The Antitrust Prohibition of Excessive Pricing*

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Abstract

We examine the implications of prohibiting excessive pricing by a dominant firm in a model in where an incumbent is a monopoly in period 1 but may compete with an entrant in period 2. The pre-entry price may retrospectively be deemed excessive if it exceeds the post-entry price, in which case the incumbent may pay a fine proportional to its pre-entry excess revenue. We show that using this retrospective benchmark induces the incumbent to expand output in period 1, but cut it in period 2 if entry takes place. The latter effect facilitates entry. Overall, the prohibition of excessive pricing benefits consumers, especially when the probability of entry is high.

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1 Introduction

Excessive pricing by a dominant firm is probably the most blatant form of abuse of dominant position from the perspective of consumers and is prohibited in many countries. In the EU, the prohibition stems from Article 102 of the Treaty of the Functioning of the European Union, which stipulates, among other things, that “imposing unfair purchase or selling prices” is an abuse of dominant position, which the article condemns. Courts have interpreted this stipulation as including a prohibition of “excessive pricing.”¹ A similar prohibition exists in many other countries, including all OECD countries, except the U.S., Canada, Australia, New Zealand and Mexico.²

One of the main obstacles to an effective implementation of the prohibition of excessive pricing is the lack of a commonly agreed upon definition of what constitutes an “excessive price,” or a generally agreed upon methodology on how to assess it. In practice, antitrust authorities and plaintiffs in excessive pricing cases often base their claims on a comparison of the dominant firm’s price with some competitive benchmark, such as the firm’s own price in other time periods, different geographical markets, or different market segments.³ In a recent example, the British CMA imposed in December 2016 a £84.2 million fine on Pfizer and a £5.2 million fine on its distributor, Flynn Pharma, for charging an excessive price for phenytoin sodium capsules, which are used to treat epilepsy. The claim was based on a price hike following the de-branding of the drug, which meant that it was no longer subject to price regulation.⁴ Similarly, the Italian Market Competition Authority fined Aspen over €5 million in September 2016 for charging excessive prices for four anti-cancer drugs; Aspen raised their prices by 300% to 1,500% after acquiring the rights to commercialize them from GlaxoSmithKline.⁵

¹In the landmark General Motors case in 1975, the European Court of Justice held that a dominant firm’s price is unfair if it is “excessive in relation to the economic value of the service provided.” See Case 26/75, General Motors Continental v. Commission [1975] ECR 1367, at para. 12. The court did not clarify however what the “economic value of the service provided” is, or indeed, how to measure it. The court reiterated this position in the United Brands case in 1978 and held that "charging a price which is excessive because it has no reasonable relation to the economic value of the product supplied would be... an abuse" Case 27/76, United Brands v. Commission [1978] ECR 207, at para. 250.


³Motta and de Streel (2006) document various benchmarks used by the European Commission, including substantial differences between the dominant firm’s prices across different geographic markets, or relative to the prices of smaller rivals. The OFT (2004) suggests similar benchmarks, including prices in other time periods, or the prices of the same products in different markets, or the underlying costs when it is possible to measure them in an economically meaningful way.

⁴See https://www.gov.uk/government/news/cma-fines-pfizer-and-flynn-90-million-for-drug-price-hike-to-nhs. After patents expired in September 2012, Pfizer sold the rights for distribution of the drug in the UK to Flynn Pharma, which in turn de-branded the drug, in order to avoid price regulation, and raised its price to the British National Health Services from £2.83 to £67.50, before reducing it to £54 in May 2014.

⁵See https://www.natlawreview.com/article/italy-s-agcm-market-competition-authority-fines-aspen-eur-5-
In this paper, we consider a retrospective benchmark: the price that prevails after a rival enters the market and starts competing with the dominant firm is used as a benchmark to assess whether the firm's pre-entry price was excessive. Such a retrospective benchmark was in fact used in two class actions in Israel. The first class action, approved by the Israeli District Court, alleged that the merchant fees charged by the incumbent acquirer of Visa cards were excessive prior to the entry of a new credit card company in 1998. The allegation was based on the fact that following entry, merchant fees dropped from more than 4% to approximately 2% shortly after entry. The Israeli Supreme Court, however, dismissed the claim on appeal, mainly because the entrant went out of business shortly after entering the market, implying that the post-entry price was not a valid benchmark.

Similarly, the Israeli District Court approved a class action against the former telecom monopoly Bezeq, alleging that the prices it charged for international phone calls were approximately 400% higher before two rivals entered the market in 1997 when it was liberalized. The Israeli Supreme Court ultimately dismissed this claim as well, this time on the grounds that before liberalization, prices were set by regulators, meaning that Bezeq did not abuse its dominant position.

Using the post-entry price as a benchmark to assess whether the pre-entry price was excessive is relatively easy, and, as we shall see below, can benefit consumers by restraining the dominant firm’s pre-entry behavior and by encouraging entry. This benchmark is likely to be used by plaintiffs in class actions when they sue for damages due to excessive pricing. Although we are not aware of cases where antitrust agencies have used such a benchmark, we suspect that this may be due to the tendency of antitrust agencies to focus on ongoing abuses of dominant position, rather than going after past abuses. This tendency, however, overlooks the pro-competitive effects of

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The European Commission recently announced that it is opening an investigation against Aspen for excessive pricing of the drugs outside of Italy. See http://europa.eu/rapid/press-release_IP-17-1323_en.htm. Another recent example is a class action in Israel alleging that Dead Sea Works Ltd, which is a monopoly in the supply of potash, participated in an international potash cartel and charged farmers an excessive price for potash. The Tel Aviv district court approved a settlement based on the fact that Dead Sea Works raised its price from $200 per ton in 2007 to $1,000 per ton in 2008-9. See T"Z 41838-09-14 Weinstein v. Dead Sea Works, Inc., at para. 13.  
6See BS"A (T"A) 106462/98 Howard Rice v. Cartisei Ashrai Leisrael Ltd.. Tk-Mh 2003(1). 
7See CC 3105/03 Isracard Ltd. v. Howard Rice, P"D N"T(5) 701, at para. 22. 
9See PCC 729/04 Bezeq Beinleumi Ltd. v. Kay Machshava (Nevo 26.4.2010). 
10Although private antitrust enforcement is still uncommon in the EU, there seems to be a change in this respect (see Bovis and Clarke, 2015).
11A case in point is the Israeli Antitrust Authority, which states in its revised guidelines on excessive pricing (Guideline 1/17) that it will not intervene if competition corrected or is likely to correct the high price. Similarly, in the UK, the OFT closed in 2002 an excessive pricing investigation against condom manufacturer Durex, because new entrants, such as Trojan condoms, have entered the market. The OFT reasoned that “any potential remedies such as a price cap could stifle such entry and hinder rather than help the competitive process” (see Niels et al., 2011).
the prohibition of excessive pricing that we highlight in this paper.\textsuperscript{12}

Our analysis is based on a two-period model, in which an incumbent is a monopoly in period 1, but may face competition from an entrant in period 2. Absent entry, the incumbent acts in period 2 as a monopoly. But if entry occurs, the incumbent and entrant produce homogenous products, and compete by setting quantities.\textsuperscript{13} Under the retrospective benchmark which we consider, the incumbent anticipates that if, following entry, the price drops below the period 1 price, the latter may be deemed excessive, in which case the incumbent will have to pay a fine which is proportional to the excess revenue it made in period 1. Consequently, the incumbent has an incentive to expand its period 1 output to lower the pre-entry price, but at the same time, it also has an incentive to cut its period 2 output if entry occurs, to prevent the post-entry price from dropping by too much. The incumbent may in fact expand output in period 1 to the point where the resulting pre-entry price is just equal to the post-entry price, and hence is no longer excessive. This incentive is particularly strong when the expected fines and the discounted probability of entry are large. Although the entrant responds to the incumbent’s softer behavior by expanding its own output, aggregate output in period 2 is nonetheless lower than it would be absent a prohibition of excessive pricing.

Our analysis has a number of interesting implications. First, the retrospective benchmark that we consider involves a trade-off: the pre-entry output is higher while the post-entry output is lower than they would be absent prohibition of excessive pricing. We show, however, that the expansion of pre-entry output exceeds the contraction of post-entry output, and moreover, when demand is linear, the discounted expected consumers’ surplus is higher than it would be absent a prohibition of excessive pricing. Hence, ignoring other real-life considerations which are not included in our model, such as the incentive to invest, or raise the quality and variety of products, the prohibition of excessive pricing is pro-competitive and benefits consumers.

Second, the incumbent in our model has a stronger incentive to expand its pre-entry output when the discounted probability of entry high. Consequently, the retrospective benchmark that we consider is particularly beneficial to consumers when the incumbent faces a high probability

\textsuperscript{12}In other areas of antitrust, most notably cartel cases, antitrust agencies do not hesitate to take a retrospective approach, even if the cartels had already broke down. Moreover, antitrust agencies have recently began to conduct retrospective merger reviews (e.g., Farrell, Paultler, and Vita, 2009). Hence, at least in principle, there should be no reason to rule out a retrospective approach when it comes to excessive pricing.

\textsuperscript{13}The assumption that the products are homogenous is a reasonable approximation for the markets in the two Israeli class actions mentioned earlier: the acquiring market for Visa credit cards and the market for international phone calls. It is also interesting to note that currently, there are 10 pending class action law suits in Israel alleging excessive pricing. Among the products involved in these cases, are cocoa powder, margarine, white cheese, heavy cream, and green tea. Arguably, these products are fairly homogenous.
of entry. This result is interesting because it is often argued that there is no need to intervene in excessive pricing cases when the probability of entry is high, since then the “market will correct itself.”\textsuperscript{14} This argument, however, ignores the harm to consumers before entry occurs. Once this harm is accounted for, the type of intervention that we examine - suing firm 1 for damages if and when entry occurs - benefits consumers, particularly when the probability of entry is high.

Third, the fact that the incumbent cuts its post-entry output makes entry more profitable and hence promotes it. This result stands in sharp contrast to the often made claims that the prohibition of excessive pricing discourages entry by inducing the incumbent to lower its price.\textsuperscript{15} Although in our model the incumbent indeed expands output before entry occurs and hence charges a lower price, what matters for entry is not the incumbent’s pre-entry behavior, but rather its post-entry behavior. And, as we show, under a retrospective benchmark, the incumbent has an incentive to cut its output following entry to prevent the post-entry price from falling by too much below the pre-entry price and thereby lower the fine it may have to pay.

As far as we know, our paper is the first to examine the competitive implications of the prohibition of excessive prices in the context of a formal economic model. O’Donoghue and Padilla (2006), Motta and de Streele (2003), and Green (2006) review and critically examine the case law and policy issues. They also review different possible benchmarks that can be used to assess if prices are excessive, and discuss their potential drawbacks. Gal (2004) compares the EU and U.S. antitrust laws that apply to the prohibition of excessive pricing and explains the difference between the two systems. Ezrachi and Gilo (2010a, 2010b) critically discuss the main grounds for the reluctance of some antitrust agencies and courts to intervene in excessive pricing cases. Ezrachi and Gilo (2009) also discuss the retrospective benchmark that we consider in this paper, but do it in the context of a legal policy paper without a formal model.

Our analysis is related to the literature on most-favoured-customer (MFN) clauses, which guarantee past consumers a rebate if the price falls in the future. Cooper (1986), Neilson and Winter

\textsuperscript{14}For instance, the OECD competition committee (OECD, 2011) emphasizes that “The existence of high and non-transitory structural entry barriers are probably considered the most important single requirement for conducting an excessive price case.” It also adds that “This requirement is based on the fundamental proposition that competition authorities should not intervene in markets where it is likely that normal competitive forces over time eliminate the possibilities of a dominant company to charge high prices.” Likewise, O’Donoghue and Padilla (2006) write that “The key consideration is to limit intervention to cases in which entry barriers are very high and, therefore, where there is a reasonable prospect that consumers could be exploited.” (p. 635–636). Similarly, Motta and de Streele (2006) write that “exploitative practices are self-correcting because excessive prices will attract new entrants.” (p. 15)

\textsuperscript{15}For instance, Areeda and Hovenkamp (2001) write: “While permitting the monopolist to charge its profit-maximizing price encourages new competition, forcing it to price at a judicially administered competitive level would discourage entry and thus prolong the period of such pricing” (para. 720b). Similar arguments appear in Whish (2003, p. 688–689) and in Economic Advisory Group on Competition Policy (2005, p. 11).
(1993), and Schnitzer (1994) show that competing firms have an incentive to adopt retroactive MFN’s in order to facilitate collusion (MFN’s make firms reluctant to cut future prices in order to avoid paying rebates to past consumers). Although the fine that the dominant firm may have to pay when the post-entry price falls is akin to a rebate to past consumers, the dominant firm is much better off without it, since then it is free to exploit its monopoly power prior to entry, and can respond optimally to entry if it occurs. Moreover, since the prohibition of excessive pricing restrains the dominant firm’s behavior, it is pro-competitive, contrary to MFN’s which facilitate collusion.

Our analysis is also related to the literature that proposes legal rules to deter predatory pricing. Like our paper, this literature also proposes an antitrust policy based on the response of a dominant firm to entry. Williamson (1977) proposes that following entry, the dominant firm will not be able to raise output above the pre-entry level for 12 – 18 months. Similarly, Edlin (2002) proposes to block a dominant firm from significantly cutting its price for a period of 12 – 18 months following substantial entry into its market. Both rules prevent predation. Baumol (1979) argues that the dominant firm should not be prevented from reacting to entry, and proposes instead that it will not be allowed to raise its price if and when the entrant exits the market, unless this is justified by cost or demand changes. This rule prevents recoupment.

The rest of the paper is organized as follows: Section 2 presents the model. Sections 3 and 4 analyze the equilibrium. Section 5 considers the linear demand case which allows us to obtain closed-form solutions, which we use for comparative statics and welfare analysis. In section 6, we show that our analysis applies, with minimal modifications, to the case of a contemporaneous benchmark, whereby the dominant firm’s price is compared to the price that it charges in another market. We conclude in Section 7. The Appendix contains some technical proofs.

2 The model

There are two time periods. In period 1, firm 1 operates as a monopoly. In period 2, firm 1 continues to operate as a monopoly with probability $1 - \alpha$. With probability $\alpha$, firm 2 enters the market and competes with firm 1. We assume that firms 1 and 2 produce homogenous products, and compete by setting quantities. The assumption that products are homogenous is a reasonable approximation for the two Israeli class actions mentioned in the Introduction, as well as several other class actions that are currently pending in court. For simplicity, we assume that both firms
have the same constant marginal cost $c$ and denote the (downward sloping) inverse demand function by $p(Q)$, where $Q$ is the aggregate output level. To ensure that the market is viable, we assume that $p(0) > c$. The intertemporal discount factor is $\delta$.

The prohibition of excessive pricing is enforced in period 2 as follows: if entry occurs and the period 2 price, $p_2$, falls below the period 1 price, $p_1$, a court rules that $p_1$ was excessive probability $\gamma$. The parameter $\gamma$ reflects various legal factors, including the stringency of antitrust enforcement against excessive pricing, the availability of data on prices and quantities needed to support the case, and potential defenses that the dominant firm may have for its high prices, such as the need to recoup large investments. When $p_1$ is deemed excessive, firm 1 has to pay a fine which is proportional to its excessive revenue in period 1 and is given by $\tau (p_1 - p_2) Q_1$, where $\tau > 0$, and $Q_1$ is firm 1’s output in period 1. To ensure that we have interior solutions, we will make the following assumptions:

A1 $p'(Q) + p''(Q)(1 + \gamma)Q < 0$

A2 $\gamma \tau < 1$

Assumption A1 is a modified version of the standard assumption that $p'(Q) + p''(Q)(1 + \gamma)Q < 0$. It is stronger because $\gamma \tau > 0$, but like the standard assumption, it also holds when the demand function is concave or not too convex. The assumption ensures that the marginal revenue functions are downward sloping. Assumption A2 ensures that the expected fine that firm 1 pays is not so large that firm 1 wishes to exit in period 2 when firm 2 enters.\footnote{For example, in the Israeli cases mentioned in the Introduction, $\tau$ was equal to 1 as plaintiffs were suing for the actual damages. Since $\gamma < 1$, $\gamma \tau$ was indeed below 1.}

In the next two sections we characterize the equilibrium in our model. We begin in Section 3 by considering the equilibrium in period 2, and then we turn to period 1 in Section 4.

3 The equilibrium in period 2

Absent entry in period 2, the court cannot evaluate whether $p_1$ was excessive. Hence, firm 1 simply maximizes its period 2 profit by producing the monopoly output, $Q^M$, defined implicitly by $MR(Q) \equiv p(Q) + p'(Q)Q = c$ ("M" stands for "Monopoly").\footnote{Note that if $Q_1 > Q^M$, the price in period 2, $p_2$, will exceed that in period 1, $p_1$. However, absent entry, there is no competitive benchmark in either period, so it is hard to make the case that $p_2$ is excessive. While in the}
Then, \( \pi_2 \) function is given by

\[
\pi_2 (q_1, q_2) = \begin{cases} 
(p(q_1 + q_2) - c)q_1, & q_1 + q_2 \leq Q_1, \\
(p(q_1 + q_2) - c)q_1 - \gamma \tau [p(Q_1) - p(q_1 + q_2)]Q_1, & q_1 + q_2 > Q_1.
\end{cases}
\]

Now suppose that firm 2 enters in period 2 and let \( q_1 \) and \( q_2 \) be the resulting output levels. Given firm 1’s output in period 1, \( Q_1 \), firm 1 can be found liable for having charged an excessive price in period 1 if and only if \( q_1 + q_2 > Q_1 \) (output in period 2 exceeds firm 1’s output in period 1) because then \( p(q_1 + q_2) < p(Q_1) \). Recalling that firm 1 is found liable with probability \( \gamma \) and the fine it pays in this case is equal to \( \tau (p_1 - p_2)Q_1 \), where \( p_1 = p(Q_1) \) and \( p_2 = p(q_1 + q_2) \), the period 2 profits of firms 1 and 2 are given by

\[
\pi_1 (q_1, q_2) = \begin{cases} 
(p(q_1 + q_2) - c)q_1, & q_1 + q_2 \leq Q_1, \\
(p(q_1 + q_2) - c)q_1 - \gamma \tau [p(Q_1) - p(q_1 + q_2)]Q_1, & q_1 + q_2 > Q_1.
\end{cases}
\]

Note that \( \pi_1 (q_1, q_2) \) is continuous at \( q_1 + q_2 = Q_1 \); in the Appendix we prove that Assumption A1 ensures that \( \pi_1 (q_1, q_2) \) is piecewise concave in \( q_1 \) (i.e., both when \( q_1 + q_2 \leq Q_1 \), as well as when \( q_1 + q_2 > Q_1 \)), and \( \pi_2 (q_1, q_2) \) is concave in \( q_2 \).

The next result characterizes the best-response functions of the two firms in period 2.

**Lemma 1:** (The best-response functions under entry) Suppose that firm 2 enters in period 2. Then, firm 2’s best-response function is given by \( BR_2(q_1) = r^C_2(q_1) \), while firm 1’s best-response function is given by

\[
BR_1(q_2) = \begin{cases} 
Q_1 - q_2, & p(Q_1) + p'(Q_1)(Q_1 - q_2) < c, \\
r^C_1(q_2), & p(Q_1) + p'(Q_1)(Q_1 - q_2) > c > p(Q_1) + p'(Q_1)((1 + \gamma \tau)Q_1 - q_2), \\
r^E_1(q_2), & p(Q_1) + p'(Q_1)((1 + \gamma \tau)Q_1 - q_2) > c,
\end{cases}
\]

where \( r^C_1(q_j) \) is the “Cournot” best-response function (“C” stands for “Cournot”), defined implicitly by

\[
p(q_i + q_j) + p'(q_i + q_j)q_i = c,
\]

and \( r^E_1(q_2) \) is firm 1’s best-response function against \( q_2 \) when \( p_1 \) is excessive (“E” stands for
“Excessive”), defined implicitly by

\[ p(q_1 + q_2) + p'(q_1 + q_2) (q_1 + \gamma \tau Q_1) = c. \]  

(4)

Assumption A1 ensures that both best-response functions are downward sloping in the \((q_1, q_2)\) space and \(BR'_1(\cdot) \leq -1 \leq BR'_2(\cdot) < 0\), with \(BR'_1(\cdot) = -1\) only when \(q_1 + q_2 = Q_1\).

**Proof:** Since firm 2’s profit is the traditional Cournot profit, \(BR_2(q_1) = r^C_2(q_1)\), where \(r^C_2(q_1)\) is defined by (3). To characterize \(BR_1(q_2)\), note that

\[
\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} = \begin{cases} 
  p(q_1 + q_2) + p'(q_1 + q_2) q_1 - c, & q_1 + q_2 \leq Q_1, \\
  p(q_1 + q_2) + p'(q_1 + q_2) (q_1 + \gamma \tau Q_1) - c, & q_1 + q_2 > Q_1.
\end{cases}
\]  

(5)

Note that since \(p'(q_1 + q_2) < 0\), \(\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} < 0\) as \(q_1 + q_2\) approaches \(Q_1\) from below also implies that \(\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} < 0\) as \(q_1 + q_2\) approaches \(Q_1\) from above. Together with the fact that \(\pi_1(q_1, q_2)\) is continuous at \(q_1 + q_2 = Q_1\) and piecewise concave, it follows that

(i) \(\pi_1(q_1, q_2)\) attains a maximum at \(q_1 < Q_1 - q_2\) if \(\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} < 0\) as \(q_1 + q_2\) approaches \(Q_1\) from below, i.e., when \(p(Q_1) + p'(Q_1) (Q_1 - q_2) < c\),

(ii) \(\pi_1(q_1, q_2)\) attains a maximum at \(q_1 > Q_1 - q_2\) if \(\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} > 0\) as \(q_1 + q_2\) approaches \(Q_1\) from above, i.e., when \(p(Q_1) + p'(Q_1) ((1 + \gamma \tau) Q_1 - q_2) > c\),

(iii) \(\pi_1(q_1, q_2)\) attains a maximum at \(q_1 = Q_1 - q_2\) if \(\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} > 0\) as \(q_1 + q_2\) approaches \(Q_1\) from below, and \(\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} < 0\) as \(q_1 + q_2\) approaches \(Q_1\) from above, i.e., when \(p(Q_1) + p'(Q_1) (Q_1 - q_2) > c > p(Q_1) + p'(Q_1) ((1 + \gamma \tau) Q_1 - q_2)\).

In case (i), \(p_1\) is not excessive, and firm 1’s best-response function is defined by \(r^C_1(q_2)\). In case (ii), \(p_1\) is excessive and firm 1’s best-response function is defined by \(r^E_1(q_2)\). And in case (iii), firm 1 sets \(q_1\) to ensure that \(q_1 + q_2 = Q_1\); this ensures that \(p_1\) is not deemed excessive.

To study the slopes of the best-response functions in the \((q_1, q_2)\) space, notice first that

\[
BR'_2(\cdot) = -\frac{\partial^2 \pi_2(q_1^*, q_2^*)}{\partial q_1\partial q_2} = -\frac{p' + p'' q_2}{2p' + p'' q_2},
\]
where the arguments of $p'$ and $p''$ are suppressed to ease notation. Assumption A1 is sufficient to ensure that $-1 \leq BR_2'(\cdot) < 0$. The proof that $BR_1'(\cdot) < -1$ when $BR_1(q_2) = r^C_1(q_2)$ is similar. When $BR_1(q_2) = Q_1 - q_2$, it is obvious that $BR_1'(\cdot) = -1$. Finally, when $BR_1(q_2) = r^E_1(q_2)$, then

$$BR_1'(\cdot) = \frac{\partial^2 \pi_1(q_1, q_2)}{\partial q_1 \partial q_2} = -\frac{2p' + p''(q_1 + \gamma \tau Q_1)}{p' + p''(q_1 + \gamma \tau Q_1)} < -1,$$

where the inequality is implied by Assumption A1.

The best-response function of firm 1 is illustrated in Figure 1.\(^{18}\) The figure shows the Cournot best-response function of firm 1, $r^C_1(q_2)$, as well as its best-response function when $p_1 > p_2$, $r^E_1(q_2)$. The latter lies everywhere below $r^C_1(q_2)$ because when $p_1 > p_2$, firm 1 has, in expectation, an extra marginal cost. This cost arises because an increase in $q_1$ lowers $p_2$ and therefore increases the excessive revenue, $[p(Q_1) - p(q_1 + q_2)]Q_1$, on which firm 1 pays a fine if found liable in court. When $q_1 + q_2 = Q_1$ lies above $r^C_1(q_2)$, the aggregate output in period 2, $r^C_1(q_2) + q_2$, falls short of the output in period 1, $Q_1$, so $p_2 > p_1$, meaning that $p_1$ is not excessive. Hence, the best-response of firm 1 is given by $r^C_1(q_2)$. By contrast, when $q_1 + q_2 = Q_1$ lies below $r^E_1(q_2)$, the aggregate output in period 2, $r^E_1(q_2) + q_2$, exceeds $Q_1$, so now $p_1$ is excessive and the best-response function of firm 1 is given by $r^E_1(q_2)$. And, when $q_1 + q_2 = Q_1$ lies below $r^C_1(q_2)$ but above $r^E_1(q_2)$, firm 1 sets $q_1$ such that $q_1 + q_2 = Q_1$ to ensure that $p_1 = p_2$. Note that in this case, $p_1$ is not excessive, but firm 1 cannot play its Cournot best response against $q_2$ because then $p_1$ will be deemed excessive. In other words, firm 1 is constrained in this case to keep $q_1$ below the $q_1 + q_2 = Q_1$ line to ensure that $p_1$ is not retrospectively deemed excessive.

Overall then, the best-response function of firm 1 is given by the thick downward sloping line in Figure 1. The figure shows three different cases, depending on how large $Q_1$ is.

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\(^{18}\)The best-response functions in Figures 1 and 2 are drawn as linear only for convenience; in general they need not be linear. It the following analysis, however, we do not rely on the linearity of the best-response functions.
Figure 1: The best-response function of firm 1 in period 2

We denote the Nash equilibrium in period 2 following entry by \((q_1^*, q_2^*)\). In the next result, we show that \(q_1^* > 0\) and \(q_2^* > 0\).

**Lemma 2:** (Firm 1 is always active in period 2) Both firms are active in the market when firm 2 enters.

**Proof:** See the Appendix.

Lemma 2 ensures that \(r_1^E(Q^M) > 0\). Since Lemma 1 implies that \(r_1^E(q_2)\) is steeper than \(r_2^C(q_1)\), the two curves intersect, when \(p_1\) is excessive, at the interior of the \((q_1, q_2)\) space. As Figure 2 illustrates, three types of equilibria can emerge, depending on how high \(Q_1\) is.

Figure 2: the Nash equilibrium in period 2
The first type of equilibrium, illustrated in Figure 2a, is the Cournot equilibrium, \((q_1^C, q_2^C)\). It is attained when \(Q_1\) exceeds the aggregate Cournot output in period 2. Then, \(p_1\) is not excessive, so the best-response function of firm 1 is indeed given by the Cournot best-response function, \(r_1^C(q_2)\). Since the Cournot best-response functions are symmetric, the Cournot equilibrium lies on the diagonal in the \((q_1, q_2)\) space.

The second type of equilibrium emerges when \(Q_1\) is below the aggregate Cournot output, but above the aggregate output when \(p_1\) is excessive (the latter is attained at the intersection of \(r_1^F(q_2)\) and \(r_2^C(q_1)\)). As Figure 2b illustrates, firm 1 sets in this case \(q_1 = Q_1 - q_2\), to ensure that \(p_1 = p_2\), so \(p_1\) is not excessive. The equilibrium then, \((q_1^*, q_2^*)\), is defined by the intersection of \(q_1 + q_2 = Q_1\) with \(r_2^C(q_1)\). Since \(q_1 + q_2 = Q_1\) passes below the Cournot equilibrium point, \((q_1^C, q_2^C)\), the equilibrium point \((q_1^*, q_2^*)\) lies above the diagonal in the \((q_1, q_2)\) space, meaning that \(q_2^* > q_1^*\).

The third equilibrium, illustrated in Figure 2c, arises when \(Q_1\) is even lower than the aggregate output produced when \(r_1^F(q_2)\) and \(r_2^C(q_1)\) intersect. Now firm 1 plays a best response against \(q_2\), despite the fact that the resulting price renders \(p_1\) excessive. The equilibrium then is defined by the intersection of \(r_1^F(q_2)\) and \(r_1^C(q_1)\). Since \(r_1^F(q_2) < r_1^C(q_1)\), the equilibrium point again lies above the diagonal in the \((q_1, q_2)\) space, so once again, \(q_2^* > q_1^*\).

In Lemma 6 below, we will prove that firm 1 sets \(Q_1\) such that \(Q^M \leq Q_1 \leq q_1^C + q_2^C\), i.e., between the monopoly output and the aggregate Cournot output. Intuitively, without an antitrust prohibition of excessive prices, firm 1 will set in period 1 the monopoly output, \(Q^M\). When excessive pricing is prohibited, i.e., when \(\gamma \tau > 0\), firm 1 has an incentive to expand \(Q_1\) above \(Q^M\) in order to lower \(p_1\) and therefore the fine that it may have to pay in period 2 should entry occur; hence, \(Q^M \leq Q_1\). The aggregate output in period 2 is at most \(q_1^C + q_2^C\) and is below that when the equilibrium in period 2 is as in Figures 2b or 2c. Hence, \(Q_1 = q_1^C + q_2^C\) is sufficient to ensure that \(p_1\) is not excessive, so firm 1 will never wish to expand \(Q_1\) above \(q_1^C + q_2^C\).

But since \(Q_1 \leq q_1^C + q_2^C\), we never obtain the type of equilibrium illustrated in Figure 2a. Since in the equilibria illustrated in Figures 2b and 2c the best-response of firm 1 lies below its Cournot best-response function, the Nash equilibrium in period 2 is attained in the \((q_1, q_2)\) space below a 45° line that passes through \(q_1^C + q_2^C\). That is, \(q_1^* + q_2^* \leq q_1^C + q_2^C\), with equality holding only when \(\gamma \tau = 0\), in which case \(r_1^F(q_2) = r_1^C(q_1)\).

**Lemma 3:** (The Nash equilibrium in period 2 under entry) The Nash equilibrium in period 2 when firm 2 enters, \((q_1^*, q_2^*)\), is defined implicitly by the intersection of \(r_1^F(q_2)\) and \(r_2^C(q_1)\) if \(p_1\) is
excessive, and by the intersection of \( q_1 + q_2 = Q_1 \) and \( r_2^C(q_1) \) if \( p_1 \) is not excessive. Either way, 
\[ q_1^* \leq q_1^C = q_2^C \leq q_2^* \quad \text{and} \quad q_1^* + q_2^* \leq q_1^C + q_2^C, \]
with equalities holding only when \( \gamma \tau = 0 \).

Lemma 3 implies that when firm 2 enters in period 2, the period 1 output level, \( Q_1 \), matters: 
either \( p_1 \) is excessive and firm 1 pays in expectation a fine that depends on \( Q_1 \), or firm 1 chooses 
its output in period 2 such that \( q_1 + q_2 = Q_1 \) to ensure that \( p_1 \) is not excessive. Either way, in 
equilibrium, \( q_1 \) and \( q_2 \) depend on \( Q_1 \).

An important implication of Lemma 3 is that whenever \( \gamma \tau > 0 \), 
\[ \pi_2(q_1^*, q_2^*) > \pi_2(q_1^C, q_2^C), \]
where the first inequality follows by revealed preferences and the second follows because 
\( q_1^* < q_1^C \). Since firm 2 makes more money when firm 1 is subject to a prohibition of excessive 
pricing, the prohibition encourages entry, contrary to what many scholars claim. As we mention in 
the Introduction, the claims that the prohibition of excessive pricing discourages entry is based on 
the idea that a high pre-entry price will attract entry, while a low pre-entry price may discourage 
it. However, entrants base their entry decisions on the anticipated behavior of incumbents after 
entry takes place, not before it does. As the analysis above shows, the fine that firm 1 may have 
to pay in period 2 softens its behavior in period 2 and therefore encourages entry.

Whether the Nash equilibrium in period 2 is such that \( p_1 \) is excessive (as in Figure 2c) or is 
not excessive (as in Figure 2b) depends on the size of \( Q_1 \). Let \( \overline{Q}_1 \) be the critical value of \( Q_1 \) such 
that \( p_1 \) is excessive if \( Q_1 < \overline{Q}_1 \) and is not excessive if \( Q_1 \geq \overline{Q}_1 \). Note that \( \overline{Q}_1 \) is attained when 
\( q_1 + q_2 = Q_1 \) passes through the intersection of \( r_1^F(q_2) \) and \( r_2^C(q_1) \). Hence, \( \overline{Q}_1 \) has to satisfy (3) 
when \( q_i = q_2 \), (4), and \( q_1 + q_2 = \overline{Q}_1 \). Substituting for \( q_2 \) from the last equation into (3) and (4) 
yields 
\[ p(\overline{Q}_1) + p'(\overline{Q}_1)(\overline{Q}_1 - q_1) = c, \]
and 
\[ p(\overline{Q}_1) + p'(\overline{Q}_1)(q_1 + \gamma \tau \overline{Q}_1) = c. \]
Adding the two equations and simplifying, \( \overline{Q}_1 \) is implicitly defined by the equation 
\[ p(\overline{Q}_1) + p'(\overline{Q}_1) \left( \frac{1 + \gamma \tau}{2} \right) \overline{Q}_1 = c. \]

**Lemma 4:** (The properties of \( \overline{Q}_1 \) ) \( Q^M < \overline{Q}_1 < q_1^C + q_2^C \) and \( \overline{Q}_1 \) is decreasing with the size of the 
extected fine, \( \gamma \tau \). The market shares of firms 1 and 2 when \( Q_1 = \overline{Q}_1 \) are \( \frac{1 - \gamma \tau}{2} \) and \( \frac{1 + \gamma \tau}{2} \).
Lemma 4: See the Appendix.

Lemma 4 provides a lower and upper bound on $Q_1$, which is the critical value of $Q_1$ that delineates equilibria in which $p_1$ is excessive from equilibria in which $p_1$ is not excessive. Lemma 4 also implies that as the expected fine, $\gamma\tau$, increases, the range of parameters for which $p_1$ is excessive (which happens when $Q_1 < Q_1$) shrinks. That is, an increase in $\gamma\tau$ makes it more likely that firm 1 will set $Q_1$ such that given the ensuing equilibrium in period 2, $p_1$ will not be deemed excessive. Notice that at the limit, as $\gamma\tau$ approaches 1, (6) coincides with the first order condition for $Q^M$, implying that $Q_1 = Q^M$. Since $Q_1 < Q^M$, it follows that as $\gamma\tau$ approaches 1, $p_1$ will always be excessive if entry occurs in period 2.

We conclude this section by studying the effect of $Q_1$ on the Nash equilibrium in period 2.

Lemma 5: (The effect of $Q_1$ on the equilibrium in period 2) $-\gamma\tau < \frac{\partial q^*_1}{\partial Q_1} < 0 < \frac{\partial q^*_2}{\partial Q_1} < \gamma\tau$ and $\frac{\partial (q^*_1 + q^*_2)}{\partial Q_1} < -\gamma\tau$ if $Q_1 < \overline{Q}_1$ ($p_1$ is excessive), and $\frac{\partial q^*_1}{\partial Q_1} > 1$, $\frac{\partial q^*_2}{\partial Q_1} < 0$, and $\frac{\partial (q^*_1 + q^*_2)}{\partial Q_1} = 1$ if $Q_1 \geq \overline{Q}_1$ ($p_1$ is not excessive).

Lemma 5 shows that the Nash equilibrium output of firm 1 in period 2, $q^*_1$, is a U-shaped function of $Q_1$: $q^*_1$ decreases with $Q_1$ so long as $Q_1 < \overline{Q}_1$, but once $Q_1 \geq \overline{Q}_1$, a further increase in $Q_1$ leads to an increase in $q^*_1$. The intuition for this non-monotonic relationship between $q^*_1$ and $Q_1$ is as follows: whenever $Q_1 < \overline{Q}_1$, $p_1$ is excessive. Firm 1 has an incentive to limit $q^*_1$ in order to keep $p_2$ high, and thereby lower the expected fine it has to pay. This incentive becomes stronger as $Q_1$ increases because the fine is proportional to $Q_1$. However, once $Q_1 \geq \overline{Q}_1$, firm 1 chooses $q^*_1$ such that $q^*_1 + q^*_2 = Q_1$, so now $q^*_1$ increases with $Q_1$.

4 The equilibrium in period 1

In period 1, firm 1 chooses $Q_1$ in order to maximize the discounted sum of its period 1 and period 2 profits:

$$\Pi_1(Q_1) = (p(Q_1) - c)Q_1 + \delta \left[ (1 - \alpha) \pi^M + \alpha \pi_1(q^*_1, q^*_2) \right], \quad (7)$$

where $(p(Q_1) - c)Q_1$ is firm 1’s profit in period 1, $\pi^M \equiv \pi_1(q^*_1, 0)$ is firm 1’s monopoly profit in period 2 absent entry, $\pi_1(q^*_1, q^*_2)$ is firm 1’s profit in period 2 when entry occurs, and $\delta$ is the intertemporal discount factor. In the next lemma, we establish a useful bound on $Q_1$:
Lemma 6: (A bound on $Q_1$) The period 1 output of firm 1, $Q_1$, is between the monopoly output and the aggregate Cournot output: $Q^M \leq Q_1 \leq q_1^C + q_2^C$.

Proof: Suppose by way of negation that $Q_1 < Q^M$. Then, firm 1 can raise $Q_1$ slightly towards $Q^M$ and make more money in period 1. Moreover, $p_1$ falls and hence is less likely to be deemed excessive in period 2. Therefore, $Q_1 < Q^M$ cannot be optimal.

Next, suppose by way of negation that $Q_1 > q_1^C + q_2^C$. Then firm 1 can raise its period 1 profit by lowering $Q_1$ slightly towards $q_1^C + q_2^C$, without rendering $p_1$ excessive (the aggregate output in period 2 can be at most equal to the aggregate Cournot level, $q_1^C + q_2^C$). Hence, $Q_1 > q_1^C + q_2^C$ is not optimal either.

Since Lemma 6 shows that $Q_1 \leq q_1^C + q_2^C$, the equilibrium is attained in the $(q_1, q_2)$ space on the $q_1 + q_2 = Q_1$ line (as in Figure 1b) or below it (as in Figure 1c). The discounted expected profit of firm 1 can be rewritten as:

$$
\Pi_1(Q_1) = \begin{cases} 
(p(Q_1) - c)Q_1 + \delta (1 - \alpha) \pi^M + \delta \alpha ([p(q_1^* + q_2^*) - c]q_1^* - \gamma \tau (p(Q_1) - p(q_1^* + q_2^*))Q_1), & Q^M \leq Q_1 < \overline{Q}_1, \\
(p(Q_1) - c)Q_1 + \delta (1 - \alpha) \pi^M + \delta \alpha (p(Q_1) - c) q_1^*, & Q_1 \geq \overline{Q}_1,
\end{cases}
$$

where $(q_1^*, q_2^*)$ is defined by the intersection of $r_1^C(q_2)$ and $r_2^C(q_1)$ if $Q_1 < \overline{Q}_1$ and by the intersection of $q_1 + q_2 = \overline{Q}_1$ and $r_2^C(q_1)$ if $\overline{Q}_1 \leq Q_1 \leq q_1^C + q_2^C$. Note that at $\overline{Q}_1$, $q_1^* + q_2^* = \overline{Q}_1$, so $p(\overline{Q}_1) = p(q_1^* + q_2^*)$. Moreover, recalling that $\overline{Q}_1$ is attained when $q_1 + q_2 = Q_1$ passes through the intersection of $r_1^C(q_2)$ and $r_2^C(q_1)$, it follows that at $\overline{Q}_1$, $q_1^*$ and $q_2^*$ are equal at the first and second lines of (8); hence, $\Pi_1(Q_1)$ is continuous at $Q_1 = \overline{Q}_1$.

Let $Q_1^*$ denote the optimal choice of $Q_1$. In order to characterize $Q_1^*$, we shall make the following assumption:

A3 $\Pi_1(Q_1)$ is piecewise concave (i.e., concave in each of its two relevant segments)

In the next section, we show that Assumption A3 holds when the demand function is linear, provided that $\delta \alpha$, which is the discounted probability of entry in period 2, is below 0.9. Indeed, it is easy to see that when $\alpha = 0$, $\Pi_1(Q_1)$ is concave by Assumption A1; by continuity this is also true so long as $\alpha$ is not too large. Given Assumption A3, we can now establish the following result:
Proposition 1: (The choice of $Q^*_1$) $Q^M < Q^*_1$. Let $\frac{\partial q_1^*}{\partial Q_1^*}$ be the derivative of $q_1^*$ with respect to $Q_1$ when $Q_1 \geq \overline{Q}_1$ ($p_1$ is not excessive) and $\frac{\partial q_2^*}{\partial Q_1^*}$ the derivative of $q_2^*$ with respect to $Q_1$ when $Q_1 < \overline{Q}_1$ ($p_1$ is excessive). Then,

(i) $Q^*_1 < \overline{Q}_1$ (firm 1 chooses its period-1 output such that $p_1$ ends up being excessive) if $\frac{\partial q_1^*}{\partial Q_1^*} < \frac{(1+\delta_0)(1-\gamma r)}{\delta_0(1+\gamma r)}$ and $\frac{\partial q_2^*}{\partial Q_1^*} > \frac{\gamma r(1+2\delta_0)-1}{\delta_0(1+\gamma r)}$. Both inequalities hold when $\delta_0$ is sufficiently small. Moreover, $\delta_0 < \frac{1-\gamma r}{2\gamma r}$ is necessary for the first inequality and sufficient for the second.

(ii) $Q^*_1 > \overline{Q}_1$ (firm 1 chooses its period-1 output such that $p_1$ will not end up being excessive) if $\frac{\partial q_1^*}{\partial Q_1^*} \geq \frac{(1+\delta_0)(1-\gamma r)}{\delta_0(1+\gamma r)}$, and $\frac{\partial q_2^*}{\partial Q_1^*} > \frac{\gamma r(1+2\delta_0)-1}{\delta_0(1+\gamma r)}$. $\delta_0 > \frac{1-\gamma r}{2\gamma r}$ is sufficient for the first inequality and necessary for the second inequality.

(iii) Firm 1’s problem has two local optima, one below and one above $\overline{Q}_1$ if $\frac{\partial q_1^*}{\partial Q_1^*} > \frac{(1+\delta_0)(1-\gamma r)}{\delta_0(1+\gamma r)}$, and $\frac{\partial q_2^*}{\partial Q_1^*} < \frac{\gamma r(1+2\delta_0)-1}{\delta_0(1+\gamma r)}$, where $\delta_0 > \frac{1-\gamma r}{2\gamma r}$ is sufficient for both inequalities.

Proof: First, we evaluate $\Pi'_1(Q_1)$ at $Q^M$. Since $\overline{Q}_1 > Q^M$ by Lemma 4, $p_1$ is excessive when $Q_1 = Q^M$, so $\Pi_1(Q_1)$ is given by the first line of (8). Differentiating the expression and using the envelope theorem (by (5), the derivative of the square bracketed term with respect $q_1^*$ vanishes), yields

$$\Pi'_1(Q_1) = MR(Q_1) - c - \delta_0 \left[ \gamma r (MR(Q_1) - p(q_1^* + q_2^*)) - p'(q_1^* + q_2^*)(q_1^* + \gamma r Q_1^*) \frac{\partial q_2^*}{\partial Q_1^*} \right],$$

(9)

where $\frac{\partial q_2^*}{\partial Q_1^*}$ is the derivative of $q_2^*$ with respect to $Q_1$ when $Q_1 < \overline{Q}_1$. Evaluating $\Pi'_1(Q_1)$ at $Q^M$ and noting that by definition, $MR(Q^M) = c$,

$$\Pi'_1(Q^M) = -\delta_0 \left[ \gamma r (c - p(q_1^* + q_2^*)) - p'(q_1^* + q_2^*)(q_1^* + \gamma r Q_1^*) \frac{\partial q_2^*}{\partial Q_1^*} \right]$$

$$= -\delta_0 p'(q_1^* + q_2^*)(q_1^* + \gamma r Q_1^*) \left[ \gamma r - \frac{\partial q_2^*}{\partial Q_1^*} \right] > 0,$$

where the second equality follows by substituting for $p(q_1^* + q_2^*) - c$ from (4) and the inequality follows from Lemma 5 which shows that when $Q_1 < \overline{Q}_1$, $\frac{\partial q_2^*}{\partial Q_1^*} < \gamma r$. Since $\Pi'_1(Q^M) > 0$, $Q'_1 > Q^M$.

Second, we examine whether firm 1 has an incentive to raise $Q^*_1$ all the way to the point where $p_1$ is no longer excessive, i.e., above $\overline{Q}_1$. To this end, we first evaluate $\Pi'_1(\overline{Q}_1)$ as $\overline{Q}_1$ approaches $\overline{Q}_1$ from below. Using $\Pi'_1(\overline{Q}_1)$ to denote the derivative of $\Pi_1(Q_1)$ as $Q_1$ approaches
\( Q_1 \) from below, and recalling that when \( Q_1 < Q_1 \), \( \Pi'_1 (Q_1) \) is given by (9), we get

\[
\Pi'_1 \left( Q_1 \right) = MR \left( Q_1 \right) - c - \delta \alpha \left[ \gamma \tau \left( MR(Q_1) - p (q_1^* + q_2^*) \right) - p' (q_1^* + q_2^*) \left( q_1^* + \gamma \tau Q_1 \right) \frac{\partial q_1^*}{\partial Q_1} \right]
\]

\[
= MR \left( Q_1 \right) - c - \delta \alpha \left[ \gamma \tau \left( MR(Q_1) - p (Q_1) \right) - p' \left( Q_1 \right) \left( q_1^* + \gamma \tau Q_1 \right) \frac{\partial q_1^*}{\partial Q_1} \right] \tag{10}
\]

\[
= p' \left( Q_1 \right) \left( \frac{1 - \gamma \tau}{2} \right) Q_1 - \delta \alpha \left[ \gamma \tau p' \left( Q_1 \right) Q_1 - p' \left( Q_1 \right) \left( q_1^* + \gamma \tau Q_1 \right) \frac{\partial q_1^*}{\partial Q_1} \right]
\]

where the second equality follows because by definition, \( q_1^* + q_2^* = Q_1 \), the third equality follows by using (6), and the fifth equality follows since by Lemma 4, \( \frac{\partial q_1^*}{\partial Q_1} = \frac{1 - \gamma \tau}{2} \) when \( Q_1 = Q_1 \). Since \( p' \left( Q_1 \right) < 0 \), (10) implies that \( \Pi'_1 \left( Q_1 \right) \) has the same sign as \( \frac{\gamma \tau (1+2\delta \alpha) - 1}{\delta \alpha (1+\gamma \tau)} - \frac{\partial q_1^*}{\partial Q_1} \). Note that by Lemma 5, \( 0 < \frac{\partial q_1^*}{\partial Q_1} < \gamma \tau < 1 \) and also note that \( \frac{\gamma \tau (1+2\delta \alpha) - 1}{\delta \alpha (1+\gamma \tau)} \) is increasing with \( \delta \alpha \) from \(-\infty \) when \( \delta \alpha = 0 \) to \( \frac{3\tau\gamma - 1}{1+\gamma \tau} \) when \( \delta \alpha = 1 \). Hence, \( \Pi'_1 \left( Q_1 \right) \leq 0 \) when \( \delta \alpha \) is sufficiently small and moreover, \( \Pi'_1 \left( Q_1 \right) \leq 0 \) for all \( \delta \alpha \) when \( \gamma \tau \leq \frac{1}{3} \). In particular, \( \delta \alpha \leq \frac{1 - \gamma \tau}{2\gamma \tau} \) is sufficient to ensure that \( \frac{\gamma \tau (1+2\delta \alpha) - 1}{\delta \alpha (1+\gamma \tau)} \leq 0 \), in which case, \( \Pi'_1 \left( Q_1 \right) \leq 0 \). By continuity then, \( \Pi'_1 \left( Q_1 \right) \) is decreasing with \( \delta \alpha \) when \( \delta \alpha \) does not exceed \( \frac{1 - \gamma \tau}{2\gamma \tau} \), and \( \delta \alpha > \frac{1 - \gamma \tau}{2\gamma \tau} \).

Next, we evaluate \( \Pi'_1 \left( Q_1 \right) \) as \( Q_1 \) approaches \( Q_1 \) from above. Recalling that when \( Q_1 \geq Q_1 \), \( \Pi_1 \left( Q_1 \right) \) is given by the second line of (8), we get

\[
\Pi'_1 \left( Q_1 \right) = MR \left( Q_1 \right) - c + \delta \alpha \left[ p' \left( Q_1 \right) q_1^* + \left( p \left( Q_1 \right) - c \right) \frac{\partial q_1^*}{\partial Q_1} \right], \tag{11}
\]

where \( \frac{\partial q_1^*}{\partial Q_1} \) is the derivative of \( q_1^* \) with respect to \( Q_1 \) when \( Q_1 > Q_1 \).

Using \( \Pi'_1 \left( Q_1 \right) \) to denote the derivative of \( \Pi_1 \left( Q_1 \right) \) as \( Q_1 \) approaches \( Q_1 \) from above, using
addition, sufficient for $< 1 @q from Lemma 5 that $p Q_1, q_1^{*} = 1 - \gamma \tau \over 2$,

$$
\Pi'_1 \left( \overline{Q}_1^+ \right) = MR(\overline{Q}_1) - c + \delta \alpha \left[ p'(\overline{Q}_1) q_1^{*} + (p(\overline{Q}_1) - c) \frac{\partial q_1^{*}}{\partial \overline{Q}_1^+} \right] = p'(\overline{Q}_1) \left( \frac{1 - \gamma \tau}{2} \right) \overline{Q}_1 + \delta \alpha \left[ p'(\overline{Q}_1) q_1^{*} + (p(\overline{Q}_1) - c) \frac{\partial q_1^{*}}{\partial \overline{Q}_1^+} \right] = p'(\overline{Q}_1) \left( \frac{1 - \gamma \tau}{2} \right) \overline{Q}_1 + \delta \alpha \left[ p'(\overline{Q}_1) q_1^{*} - p'(\overline{Q}_1) \left( \frac{1 + \gamma \tau}{2} \right) \overline{Q}_1 \frac{\partial q_1^{*}}{\partial \overline{Q}_1^+} \right] = -\delta \alpha \left( \frac{1 + \gamma \tau}{2} \right) p'(\overline{Q}_1) \overline{Q}_1 \left[ \frac{\partial q_1^{*}}{\partial \overline{Q}_1^+} - \frac{(1 + \delta \alpha)(1 + \gamma \tau)}{\delta \alpha (1 + \gamma \tau)} \right].
$$

(12)

Since $p'(\overline{Q}_1) < 0$, (12) implies that $\Pi'_1 \left( \overline{Q}_1^+ \right)$ has the same sign as $\frac{\partial q_1^{*}}{\partial \overline{Q}_1^+} - \frac{(1 + \delta \alpha)(1 + \gamma \tau)}{\delta \alpha (1 + \gamma \tau)}$. Recall from Lemma 5 that $\frac{\partial q_1^{*}}{\partial \overline{Q}_1^+} > 1$. Since $\frac{(1 + \delta \alpha)(1 - \gamma \tau)}{\delta \alpha (1 + \gamma \tau)}$ is decreasing with $\delta \alpha$ from $\infty$ when $\delta \alpha = 0$ to $2(1 - \gamma \tau) < 2$ when $\delta \alpha = 1$, it follows that $\Pi'_1 \left( \overline{Q}_1^+ \right) \leq 0$ for $\delta \alpha$ sufficiently small. In particular, since $\frac{\partial q_1^{*}}{\partial \overline{Q}_1^+} > 1$, a necessary condition for $\Pi'_1 \left( \overline{Q}_1^+ \right) \leq 0$ is $\frac{(1 + \delta \alpha)(1 - \gamma \tau)}{\delta \alpha (1 + \gamma \tau)} > 1$, which is equivalent to $\delta \alpha < \frac{1 - \gamma \tau}{2 \gamma \tau}$. In turn, $\Pi'_1 \left( \overline{Q}_1^+ \right) \leq 0$ implies $Q_1^{*} \leq \overline{Q}_1$.

By contrast, recalling that $\frac{\partial q_1^{*}}{\partial \overline{Q}_1^+} > 1$, it follows that $\frac{(1 + \delta \alpha)(1 - \gamma \tau)}{\delta \alpha (1 + \gamma \tau)} < 1$, or $\delta \alpha > \frac{1 - \gamma \tau}{2 \gamma \tau}$, is sufficient for $\frac{\partial q_1^{*}}{\partial \overline{Q}_1^+} > \frac{(1 + \delta \alpha)(1 - \gamma \tau)}{\delta \alpha (1 + \gamma \tau)}$. Then, $\Pi'_1 \left( \overline{Q}_1^+ \right) > 0$, in which case $Q_1^{*} > \overline{Q}_1$, provided that in addition, $\Pi'_1 \left( \overline{Q}_1 \right) \geq 0$.

Altogether then, the analysis of $\Pi'_1 \left( \overline{Q}_1^+ \right)$ and $\Pi'_1 \left( \overline{Q}_1 \right)$ implies that there are four possible cases that can arise:

(i) $\Pi'_1 \left( \overline{Q}_1 \right) < 0$ and $\Pi'_1 \left( \overline{Q}_1^+ \right) < 0$, so $Q_1^{*} < \overline{Q}_1$, when

$$
\frac{\partial q_1^{*}}{\partial \overline{Q}_1^+} < \frac{(1 + \delta \alpha)(1 - \gamma \tau)}{\delta \alpha (1 + \gamma \tau)}, \quad \text{and} \quad \frac{\partial q_2}{\partial \overline{Q}_1^+} > \frac{\gamma \tau (1 + 2 \delta \alpha) - 1}{\delta \alpha (1 + \gamma \tau)}.
$$

Both inequalities hold when $\delta \alpha$ is sufficiently small. Moreover, $\delta \alpha < \frac{1 - \gamma \tau}{2 \gamma \tau}$ is necessary for the first inequality and sufficient for the second.

(ii) $\Pi'_1 \left( \overline{Q}_1 \right) \geq 0$ and $\Pi'_1 \left( \overline{Q}_1^+ \right) < 0$, so $Q_1^{*} = \overline{Q}_1$, when

$$
\frac{\partial q_1^{*}}{\partial \overline{Q}_1^+} < \frac{(1 + \delta \alpha)(1 - \gamma \tau)}{\delta \alpha (1 + \gamma \tau)}, \quad \text{and} \quad \frac{\partial q_2}{\partial \overline{Q}_1^+} \leq \frac{\gamma \tau (1 + 2 \delta \alpha) - 1}{\delta \alpha (1 + \gamma \tau)}.
$$

Both inequalities cannot hold simultaneously however because $\delta \alpha < \frac{1 - \gamma \tau}{2 \gamma \tau}$ is necessary for the
first inequality, but when it holds, \( \frac{\gamma \tau (1 + 2\delta \alpha) - 1}{\delta \alpha (1 + \gamma \tau)} < 0 \), because

\[
\gamma \tau (1 + 2\delta \alpha) - 1 < \gamma \tau \left(1 + 2 \left(\frac{1 - \gamma \tau}{2 \gamma \tau}\right)\right) - 1 < 0.
\]

Since \( \frac{\partial q_2^*}{\partial Q_1} > 0 \), we cannot have \( \frac{\partial q_2^*}{\partial Q_1} \geq \frac{\gamma \tau (1 + 2\delta \alpha) - 1}{\delta \alpha (1 + \gamma \tau)} \).

(iii) \( \Pi_1' \left(Q_1^{-}\right) \geq 0 \) and \( \Pi_1' \left(Q_1^{+}\right) > 0 \), so \( Q_1^+ > Q_1 \), when

\[
\frac{\partial q_1^*}{\partial Q_1^+} \geq \frac{(1 + \delta \alpha) (1 - \gamma \tau)}{\delta \alpha (1 + \gamma \tau)}, \quad \text{and} \quad \frac{\partial q_2^*}{\partial Q_1} > \frac{\gamma \tau (1 + 2\delta \alpha) - 1}{\delta \alpha (1 + \gamma \tau)}.
\]

\[\delta \alpha > \frac{1 - \gamma \tau}{2 \gamma \tau}\] is sufficient for the first inequality and necessary for the second.

(iv) \( \Pi_1' \left(Q_1^{-}\right) < 0 \) and \( \Pi_1' \left(Q_1^{+}\right) > 0 \), in which case there are two local optima, one below and one above \( Q_1 \). This case arises when

\[
\frac{\partial q_1^*}{\partial Q_1^+} > \frac{(1 + \delta \alpha) (1 - \gamma \tau)}{\delta \alpha (1 + \gamma \tau)}, \quad \text{and} \quad \frac{\partial q_2^*}{\partial Q_1} < \frac{\gamma \tau (1 + 2\delta \alpha) - 1}{\delta \alpha (1 + \gamma \tau)}.
\]

\[\delta \alpha > \frac{1 - \gamma \tau}{2 \gamma \tau}\] is sufficient for both inequalities. ■

Intuitively, when \( Q_1 = Q^M \), firm 1 maximizes its profit in period 1, but then, if entry takes place in period 2, the aggregate output in period 2 exceeds \( Q^M \), so \( p_1 \) is rendered excessive and firm 1 may have to pay a fine (by Lemma 4, \( Q^M < Q_1 \), so at \( Q_1 = Q^M \), \( p_1 \) is excessive). Raising \( Q_1 \) slightly above \( Q^M \) entails a second order loss of profits in period 1, but has a first order beneficial effect on the expected fine that firm 1 pays in period 2. Hence, firm 1 sets \( Q_1 \) above \( Q^M \), implying that the prohibition of excessive pricing has a pro-competitive effect on the pre-entry behavior of firm 1, even if eventually, firm 1 is not found liable in period 2.

A further increase in \( Q_1 \) involves a trade-off: firm 1 loses money in period 1 as it expands \( Q_1 \) above the monopoly level, but it lowers the expected fine in period 2 if entry occurs. Once \( Q_1 \geq Q_1 \), firm 1 ensures that \( p_1 \) will not be deemed excessive by setting \( q_1 \) such that \( q_1 + q_2 = Q_1 \), to ensure that \( p_1 = p_2 \). Proposition 1 shows that when firm 1 expands \( Q_1 \) to the point where \( p_1 \) is no longer excessive, it actually expands it beyond \( Q_1 \), despite the fact that the expansion entails a loss of profit in period 1 (as \( Q_1 \) moves further away above \( Q^M \)). The reason why firm 1 expands \( Q_1 \) is that doing so allows it to raise \( q_1 \) closer to its Cournot best-response function in period 2 without rendering \( p_1 \) excessive.
Proposition 1 shows that, so long as $\alpha$, which represents the discounted probability of entry, is not too large, firm 1 sets $Q_1 < \overline{Q}_1$, in which case $p_1$ is excessive. Firm 1 then still expands $Q_1$ above $Q^M$ to lower the expected fine it pays in period 2, but does not expand it all the way to the point where $p_1$ is no longer excessive. By contrast, when $\alpha$ is large relative to $\frac{1-\gamma\tau}{2\gamma\tau} > 1$, so $\alpha$ can never be large enough to ensure that $Q_1^* > \overline{Q}_1$.

Raising $Q_1$ benefits consumers in period 1, but whenever $Q_1 < \overline{Q}_1$ (p1 is excessive), it has a negative effect in period 2 because then, as Lemma 5 above shows, firm 1’s output, $q_1^*$, as well as aggregate output, $q_1^* + q_2^*$, are both decreasing with $Q_1$. As we already mentioned, firm 1 limits $q_1^*$ in this case to raise $p_2$, and hence lower the gap between $p_2$ and $p_1$, and thereby the fine it has to pay when found liable in court. When $Q_1 > \overline{Q}_1$ (p1 is not excessive), firm 1 sets $q_1$ such that $q_1^* + q_2^* = Q_1$. Hence, an increase in $Q_1$ also leads to an increase in aggregate output in period 2. However, since Lemma 3 shows that $q_1^* + q_2^* < q_1^C + q_2^C$, the equilibrium in period 2 is less competitive than it would be without the prohibition on excessive pricing.

The implication then is that the prohibition of excessive pricing involves a trade-off between restraining the monopoly power of firm 1 in period 1, and leading to a less competitive outcome in period 2 when entry takes place. Using Lemma 5 that shows that \( \frac{\partial(q_1^* + q_2^*)}{\partial Q_1} < -\gamma\tau \) if $Q_1 < \overline{Q}_1$ and \( \frac{\partial(q_1^* + q_2^*)}{\partial Q_1} = 1 \) if $Q_1 \geq \overline{Q}_1$, we have the following result.

**Proposition 2:** (The effect of the prohibition of excessive pricing on output) The prohibition of excessive pricing raises the period 1 output from $Q^M$ to $Q_1^*$, but lowers the period 2 output from $q_1^C + q_2^C$ to $q_1^* + q_2^*$. When $Q_1 < \overline{Q}_1$ (p1 is excessive), the expansion of output in period 1 exceeds the contraction of aggregate output in period 2. When $Q_1 > \overline{Q}_1$ (p1 is not excessive), the expansion of output in period 1 is equal to the contraction of aggregate output in period 2.

Proposition 2 implies that so long as $p_1$ is excessive, the prohibition of excessive pricing has a bigger (positive) effect on output in period 1 than a (negative) effect on output in period 2. Now recall that by Assumption A1, demand is either concave or not too convex. If demand is either concave or linear, the bigger expansion of output in period 1 than the contraction of output in period 2 implies that $p_1$ decreases more than $p_2$ increases. By continuity, $p_1$ also decreases more than $p_2$ increases, so long as demand is not too convex.

**Proposition 3:** (Comparative statics of $Q_1^*$) $Q_1^*$ increases with the discounted probability of entry in period 2, $\alpha$, but is independent of $\gamma\tau$ when $Q_1^* > \overline{Q}_1$ (p1 is not excessive).
**Proof:** See the Appendix.

Intuitively, when \( Q_1^* < Q_1 \) (\( p_1 \) is excessive), an increase in \( \delta \alpha \) implies that entry is more likely, in which case firm 1 may have to pay a fine. Hence firm 1 has a stronger incentive to expand \( Q_1 \) and thereby lower the expected fine it pays. However, when \( Q_1^* > Q_1 \), \( p_1 \) is not excessive because firm 1 keeps its period 2 output below its Cournot best-response function to ensure that \( q_1^* + q_2^* = Q_1 \). An increase in \( Q_1 \) relaxes this constraint and allows firm 1 to move closer to its Cournot best-response function. Consequently, an increase in \( \delta \alpha \), which makes it more likely that \( q_1^* \) will be constrained in period 2, induces firm 1 to expand \( Q_1 \).

The fact that firm 1 expands \( Q_1 \) as \( \delta \alpha \) increases, means that consumers benefit from the prohibition of excessive pricing before entry takes place, especially when the probability of entry is high. This result is interesting because, as mentioned in the Introduction, it is often argued that when the probability of entry is high, there is no reason to intervene in excessive pricing cases, since “the market will correct itself.” This argument, however, ignores the harm to consumers before entry occurs and simply says that this harm is not going to last for a long time. While this is true, our analysis shows that nonetheless, the retrospective benchmark that we consider restrains the dominant firm’s pre-entry behavior and is therefore pro-competitive, particularly when the probability of entry is high.

Proposition 3 also shows that, as might be expected, the expected fines, \( \gamma \tau \), have no effect on \( Q_1^* \) when \( p_1 \) is not excessive. When \( p_1 \) in excessive, i.e., \( Q_1^* < Q_1 \), the effect of \( \gamma \tau \) on \( Q_1^* \) is less clear since (9) shows that \( \gamma \tau \) affects \( Q_1^* \) both directly, as well as indirectly via its effect on the equilibrium output levels in period 2, \( q_1^* \) and \( q_2^* \). In the next section, we will examine the effect of \( \gamma \tau \) on \( Q_1^* \) when \( Q_1^* < Q_1 \) under the assumption that demand is linear.

## 5 The linear demand case

In this section, we will derive additional results by assuming that the demand function is linear and given by \( p = a - Q \). To simplify the exposition, let \( A = a - c \). The advantage of assuming that demand is linear is that it allows us to obtain closed-form solutions which facilitate the analysis.
5.1 Period 2

Absent entry in period 2, firm 1 simply produces the monopoly output, \( Q^M = \frac{A}{2} \), and earns the monopoly profit,

\[
\pi^M = \left( \frac{A}{2} \right)^2.
\] (13)

If entry takes place, \( p_1 \) can be deemed excessive if it exceeds \( p_2 \), i.e., \( a - (q_1 + q_2) < a - Q_1 \), or \( q_1 + q_2 > Q_1 \). Hence, the period 2 profits of the two firms are,

\[
\pi_1 (q_1, q_2) = \begin{cases} 
(A - q_1 - q_2)q_1, & q_1 + q_2 \leq Q_1, \\
(A - q_1 - q_2)q_1 - \gamma \tau [(a - Q_1) - (a - q_1 - q_2)] Q_1, & q_1 + q_2 > Q_1,
\end{cases}
\]

and

\[
\pi_2 (q_1, q_2) = (A - q_1 - q_2)q_2.
\]

The best-response function of firm 2 is defined by the familiar Cournot best-response function,

\[
BR_2(q_1) = r^C_2 (q_1) = \frac{A - q_1}{2}.
\]

The best-response function of firm 1 is equal to the Cournot best-response function \( r^C_1 (q_2) = \frac{A - q_2}{2} \) if \( q_1 + q_2 \leq Q_1 \), i.e., if \( \frac{A - q_2}{2} + q_2 = \frac{A + q_2}{2} \leq Q_1 \). If \( q_1 + q_2 > Q_1 \), \( p_1 \) is deemed excessive with probability \( \gamma \), so the best-response function of firm 1 maximizes the second line of \( \pi_1 (q_1, q_2) \) and hence is given by \( r^E_1 (q_2) = \frac{A - q_2 - \gamma \tau q_2}{2} \). But then \( r^E_1 (q_2) + q_2 \geq Q_1 \) only if \( \frac{A - q_2}{2} - \frac{\gamma \tau Q_1}{2} + q_2 > Q_1 \), or \( \frac{A + q_2}{2 + \gamma \tau Q_1} > Q_1 \). And if \( \frac{A + q_2}{2 + \gamma \tau Q_1} \leq Q_1 < \frac{A + q_2}{2} \), the best-response function of firm 1 is \( Q_1 - q_2 \). Using the definitions of \( r^C_1 (q_2) \) and \( r^E_1 (q_2) \), and rearranging terms, we have:

\[
BR_1(q_2) = \begin{cases} 
r^C_1 (q_2) = \frac{A - q_2}{2}, & \frac{A + q_2}{2} \leq Q_1, \\
Q_1 - q_2, & \frac{A + q_2}{2 + \gamma \tau Q_1} \leq Q_1 < \frac{A + q_2}{2}, \\
r^E_1 (q_2) = \frac{A - q_2}{2} - \frac{\gamma \tau Q_1}{2}, & Q_1 < \frac{A + q_2}{2 + \gamma \tau Q_1}.
\end{cases}
\]

It can be easily checked that this expression coincides with the best-response function of firm 1 characterized in Lemma 1 when \( p = a - Q \).

We are now ready to characterize the Nash equilibrium in period 2.

**Lemma 7:** *(The post-entry Nash equilibrium in the linear demand case)* The Nash equilibrium in period 2 when firm 2 enters is \( \left( \frac{A - 2\gamma Q_1}{3}, \frac{A + \gamma \tau Q_1}{3} \right) \) if \( Q_1 < \frac{2A}{3 + \gamma \tau} \) and \( (2Q_1 - A, A - Q_1) \) if
\[ \frac{2A}{3+\gamma \tau} \leq Q_1 < \frac{2A}{\gamma}. \]

Proof: See the Appendix.

Note that at \( Q_1 = \frac{2A}{3+\gamma \tau}, \quad \frac{A-2\gamma \tau Q_1}{3} = 2Q_1 - A \) and \( \frac{A+\gamma \tau Q_1}{3} = A - Q_1 \). Also note that at \( Q_1 = \frac{2A}{3+\gamma \tau}, \quad \frac{q_1}{Q_1} = \frac{2Q_1-A}{Q_1} = 1-\frac{\gamma \tau}{2} \) and \( \frac{q_2}{Q_1} = \frac{A-Q_1}{Q_1} = \frac{1+\gamma \tau}{2} \), as proved in Lemma 4 for the general case. It is also worth noting that when \( \gamma \tau = 0 \) (excessive pricing is not prohibited), we obtain the Cournot outcome.

The period 1 price, \( p_1 \), is excessive when \( Q_1 \leq \frac{2A}{3+\gamma \tau} \). When \( \frac{2A}{3+\gamma \tau} \leq Q_1 < \frac{2A}{\gamma} \), firm 1 sets \( q_1 \) such that \( p_2 = p_1 \), to ensure that \( p_1 \) will not be excessive. The critical value of \( Q_1 \) below which \( p_1 \) is excessive is \( \frac{2A}{3+\gamma \tau} \). It is easy to verify that this expression satisfies equation (6) when \( p = a - Q \).

Substituting the equilibrium values of \( q_1 \) and \( q_2 \) into \( \pi_1(q_1, q_2) \) yields:

\[
\pi_1(q_1^*, q_2^*) = \begin{cases} 
(\frac{4}{3})^2 - \frac{\gamma \tau Q_1(7A-(9+\gamma \tau)Q_1)}{g} & Q_1 \leq \frac{2A}{3+\gamma \tau}, \\
(A-Q_1)(2Q_1-A) & \frac{2A}{3+\gamma \tau} \leq Q_1 < \frac{2A}{\gamma}. 
\end{cases}
\tag{14}
\]

5.2 Period 1

Next, we consider firm 1’s problem in period 1. Using (13) and (14), the expected profit of firm 1 in period 1 is

\[
\Pi_1(Q_1) = \begin{cases} 
(A-Q_1)Q_1 + \delta (1-\alpha) (\frac{4}{3})^2 + \delta \alpha \left[ (\frac{4}{3})^2 - \frac{\gamma \tau Q_1(7A-(9+\gamma \tau)Q_1)}{g} \right] & Q_1 < \frac{2A}{3+\gamma \tau}, \\
(A-Q_1)Q_1 + \delta (1-\alpha) (\frac{4}{3})^2 + \delta \alpha (A-Q_1)(2Q_1-A) & \frac{2A}{3+\gamma \tau} \leq Q_1 \leq \frac{2A}{\gamma}. 
\end{cases}
\tag{15}
\]

where \( \Pi_1(Q_1) \) is continuous at \( Q_1 = \frac{2A}{3+\gamma \tau} = Q_1 \). Differentiating with respect to \( Q_1 \) yields,

\[
\Pi'_1(Q_1) = \begin{cases} 
A - 2Q_1 - \frac{\delta \alpha g}{\gamma \tau} [7A - 2(9 + \gamma \tau)Q_1] & Q_1 < \frac{2A}{3+\gamma \tau}, \\
A - 2Q_1 + \delta \alpha (3A - 4Q_1) & \frac{2A}{3+\gamma \tau} \leq Q_1 \leq \frac{2A}{\gamma}. 
\end{cases}
\tag{16}
\]

Note that \( \Pi''_1(Q_1) < 0 \) for \( \frac{2A}{3+\gamma \tau} \leq Q_1 \leq \frac{2A}{\gamma} \) and \( \Pi''_1(Q_1) \leq 0 \) for \( Q_1 < \frac{2A}{3+\gamma \tau} \), provided that \( \delta \alpha \leq \frac{9}{\gamma \tau (9+\gamma \tau)} \), where \( \frac{9}{\gamma \tau (9+\gamma \tau)} > 0.9 \) since \( \gamma \tau < 1 \) by Assumption A2. Consequently, \( \delta \alpha \leq 0.9 \) is sufficient to ensure that \( \delta \alpha \leq \frac{9}{\gamma \tau (9+\gamma \tau)} \), in which case \( \Pi_1(Q_1) \) is piecewise concave. In what follows we will therefore make the following assumption.

A4 \( \delta \alpha \leq 0.9 \) (\( \Pi_1(Q_1) \) is concave in each of its two relevant segments)
In the next proposition we characterize $Q_1^*$ and the resulting equilibrium in period 2.

**Proposition 4: (The equilibrium in the linear demand case) Suppose that $p = a - Q$. Then,**

$$Q_1^* = \begin{cases} 
\frac{A}{2} \left[\frac{9 - 7\delta\alpha \gamma}{9 - \delta\alpha \gamma (9 + \gamma\tau)}\right], & \delta\alpha < Z(\gamma\tau), \\
\frac{A}{2} \left[\frac{1 + 3\delta\alpha}{1 + 2\delta\alpha}\right], & \delta\alpha \geq Z(\gamma\tau),
\end{cases}$$  \hspace{1cm} (17)

where

$$Z(\gamma\tau) = \frac{1 + 7\gamma\tau (2 - \gamma\tau) - (1 + \gamma\tau) \sqrt{1 + 5\gamma\tau (2 + \gamma\tau)}}{2\gamma\tau (1 + 11\gamma\tau)}. \hspace{1cm} (18)$$

$p_1$ is excessive if $\delta\alpha < Z(\gamma\tau)$, but not otherwise. Note that so long as $0 < \delta\alpha < 1$, $\frac{A}{2} < Q_1^* < \frac{2A}{3}$, implying that $Q_1^*$ is above the monopoly level, but below the aggregate Cournot level. The resulting equilibrium quantities in period 2 are given by

$$q_1^* = \begin{cases} 
\frac{A(3(1 - \gamma\tau) - \delta\alpha \gamma (3 - \gamma\tau))}{9 - \delta\alpha\gamma(9 + \gamma\tau)}, & \delta\alpha < Z(\gamma\tau), \\
\frac{\delta\alpha A}{1 + 2\delta\alpha}, & \delta\alpha \geq Z(\gamma\tau),
\end{cases}$$ \hspace{1cm} (19)

and

$$q_2^* = \begin{cases} 
\frac{3A(2 + \gamma\tau)(1 - \delta\alpha \gamma\tau)}{2(9 - \delta\alpha \gamma(9 + \gamma\tau))}, & \delta\alpha < Z(\gamma\tau), \\
\frac{A(1 + \delta\alpha)}{2(1 + 2\delta\alpha)}, & \delta\alpha \geq Z(\gamma\tau),
\end{cases}$$ \hspace{1cm} (20)

**Proof:** See the Appendix.

Proposition 4 fully characterizes the equilibrium and allow us to provide the precise conditions under which $p_1$ is excessive or not and examine how the equilibrium responds to changes in discount probability of entry, $\delta\alpha$, and the expected fine, $\gamma\tau$. When $\delta\alpha < Z(\gamma\tau)$ ($\delta\alpha$ is small) firm 1 sets $Q_1$ such that $p_1$ ends up being excessive. But when $\delta\alpha \geq Z(\gamma\tau)$ ($\delta\alpha$ is large), firm 1 sets $Q_1$ such that $p_1$ is not excessive. Plotting $Z(\gamma\tau)$ with Mathematica reveals that $Z'(\gamma\tau) < 0$, with $Z(1) = 0$. Hence, the latter case becomes more likely as $\gamma\tau$ increases. The implication is that either an increase in the discounted probability of entry, $\delta\alpha$, or an increase in the expected fines, $\gamma\tau$, that firm 1 pays when $p_1$ is excessive, induce firm 1 to expand $Q_1$ to ensure that $p_1$ is not excessive.

Moreover, when $\delta\alpha < Z(\gamma\tau)$ ($p_1$ is excessive):

$$\frac{\partial Q_1^*}{\partial (\gamma\tau)} = \frac{A(18\delta\alpha (1 + \gamma\tau) - 7(\delta\alpha \gamma\tau)^2)}{2(9 - \delta\alpha \gamma(9 + \gamma\tau))^2} > 0, \hspace{1cm} \frac{\partial Q_1^*}{\partial (\delta\alpha)} = \frac{9A\gamma\tau (2 + \gamma\tau)}{2(9 - \delta\alpha \gamma(9 + \gamma\tau))^2} > 0.$$
And when $\delta \alpha > Z(\gamma \tau)$ ($p_1$ is not excessive):

$$\frac{\partial Q_1^*}{\partial (\gamma \tau)} = 0, \quad \frac{\partial Q_1^*}{\partial (\delta \alpha)} = \frac{A}{2(1 + 2\delta \alpha)^2} > 0.$$ 

The fact that $\frac{\partial Q_1^*}{\partial (\delta \alpha)} > 0$ was already established in Proposition 3 for the general case. The analysis here shows however that $Q_1^*$ also increases as the per-unit expected fine, $\gamma \tau$, increases when $\delta \alpha < Z(\gamma \tau)$ ($p_1$ is excessive). Intuitively, an increase in $\gamma \tau$ implies that firm 1 would have to pay higher fines when entry occurs, so firm 1 has a stronger incentive to expand $Q_1$ in order to lower the total expected fine it pays.

### 5.3 Welfare in the linear demand case

We now turn to the welfare implications of using the post-entry price as a benchmark to assess whether the dominant firm's pre-entry price was excessive. When demand is linear, consumers' surplus, given an aggregate output $Q$, is

$$CS(Q) = \int_0^Q (a - x) \, dx - (a - Q) Q = \frac{Q^2}{2}.$$ 

Absent entry, firm 1 is a monopoly in period 2 and produces the monopoly output $\frac{4}{3}$. With entry, the equilibrium in period 2 is characterized in Lemma 7. The aggregate output is $\frac{2A - \gamma \tau Q_1^*}{3}$ if $\delta \alpha < Z(\gamma \tau)$ ($p_1$ is excessive), and $Q_1^*$ if $\delta \alpha \geq Z(\gamma \tau)$ ($p_1$ is not excessive), where $Q_1^*$ is given by (17). (In the latter case, $p_1$ is not excessive precisely because firm 1 sets $q_1$ such that $q_1^2 + q_2^2 = Q_1^*$). Since the probability of entry is $\alpha$, the overall expected discounted consumers’ surplus over the two periods is given by

$$CS(Q_1^*) = \begin{cases} \frac{(Q_1^*)^2}{2} + \frac{\delta(1-\alpha)}{2} \left( \frac{A}{2} \right)^2 + \frac{\delta \alpha}{2} \left( \frac{2A - \gamma \tau Q_1^*}{3} \right)^2, & \delta \alpha < Z(\gamma \tau), \\
\frac{(Q_1^*)^2}{2} + \frac{\delta(1-\alpha)}{2} \left( \frac{A}{2} \right)^2 + \frac{\delta \alpha (Q_1^*)^2}{2}, & \delta \alpha \geq Z(\gamma \tau), \end{cases}$$ 

(21)

It is obvious that $CS(Q_1^*)$ increases with $\delta \alpha$ because total output in period 2 when entry exceeds total output absent entry. In the next proposition we examine how $CS(Q_1^*)$ is affected by $\gamma \tau$.

**Proposition 5:** *(The comparative statics of consumers’ surplus with respect to $\gamma \tau$ in the linear demand case)* Suppose that $p = a - Q$. Then, an increase in the expected per-unit fine, $\gamma \tau$, raises
consumers’ surplus when \( \delta \alpha < Z(\gamma \tau) \) \( (p_1 \) is excessive) and has no effect when \( \delta \alpha \geq Z(\gamma \tau) \) \( (p_1 \) is not excessive).

**Proof:** See the Appendix.

We have already shown that the prohibition of excessive pricing involves a trade-off between a higher output in period 1, and a lower output in period 2. Proposition 5 allows us to examine the overall effect on consumers’ surplus. To understand the proposition, note that when \( \gamma \tau = 0 \), we obtain the same outcome as if there is no prohibition of excessive pricing. Hence we can evaluate the welfare implications of the prohibition of excessive pricing by comparing consumers’ surplus when \( \gamma \tau > 0 \) and when \( \gamma \tau = 0 \). Since the proposition shows that consumers’ surplus increases with \( \gamma \tau \) when \( p_1 \) is excessive and is not affected by \( \gamma \tau \) when \( p_1 \) is not excessive, we can conclude that the prohibition benefits consumers, and more so as \( \gamma \tau \) increases.

It should be pointed out that whether in reality it is a good idea to raise \( \gamma \tau \), also depends on additional factors which our model abstracts from, like the cost of detecting excessive prices, the legal costs involved with court cases, and the potential effect of the prohibition of excessive prices on the firm’s incentive to invest. Hence, Proposition 5 should be interpreted cautiously. Still, the proposition shows that, at least in the context of the linear demand case, the effects that we identify - a decrease in \( p_1 \) and an increase in \( p_2 \) - ultimately benefit consumers on balance.

### 6 Contemporaneous benchmarks for excessive pricing

There are many cases in which the benchmark to assess the price of a dominant firm is not the price that the firm is charging in another period, as we have considered so far, but rather the price that it is charging contemporaneously in another market.\(^{19}\) To explore this case, we shall use again the linear demand case, but will now assume that the firm is a monopoly in market 1 but faces competition from firm 2 in market 2. Then, \( Q_1 \) is the firm’s output in market 1, \( q_1 \) is its output in market 2, and \( q_2 \) is firm 2’s output in market 2. The model then is identical to the model presented earlier, except that instead of having two time periods, we have two markets. The main

\(^{19}\)For example, in British Leyland Public Ltd. Co. v. Commission [1986], the European court determined that the price charged for providing traders with certificates that left-hand drive vehicles conform to an approved type was excessive by comparing it to the price for certificates for right-hand drive vehicles. In the NAPP case, the OFT has determined that the price charged to community pharmacies in the U.K. for sustained release morphine was excessive by comparing it to the price charged to hospitals. See “Napp Pharmaceutical Holdings Limited And Subsidiaries (Napp),” Decision of the Director General of Fair Trading, No Ca98/2/2001, 30 March 2001.
implications in terms of the analysis is that now, \( Q_1 \), \( q_1 \), and \( q_2 \), are set simultaneously, instead of \( Q_1 \) being set before \( q_1 \) and \( q_2 \). The profit functions of firms 1 and 2 are given by

\[
\Pi_1(Q_1, q_1, q_2) = \begin{cases} 
(A - Q_1) Q_1 + (A - q_1 - q_2) q_1, & q_1 + q_2 \leq Q_1, \\
(A - Q_1) Q_1 + (A - q_1 - q_2) q_1 - \gamma \tau [(a - Q_1) - (a - q_1 - q_2)] Q_1, & q_1 + q_2 > Q_1,
\end{cases}
\]

and

\[
\Pi_2(q_1, q_2) = (A - q_1 - q_2) q_2.
\]

We characterize the Nash equilibrium in the next proposition.

**Proposition 6:** *(The equilibrium in the contemporaneous excessive pricing case)* Suppose that \( p = a - Q \), firm 1 is a monopoly in market 1, but competes with firm 2 in market 2. Then, the equilibrium is given by

\[
Q_1^* = \frac{A (3 - 2 \gamma \tau)}{6 - \gamma \tau (6 - 6 \gamma \tau)}, \quad q_1^* = \frac{A (2 - \gamma \tau (4 - 6 \gamma \tau))}{6 - \gamma \tau (6 - 6 \gamma \tau)}, \quad q_2^* = \frac{A (2 - \gamma \tau (1 + 6 \gamma \tau))}{6 - \gamma \tau (6 - 6 \gamma \tau)}.
\]

(22)

if \( \gamma \tau < \frac{1}{3} \) and by

\[
Q_1^* = \frac{3 A}{5}, \quad q_1^* = \frac{A}{5}, \quad q_2^* = \frac{2 A}{5}.
\]

(23)

if \( \gamma \tau \geq \frac{1}{3} \). \( p_1 \) is excessive only when \( \gamma \tau < \frac{1}{3} \).

**Proof:** See the Appendix.

When \( p_1 \) is excessive, an increase in \( \gamma \tau \) raises \( Q_1^* \), but lowers \( q_1^* + q_2^* \). Hence, an increase in \( \gamma \tau \) benefits consumers in market 1, but harms consumers in market 2. To find the overall effect on consumers, note that given Proposition 6, the aggregate expected consumers’ surplus across the two markets is

\[
CS(Q_1^*, q_1^*, q_2^*) = \begin{cases} 
\frac{1}{2} \left( \frac{A(3 - 2 \gamma \tau)}{6 - \gamma \tau (6 - 6 \gamma \tau)} \right)^2 + \frac{1}{2} \left( \frac{A(4 - 5 \gamma \tau)}{6 - \gamma \tau (6 - 6 \gamma \tau)} \right)^2, & \gamma \tau < \frac{1}{3}, \\
2 \times \frac{1}{2} \left( \frac{3 A}{5} \right)^2, & \gamma \tau \geq \frac{1}{3}.
\end{cases}
\]

The expression in the top line is a U-shaped function of \( \gamma \tau \) and attains its highest value in the interval \( [0, \frac{1}{3}] \) at \( \gamma \tau = \frac{1}{3} \), where its value is \( \frac{9 A^2}{25} \), similar to its value in the bottom line (these properties can be verified with Mathematica). Hence, even in the contemporaneous excessive pricing case, increasing \( \gamma \tau \) is overall beneficial to consumers, although it should be emphasized that not all consumers benefit: consumers in market 2 are harmed. Moreover, note that when
\( \gamma \tau = 0 \), our model yields the same outcome as if there is no prohibition of excessive pricing. Since consumers’ surplus is higher when \( \gamma \tau = \frac{1}{3} \) than when \( \gamma \tau = 0 \), the prohibition of excessive pricing is overall beneficial for consumers.

The contemporaneous benchmark that we consider here resembles contemporaneous MFN’s, which prevent firms from offering selective discounts to some consumers. Besanko and Lyon (1992) show that contemporaneous MFN’s relax price competition, though in equilibrium firms may not wish to adopt them unilaterally. Their model differs from ours in that they assume that firms compete with each other in both markets (the market for shoppers and the market for non-shoppers in their model), while in our model, firm 1 is a monopoly in market 1. Moreover, they assume that under an MFN, a firm cannot price discriminate, while in our model, firm 1 can still price discriminate, but if it does, it may have to pay a fine with probability \( \gamma \). In terms of results, Basenko and Lyon (1992) show that MFN’s harm consumers because they raise the average price across the two markets, while in our framework a prohibition of excessive pricing is actually beneficial to consumers.

Finally, it should be noted that although we assume that demand and costs are the same in markets 1 and 2, in real life cases, they may differ across markets. In that case one would not be able to use the price in one market as a benchmark for the competitive price in another market without further analysis. For instance, instead of simply comparing prices across markets, one may have to compare price cost margins, though that opens the door for the need to establish the relevant cost in each market, which is typically complicated and often contentious.

7 Conclusion

We have examined the competitive effects of the prohibition of excessive pricing by a dominant firm. One problem when implementing this prohibition is to establish an appropriate competitive benchmark to assess whether the dominant firm’s price is indeed excessive. In this paper we have studied one such benchmark: the price that prevails once there is entry into the market is used to determine whether the pre-entry price was excessive. If the post-entry price is lower than the pre-entry price, the dominant firm may pay have to pay a fine proportional to the excess revenue it made in the pre-entry period.

We find that this benchmark induces the dominant firm to expand output before there is entry, but then it also induces it to cut output once entry occurs. While the entrant responds to
the soft post-entry behavior of the dominant firm by expanding output, total output is lower in the
post-entry level than it would be absent a prohibition of excessive pricing. Yet the expansion of
output before there is entry exceeds the contraction post entry, so at least when demand is linear,
the prohibition benefits consumers.

Although our results suggest that the prohibition of excessive pricing is pro competitive and
benefits consumers, it is worth bearing in mind that our analysis abstracts from many considerations
which are important in real-life cases. For example, our model does not take into account the
incentive of firms to invest in R&D, advertising, or product quality, hold inventories, offer a variety
of products, choose locations and other non-price decisions that affect competition and consumer
welfare. Our model also abstracts from the cost of litigation and other legal costs, as well as from
demand and cost fluctuations which make it harder to compare prices across different time periods.
Hence, more research is needed before we fully understand the competitive implications of the
prohibition of excessive pricing. Our results show, however, that absent other considerations, the
effect of the prohibition on output is beneficial to consumers. Moreover, we show that the prohibition
of excessive pricing actually promotes entry into the market as it induces the incumbent firm to
behave more softly once entry takes place.

Our analysis can be extended in a number of ways. We mention a few. First, it is possible
to endogenize the probability of entry, \( \alpha \), by assuming that the entrant has to bear an entry cost,
drawn from some known interval; entry then occurs only if the entrant’s profit exceeds its entry
cost. In such a setting, the dominant firm will have to take into account the effects of its pre-
entry output on the post-entry equilibrium and hence on the probability of entry. Second, one can
consider the case where incumbent and entrant have different marginal costs. Then it is possible
to ask whether the prohibition of excessive pricing promotes efficient entry more than it promotes
inefficient entry. Third, it is possible to assume that the probability that the dominant firm is
convicted, \( \gamma \), increases with the gap between the pre- and post-entry prices. For instance, \( \gamma = 0 \)
if the gap is below \( \Delta \), but \( \gamma > 0 \) if the gap exceeds \( \Delta \). It should be interesting to examine how
consumers’ surplus changes with \( \Delta \). Fourth, one can study a model in which the marginal cost of
firm 1 is private information. In that case, the period 1 output will be a signal for firm 1’s cost,
which will affect the incentive to charge an excessive price. We leave these extensions and others
for future research.
8 Appendix

Following are the proofs that $\pi_i(q_i, q_j)$ is concave in $q_i$ and the proofs of Lemmas 2, 4, 5, and 7 and Propositions 3-6.

Claim: $\pi_i(q_i, q_j)$ is concave in $q_i$.

Proof: Differentiating the first line in (1) yields:

$$\frac{\partial^2 \pi_1(q_1, q_2)}{\partial q_1^2} = 2p'(q_1 + q_2) + p''(q_1 + q_2)q_1.$$ 

If $p''(\cdot) \leq 0$, we are done. If $p''(q_1 + q_2) > 0$,

$$\frac{\partial^2 \pi_1(q_1, q_2)}{\partial q_1^2} < 2p'(q_1 + q_2) + p''(q_1 + q_2)(q_1 + q_2) < 0,$$

where the last inequality follows from Assumption A1. Hence, $\pi_1(q_1, q_2)$ is concave in $q_1$ when $q_1 + q_2 \leq Q_1$. The proof that $\pi_2(q_1, q_2)$ is concave in $q_2$ is identical.

Differentiating the second line in (1) yields:

$$\frac{\partial^2 \pi_1(q_1, q_2)}{\partial q_2^2} < 2p'(q_1 + q_2) + p''(q_1 + q_2)(q_1 + \gamma\tau Q_1).$$

Again, if $p''(\cdot) \leq 0$, we are done. If $p''(q_1 + q_2) > 0$,

$$\frac{\partial^2 \pi_1(q_1, q_2)}{\partial q_1^2} < 2p'(q_1 + q_2) + p''(q_1 + q_2)(q_1 + \gamma\tau (q_1 + q_2))$$

$$< 2p'(q_1 + q_2) + p''(q_1 + q_2)(1 + \gamma\tau)(q_1 + q_2)$$

$$< 0,$$

where the first inequality follows because $q_1 + q_2 > Q_1$, and the last inequality follows from Assumption A1. Hence, $\pi_1(q_1, q_2)$ is also concave in $q_1$ when $q_1 + q_2 > Q_1$.

Proof of Lemma 2: Suppose that $p_1$ is excessive and assume by way of negation that firm 1 exits the market when firm 2 enters. Then firm 2 is a monopoly in period 2 and produces the monopoly output, since $r_C^2(0) = Q^M$. For $p_1$ to be excessive, it must be that $Q_1 < Q^M$. Moreover, given Assumption A2, $\gamma\tau Q_1 < Q_1 \leq Q^M$. Since $p_1$ is excessive, firm 1’s best-response function against $q_2$ is $r_E^2(q_2)$. Evaluating the second line in (5) at $(0, Q^M)$, and noting that $Q^M$ is defined implicitly
by the first-order condition $p(Q^M) + p'(Q^M)Q^M = c,
\frac{\partial \pi_1 (0, Q^M)}{\partial q_1} = p(Q^M) - c + \gamma \tau p' (Q^M) Q_1 = -p'(Q^M) (Q^M - \gamma \tau Q_1) > 0,
$
where the inequality follows since $p'(\cdot) < 0$ and $Q^M > \gamma \tau Q_1$. Hence, $r^E_1 (Q^M) > 0$, contradicting the assumption that firm 1 exits the market. 

**Proof of Lemma 4:** The monopoly output, $Q^M$ is defined by

$$\pi' (Q_1) = p (Q_1) + p' (Q_1) Q_1 - c = 0.$$ 

Evaluating $\pi' (Q_1)$ at $Q_1$ and using (6),

$$\pi' (\overline{Q}) = p (\overline{Q}_1) + p' (\overline{Q}_1) \overline{Q}_1 - c = -p' (\overline{Q}_1) \left( \frac{\gamma \tau - 1}{2} \right) \overline{Q}_1 < 0,$$

where the inequality follows by Assumption A2. Hence, $\overline{Q} > Q^M$.

The Cournot equilibrium is defined by $\pi'_1 (q_1, q_2) = 0$ and $\pi'_2 (q_1, q_2) = 0$, where

$$\pi'_i (q_1, q_2) = p (q_1 + q_2) + p' (q_1 + q_2) q_i - c.$$

Since the equilibrium is symmetric, $q^C_1 = q^C_2 = q^C$,

$$\pi'_i (q^C_1, q^C_2) = p (2q^C) + p' (2q^C) q^C - c = 0.$$

Evaluating this equation at $\overline{Q}_1$ and using (6),

$$\pi'_i (\overline{Q}) = p (\overline{Q}_1) + p' (\overline{Q}_1) \frac{\overline{Q}_1}{2} - c = -\gamma \tau p' (\overline{Q}_1) \frac{\overline{Q}_1}{2} > 0.$$

Hence, $\overline{Q} < q^C_1 + q^C_2$.

Next, differentiating equation (6) with respect to $\overline{Q}_1$ and $\gamma \tau$, and rearranging terms,

$$\frac{\partial \overline{Q}_1}{\partial (\gamma \tau)} = \frac{-\frac{p' (\overline{Q}_1) \overline{Q}_1}{2}}{p' (\overline{Q}_1) \left( 1 + \frac{1 + \gamma \tau}{2} \right) + p'' (\overline{Q}_1) \left( \frac{1 + \gamma \tau}{2} \right) \overline{Q}_1}.$$
Assumption A1 ensure that the denominator is negative and hence, $\frac{\partial Q_1}{\partial (\gamma \tau)} < 0$.

Finally, recall that $Q_1$ satisfies both (4) and (3) when $q_i = q_2$. Subtracting the latter from the former, using the fact that $q_1 + q_2 = Q_1$, and rearranging yields,

$$p' (Q_1) [q_1^* + \gamma \tau Q_1 - q_2^*] = 0, \quad \Rightarrow \quad 2p' (Q_1) \frac{q_1^*}{Q_1} - \frac{1 - \gamma \tau}{2} = 0.$$  

Hence, $\frac{q_1^*}{Q_1} = -\frac{1 - \gamma \tau}{2}$ when $Q_1 = Q_1$, and since $q_1 + q_2 = Q_1$, $\frac{q_2^*}{Q_1} = \frac{1 + \gamma \tau}{2}$. We now prove Lemma 5: First, suppose that $Q_1 < Q_1$. Then the Nash equilibrium is defined by the intersection of $r_1^C (q_2)$ and $r_2^C (q_1)$. Fully differentiating this system yields the following comparative statics matrix

$$
\begin{bmatrix}
2p' + p'' (q_1^* + \gamma \tau Q_1) & p' + p'' (q_1^* + \gamma \tau Q_1) \\
p' + p'' q_2^* & 2p' + p'' q_2^*
\end{bmatrix} \times
\begin{bmatrix}
\frac{\partial q_1^*}{\partial Q_1} \\
\frac{\partial q_2^*}{\partial Q_1}
\end{bmatrix} = \begin{bmatrix}-\gamma \tau p' \\
0\end{bmatrix},
$$

where the arguments of $p'$ and $p''$ are suppressed to ease notation. Hence,

$$\frac{\partial q_1^*}{\partial Q_1} = -\frac{\gamma \tau p' (2p' + p'' q_2^*)}{(2p' + p'' (q_1^* + \gamma \tau Q_1)) (2p' + p'' q_2) - (p' + p'' (q_1^* + \gamma \tau Q_1)) (p' + p'' q_2^*)}$$

$$= -\frac{\gamma \tau (2p' + p'' q_2^*)}{3p' + p'' (q_1^* + q_2^* + \gamma \tau Q_1)},$$

and

$$\frac{\partial q_2^*}{\partial Q_1} = \frac{\gamma \tau p' (p' + p'' q_2^*)}{(2p' + p'' (q_1^* + \gamma \tau Q_1)) (2p' + p'' q_2) - (p' + p'' (q_1^* + \gamma \tau Q_1)) (p' + p'' q_2^*)}$$

$$= \frac{\gamma \tau (p' + p'' q_2^*)}{3p' + p'' (q_1^* + q_2^* + \gamma \tau Q_1)}.$$

Since $\frac{p' + p'' q_2^*}{3p' + p'' (q_1^* + q_2^* + \gamma \tau Q_1)} = \frac{1}{1 + \frac{2p' + p'' (q_1^* + q_2^* + \gamma \tau Q_1)}{p' + p'' q_2^*}} < 1$ by Assumption A1, $-\gamma \tau < \frac{\partial q_1^*}{\partial Q_1} < 0 < \frac{\partial q_2^*}{\partial Q_1} < \gamma \tau$. Moreover, using Assumption A1, it follows that whenever $Q_1 < Q_1$,

$$\frac{\partial (q_1^* + q_2^*)}{\partial Q_1} = \frac{-\gamma \tau (2p' + p'' q_2^*)}{3p' + p'' (q_1^* + q_2^* + \gamma \tau Q_1)} + \frac{\gamma \tau (p' + p'' q_2^*)}{3p' + p'' (q_1^* + q_2^* + \gamma \tau Q_1)}$$

$$= \frac{-p' \gamma \tau}{3p' + p'' (q_1^* + q_2^* + \gamma \tau Q_1)} < -\gamma \tau.$$

Now suppose that $Q_1 \geq Q_1$. Then the Nash equilibrium is defined by the intersection of $q_1 = Q_1 - q_2$ and $r_2^C (q_1)$. Fully differentiating this system, yields the following comparative statics
Proof of Proposition 3: Suppose that $Q'_1 < \overline{Q}_1$. Then $Q'_1$ is implicitly defined by $\Pi'_1(Q_1) = 0$, where $\Pi'_1(Q_1)$ is given by (9). Fully differentiating the equation, using Assumption A3, and the fact that $\Pi'_1(Q_1) = 0$,

$$
\frac{\partial Q'_1}{\partial (\delta \alpha)} = \frac{\gamma \tau (MR(Q_1) - p(q'_1 + q'_2)) - p'(q'_1 + q'_2)(q'_1 + \gamma \tau Q_1) \frac{\partial q'_2}{\partial Q_1}}{\Pi''_1(Q_1)} = MR(Q_1) - c \frac{\partial q'_2}{\partial Q_1} > 0,
$$

where the inequality follows because $\Pi''_1(Q_1) < 0$ and because $Q'_1 > Q^M$ implies that $MR(Q_1) < c$.

Likewise, when $Q'_1 > \overline{Q}_1$, $Q'_1$ is implicitly defined by $\Pi'_1(Q_1) = 0$, where $\Pi'_1(Q_1)$ is given
by (11). Fully differentiating the equation, using Assumption A3, and the fact that \( \Pi'_1(Q_1) = 0 \),

\[
\frac{\partial Q_1'}{\partial (\delta \alpha)} = - \frac{p'(Q_1) q_1' + (p(Q_1) - c) \frac{\delta q_1}{\delta Q_1}}{\Pi''_1(Q_1)} = MR(Q_1) - c \frac{\delta Q_1'}{\delta \alpha} > 0,
\]

where the inequality follows because \( \Pi''_1(Q_1) < 0 \) and because \( Q_1^* > Q^M \) implies that \( MR(Q_1) < c \).

As for \( \gamma \tau \), note that when \( Q_1^* > \overline{Q}_1 \), \( \Pi_1(Q_1) \) is independent of \( \gamma \tau \).  \( \blacksquare \)

**Proof of Proposition 4:** Evaluating \( \Pi'_1(Q_1) \) as \( Q_1 \) approaches \( \overline{Q}_1 \equiv \frac{2A}{3 + \gamma \tau} \) from below and noting that in this case, \( \Pi'_1(Q_1) \) is given by the first line in (16) yields,

\[
\Pi'_1(\overline{Q}_1) = A - \frac{4A}{3 + \gamma \tau} - \frac{\delta \alpha \gamma \tau}{9} \left[ 7A - \frac{4A (9 + \gamma \tau)}{3 + \gamma \tau} \right]
\]

\[
= A \frac{A}{9 (3 + \gamma \tau)} \left[ 27 + 9 \gamma \tau - 36 - 7 \delta \alpha \gamma \tau (3 + \gamma \tau) + 4 \delta \alpha \gamma \tau (9 + \gamma \tau) \right]
\]

\[
= \frac{3A \gamma \tau (5 - \gamma \tau)}{9 (3 + \gamma \tau)} \left[ \delta \alpha - \frac{3 (1 - \gamma \tau)}{\gamma \tau (5 - \gamma \tau)} \right].
\]

Likewise, evaluating \( \Pi'_1(Q_1) \) when \( Q_1 \) approaches \( \overline{Q}_1 \equiv \frac{2A}{3 + \gamma \tau} \) from above and noting that in this case \( \Pi'_1(Q_1) \) is given by the second line in (16),

\[
\Pi'_1(\overline{Q}_1) = A - \frac{4A}{3 + \gamma \tau} + \delta \alpha \left( 3A - \frac{8A}{3 + \gamma \tau} \right)
\]

\[
= A \left[ 3 + \gamma \tau \right] \left[ 3 + \gamma \tau - 4 + 9 \delta \alpha + 3 \delta \alpha \gamma \tau - 8 \delta \alpha \right]
\]

\[
= A \left[ 1 + 3 \gamma \tau \right] \left[ \delta \alpha - \frac{1 - \gamma \tau}{1 + 3 \gamma \tau} \right].
\]

Noting that

\[
\frac{3 (1 - \gamma \tau)}{\gamma \tau (5 - \gamma \tau)} - \frac{1 - \gamma \tau}{1 + 3 \gamma \tau}
\]

\[
= \frac{(1 - \gamma \tau) (3 + 4 \gamma \tau + (\gamma \tau)^2)}{\gamma \tau (5 - \gamma \tau) (1 + 3 \gamma \tau)} > 0,
\]

there are now three cases to consider:

(i) If \( \delta \alpha < \frac{1 - \gamma \tau}{1 + 3 \gamma \tau} < \frac{3 (1 - \gamma \tau)}{\gamma \tau (5 - \gamma \tau)} \), then \( \Pi'_1(\overline{Q}_1) < 0 \) and \( \Pi'_1(\overline{Q}_1) < 0 \), so \( Q_1^* < \overline{Q}_1 \). Then, \( \Pi'_1(Q_1) \) is given by the first line in (16). Setting it equal to 0 and solving yields the expression in
the first line of (17). Note that

\[
\frac{2A}{3 + \gamma} - \frac{A}{2} \left[ \frac{9 - 7 \delta \alpha \gamma \tau}{9 - \delta \alpha \gamma (9 + \gamma \tau)} \right] = \frac{3A \gamma \tau (5 - \gamma \tau)}{2} \left[ \frac{3(1 - \gamma \tau)}{\gamma \tau (5 - \gamma \tau)} - \delta \alpha \right]
\]

Hence, \( A \left[ \frac{9 - 7 \delta \alpha \gamma \tau}{9 - \delta \alpha \gamma (9 + \gamma \tau)} \right] < Q_1 \equiv \frac{2A}{3 + \gamma} \) whenever \( \delta \alpha < \frac{3(1 - \gamma \tau)}{\gamma \tau (5 - \gamma \tau)} \) (otherwise, \( Q_1^* = Q_1 \)).

(ii) If \( \delta \alpha > \frac{3(1 - \gamma \tau)}{\gamma \tau (5 - \gamma \tau)} > \frac{1 - \gamma \tau}{1 + 3 \gamma \tau} \), then \( \Pi'_1 \left( Q_1^* \right) > 0 \) and \( \Pi'_1 \left( Q_1^* \right) > 0 \), so \( Q_1^* > Q_1 \). Now \( \Pi'_1 \) is given by the second line in (16); setting it equal to 0 and solving yields the expression in the second line of (17). Note that

\[
\frac{A}{2} \left[ \frac{1 + 3 \delta \alpha}{1 + 2 \delta \alpha} \right] - \frac{2A}{3 + \gamma} = \frac{A (1 + 3 \gamma \tau)}{2} \left[ \frac{\delta \alpha - \frac{1 - \gamma \tau}{1 + 3 \gamma \tau}}{1 + 2 \delta \alpha (3 + \gamma \tau)} \right].
\]

Hence, \( \delta \alpha \geq \frac{1 - \gamma \tau}{1 + 3 \gamma \tau} \) ensures that \( A \left[ \frac{1 + 3 \delta \alpha}{1 + 2 \delta \alpha} \right] \geq Q_1 \equiv \frac{2A}{3 + \gamma} \) (when \( \delta \alpha < \frac{1 - \gamma \tau}{1 + 3 \gamma \tau}, Q_1^* = Q_1 \)).

(iii) If \( \frac{1 - \gamma \tau}{1 + 3 \gamma \tau} \leq \delta \alpha < \frac{3(1 - \gamma \tau)}{\gamma \tau (5 - \gamma \tau)} \), then \( \Pi'_1 \left( Q_1^* \right) < 0 \leq \Pi'_1 \left( Q_1^* \right) \), so both \( A \left[ \frac{9 - 7 \delta \alpha \gamma \tau}{9 - \delta \alpha \gamma (9 + \gamma \tau)} \right] \) and \( \frac{A}{2} \left[ \frac{1 + 3 \delta \alpha}{1 + 2 \delta \alpha} \right] \) are local maxima. To determine which is a global maximum, note that when \( \delta \alpha \) is close to \( \frac{1 - \gamma \tau}{1 + 3 \gamma \tau} \), \( \Pi'_1 \left( Q_1^* \right) \) goes to 0, while \( \Pi'_1 \left( Q_1^* \right) \) < 0. Hence, \( A \left[ \frac{9 - 7 \delta \alpha \gamma \tau}{9 - \delta \alpha \gamma (9 + \gamma \tau)} \right] \) is a global maximum. By contrast, when \( \delta \alpha \) goes to \( \frac{3(1 - \gamma \tau)}{\gamma \tau (5 - \gamma \tau)} \), \( \Pi'_1 \left( Q_1^* \right) \) goes to 0, while \( \Pi'_1 \left( Q_1^* \right) \) > 0, so \( \frac{A}{2} \left[ \frac{1 + 3 \delta \alpha}{1 + 2 \delta \alpha} \right] \) is a global maximum. Substituting \( Q_1 = A \left[ \frac{9 - 7 \delta \alpha \gamma \tau}{9 - \delta \alpha \gamma (9 + \gamma \tau)} \right] \) into the first line of (15) and \( Q_1 = A \left[ \frac{1 + 3 \delta \alpha}{1 + 2 \delta \alpha} \right] \) into the second line of (15) and comparing the resulting expressions, reveals that \( \frac{A}{2} \left[ \frac{9 - 7 \delta \alpha \gamma \tau}{9 - \delta \alpha \gamma (9 + \gamma \tau)} \right] \) is a global maximum if \( \delta \alpha < Z(\gamma \tau) \), whereas \( \frac{A}{2} \left[ \frac{1 + 3 \delta \alpha}{1 + 2 \delta \alpha} \right] \) is a global maximum if \( \delta \alpha > Z(\gamma \tau) \), where \( Z(\gamma \tau) \) is defined in the proposition.

The equilibrium values in period 2 are obtained by substituting \( Q_1^* \) into the period 2 equilibrium, which is \( \left( \frac{A - 2 \gamma \tau Q_1}{3}, \frac{A + \gamma \tau Q_1}{3} \right) \) if \( Q_1 < \frac{2A}{3 + \gamma} \) and \( (2Q_1 - A, A - Q_1) \) if \( \frac{2A}{3 + \gamma} < Q_1 < \frac{2A}{3} \).

\( \blacksquare \)

**Proof of Proposition 5:** Note from (21) that \( CS(Q_1^*) \) depends on \( \gamma \tau \) only through \( Q_1^* \). But when \( \delta \alpha \geq Z(\gamma \tau) \), (17) shows that \( Q_1^* \) is independent of \( \gamma \tau \), so \( \frac{\partial CS(Q_1^*)}{\partial (\gamma \tau)} = 0 \). Next suppose that \( \delta \alpha < Z(\gamma \tau) \). Then,

\[
\frac{\partial CS(Q_1^*)}{\partial (\gamma \tau)} = \left[ Q_1^* - \delta \alpha \left( \frac{2A - \gamma \tau Q_1^*}{3} \right) \frac{\gamma \tau}{3} \right] \frac{\partial Q_1^*}{\partial (\gamma \tau)} = \frac{1}{9} \left[ (9 + \delta \alpha (\gamma \tau)^2) Q_1^* - 2 \delta \alpha \gamma \tau A \right] \frac{\partial Q_1^*}{\partial (\gamma \tau)},
\]

35
where \( \frac{\partial Q^*_1}{\partial (\gamma \tau)} > 0 \). Using (17), the square bracketed expression is given by

\[
\left(9 + \delta \alpha (\gamma \tau)^2\right) \frac{A}{2} \left[\frac{9 - 7\delta \alpha \gamma \tau}{9 - \delta \alpha \gamma \tau (9 + \gamma \tau)}\right] - 2\delta \alpha \gamma \tau A \]

\[
= A \left[\frac{(9 - 7\delta \alpha \gamma \tau) (9 + \delta \alpha (\gamma \tau)^2) - 4\delta \alpha \gamma \tau (9 - \delta \alpha \gamma \tau (9 + \gamma \tau))}{2 (9 - \delta \alpha \gamma \tau (9 + \gamma \tau))}\right],
\]

which is positive for all \( \delta \alpha < 0.9 \) (this is verified with Mathematica), and hence for all \( \delta \alpha < Z(\gamma \tau). \)\(^{20}\) Hence, \( \frac{\partial CS(Q^*_1)}{\partial (\gamma \tau)} > 0. \]

**Proof of Proposition 6:** Suppose first that \( Q_1 < q_1 + q_2 \). Then, the Nash equilibrium is defined by the following first-order conditions:

\[
\frac{\partial \Pi_1 (Q_1, q_1, q_2)}{\partial Q_1} = A - 2Q_1 - \gamma \tau [q_1 + q_2 - Q_1] + \gamma \tau Q_1 = 0,
\]

\[
\frac{\partial \Pi_1 (Q_1, q_1, q_2)}{\partial q_1} = A - 2q_1 - q_2 - \gamma \tau Q_1 = 0,
\]

and

\[
\frac{\partial \Pi_2 (\cdot, q_1, q_2)}{\partial q_2} = A - q_1 - 2q_2 = 0.
\]

Solving the three equations yields (22). Note that \( Q^*_1 \) and \( q^*_2 \) are increasing with \( \gamma \tau \), while \( q^*_1 \) is decreasing with \( \gamma \tau \). Moreover, \( q_1^* + q_2^* \) is decreasing with \( \gamma \tau \) and \( Q_1^* < q_1^* + q_2^* \) whenever \( \gamma \tau < \frac{1}{3} \) and \( Q_1^* \geq q_1^* + q_2^* \) when \( \gamma \tau \geq \frac{1}{3} \).

Suppose then that \( \gamma \tau \geq \frac{1}{3} \). If \( Q_1 \geq q_1 + q_2 \), \( p_1 \) is not excessive, so firm 1’s profit is given by the top line of \( \Pi_1 (Q_1, q_1, q_2) \). But then we get the monopoly outcome in market 1 and the Cournot output in market 2, so the aggregate output levels are \( \frac{2}{3} \) in market 1 and \( \frac{2A}{3} \) in market 2. Since \( \frac{4}{3} < \frac{2A}{3} \), \( p_1 \) is in fact excessive, a contradiction. Hence, to ensure that \( p_1 \) is not excessive, firm 1 must set \( q_1 \) and \( Q_1 \) such that \( q_1 + q_2 = Q_1 \). Its profit then becomes

\[
\Pi_1 (Q_1, q_1, q_2) = (A - q_1 - q_2) (q_1 + q_2) + (A - q_1 - q_2) q_1.
\]

\(^{20}\)The expression depends on two parameters: \( \delta \alpha \) and \( \gamma \tau \). A three dimensional plot shows that the expression is positive for all \( \delta \alpha < 0.9 \).
The resulting Nash equilibrium is therefore defined by the following first-order conditions:

\[ \frac{\partial \Pi_1 (Q_1, q_1, q_2)}{\partial Q_1} = 2A - 4q_1 - 3q_2 = 0, \]

and

\[ \frac{\partial \Pi_2 (q_1, q_2)}{\partial q_2} = A - q_1 - 2q_2 = 0. \]

Solving, and using the fact that \( q_1 + q_2 = Q_1 \), yields (22). ■
9 References


