $Q_1 < \overline{Q}_1$. Evaluating $\Pi_1'(Q_1)$ at Q^M and noting that by definition, $MR(Q^M) = c$,

$$\begin{aligned}\Pi_1'\left(Q^M\right) &= -\delta\alpha\left[\gamma\tau(c - p(q_1^* + q_2^*)) - p'(q_1^* + q_2^*)\left(q_1^* + \gamma\tau Q_1^M\right)\frac{\partial q_2^*}{\partial Q_1^-}\right] \\ &= -\delta\alpha p'(q_1^* + q_2^*)\left(q_1^* + \gamma\tau Q_1^M\right)\left[\gamma\tau - \frac{\partial q_2^*}{\partial Q_1^-}\right] > 0,\end{aligned}$$

where the second equality follows by substituting for $p(q_1^* + q_2^*) - c$ from (2) and the inequality follows from Lemma 5 which shows that when $Q_1 < \overline{Q}_1$, $\frac{\partial q_2^*}{\partial Q_1^-} < \gamma\tau$. Since $\Pi_1'\left(Q^M\right) > 0$, $Q_1^* > Q^M$.

Second, we examine whether firm 1 has an incentive to raise Q_1^* all the way to the point where p_1 is no longer excessive, i.e., above $\overline{Q}_1$. To this end, we first evaluate $\Pi_1'\left(\overline{Q}_1\right)$ as $\overline{Q}_1$ approaches $\overline{Q}_1$ from below. Using $\Pi_1'\left(\overline{Q}_1^-\right)$ to denote the derivative of $\Pi_1(Q_1)$ as Q_1 approaches $\overline{Q}_1$ from below, and recalling that when $Q_1 < \overline{Q}_1$, $\Pi_1'(Q_1)$ is given by (14), we get

$$\begin{aligned}\Pi_1'\left(\overline{Q}_1^-\right) &= MR(\overline{Q}_1) - c - \delta\alpha\left[\gamma\tau(MR(\overline{Q}_1) - p(q_1^* + q_2^*)) - p'(q_1^* + q_2^*)(q_1^* + \gamma\tau\overline{Q}_1)\frac{\partial q_2^*}{\partial Q_1^-}\right] \\ &= MR(\overline{Q}_1) - c - \delta\alpha\left[\gamma\tau(MR(\overline{Q}_1) - p(\overline{Q}_1)) - p'(\overline{Q}_1)(q_1^* + \gamma\tau\overline{Q}_1)\frac{\partial q_2^*}{\partial Q_1^-}\right] \\ &= p'(\overline{Q}_1)\left(\frac{1-\gamma\tau}{2}\right)\overline{Q}_1 - \delta\alpha\left[\gamma\tau p'(\overline{Q}_1)\overline{Q}_1 - p'(\overline{Q}_1)(q_1^* + \gamma\tau\overline{Q}_1)\frac{\partial q_2^*}{\partial Q_1^-}\right] \\ &= p'(\overline{Q}_1)\overline{Q}_1\left[\frac{1-\gamma\tau(1+2\delta\alpha)}{2} + \delta\alpha\left(\frac{q_1^*}{\overline{Q}_1} + \gamma\tau\right)\frac{\partial q_2^*}{\partial Q_1^-}\right] \\ &= p'(\overline{Q}_1)\overline{Q}_1\left[\frac{1-\gamma\tau(1+2\delta\alpha)}{2} + \delta\alpha\left(\frac{1+\gamma\tau}{2}\right)\frac{\partial q_2^*}{\partial Q_1^-}\right] \\ &= -\delta\alpha\left(\frac{1+\gamma\tau}{2}\right)p'(\overline{Q}_1)\overline{Q}_1\left[\frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)} - \frac{\partial q_2^*}{\partial Q_1^-}\right],\end{aligned} \tag{15}$$

where the second equality follows because by definition, $q_1^* + q_2^* = \overline{Q}_1$, and the third equality follows by using (3). As for the fifth equality, recall that $\overline{Q}_1$ satisfies both (2) and (1) when $q_i = q_2$. Subtracting the latter from the former, using the fact that $q_1 + q_2 = \overline{Q}_1$, and rearranging yields,

$$p'(\overline{Q}_1)\left[q_1^* + \gamma\tau\overline{Q}_1 - q_2^*\right] = 0, \qquad \Rightarrow \qquad 2p'(\overline{Q}_1)\overline{Q}_1\left[\frac{q_1^*}{\overline{Q}_1} - \frac{1-\gamma\tau}{2}\right] = 0. \tag{16}$$

Hence, $\frac{q_1^*}{\overline{Q}_1} = \frac{1-\gamma\tau}{2}$ when $Q_1 = \overline{Q}_1$. Since $p'(\overline{Q}_1) < 0$, (15) implies that $\Pi_1'\left(\overline{Q}_1^-\right)$ has the same sign as $\frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)} - \frac{\partial q_2^*}{\partial Q_1^-}$. Note that by Lemma 5, $0 < \frac{\partial q_2^*}{\partial Q_1^-} < \gamma\tau < 1$ and also note that $\frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)}$ is increasing with $\delta\alpha$ from $-\infty$ when $\delta\alpha = 0$ to $\frac{3\gamma\tau-1}{1+\gamma\tau}$ when $\delta\alpha = 1$. Hence, $\Pi_1'\left(\overline{Q}_1^-\right) \leq 0$ when $\delta\alpha$ is sufficiently small and moreover, $\Pi_1'\left(\overline{Q}_1^-\right) \leq 0$ for all $\delta\alpha$ when $\gamma\tau \leq \frac{1}{3}$. In particular, $\delta\alpha \leq \frac{1-\gamma\tau}{2\gamma\tau}$ is sufficient to ensure that $\frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)} \leq 0$, in which case, $\Pi_1'\left(\overline{Q}_1^-\right) \leq 0$. By continuity then, $\Pi_1'\left(\overline{Q}_1^-\right) < 0$ when $\delta\alpha$ does not exceed $\frac{1-\gamma\tau}{2\gamma\tau}$ by too much. By contrast, a necessary condition for $\Pi_1'\left(\overline{Q}_1^-\right) > 0$ is $\frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)} > 0$, or $\delta\alpha > \frac{1-\gamma\tau}{2\gamma\tau}$.

Next, we evaluate $\Pi_1'(Q_1)$ as Q_1 approaches $\overline{Q}_1$ from above. Recalling that when $Q_1 \geq \overline{Q}_1$, $\Pi_1(Q_1)$ is given by the second line of (4), we get

$$\Pi_1'(Q_1) = MR(Q_1) - c + \delta\alpha\left[p'(Q_1)q_1^* + (p(Q_1) - c)\frac{\partial q_1^*}{\partial Q_1^+}\right], \tag{17}$$

where $\frac{\partial q_1^*}{\partial Q_1^+}$ is the derivative of q_1^* with respect to Q_1 when $Q_1 > \overline{Q}_1$.

Using $\Pi_1'\left(\overline{Q}_1^+\right)$ to denote the derivative of $\Pi_1(Q_1)$ as Q_1 approaches $\overline{Q}_1$ from above, using (3), and recalling from (16) that when $Q_1 = \overline{Q}_1$, $\frac{q_1^*}{\overline{Q}_1} = \frac{1-\gamma\tau}{2}$,

$$\begin{aligned}
\Pi_1'\left(\overline{Q}_1^+\right) &= MR(\overline{Q}_1) - c + \delta\alpha\left[p'(\overline{Q}_1)q_1^* + (p(\overline{Q}_1) - c)\frac{\partial q_1^*}{\partial Q_1^+}\right] \\
&= p'(\overline{Q}_1)\left(\frac{1-\gamma\tau}{2}\right)\overline{Q}_1 + \delta\alpha\left[p'(\overline{Q}_1)q_1^* + (p(\overline{Q}_1) - c)\frac{\partial q_1^*}{\partial Q_1^+}\right] \\
&= p'(\overline{Q}_1)\left(\frac{1-\gamma\tau}{2}\right)\overline{Q}_1 + \delta\alpha\left[p'(\overline{Q}_1)q_1^* - p'(\overline{Q}_1)\left(\frac{1+\gamma\tau}{2}\right)\overline{Q}_1\frac{\partial q_1^*}{\partial Q_1^+}\right] \\
&= p'(\overline{Q}_1)\overline{Q}_1\left[\frac{1-\gamma\tau}{2} + \delta\alpha\left(\frac{q_1^*}{\overline{Q}_1} - \left(\frac{1+\gamma\tau}{2}\right)\frac{\partial q_1^*}{\partial Q_1^+}\right)\right] \\
&= -\delta\alpha\left(\frac{1+\gamma\tau}{2}\right)p'(\overline{Q}_1)\overline{Q}_1\left[\frac{\partial q_1^*}{\partial Q_1^+} - \frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)}\right].
\end{aligned} \tag{18}$$

Since $p'(\overline{Q}_1) < 0$, (18) implies that $\Pi_1'\left(\overline{Q}_1^+\right)$ has the same sign as $\frac{\partial q_1^*}{\partial Q_1^+} - \frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)}$. Recall from Lemma 5 that $\frac{\partial q_1^*}{\partial Q_1^+} > 1$. Since $\frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)}$ is decreasing with $\delta\alpha$ from ∞ when $\delta\alpha = 0$ to $\frac{2(1-\gamma\tau)}{1+\gamma\tau} < 2$ when $\delta\alpha = 1$, it follows that $\Pi_1'\left(\overline{Q}_1^+\right) \leq 0$ for $\delta\alpha$ sufficiently small. In particular, since $\frac{\partial q_1^*}{\partial Q_1^+} > 1$, a necessary condition for $\Pi_1'\left(\overline{Q}_1^+\right) \leq 0$ is $\frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)} > 1$, which is equivalent to $\delta\alpha < \frac{1-\gamma\tau}{2\gamma\tau}$. In turn, $\Pi_1'\left(\overline{Q}_1^+\right) \leq 0$ implies $Q_1^* \leq \overline{Q}_1$.

By contrast, recalling that $\frac{\partial q_1^*}{\partial Q_1^+} > 1$, it follows that $\frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)} < 1$, or $\delta\alpha > \frac{1-\gamma\tau}{2\gamma\tau}$, is sufficient for $\frac{\partial q_1^*}{\partial Q_1^+} > \frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)}$. Then, $\Pi_1'\left(\overline{Q}_1^+\right) > 0$, in which case $Q_1^* > \overline{Q}_1$, provided that in addition, $\Pi_1'\left(\overline{Q}_1^-\right) \geq 0$.

Altogether then, the analysis of $\Pi_1'\left(\overline{Q}_1^+\right)$ and $\Pi_1'\left(\overline{Q}_1^-\right)$ implies that there are four possible cases that can arise:

(i) $\Pi_1'\left(\overline{Q}_1^-\right) < 0$ and $\Pi_1'\left(\overline{Q}_1^+\right) < 0$, so $Q_1^* < \overline{Q}_1$, when

$$\frac{\partial q_1^*}{\partial Q_1^+} < \frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)}, \qquad \text{and} \qquad \frac{\partial q_2^*}{\partial Q_1^-} > \frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)}.$$

Both inequalities hold when $\delta\alpha$ is sufficiently small. Moreover, $\delta\alpha < \frac{1-\gamma\tau}{2\gamma\tau}$ is necessary for the first inequality and sufficient for the second.

(ii) $\Pi_1'\left(\overline{Q}_1^-\right) \geq 0$ and $\Pi_1'\left(\overline{Q}_1^+\right) < 0$, so $Q_1^* = \overline{Q}_1$, when

$$\frac{\partial q_1^*}{\partial Q_1^+} < \frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)}, \qquad \text{and} \qquad \frac{\partial q_2^*}{\partial Q_1^-} \leq \frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)}.$$

Both inequalities cannot hold simultaneously, however, because $\delta\alpha < \frac{1-\gamma\tau}{2\gamma\tau}$ is necessary for the first inequality, but when it holds, $\frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)} < 0$, because

$$\gamma\tau(1+2\delta\alpha)-1 < \gamma\tau\left(1+2\left(\frac{1-\gamma\tau}{2\gamma\tau}\right)\right)-1 < 0.$$

Since $\frac{\partial q_2^*}{\partial Q_1^-} > 0$, we cannot have $\frac{\partial q_2^*}{\partial Q_1^-} \leq \frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)}$.

(iii) $\Pi_1'\left(\overline{Q}_1^-\right) \geq 0$ and $\Pi_1'\left(\overline{Q}_1^+\right) > 0$, so $Q_1^* > \overline{Q}_1$, when

$$\frac{\partial q_1^*}{\partial Q_1^+} \geq \frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)}, \qquad \text{and} \qquad \frac{\partial q_2^*}{\partial Q_1^-} > \frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)}.$$

$\delta\alpha > \frac{1-\gamma\tau}{2\gamma\tau}$ is sufficient for the first inequality and necessary for the second.

(iv) $\Pi_1'\left(\overline{Q}_1^-\right) < 0$ and $\Pi_1'\left(\overline{Q}_1^+\right) > 0$, in which case there are two local optima, one below and one above $\overline{Q}_1$. This case arises when

$$\frac{\partial q_1^*}{\partial Q_1^+} > \frac{(1+\delta\alpha)(1-\gamma\tau)}{\delta\alpha(1+\gamma\tau)}, \qquad \text{and} \qquad \frac{\partial q_2^*}{\partial Q_1^-} < \frac{\gamma\tau(1+2\delta\alpha)-1}{\delta\alpha(1+\gamma\tau)}.$$

$\delta\alpha > \frac{1-\gamma\tau}{2\gamma\tau}$ is sufficient for both inequalities. □

Proof of Proposition 2. Suppose that $Q_1^* < \overline{Q}_1$. Then Q_1^* is implicitly defined by $\Pi_1'(Q_1) = 0$, where $\Pi_1'(Q_1)$ is given by (14). Fully differentiating the equation, using

Assumption A3, and the fact that $\Pi_1'(Q_1) = 0$,

$$\frac{\partial Q_1^*}{\partial(\delta\alpha)} = \frac{\gamma\tau(MR(Q_1) - p(q_1^* + q_2^*)) - p'(q_1^* + q_2^*)(q_1^* + \gamma\tau Q_1)\frac{\partial q_2^*}{\partial Q_1}}{\Pi_1''(Q_1)} = \frac{MR(Q_1) - c}{\delta\alpha\Pi_1''(Q_1)} > 0,$$

where the inequality follows because $\Pi_1''(Q_1) < 0$ and because $Q_1^* > Q^M$ implies that $MR(Q_1) < c$.

Likewise, when $Q_1^* > \overline{Q}_1$, Q_1^* is implicitly defined by $\Pi_1'(Q_1) = 0$, where $\Pi_1'(Q_1)$ is given by (17). Fully differentiating the equation, using Assumption A3, and the fact that $\Pi_1'(Q_1) = 0$,

$$\frac{\partial Q_1^*}{\partial(\delta\alpha)} = -\frac{p'(Q_1)q_1^* + (p(Q_1) - c)\frac{\partial q_1^*}{\partial Q_1^+}}{\Pi_1''(Q_1)} = \frac{MR(Q_1) - c}{\delta\alpha\Pi_1''(Q_1)} > 0,$$

where the inequality follows because $\Pi_1''(Q_1) < 0$ and because $Q_1^* > Q^M$ implies that $MR(Q_1) < c$.

As for $\gamma\tau$, note that when $Q_1^* > \overline{Q}_1$, $\Pi_1(Q_1)$ is independent of $\gamma\tau$. $\square$

The equilibrium in the retrospective benchmark case under quantity competition when demand is linear: Absent entry in period 2, firm 1 produces the monopoly output $\frac{A}{2}$, where $A \equiv a - c$, and earns the monopoly profit $\left(\frac{A}{2}\right)^2$. If entry takes place, p_1 can be deemed excessive if it exceeds p_2, i.e., $a - (q_1 + q_2) < a - Q_1$, or $q_1 + q_2 > Q_1$. Hence, the period 2 profits are,

$$\pi_1(q_1, q_2) = \begin{cases} (A - q_1 - q_2)q_1, & q_1 + q_2 \le Q_1, \\ (A - q_1 - q_2)q_1 - \gamma\tau[(a - Q_1) - (a - q_1 - q_2)]Q_1, & q_1 + q_2 > Q_1, \end{cases}$$

and

$$\pi_2(q_1, q_2) = (A - q_1 - q_2)q_2.$$

The best-response function of firm 2 is defined by the familiar Cournot best-response function,

$$BR_2(q_1) = r_2^C(q_1) = \frac{A - q_1}{2}.$$

The best-response function of firm 1 is equal to the Cournot best-response function $r_1^C(q_2) = \frac{A-q_2}{2}$ if $r_1^C(q_2) + q_2 \le Q_1$, i.e., if $\frac{A-q_2}{2} + q_2 = \frac{A+q_2}{2} \le Q_1$. If $r_1^C(q_2) + q_2 > Q_1$, p_1 is deemed excessive with probability γ, so the best-response function of firm 1 maximizes the second line of $\pi_1(q_1,\ q_2)$ and hence is given by $r_1^E(q_2) = \frac{A-q_2}{2} - \frac{\gamma\tau Q_1}{2}$. But then $r_1^E(q_2) + q_2 \ge Q_1$ only if $\frac{A-q_2}{2} - \frac{\gamma\tau Q_1}{2} + q_2 > Q_1$, or $\frac{A+q_2}{2+\gamma\tau Q_1} > Q_1$. And if $\frac{A+q_2}{2+\gamma\tau Q_1} \le Q_1 < \frac{A+q_2}{2}$, the best-response function of firm 1 is $Q_1 - q_2$. Using the definitions of $r_1^C(q_2)$ and $r_1^E(q_2)$, and rearranging terms, we have:

$$BR_1(q_2) = \begin{cases} r_1^C(q_2) = \frac{A-q_2}{2}, & \frac{A+q_2}{2} \leq Q_1, \\ Q_1 - q_2, & \frac{A+q_2}{2+\gamma\tau Q_1} \leq Q_1 < \frac{A+q_2}{2}, \\ r_1^E(q_2) = \frac{A-q_2}{2} - \frac{\gamma\tau Q_1}{2}, & Q_1 < \frac{A+q_2}{2+\gamma\tau Q_1}. \end{cases}$$

It can be easily checked that this expression coincides with the best-response function of firm 1 characterized in Lemma 1 when $p = a - Q$.

The Nash equilibrium in period 2 is attained at the intersection of $BR_1(q_2)$ and $BR_2(q_1)$. There are three possible cases to consider. First, if $r_1^C(q_2)$ and $r_2^C(q_1)$ intersect below the $q_1 + q_2 = Q_1$ line, we obtain the Cournot equilibrium, $\left(q_1^C, q_2^C\right) = \left(\frac{A}{3}, \frac{A}{3}\right)$. This equilibrium can arise however only if the aggregate output, $\frac{2A}{3}$, is below Q_1. But if $\frac{2A}{3} < Q_1$, firm 1 can lower Q_1 towards the monopoly output $\frac{A}{2}$ and thereby raise its period 1 profit without making p_1 excessive (since $Q_1 > \frac{2A}{3}$). Hence in equilibrium, $Q_1 \leq \frac{2A}{3}$, meaning that the Cournot outcome is not an equilibrium.

Second, suppose that $r_1^E(q_2)$ and $r_2^C(q_1)$ intersect above the $q_1 + q_2 = Q_1$ line, in which case p_1 is excessive. Then the equilibrium is given by $\left(\frac{A-2\gamma\tau Q_1}{3}, \frac{A+\gamma\tau Q_1}{3}\right)$.[28] This case can arise, however, only if the aggregate output in equilibrium, $\frac{2A-\gamma\tau Q_1}{3}$, exceeds Q_1, or equivalently if $Q_1 < \frac{2A}{3+\gamma\tau} \equiv \overline{Q}_1$. Notice that $\frac{2A}{3+\gamma\tau}$ satisfies Eq. (3) when $p = a - Q$, and is below the monopoly output $\frac{A}{2}$, as Lemma 4 shows.

Third, if $\frac{2A}{3+\gamma\tau} \leq Q_1 < \frac{2A}{3}$, the equilibrium is attained at the intersection of $r_2^C(q_1) = \frac{A-q_1}{2}$ and $q_1 = Q_1 - q_2$, and is given by $(2Q_1 - A, A - Q_1)$. Now, p_1 is not excessive since firm 1 sets q_1^* such that $p_2 = p_1$.

Next, we turn to period 1. Since p_1 is excessive when $Q_1 \leq \frac{2A}{3+\gamma\tau}$, but not when $\frac{2A}{3+\gamma\tau} \leq Q_1 < \frac{2A}{3}$, the expected discounted profit of firm 1 in period 1 is

$$\Pi_1(Q_1) = \begin{cases} (A - Q_1)Q_1 + \delta(1-\alpha)\left(\frac{A}{2}\right)^2 \\ + \delta\alpha\left[\left(\frac{A}{3}\right)^2 - \frac{\gamma\tau Q_1(7A-(9+\gamma\tau)Q_1)}{9}\right], & Q_1 < \frac{2A}{3+\gamma\tau}, \\ (A - Q_1)Q_1 + \delta(1-\alpha)\left(\frac{A}{2}\right)^2 \\ + \delta\alpha(A - Q_1)(2Q_1 - A), & \frac{2A}{3+\gamma\tau} \leq Q_1 \leq \frac{2A}{3}. \end{cases} \tag{19}$$

Note that $\Pi_1(Q_1)$ is continuous at $Q_1 = \frac{2A}{3+\gamma\tau} \equiv \overline{Q}_1$. Firm 1 sets Q_1 to maximize this expression. Differentiating $\Pi_1(Q_1)$ yields,

$$\Pi_1'(Q_1) = \begin{cases} A - 2Q_1 - \frac{\delta\alpha\gamma\tau}{9}[7A - 2(9+\gamma\tau)Q_1], & Q_1 < \frac{2A}{3+\gamma\tau}, \\ A - 2Q_1 + \delta\alpha(3A - 4Q_1), & \frac{2A}{3+\gamma\tau} \leq Q_1 \leq \frac{2A}{3}. \end{cases} \tag{20}$$

Note that $\Pi_1''(Q_1) < 0$ for $\frac{2A}{3+\gamma\tau} \leq Q_1 \leq \frac{2A}{3}$ and $\Pi_1''(Q_1) \leq 0$ for $Q_1 < \frac{2A}{3+\gamma\tau}$, provided that $\delta\alpha \leq \frac{9}{\gamma\tau(9+\gamma\tau)} < 0.9$, where the last inequality follows since $\gamma\tau < 1$ by Assumption A2. Consequently, $\delta\alpha < 0.9$ is sufficient to ensure that $\Pi_1(Q_1)$ is piecewise concave.

[28] Since $\gamma\tau \leq 1$, $Q_1 < \frac{A}{2\gamma\tau}$, which implies in turn that $q_1^* = \frac{A-2\gamma\tau Q_1}{3} > 0$.

Evaluating $\Pi_1'(Q_1)$ as Q_1 approaches $\overline{Q}_1 \equiv \frac{2A}{3+\gamma\tau}$ from below and noting that in this case, $\Pi_1'(Q_1)$ is given by the first line in (20) yields,

$$\begin{aligned}\Pi_1'\left(\overline{Q}_1^-\right) &= A - \frac{4A}{3+\gamma\tau} - \frac{\delta\alpha\gamma\tau}{9}\left[7A - \frac{4A(9+\gamma\tau)}{3+\gamma\tau}\right] \\ &= \frac{A}{9(3+\gamma\tau)}[27 + 9\gamma\tau - 36 - 7\delta\alpha\gamma\tau(3+\gamma\tau) + 4\delta\alpha\gamma\tau(9+\gamma\tau)] \\ &= \frac{3A\gamma\tau(5-\gamma\tau)}{9(3+\gamma\tau)}\left[\delta\alpha - \frac{3(1-\gamma\tau)}{\gamma\tau(5-\gamma\tau)}\right].\end{aligned}$$

Likewise, evaluating $\Pi_1'(Q_1)$ when Q_1 approaches $\overline{Q}_1 \equiv \frac{2A}{3+\gamma\tau}$ from above and noting that in this case $\Pi_1'(Q_1)$ is given by the second line in (20),

$$\begin{aligned}\Pi_1'\left(\overline{Q}_1^+\right) &= A - \frac{4A}{3+\gamma\tau} + \delta\alpha\left(3A - \frac{8A}{3+\gamma\tau}\right) \\ &= \frac{A}{3+\gamma\tau}[3 + \gamma\tau - 4 + 9\delta\alpha + 3\delta\alpha\gamma\tau - 8\delta\alpha] \\ &= \frac{A(1+3\gamma\tau)}{3+\gamma\tau}\left[\delta\alpha - \frac{1-\gamma\tau}{1+3\gamma\tau}\right].\end{aligned}$$

Noting that

$$\frac{3(1-\gamma\tau)}{\gamma\tau(5-\gamma\tau)} - \frac{1-\gamma\tau}{1+3\gamma\tau} = \frac{(1-\gamma\tau)\left(3 + 4\gamma\tau + (\gamma\tau)^2\right)}{\gamma\tau(5-\gamma\tau)(1+3\gamma\tau)} > 0,$$

there are now three cases to consider.

(i) If $\delta\alpha < \frac{1-\gamma\tau}{1+3\gamma\tau} < \frac{3(1-\gamma\tau)}{\gamma\tau(5-\gamma\tau)}$, then $\Pi_1'\left(\overline{Q}_1^-\right) < 0$ and $\Pi_1'\left(\overline{Q}_1^+\right) < 0$, so $Q_1^* < \overline{Q}_1$. Then, $\Pi_1'(Q_1)$ is given by the first line in (20). Setting it equal to 0 and solving, the equilibrium output of firm 1, Q_1^*, is given by the first line of (22) below. Noting that

$$\frac{2A}{3+\gamma\tau} - \frac{A}{2}\left[\frac{9 - 7\delta\alpha\gamma\tau}{9 - \delta\alpha\gamma\tau(9+\gamma\tau)}\right] = \frac{3A\gamma\tau(5-\gamma\tau)\left[\frac{3(1-\gamma\tau)}{\gamma\tau(5-\gamma\tau)} - \delta\alpha\right]}{2(3+\gamma\tau)(9 - \delta\alpha\gamma\tau(9+\gamma\tau))},$$

it follows that $\frac{A}{2}\left[\frac{9-7\delta\alpha\gamma\tau}{9-\delta\alpha\gamma\tau(9+\gamma\tau)}\right] < \frac{2A}{3+\gamma\tau}$ whenever $\delta\alpha < \frac{3(1-\gamma\tau)}{\gamma\tau(5-\gamma\tau)}$ (otherwise, $Q_1^* = \frac{2A}{3+\gamma\tau}$).

(ii) If $\delta\alpha > \frac{3(1-\gamma\tau)}{\gamma\tau(5-\gamma\tau)} > \frac{1-\gamma\tau}{1+3\gamma\tau}$, then $\Pi_1'\left(\overline{Q}_1^-\right) > 0$ and $\Pi_1'\left(\overline{Q}_1^+\right) > 0$, so $Q_1^* > \overline{Q}_1$. Now $\Pi_1'(Q_1)$ is given by the second line in (20); setting it equal to 0 and solving, reveals that Q_1^* is given by the second line of (22). Note that

$$\frac{A}{2}\left[\frac{1+3\delta\alpha}{1+2\delta\alpha}\right] - \frac{2A}{3+\gamma\tau} = \frac{A(1+3\gamma\tau)\left[\delta\alpha - \frac{1-\gamma\tau}{1+3\gamma\tau}\right]}{2(1+2\delta\alpha)(3+\gamma\tau)}.$$

Hence, $\delta\alpha \geq \frac{1-\gamma\tau}{1+3\gamma\tau}$ ensures that $\frac{A}{2}\left[\frac{1+3\delta\alpha}{1+2\delta\alpha}\right] \geq \frac{2A}{3+\gamma\tau}$ (when $\delta\alpha < \frac{1-\gamma\tau}{1+3\gamma\tau}$, $Q_1^* = \frac{2A}{3+\gamma\tau}$).

(iii) If $\frac{1-\gamma\tau}{1+3\gamma\tau} \leq \delta\alpha < \frac{3(1-\gamma\tau)}{\gamma\tau(5-\gamma\tau)}$, then $\Pi_1'\left(\overline{Q_1^-}\right) < 0 \leq \Pi_1'\left(\overline{Q_1^+}\right)$, so both $\frac{A}{2}\left[\frac{9-7\delta\alpha\gamma\tau}{9-\delta\alpha\gamma\tau(9+\gamma\tau)}\right]$ and $\frac{A}{2}\left[\frac{1+3\delta\alpha}{1+2\delta\alpha}\right]$ are local maxima. To determine which is a global maximum, note that when $\delta\alpha$ is close to $\frac{1-\gamma\tau}{1+3\gamma\tau}$, $\Pi_1'\left(\overline{Q_1^+}\right)$ goes to 0, while $\Pi_1'\left(\overline{Q_1^-}\right) < 0$. Hence, $\frac{A}{2}\left[\frac{9-7\delta\alpha\gamma\tau}{9-\delta\alpha\gamma\tau(9+\gamma\tau)}\right]$ is a global maximum. By contrast, when $\delta\alpha$ goes to $\frac{3(1-\gamma\tau)}{\gamma\tau(5-\gamma\tau)}$, $\Pi_1'\left(\overline{Q_1^-}\right)$ goes to 0, while $\Pi_1'\left(\overline{Q_1^+}\right) > 0$, so $\frac{A}{2}\left[\frac{1+3\delta\alpha}{1+2\delta\alpha}\right]$ is a global maximum. Substituting $Q_1 = \frac{A}{2}\left[\frac{9-7\delta\alpha\gamma\tau}{9-\delta\alpha\gamma\tau(9+\gamma\tau)}\right]$ into the first line of (19) and $Q_1 = \frac{A}{2}\left[\frac{1+3\delta\alpha}{1+2\delta\alpha}\right]$ into the second line of (19) and comparing the resulting expressions, reveals that $\frac{A}{2}\left[\frac{9-7\delta\alpha\gamma\tau}{9-\delta\alpha\gamma\tau(9+\gamma\tau)}\right]$ is a global maximum if $\delta\alpha < Z(\gamma\tau)$, whereas $\frac{A}{2}\left[\frac{1+3\delta\alpha}{1+2\delta\alpha}\right]$ is a global maximum if $\delta\alpha > Z(\gamma\tau)$, where

$$Z(\gamma\tau) \equiv \frac{1+7\gamma\tau(2-\gamma\tau)-(1+\gamma\tau)\sqrt{1+5\gamma\tau(2+\gamma\tau)}}{2\gamma\tau(1+11\gamma\tau)}. \tag{21}$$

Note that $Z(1) = 0$. Plotting $Z(\gamma\tau)$ with Mathematica reveals that $Z'(\gamma\tau) < 0$.

Overall then, the equilibrium output of firm 1 in period 1 is given by

$$Q_1^* = \begin{cases} \frac{A}{2}\left[\frac{9-7\delta\alpha\gamma\tau}{9-\delta\alpha\gamma\tau(9+\gamma\tau)}\right], & \delta\alpha < Z(\gamma\tau), \\ \frac{A}{2}\left[\frac{1+3\delta\alpha}{1+2\delta\alpha}\right], & \delta\alpha \geq Z(\gamma\tau), \end{cases} \tag{22}$$

where p_1 is excessive if $\delta\alpha < Z(\gamma\tau)$, but not otherwise.

Note that so long as $0 < \delta\alpha < 1$, $\frac{A}{2} < Q_1^* < \frac{2A}{3}$, implying that Q_1^* is above the monopoly level, but below the aggregate Cournot level. Also note that $\frac{\partial Q_1^*}{\partial(\delta\alpha)} > 0$, as we already proved in Proposition 2, and moreover $\frac{\partial Q_1^*}{\partial(\gamma\tau)} = \frac{A\left(18\delta\alpha(1+\gamma\tau)-7(\delta\alpha\gamma\tau)^2\right)}{2(9-\delta\alpha\gamma\tau(9+\gamma\tau))^2} > 0$ when $\delta\alpha < Z(\gamma\tau)$ (p_1 is excessive) and $\frac{\partial Q_1^*}{\partial(\gamma\tau)} = 0$ when $\delta\alpha > Z(\gamma\tau)$ (p_1 is not excessive). □

Proof of Proposition 3. First, note that $CS(Q_1^*)$ increases with $\delta\alpha$ because total output in period 2 is higher under entry and because Q_1^* increases with $\delta\alpha$ as we proved in Proposition 2 (this also follows from (22)).

Second, note from (5) that $CS(Q_1^*)$ depends on $\gamma\tau$ only through Q_1^*. But when $\delta\alpha \geq Z(\gamma\tau)$, (22) shows that Q_1^* is independent of $\gamma\tau$, so $\frac{\partial CS(Q_1^*)}{\partial(\gamma\tau)} = 0$.

Finally, suppose that $\delta\alpha < Z(\gamma\tau)$. Then,

$$\begin{aligned} \frac{\partial CS(Q_1^*)}{\partial(\gamma\tau)} &= \left[Q_1^* - \delta\alpha\left(\frac{2A-\gamma\tau Q_1^*}{3}\right)\frac{\gamma\tau}{3}\right]\frac{\partial Q_1^*}{\partial(\gamma\tau)} \\ &= \frac{1}{9}\left[\left(9+\delta\alpha(\gamma\tau)^2\right)Q_1^* - 2\delta\alpha\gamma\tau A\right]\frac{\partial Q_1^*}{\partial(\gamma\tau)}, \end{aligned}$$

where $\frac{\partial Q_1^*}{\partial(\gamma\tau)} > 0$. Using (22), the square bracketed expression is given by

$$\left(9+\delta\alpha(\gamma\tau)^2\right)\frac{A}{2}\left[\frac{9-7\delta\alpha\gamma\tau}{9-\delta\alpha\gamma\tau(9+\gamma\tau)}\right]-2\delta\alpha\gamma\tau A$$
$$=\frac{A\left[(9-7\delta\alpha\gamma\tau)\left(9+\delta\alpha(\gamma\tau)^2\right)-4\delta\alpha\gamma\tau(9-\delta\alpha\gamma\tau(9+\gamma\tau))\right]}{2(9-\delta\alpha\gamma\tau(9+\gamma\tau))},$$

which is positive for all $\delta\alpha < 0.9$ (this is verified with Mathematica), and hence for all $\delta\alpha < Z(\gamma\tau)$.[29] Hence, $\frac{\partial CS(Q_1^*)}{\partial(\gamma\tau)} > 0$. □

Proof of Proposition 5. First, we rewrite firm 1's profit as

$$\Pi(p_1) = Q(p_1)(p_1-c)+\delta(1-\alpha)\pi^M+\delta\alpha\int_{\underline{p}(p_1)}^{p^M} Q(c_2)(c_2-c)dF(c_2)$$
$$-\delta\alpha\gamma\tau\int_{\underline{p}(p_1)}^{p_1}(p_1-c_2)Q(p_1)dF(c_2).$$

Recalling that $\underline{\mathrm{p}}(p_1)$ is defined implicitly by $Q\left(\underline{p}(p_1)\right)\left(\underline{p}(p_1)-c\right)=\gamma\tau\left(p_1-\underline{p}(p_1)\right)Q(p_1)$, the first-order condition for p_1 is given by

$$\begin{aligned}\Pi'(p_1) &= Q(p_1)+Q'(p_1)(p_1-c)\\ &\quad -\delta\alpha\left[Q\left(\underline{p}(p_1)\right)\left(\underline{p}(p_1)-c\right)-\gamma\tau Q(p_1)\left(p_1-\underline{p}(p_1)\right)\right]f\left(\underline{p}(p_1)\right)\underline{p}'(p_1)\\ &\quad -\delta\alpha\gamma\tau\int_{\underline{p}(p_1)}^{p_1}[Q(p_1)+Q'(p_1)(p_1-c_2)]dF(c_2)\\ &= Q(p_1)+Q'(p_1)(p_1-c)-\delta\alpha\gamma\tau\int_{\underline{p}(p_1)}^{p_1}[Q(p_1)+Q'(p_1)(p_1-c_2)]dF(c_2)=0.\end{aligned}$$

Consistent with Proposition 2, the first-order condition implies that p_1 is decreasing with $\delta\alpha$, which is the discounted probability that firm 2 is born.

Evaluating $\Pi'(p_1)$ at p^M and recalling that p^M is implicitly defined by $Q\left(p^M\right)+Q'\left(p^M\right)\left(p^M-c\right)=0$, yields

$$\begin{aligned}\Pi'\left(p^M\right) &= -\delta\alpha\gamma\tau\int_{\underline{p}(p^M)}^{p^M}\left[Q\left(p^M\right)+Q'\left(p^M\right)\left(p^M-c_2\right)\right]dF(c_2)\\ &= \delta\alpha\gamma\tau Q'\left(p^M\right)\int_{\underline{p}(p^M)}^{p^M}(c_2-c)dF(c_2)<0.\end{aligned}$$

Hence, the prohibition of excessive pricing induces firm 1 to lower p_1 below p^M and therefore benefits consumers in period 1. □

[29] The expression depends on two paramaters: $\delta\alpha$ and $\gamma\tau$. A three dimensional plot shows that the expression is positive for all $\delta\alpha < 0.9$.

Proof of Proposition 6. Suppose first that $p_1 > p_{12}$ which is equivalent to $Q_1 < q_1 + bq_2$. Then, the Nash equilibrium is defined by the following first-order conditions:

$$\frac{\partial \Pi_1(Q_1, q_1, q_2)}{\partial Q_1} = A - 2Q_1 - \gamma\tau[q_1 + bq_2 - Q_1] + \gamma\tau Q_1 = 0,$$

$$\frac{\partial \Pi_1(Q_1, q_1, q_2)}{\partial q_1} = A - 2q_1 - bq_2 - \gamma\tau Q_1 = 0,$$

and

$$\frac{\partial \Pi_2(, q_1, q_2)}{\partial q_2} = A - 2q_2 - bq_1 = 0.$$

Solving the three equations yields (10) and (11). Straightforward computations reveal that $Q_1^* < q_1^* + bq_2^*$, i.e., p_1 is excessive, if and only if $\gamma\tau < \frac{b(2-b)}{4+b-2b^2}$. Moreover, substituting (10) and (11) into (9) and differentiating with respect to $\gamma\tau$ reveals that p_1 is decreasing with $\gamma\tau$, while p_{21} and p_{22} are increasing with $\gamma\tau$.

Suppose then that $\gamma\tau \geq \frac{b(2-b)}{4+b-2b^2}$, so p_{12} is not excessive. Then, firm 1's profit is given by the top line of $\Pi_1(Q_1,\ q_1,\ q_2)$, and in equilibrium, $Q_1^* = \frac{A}{2}$, $q_1^* = q_2^* = \frac{A}{2+b}$. But then $p_1 = \frac{A}{2+b} < \frac{A}{2}$, so p_1 is in fact excessive, a contradiction. Hence, to ensure that p_1 is not excessive, firm 1 must set Q_1 and q_1 such that $Q_1 = q_1 + bq_2$. Its profit then becomes

$$\Pi_1(Q_1, q_1, q_2) = (A - q_1 - bq_2)(q_1 + bq_2) + (A - q_1 - bq_2)q_1.$$

The resulting Nash equilibrium is therefore defined by the following first-order conditions:

$$\frac{\partial \Pi_1(Q_1, q_1, q_2)}{\partial Q_1} = 2A - 4q_1 - 3bq_2 = 0,$$

and

$$\frac{\partial \Pi_2(, q_1, q_2)}{\partial q_2} = A - 2q_2 - bq_1 = 0.$$

Solving, and using the fact that $Q_1 = q_1 + bq_2$, yields (12). □

The equilibrium in the contemporaneous benchmark case under price competition: Repeating the same steps as in Proposition 6, the Nash equilibrium when firms set prices is given by

$$p_1^* = \frac{A\Big((2-b^2)H - (2-b)\gamma\tau + b^2\Big(1 + (\gamma\tau)^2\Big)\Big)}{2((1-b^2)H + (2+b^2)(1-\gamma\tau))},$$

$$p_{12}^* = \frac{A(1-b)(1-\gamma\tau)(2+\gamma\tau+b(1+\gamma\tau))}{(1-b^2)H + (2+b^2)(1-\gamma\tau)},$$

$$p_{22}^* = \frac{A(1-b)((1+b)H + 2(1-\gamma\tau) + b\gamma\tau(1+b))}{2((1-b^2)H + (2+b^2)(1-\gamma\tau))},$$

where $H \equiv 2 - 2\gamma\tau - (\gamma\tau)^2$ if $\gamma\tau < \frac{b(2+b)}{4+b-2b^2}$ and by

$$p_1^* = p_{12}^* = \frac{A(1-b)(4+3b)}{8-5b^2}, \qquad p_{22}^* = \frac{A(1-b)(4+3b-b(1+b))}{8-5b^2}.$$

if $\gamma\tau \geq \frac{b(2+b)}{4+b-2b^2}$. When $\gamma\tau < \frac{b(2+b)}{4+b-2b^2}$, p_1 exceeds p_{12}, in which case p_1 is excessive. Differentiating the equilibrium prices with respect to $\gamma\tau$, reveals that an increase in $\gamma\tau$ leads to a decrease in p_1 and an increase in p_{12} and p_{22}. $\square$

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