Capital structure with countervailing incentives

Yossef Spiegel*
and
Daniel F. Spulber**

The regulated firm's choice of capital structure is affected by countervailing incentives: the firm wishes to signal high value to capital markets to boost its market value while also signalling high cost to regulators to induce rate increases. When the firm's investment is large, countervailing incentives lead both high- and low-cost firms to choose the same capital structure in equilibrium, thus decoupling capital structure from private information. When investment is small or medium-sized, the model may admit separating equilibria in which high-cost firms issue greater equity and low-cost firms rely more on debt financing.

1. Introduction

The capital structure of regulated firms is a key determinant of regulated rates. Under traditional cost-of-service regulation and some forms of price-cap regulation, commissions set regulated rates so as to ensure firms a "fair" rate of return on their equity (see e.g., Bonbright, Danielson, and Kamerschen, 1988; Phillips, 1988; and Spulber, 1989). Consequently, regulated firms have an incentive to choose their capital structure in anticipation of its effect on their rates. Given the size and political sensitivity of the regulated sector and the fact that the stocks of regulated firms are so widely held, it is clear that an understanding of strategic interaction between the firm's capital structure and the rate setting process is needed. In this article, we examine

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* Tel Aviv University; spiegel@post.tau.ac.il.
** Northwestern University; jems@nwu.edu.

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1 For example, the Federal Communications Commission sets price caps on interstate access rates to ensure local exchange carriers a rate of return of 11.25% on their investments (FCC, CC Docket 89-624). Similarly, the FCC has established an interim industrywide rate-of-return factor of 11.25% for cable television cost-of-service proceedings (FCC, MM Docket No. 93-215).

2 The regulated public utilities sector in the United States, including telecommunications, electricity, natural gas, and sanitary services, accounted for about 5% of gross domestic product in 1994 (Bureau of Economic Analysis, 1996).

3 Among the New York Times list of favorite stocks, which reports the fifteen issues with the most shareholders, ten are stocks of regulated utilities.
two key aspects of the financing strategies of regulated firms. First, due to the limited commitment ability of regulators, a regulated firm may have an incentive to become leveraged, since debt may deter regulators from lowering rates because they seek to minimize the likelihood that the firm will go bankrupt and incur a deadweight loss. Second, asymmetries in the information that regulators, investors, and the regulated firm possess about the firm’s costs significantly complicate the leverage effect. Recognizing the information conveyed by its capital structure fundamentally alters the financing incentives of the regulated firm.

In the last decade, a large literature has emerged that studies optimal rate regulation under asymmetric information (e.g., Baron and Myerson, 1982; Laffont and Tirole, 1986; Lewis and Sappington, 1988; and Spulber, 1989). These models assume that regulators can precommit to optimal regulatory mechanisms and apply the principal-agent framework to derive incentive schedules. But are the commitments of regulators credible? Historically, the courts have given regulators a great deal of leeway in setting rates. According to the Supreme Court in the landmark Hope Natural Gas case of 1944, a regulatory agency is “not bound to the use of any single formula or combination of formulae in determining rates,” since it is the net effect that matters. Since regulatory agencies can exercise substantial discretion in setting rates, and since the commissioners change over time, their commitment ability is limited. Through prudence reviews and rate rehearings, regulators are able to change what capital expenditures are allowed in the rate base as well as the allowed rate of return on capital. Moreover, as deregulation proceeds in electric power, natural gas, and telecommunications, many state and federal regulatory agencies are questioning whether or not they are bound by any “regulatory contract.”

In Spiegel and Spulber (1994) we studied the strategic interaction between the firm’s capital structure and the rate-setting process, finding that rate regulation induces firms to become leveraged. An important aspect of rate regulation missing from that study was the presence of asymmetric information. This aspect is the main focus of the current article. To explore the effects on capital structure of limited commitment under asymmetric information, we follow Banks (1992) and Besanko and Spulber (1992) by modelling the regulatory process as a sequential game between a firm and a regulator. In the first stage of this game, the firm chooses its capital structure by issuing a mix of equity and debt to outside investors in order to raise funds to invest in a project. In the second stage, the firm’s securities are priced in the capital market according to the expectations of outside investors about the outcome of the regulatory process. In the third stage, the regulator chooses rates to maximize a welfare function defined over consumers’ surplus and firm’s profits. The fact that the regulator moves after the firm reflects the lack of regulatory commitment to rates. The regulator responds to an increase in the firm’s debt level by increasing rates, thereby reducing the probability that the firm will go bankrupt. Anticipating the regulator’s response, the firm chooses an optimal debt target by trading off the benefits of having higher rates (a leverage effect) against the increase in its expected cost of bankruptcy.

Under asymmetric information about its expected costs, the firm can use its capital structure not only to create a leverage effect, but also to signal its private information. But unlike the typical Spence-style signalling model, the firm signals to two receivers: the regulator and outside investors. We show that the presence of two receivers creates

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4 In United Railways, the Supreme Court stated in 1930 that “[w]hat will formulate a fair return in a given case is not capable of exact mathematical demonstration.” United Railways & Elec. Co. v. West, 280 U.S. 234, 249, 251 (1930).


6 See Sidak and Spulber (1996) on the problem of deregulatory takings and breach of the regulatory contract.
countervailing incentives for the firm. On the one hand, the firm wishes to signal low expected costs and therefore high profits to the capital market to boost the market value of its securities (a securities pricing effect). But on the other hand, the firm also wishes to convince the regulator that its expected costs are high because the regulated price is based on costs (a cost effect). Since the two effects work in opposite directions, our model has the interesting feature that the cost of signalling information to one receiver is due not to “burning money” but to the negative response of the second receiver.

As is common in signalling models, our model admits multiple perfect Bayesian equilibria. To eliminate equilibria supported by “unreasonable” out-of-equilibrium beliefs of the regulator and outside investors, we apply the refinement of undefeated equilibria proposed by Mailath, Okuno-Fujiwara, and Postlewaite (1993). This refinement is appealing because it requires out-of-equilibrium beliefs to be “globally” consistent, thereby avoiding the logical problems inherited in alternative belief-based refinements, in which out-of-equilibrium beliefs are adjusted separately from the beliefs at other information sets, including those along the equilibrium path. In the current model, the refinement eliminates all equilibria except for the Pareto-dominant equilibrium, that is, those that give both high- and low-cost types the highest payoffs among all equilibria.

There is evidence to suggest that regulators generally allow firms to issue new securities only if external funds are needed to cover the cost of investment in physical assets (Phillips, 1988). This implies that in our model, the size of the firm’s investment project imposes a restriction on the amount of new equity and debt that the firm can issue, and therefore on its ability to signal information. Thus the equilibrium choice of capital structure depends critically on the size of the project. When the project is small in the sense that firms cannot issue debt to the point where there is a positive leverage effect, the model may admit both a pooling equilibrium and a continuum of separating equilibria, all of which are payoff-equivalent. In the separating equilibria, firms with low probability of a cost shock (I-types) issue relatively little equity, so firms with a high probability of cost shock (h-types) have little to gain by mimicking I-types. At the same time, h-types issue relatively high levels of equity to outsiders, thereby ensuring that if I-types mimic them, they will face a significant equity-underpricing effect. In a pooling equilibrium, both types issue the same debt level that completely offsets the benefits and costs from separation for both I-types and h-types.

When the size of the project is medium, in the sense that under full information there would be a positive leverage effect for I-type firms but not for h-types, the model admits a unique undefeated separating equilibrium. In this equilibrium, I-type firms finance the project entirely with debt, thereby fully exploiting the leverage effect, while h-type firms separate themselves by issuing to outsiders enough equity. This strategy allows h-types to separate themselves because it ensures that should I-types mimic them, their equity will be sufficiently underpriced to render mimicking unprofitable.

Finally, if the project is large in the sense that both types face a positive leverage effect, the model admits a unique undefeated equilibrium, which is pooling. So long as the project is not too large, both I-type firms and h-type firms finance it entirely with debt in order to fully exploit the leverage effect. Then, the resulting regulated price does not depend on the firms’ type, so there are no countervailing incentives (only the beliefs of equityholders matter, and both types try to convince equityholders...
that their type is 1 in order to boost the market value of their equity). As a result, 1-
types cannot separate themselves and the equilibrium must be pooling. It should be
noted that this result is independent of the specific refinement we use; the application
of undefeated equilibrium only allows us to eliminate all Pareto-dominated pooling
equilibria.

When the project is larger still, the presence of countervailing incentives leads to
a unique undefeated equilibrium that is again pooling. This time however, firms use a
mix of debt and equity: they first issue debt up to a debt target, and then use equity
financing on the margin. The pooling result is due to the fact that relatively large
projects have the property that the potential gains for each type of firm from revealing
its identity to one receiver are outweighed by the loss associated with the negative
response of the second receiver; consequently, no type of firm has an incentive to
distinguish itself. Although the result depends on the refinement we use, it nonetheless
seems intuitive, since all separating equilibria in the case of relatively large projects
are Pareto dominated by the pooling equilibrium.

In practice, regulated firms make large investments in infrastructure and generally
use a mix of debt and equity to finance them. Our model shows that in such cases, the
capital structure of firms is uncorrelated with their expected values, reflecting the pool-
ing of diverse firm types. This result suggests that countervailing incentives should be
taken into account in future empirical studies of capital structure and cost of capital of
regulated firms. Moreover, this result can explain why Miller and Modigliani (1966),
in their classic study of the electric utility industry, found “no evidence of sizeable
leverage or dividend effect [on firm value] of the kind assumed in much of the tradit-
ional finance literature.”8 While this empirical result supports the Modigliani and Mil-
er irrelevance theorem (1958), it conflicts with later financial signalling models in
which capital structure conveys the firm’s private information about its value (see Harris
and Raviv (1991) for a literature survey). The countervailing incentives identified in
this article can serve to reconcile these two approaches in the case of regulated indus-
tries.

The effects of countervailing incentives on the capital structure of firms have not
been studied before, with the notable exception of Gertner, Gibbons, and Scharfstein
(1989). They examine a model in which a firm uses its capital structure to signal private
information to both the product and the capital markets. Their article, however, differs
from ours in at least two important ways. First, in their model the firm competes in an
oligopolistic product market, while in ours the firm is a regulated monopolist. Second
and more important, they assume away the possibility of bankruptcy, which plays a
key role in our analysis.

A leverage effect is identified by Taggart (1981) although not in a strategic setting.
This effect has been observed empirically by Taggart (1985) and by Dasgupta and
Nanda (1993). The effects of regulatory opportunism in a full-information setting were
considered by Spiegel and Spulber (1994) and Spiegel (1994) in the context of capital
structure and by Spiegel (1997) in the context of the choice of technology. Lewis and
Sappington (1995) examine optimal incentive regulation under asymmetric information
when the firm obtains investment funds from the capital market. Their article addresses
normative issues in an agency setting, while ours considers positive issues within a
signalling framework.

The article proceeds as follows. In Section 2 we present the basic model and define
the equilibrium concept. In Section 3 we solve the regulator’s problem, and in Section
4 we set out the capital market equilibrium. Then we fully characterize the equilibrium

8 Miller and Modigliani examine data on the electric utility industry from 1954 to 1957. They do not
account, however, for the effects of rate regulation on the firm’s capital structure choice, as our model.
strategies of the regulated firm in Section 5. Additional properties of the equilibrium are examined in Section 6. Section 7 concludes. All proofs are in the Appendix.

2. The model and the equilibrium concept

- We present a sequential model of rate regulation that examines the interaction between the regulator's pricing strategy and the capital structure of the firm. In stage 1 of the model, the firm decides whether or not to undertake a project that requires a sunk cost, \( k \). If the firm forgoes the project, it does not produce anything, and the payoffs of its equityholders and consumers are both equal to zero. If the firm undertakes the project, it issues a mix of debt and equity to finance it. Then, in stage 2, the market value of the firm's securities is determined in a perfectly competitive capital market. In stage 3, the regulator sets the regulated price, taking the capital structure of the firm as given. Finally, the firm's operating cost is realized, output is produced, and payments are made.

The sequential structure of the model allows us to examine the effects of the limited regulatory commitment to rates that, as argued in the Introduction, characterizes the regulatory framework in the United States. Moreover, the model considers the consequences of firms exercising discretion in choosing their capital structures. This conforms with general practice in which regulators limit their interference in these types of management decisions. Empirical studies suggest that regulated firms indeed exercise such discretion (Taggart, 1985; Hagerman and Ratchford, 1978).

- Consumers. The demand for the output of the project is inelastic, i.e., consumers demand a fixed quantity, which we normalize to one unit, with \( V \) representing consumers' total willingness to pay. Using \( p \) to denote the regulated price, the payoff of consumers is represented by consumers' surplus, \( CS(p) = V - p \).

- The regulated firm. The firm's operating cost, \( C \), may be subject to a shock, representing for example, a fuel price increase, equipment failure, or a cost overrun. The cost shock is equal to \( c \) and it occurs with probability \( \theta \), where \( \theta \) can be either low, \( \theta' \), or high, \( \theta^h \), \( 0 < \theta' < \theta^h < 1 \). Normalizing the firm's operating cost absent a cost shock to zero, the expected operating cost is equal to \( \theta c \). Assume that \( c < V \), so production is \textit{ex post} efficient even if the cost shock occurs. The probability \( \theta \), referred to as the firm's type, is the firm's private information.

To model the choice of capital structure, we assume that the firm is initially owned by a set of equityholders and has neither outstanding debt nor financial reserves. Let \( E \) be the market value of the new equity representing a fraction \( a \in [0, 1] \) of the firm's equity, and let \( B \) be the market value of debt with face value \( D \). The firm needs to raise funds to cover the cost of the project, \( k \leq E + B \). Evidence suggests that regulatory commissions generally do not allow regulated firms to raise external funds in excess of the costs of investment in physical assets (Phillips, 1988). Thus, the firm's budget constraint is

\[
k = E + B. \tag{1}
\]

The financial strategy of the firm is a mapping from its type, \( \theta \), to a pair \( (a(\theta), D(\theta)) \), such that (1) is satisfied. The firm chooses its financial strategy (independently of the regulator) to maximize the expected payoff of its initial equityholders, which we specify below.

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9 For example, Phillips (1988, p. 226) argues that with respect to financial decisions, "few commissions are willing to substitute their judgments for those of the management except in reorganization cases."
The firm’s earnings are \( p - C \). For a given debt obligation \( D \) and a regulated price \( p \), the firm can pay its debt if and only if \( p - C \geq D \). The firm then remains solvent, and its equityholders are the residual claimants, receiving a payoff of \( p - C - D \). If \( p - C < D \), the firm declares bankruptcy, and debtholders become the residual claimants. This reflects the concept of limited liability: the firm cannot be forced to pay debtholders more than its income. Since \( C \) equals \( c \) with probability \( \theta \) and equals zero otherwise, the probability of bankruptcy as a function of debt and the regulated price, given the firm’s type, is

\[
L(p, D|\theta) = \begin{cases} 
0, & D \leq p - c, \\
\theta, & p - c < D \leq p.
\end{cases}
\]  

When \( D < p - c \), the firm never goes bankrupt because its cash flow is sufficiently high to cover the debt payments even if the cost shock is realized. On the other hand, when \( p - c \leq D < p \), the firm goes bankrupt if and only if the cost shock is realized, an event that occurs with probability \( \theta \). Debt levels above \( p \) are dominated strategies for the firm and will never be observed in equilibrium, since equityholders are certain to get a zero payoff. In what follows, we therefore need not consider such debt levels.

Bankruptcy imposes extra costs on the firm such as legal fees and reorganization costs. We let these costs be proportional to the shortfall of earnings from the debt obligation, with a unit bankruptcy cost equal to \( t \). The expected bankruptcy costs are therefore given by

\[
T(p, D|\theta) = t \times L(P, D|\theta) \times (D - p + c).
\]  

We assume that \( t \) and \( c \) are sufficiently small so that the payoff of debtholders in the event of bankruptcy, \( p - C - t(D - p - C) \), is nonnegative in the relevant range. This ensures that the debtholders limited liability constraint is never binding. In the Appendix we provide a sufficient condition on \( t \) and \( c \) for this assumption to hold.

The expected \textit{ex post} profit of the firm as a function of the regulated price and the firm’s debt, given the firm’s type, \( \theta \), is

\[
\Pi(p, D|\theta) = p - \theta c - T(p, D|\theta).
\]  

The expected profit equals the expected earnings of the firm net of expected bankruptcy costs. It represents the combined expected \textit{ex post} return to equityholders (both old and new) and debtholders and is divided between them according to their respective claims. Since the marginal operating income is deterministic, there is no conflict of interests between equityholders and debtholders. We can treat them in the regulatory process as if they were one group (both would like \( p \) to be as high as possible). In a more general model this need not be the case. For instance, Brander and Lewis (1986, 1988) show that when the marginal operating income is stochastic, equityholders and debtholders have conflicting interests, since the firm’s expected operating income over solvent states of nature differs from the expected operating income over states of nature in which the firm goes bankrupt. Therefore, if the marginal operating income, for example, is larger in good states of nature than it is in bad states of nature, the optimal regulated price from equityholders’ point of view will be higher than the optimal regulated price from debtholders’ point of view.

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10 Throughout, taxes are assumed away. For a survey of tax-based theories of capital structure, see, e.g., Myers (1984).

11 We assume that \( k \) represents sunk costs rather than an investment in durable physical capital. This assumption means that the firm cannot use \( k \) to repay its debt when its cash flow is low.

12 Assuming instead that bankruptcy costs are constant does not change any of the results, but it complicates the analysis because the objective function of the firm becomes discontinuous.
The objective of the regulated firm is to maximize the expected payoff of its initial equityholders. If the firm undertakes the project, the payoff of initial equityholders is given by

\[ Y(p, \alpha, D|\theta) = (1 - \alpha) \times (1 - L(p, D|\theta)) \times [p - \theta c + L(p, D|\theta)c - D]. \]  

(5)

If the firm forgoes the project, the initial equityholders receive a zero payoff.

☐ The capital market. We assume that the capital market is perfectly competitive. Given the information available to outside investors, the firm’s securities will be fairly priced in equilibrium in the sense that each investor earns a zero net expected return on his investment. Thus, in what follows, we impose the competitive market constraint on the equilibrium rather than specifying an investment strategy for outside investors.

☐ The regulatory commission. The regulator chooses the regulated price to maximize the expected social welfare function \( W(p, D|\theta) = CS(p)^{(1-\gamma)}[I(p, D|\theta)] \), where \( \gamma \) is a parameter between zero and one. The parameter \( \gamma \) measures the degree to which the regulator cares about the ex post profits of the firm relative to consumer surplus. The resulting regulated price allocates the expected social surplus according to the asymmetric Nash bargaining solution for the regulatory process.

This approach follows models of the rate-setting process as a bargaining problem between consumers and investors (Spulber, 1989; Besanko and Spulber, 1992). It is also consistent with Peltzman’s (1976) political economy model of rate regulation, where \( W \) can be viewed as the regulator’s Cobb-Douglas utility function. These studies are consistent with regulatory case law, such as Hope Natural Gas, in which “[t]he fixing of ‘just and reasonable’ rates, involves a balancing of the investor’s and the consumers’ interests” that should result in rates “[w]ithin a range of reasonableness.” According to the Supreme Court decision, “[t]he return to equity owners should be commensurate with returns on investment in other enterprises having corresponding risks. That return, moreover, should be sufficient to assure confidence in the financial integrity of the enterprise, so as to maintain its credit and to attract capital.” Also, as explained by the Pennsylvania Public Utilities Commission, this range “[i]s bounded at one level by investor interest against confiscation and the need for averting any threat to the security for the capital embarked upon the enterprise. At the other level it is bounded by consumer interest against excessive and unreasonable charges for service.”

In writing the welfare function, we assume that the regulator takes into account the firm’s operating profits rather than its accounting profits, that is, we exclude the sunk cost of investment. Our model therefore represents a case of regulatory opportunism as the regulator completely ignores the firm’s sunk cost of investment. One could adopt a less extreme view of regulatory opportunism by assuming that the welfare function is given by \( CS(p)^{(s-\gamma)}([I(p, D|\theta) - sk]) \), where \( s \) is a parameter between zero and one that measures the degree of regulatory opportunism; as \( s \) increases toward one, the regulator becomes less opportunistic. However, since \( k \) in our model is constant, assuming that \( s = 0 \), as we do, does not involve any loss of generality. The use of operating profit is consistent with the notion that the regulated firm will continue to provide service as long as its operating profit is positive.

Substituting for profit and consumer surplus, the regulator’s social welfare function is

\[ \text{Federal Power Comm. v. Hope Natural Gas Co., 320 U.S. 591, 603 (1944).} \]

By increasing $p$, the regulator increases the social surplus because $T(p, D|\theta)$ is decreasing in $p$, and at the same time, he shifts part of the surplus from consumers to claimholders. To simplify the regulator’s maximization problem, we shall impose the following restriction on $\gamma$:

$$\gamma < \gamma \equiv \min \left\{ \frac{V - c}{V - \theta c}, \frac{c(1 + \theta D)}{V} \right\}.$$  

This restriction implies that the regulator is not too pro-consumer. It simplifies the exposition considerably and ensures that countervailing incentives are present at all levels of debt.

Outside investors and the regulator share a common prior, $b^0$, on the firm’s being the high type. Given $b^0$, the expected likelihood of a cost shock is $\theta = b^0 \theta^h + (1 - b^0) \theta^l$. After observing the financial strategy of the firm, outside investors and the regulator update their prior beliefs about $\theta$. Let $b^i$ and $b^R$ respectively be the posterior probability assigned by outside investors and by the regulator to the firm’s type being $\theta^h$. The outside investors’ posterior probability of a cost shock is $\theta^i = b^i \theta^h + (1 - b^i) \theta^l$, and that of the regulator is $\theta^R = b^R \theta^h + (1 - b^R) \theta^l$.

**The equilibrium concept.** We restrict attention to pure strategies. A perfect Bayesian equilibrium (PBE) in pure strategies in the three-stage asymmetric information game is a pair of strategies, $(\alpha^*(D|\theta^h), (\alpha^*(\theta, D^*(\theta)))$, a zero net expected return condition on outside investors, and a pair of belief functions, $(b^i, b^R)$ that satisfy the following four conditions:

(i) Given the financial strategy of the firm, $(\alpha, D)$, and given his posterior beliefs, $b^R$, the regulator chooses $p^*(D|\theta^h)$ to maximize expected social welfare; hence, for all $D$,

$$p^*(D|\theta^h) \in \arg\max_{p} b^R W(p, D|\theta^h) + (1 - b^R) W(p, D|\theta^l). \quad (7)$$

(ii) Given the financial strategy of the firm, $(\alpha, D)$, their correct expectations about the regulated price, $p^*(D|\theta^h)$, and their posterior beliefs, $b^i$, the market values of equity and debt, $E^* = E^*(p^*(D|\theta^h), \alpha, D|\theta^h)$ and $B^* = B^*(p^*(D|\theta^h), \alpha, D|\theta^h)$, are determined such that outside investors earn a zero net expected return on the firm’s securities.

(iii) Given its correct expectations about the regulated price, $p^*(D|\theta^h)$, and the capital market equilibrium, a $\theta$-type firm chooses its financial strategy to maximize the expected payoff of its initial equityholders, subject to the firm’s budget constraint:

$$(\alpha^*(\theta), D^*(\theta)) \in \arg\max_{(\alpha, D)} Y(p^*(D|\theta^h), \alpha, D|\theta) \quad \theta \in \{\theta^h, \theta^l\} \quad (8)$$

subject to

$$k = E^* + B^*.$$
(iv) Outside investors and the regulator have the same posterior beliefs on and off the equilibrium path, which are derived from the Bayes rule whenever it is applicable. In particular, on the equilibrium path, these beliefs are correct:

\[
\theta' = \theta^R = \begin{cases} 
\theta & \text{if } D^*(\theta') \neq D^*(\theta^0), \\
\theta^0 & \text{if } D^*(\theta') = D^*(\theta^0).
\end{cases}
\] (9)

If the top line of (9) holds, the equilibrium is separating; if the bottom line holds, it is pooling.

As is often the case in signalling models, the current model admits multiple perfect Bayesian equilibria because the belief function defined above places no restrictions on the beliefs of outside investors and the regulator off the equilibrium path. To eliminate equilibria that are supported by “unreasonable” beliefs, we apply the refinement of undefeated equilibrium due to Mailath, Okuno-Fujiwara, and Postlewaite (1993). This refinement is appealing because it ensures that any adjustment of out-of-equilibrium beliefs is consistent with beliefs at other information sets, including some information sets along the equilibrium path.\(^{15}\) Thus, undefeated equilibrium is immune to the so-called Stiglitz critique (see Cho and Kreps, 1987). This property distinguishes this refinement from other belief-based refinements such as the intuitive criterion (Cho and Kreps, 1987) and perfect sequential equilibrium (Grossman and Perry, 1986). Moreover, this refinement is not biased against pooling equilibria like other belief-based refinements (e.g., the intuitive criterion, or the D2 criterion, Banks and Sobel (1987)). The formal definition of the refinement is presented in the Appendix. Intuitively, the refinement works as follows: consider a putative equilibrium, \(\sigma\), and suppose that the capital structure is chosen by some type in an alternative equilibrium due to Mailath, Okuno-Fujiwara, and Postlewaite (1993). Then if the firm chooses the pair \((a'', D')\), outside investors and the regulator interpret this choice as a message sent by the firm. When only one type prefers \(\sigma'\) to \(\sigma\), outside investors and the regulator reason that the message was sent by this type. When both types prefer \(\sigma'\) to \(\sigma\), outside investors and the regulator find the message uninformative (both types are equally likely to have sent it), so they do not revise their prior beliefs. Finally, when one type strongly prefers \(\sigma'\) to \(\sigma\) while the other type prefers it weakly, outside investors and the regulator believe that the former type surely played \((a'', D')\), while the latter type may or may not have played it; the prior beliefs are therefore revised by increasing the weight assigned to the type that surely played \((a'', D')\).

3. The regulator’s pricing strategy

Since the equilibrium strategies are sequentially rational, we characterize the equilibrium by solving the game backwards, beginning with the regulator’s stage 3 pricing strategy. To this end, we first assume that the firm’s type is common knowledge and solve for the regulator’s pricing strategy under full information. Given the structure of our model, the pricing strategy of the regulator under asymmetric information then follows immediately from his full-information pricing strategy. Given the firm’s type, the equilibrium pricing strategy of the regulator as a function of the firm’s debt level is given by

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\(^{15}\) For a detailed discussion on the properties of undefeated equilibria and a comparison between this refinement concept and other refinements, see Mailath, Okuno-Fujiwara, and Postlewaite (1993).
where

\[
p^*(D|\theta) = \begin{cases} 
\hat{D}(\theta) + c, & D \leq \hat{D}(\theta), \\
D + c, & \hat{D}(\theta) < D < \overline{D}(\theta), \\
\hat{D}(\theta) + c + \frac{\gamma \theta t(D + c(1 - \theta))}{1 + \theta t}, & D \geq \overline{D}(\theta),
\end{cases}
\]  

(10)

The assumption that \( \gamma < \hat{\gamma} \leq (V - c)/(V - \theta c) \) ensures that \( \hat{D}(\theta) > 0 \) for all \( \theta \), while the assumption that \( V > \theta c \) implies that \( \overline{D}(\theta) > \hat{D}(\theta) \).

To understand equation (10), consider first the case where \( D \leq p - c \). Then the probability of bankruptcy is zero, so the regulator’s problem is to maximize \((V - p)^{\gamma(1 - \gamma)}(p - \theta c)\) subject to \( D < p - c \). Solving for \( p \) yields \( p^*(D|\theta) = (1 - \gamma)V + \gamma \theta c \). Using the definition of \( \hat{D}(\theta) \) yields the first line in (10). When \( D > \hat{D}(\theta) \), the constraint is binding, so \( p^*(D|\theta) = D + c \). Second, consider the case where \( D + c \leq p \leq D \). Then the probability of bankruptcy is \( \theta \), so the regulator’s problem is

\[
(V - p)^{\gamma(1 - \gamma)}(p - \theta c - \theta(D - p - c))
\]

subject to \( p - c \leq D \leq p \). Ignoring the constraint, the solution for this problem yields the third line in (10). When \( D \geq \overline{D}(\theta) \), the constraint is nonbinding. For \( D < \overline{D}(\theta) \), the constraint is binding, so \( p^*(D|\theta) = D + c \).

Figure 1 shows \( p^*(D|\theta) \) as a function of the firm’s debt level for the two possible firm types. There are several properties of \( p^*(D|\theta) \) that are worth noting. First, \( p^*(D|\theta) \) increases with the firm’s type because the regulator sets it as a markup over the firm’s expected operating costs, which in turn increase with \( \theta \). Second, \( p^*(D|\theta) \) does not depend directly on the equity share, \( \alpha \) (given his beliefs about \( \theta \), the regulator cares
about capital structure only to the extent that it affects the expected costs of bankruptcy), or on the firm’s sunk cost, k (the regulator cannot commit to rates and therefore behaves opportunistically). Third, Figure 1 shows that for debt levels below $D(\theta)$, $p^*(D | \theta)$ is sufficiently high to ensure that the firm never goes bankrupt, so in this range, $p^*(D | \theta)$ is independent of $D$. Once $D$ reaches $D(\theta)$, the regulator can no longer ignore it because then the firm would go bankrupt if the cost shock occurs. Since bankruptcy is socially costly, it is optimal for the regulator at this range to increase $p^*(D | \theta)$ at the same rate as the increase in $D$ to ensure that the probability of bankruptcy remains zero. For debt levels above $D(\theta)$, it is no longer optimal to avoid bankruptcy with certainty, since the marginal loss in consumer surplus from increasing $p^*(D | \theta)$ at the same rate as the increase in $D$ exceeds the marginal gain from keeping the probability of bankruptcy at zero. Hence, from this point on, $p^*(D | \theta)$ increases by less than the increase in debt, leaving the firm susceptible to bankruptcy.

Substituting for $p^*(D | \theta)$ in (2) and rearranging terms, the likelihood of bankruptcy is zero if $D \leq D(\theta)$, and $1$ if $D > D(\theta)$. Hence, $D(\theta)$ is the critical level of debt above which debt becomes risky (i.e., susceptible to the risk of default). In the next proposition, we study the properties of the critical debt levels, $\hat{D}(\theta)$, and $D(\theta)$.

**Proposition 1.** (i) The critical debt level, $\hat{D}(\theta)$, is increasing in the probability of the cost shock, $\theta$, and in the consumers’ willingness to pay, $V$; decreasing in the cost shock, $c$, and in the welfare weight $\gamma$; and is independent of the bankruptcy cost, $t$.

(ii) The critical debt level, $D(\theta)$, above which debt becomes risky, is increasing and concave in $\theta$, increasing in $V$ and $t$; it is decreasing in $c$ and $\gamma$. Consequently, the range of riskless debt levels becomes larger as $\theta$, $V$, and $t$ increase and as $c$ and $\gamma$ decrease.

At a first glance, it seems counterintuitive that an increase in the expected operating cost increases the range of riskless debt levels. Yet to compensate the firm for expected operating cost increases, the regulator sets the regulated price as a (weakly) increasing function of $\theta$. This in turn allows the firm to issue more debt and still remain immune to bankruptcy.

Next consider the asymmetric-information case. Since the objective function of the regulator is linear in $\theta$ and $b^\infty$, the regulator’s beliefs enter the problem only through $\theta^\infty$, which is the posterior probability of a cost shock from the regulator’s perspective. The regulator’s equilibrium pricing strategy under asymmetric information is therefore $p^* = p^*(D | \theta^\infty)$.

To see that $D(\theta)$ is concave in $\theta$, use the form of $\hat{D}(\theta)$ from equation (12), so that for any $\theta^\infty$, $\theta^\prime$, and $b^0 < 1$,

$$b^0\hat{D}(\theta^\prime) + (1 - b^0)\hat{D}(\theta^\prime) - \hat{D}(\theta^\prime) = \frac{-(\theta^\infty - \theta^\prime)^2 (1 - b^\prime)(Vt + 2c)}{(2 + \theta^\prime t)(2 + \theta^\prime t)(2 + \theta^\prime t)} < 0.$$

### 4. The capital market equilibrium

Since the capital market is competitive, debtholders and new equityholders earn a net expected return equal to the risk-free interest rate, which without a loss of generality we normalize to zero. Assuming that investors correctly anticipate the regulator’s equilibrium pricing strategy, their expectations about the likelihood of bankruptcy are represented by $L(D | \theta^\prime) = L(p^*, D | \theta^\prime)$. The equilibrium market values of new equity and debt are therefore given by

$$E^*(p^*, \alpha, D | \theta^\prime) = \alpha(1 - L(D | \theta^\prime))(p^* - \theta c + L(D | \theta^\prime)c - D)$$

and
The right side of (13) represents the share of new equityholders in the expected profits of the firm net of debt payments conditional on the firm remaining solvent. The first term on the right side of (14) represents the expected return to debtholders in the event that the firm remains solvent. The second term represents the expected return to debtholders when the firm goes bankrupt, in which case they become the residual claimants and receive the firm’s profits net of bankruptcy costs.

In equilibrium, the budget constraint of the firm must hold with equality. Substituting for \( E^*(p^*, \alpha, D | \theta) \) and \( B^*(p^*, \alpha, D | \theta) \) in the firm’s budget constraint given by equation (1) yields

\[
k = (1 - L(D | \theta')) \times [\alpha(p^* - \theta c + L(D | \theta')c - D) + D] + L(D | \theta') \times [p^* - c - \tau(D - p^* + c)].
\]

Equation (15) is the condition for a competitive equilibrium in the capital market. This equation implicitly defines a unique equity participation of new equityholders, \( \alpha^*(p^*, D | \theta) \), such that the project is fully financed:

\[
\alpha^*(p^*, D | \theta) = \frac{k - D + L(D | \theta')(1 + \tau(D - p^* + c))}{(1 - L(D | \theta'))[p^* - \theta c + L(D | \theta')c - D]}.
\]

Given \( k \), (16) implies that the firm has only one degree of freedom when it chooses a pair \((\alpha, D)\). Consequently, the financial strategy of the firm is effectively reduced to a choice of a debt level, \( D(\theta) \), with \( \alpha(\theta) \) being determined by (16).

### 5. The equilibrium capital structure

To solve for the equilibrium choice of capital structure, we substitute for \( \alpha^*(p^*, D | \theta) \) into (5) and rearrange terms to express the expected payoff of the initial equityholders of a \( \theta \)-type firm when the project is undertaken as a function of the face value of debt and the beliefs of outside investors and the regulator:

\[
Y(D, \theta', \theta^* | \theta) = \begin{cases} 
    [p^*(D | \theta^*) - \theta c - k] \times \frac{p^*(D | \theta^*) - \theta c - D}{p^*(D | \theta^*)} - \frac{k - D + L(D | \theta')(1 + \tau(D - p^* + c))}{(1 - L(D | \theta'))[p^* - \theta c + L(D | \theta')c - D]} & \text{if } D \leq \overline{D}(\theta), \\
    [p^*(D | \theta^*) - k - \tau(D - p^*(D | \theta^*) + c)] \times \frac{1 - \theta}{1 - \theta'} & \text{if } D > \overline{D}(\theta), 
\end{cases}
\]

The expected payoff function is illustrated in Figure 2 for the two types of firms, assuming that the firm’s type is common knowledge (i.e., \( \theta' = \theta^* = \theta \)). The figure shows that under full information, the expected payoff of the firm is maximized at \( \overline{D}(\theta) \), which is the largest debt level that is still riskless. The firm therefore wishes to issue as much riskless debt as possible. But since \( \alpha \geq 0 \), and \( L(D | \theta) = 0 \) for all \( D \leq \overline{D}(\theta) \), it follows from (15) that the firm can reach its debt target only if \( k \geq \overline{D}(\theta) \).

Hence, our model yields a “pecking order” theory of financing: the firm uses debt financing first until it reaches its debt target, and only then does it use equity financing to raise additional funds. Debt is used first because of its effect on the regulatory process.

Asymmetric information has an important implication for the regulated firm’s financial strategy because it conveys information to regulators and investors. In the next proposition, we distinguish the effects of prices, beliefs, and capital structure on the initial equityholders’ expected payoff function.
Proposition 2. (i) The cost effect. \( Y(D, \theta', \theta^g | \theta) \) increases with \( p^* \), which in turn increases (weakly) with \( \theta^g \), the probability that the regulator assigns to \( C = c \). As a result, for sufficiently large debt levels, each type would like to convince the regulator that its type, \( \theta \), is high to induce him to set a high regulated price.

(ii) The securities pricing effect. \( Y(D, \theta', \theta^g | \theta) \) decreases with \( \theta' \), the probability that outside investors assign to \( C = c \). Hence, each type would like to convince outside investors that its type, \( \theta \), is low.

(iii) The leverage effect. Fixing \( \theta' \) and \( \theta^g \), \( Y(D, \theta', \theta^g | \theta) \) increases in \( D \) for all \( \hat{D}(\theta) \leq D \leq \bar{D}(\theta) \) because of regulatory concern about bankruptcy.

In equilibrium, the firm chooses its financial strategy, \( D(\theta) \), by balancing the three effects identified in Proposition 2. Since the cost and the securities pricing effects work in opposite directions, the firm faces countervailing incentives: it wishes to signal high cost to the regulator but low cost to the capital market. The presence of countervailing incentives implies that the cost of signalling information to one receiver in our model is not in the form of “burning money,” as in Spence-style signalling models, but rather is due to the negative response of the other receiver.

The equilibrium financial strategy of the firm depends critically on the size of the project, \( k \). This is because \( k \) imposes a restriction on the amount of debt that the firm can issue and therefore limits its ability to signal information. We need not consider very large projects, since when \( k > \hat{D}(\theta^g) + (1 - \theta^g)c \), both types will choose to forgo the project in stage 1 of the game because undertaking it will yield their initial equityholders a negative expected payoff. We distinguish three types of projects that differ with respect to their size. If the project is small, there is no leverage effect. If the project is medium, the leverage effect dominates, leading to a separating equilibrium. Finally, if the project is large, countervailing incentives lead to a unique equilibrium that is pooling.

□ Small projects. A project is small if its set-up cost, \( k \), is less than \( \hat{D}(\theta^g) \). As Figure 1 shows, the regulated price of both types in this case is independent of the firm’s debt, so there is no leverage effect. Under full information, both types would therefore be indifferent about their debt-equity mix. Under asymmetric information, firms with a high likelihood of a cost shock (\( h \)-types) may wish to mimic firms with a low likelihood...
of a cost shock (l-types) if the latter issue equity, in order to make outside investors believe that their expected profits are higher than they really are and therefore cause their equity to be overpriced. Likewise, l-types may wish to mimic h-types to make the regulator believe that the regulated price should be raised. This suggests that the equilibrium is separating only if l-types issue relatively little equity, so that the gain to h-types from mimicking them would be small, while h-types need to issue relatively high levels of equity to outsiders to ensure that l-types will face a significant equity-underpricing effect should they mimic h-types. We now establish necessary conditions for the existence of perfect Bayesian equilibria (PBE), and characterize them.

**Proposition 3.** Suppose that $k < \frac{5}{2}$ and let $Y(\theta) = Y(D(\theta), \theta, \theta' | \theta)$ be the expected payoff of a $\theta$-type firm in a separating equilibrium. Then if $k \geq \gamma Y(\theta')/(1 - \gamma)$, there exists a continuum of separating PBE in which $D^*(\theta') = \hat{D}(\theta')$, where

$$D^*(\theta') \in [0, k - \gamma Y(\theta')/(1 - \gamma)],$$

$$D^*(\theta') \in [\max\{0, k - \gamma Y(\theta')/(1 - \gamma)\}, k], \alpha^*(\theta') \leq \gamma \leq \alpha^*(\theta'),$$

$$p^*(D(\theta) | \theta) = p^*(\theta) = \hat{D}(\theta) + c,$$ and $Y(\theta) = \hat{D}(\theta) + (1 - \theta)c - k$, $\theta \in \{\theta', \theta''\}$. In addition, if $k \geq \gamma(1 - \gamma)(V - \theta c)$, there exists a pooling PBE in which $D^*(\theta') = D^*(\theta''') = \frac{1}{2}k(1 - \gamma) - \gamma(V - \theta c)$, $\alpha^* = \gamma$, and $p^* = \hat{D}(\theta') + c$. All equilibria are payoff-equivalent and give each type its full information expected payoff.

Proposition 3 implies that when $k$ is small (but not too small), the model admits both separating and pooling PBE. Since all equilibria are payoff-equivalent, both types of firms are indifferent among them, so the refinement of undefeated equilibrium has no bite (indeed, all equilibria are equally “reasonable”). To understand why $k$ cannot be too small, note that l-types can always separate themselves; for example, when l-types finance the project entirely with debt, h-types have nothing to gain by mimicking them (there is no equity that can be overpriced), and at the same time h-types stand to lose the higher rates that they receive due to the cost effect. In contrast, h-types can separate themselves only if $k$ is sufficiently large to enable h-types to issue at least a fraction $\gamma$ of the firm’s equity to outsiders to ensure that should l-types mimic them, the negative equity underpricing they will face will outweigh the positive cost effect.

Interestingly, although Proposition 3 confirms our intuition that $\alpha^*(\theta') \geq \alpha^*(\theta)$, it may nonetheless be the case that $D^*(\theta') > D^*(\theta)$ because h-type firms are less valuable than l-type firms, so receiving a larger fraction of their equity may not be enough to compensate investors, in which case debt with a higher face value may be also needed. Hence, while our model predicts a negative correlation between expected profits and outside equity, it does not predict a necessary correlation between expected profits and debt levels. In a pooling equilibrium, both types issue the same debt level that completely offsets the benefits and costs from separation for both types and, moreover, ensures each type its full information payoffs. A pooling equilibrium can exist only if $k$ is sufficiently large to ensure the existence of such a debt level. Finally, note that when $\gamma Y(\theta')/(1 - \gamma) \leq k < \gamma(1 - \gamma)(V - \theta c)$, the equilibrium must be separating (a pooling equilibrium does not exist), while when $k < \gamma Y(\theta')/(1 - \gamma)$, there is no PBE in pure strategies.

In the absence of a leverage effect, our model becomes similar to the two-audience financial signalling model considered by Gertner, Gibbons, and Scharfstein (1988). The main result in their article (Propositions 1 and 2) states that the equilibrium is pooling if the ex ante expected profit of the firm is weakly greater under pooling than under separation, and separating if the reverse is true. Indeed, the ex ante profit of the firm

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16 Unlike equity, the market value of debt is independent of the firm’s type when $k \leq \hat{D}(\theta)$, since debt is riskless in this range, implying that the market value of debt equals its face value.
in the small-project case is the same under pooling and under separation, so their result implies ours. Although Gertner, Gibbons, and Scharfstein use the refinement concept of Farrell-Grossman-Perry or perfect sequential equilibrium (Grossman and Perry, 1986), whereas we use the refinement of undefeated equilibrium, the two refinements select the same equilibria for our small-projects case.\footnote{\text{The perfect sequential equilibrium refinement eliminates equilibria that are not immune to deviations with consistent interpretations. Interpretations are nonempty subsets of sender’s types and they are consistent; if once they are believed, they induce the receivers to choose actions such that the sender’s payoff is higher than his payoff in the putative equilibrium if and only if the sender’s type is included in the interpretation in question.}} To see that the expected profits of the firm are indeed the same under pooling and separation, note that the \textit{ex ante} profits of the firm under pooling are

\[ b^0(\hat{\theta}) + (1 - \theta^0)c + (1 - b^0)(\hat{\theta}^0) + (1 - \theta^0)c = \hat{\theta} + (1 - \theta^0)c, \]

whereas under separation they are

\[ b^0(\hat{\theta}) + (1 - \theta^0)c + (1 - b^0)(\hat{\theta}^0) + (1 - \theta^0)c = (1 - \gamma)(V - \theta c) = \hat{\theta} + (1 - \theta^0)c, \]

where the first and last equalities are implied by equations (11) and (12).

In contrast, the cases of medium and large projects, considered next, are substantially different from the model of Gertner, Gibbons, and Scharfstein (1988). This is because with medium and large projects, the leverage effect plays a crucial role in the firm’s equilibrium financial strategy.

\begin{itemize}
  \item **Medium projects.** A project is of medium size if its set-up cost is such that
  \[ \hat{\theta} < k \leq \hat{\theta}^0. \]
  If information were full, $l$-type firms could have exploited the leverage effect by issuing enough debt, whereas $h$-types could not have done so and are therefore indifferent to their capital structure. Under asymmetric information, $l$-types finance the project entirely with debt, exactly as they would have under full information, while $h$-types separate themselves by limiting the amount of debt that they issue and relying on equity financing instead.

  \begin{proposition}
  Suppose that $\hat{\theta} < k \leq \hat{\theta}^0$. Then, if $k \geq \hat{\theta}^0 Y(\theta^0) / Y(\theta^0)$, there exists a unique undefeated separating equilibrium in which
  \[ D^*(\theta^0) \leq (k Y(\theta^0) - \hat{\theta}^0 Y(\theta^0)) / (Y(\theta^0) - Y(\theta^0)), \]
  \[ \alpha^*(\theta^0) = (\hat{\theta}^0 - k) / (\theta^0 - \theta^0)c, \]
  \[ p^*(\theta^0) = \hat{\theta}^0 + c, \]
  \[ D^*(\theta) = k, \] $\alpha^*(\theta) = 0$, and $p^*(\theta) = k + c$, where $Y(\theta^0) = \hat{\theta}^0 + (1 - \theta^0)c - k$ and $Y(\theta^0) = (1 - \theta^0)c$ are the equilibrium expected payoffs of both types are equal to those that would obtain under full information.
  \end{proposition}

  Proposition 4 reveals that when the project is medium-sized, the model admits a unique undefeated equilibrium in which $l$-type firms fully exploit the leverage effect by using an all-debt financing, while $h$-types separate themselves by issuing enough equity to render mimicking by $l$-types unattractive. Unlike the small-project case, the equilibrium is now unique because the leverage effect implies that $l$-types are no longer indifferent among all separating equilibria. Thus, the undefeated equilibrium refinement eliminates all Pareto-dominated separating equilibria.

  \item **Large projects.** A project is large if its set-up cost is such that
  \[ \hat{\theta} < k \leq \bar{\theta} + (1 - \theta)c. \]
  In this case, the presence of countervailing incentives due to signalling to both regulators...
and investors causes both types of firms to pursue in equilibrium the same financial strategy. Consequently, the capital structure of firms with high or low probabilities of a cost shock coincide in equilibrium.

**Proposition 5.** Suppose that $D(\theta) < k \leq \bar{D}(\theta) + (1 - \theta)c$. Then, the model admits a unique undefeated equilibrium which is pooling. In this equilibrium, $D^* = \min\{k, D(\theta)\}$, $\alpha^* = \max\{0, (k - \bar{D}(\theta))/(1 - \theta)c\}$, and $p^* = D^* + c$.

The intuition behind Proposition 5 is as follows. When the project is such that $D(\theta) < k \leq \bar{D}(\theta)$, both types wish to finance it with as much debt as possible in order to exploit the leverage effect. The resulting regulated price is $p^* = D + c$. Since $p^*$ does not depend on the firms’ type, the regulator’s beliefs do not matter. Similarly, the beliefs of debtholders do not matter, since $p^*$ is high enough to ensure that debt is riskless (the firm can fully repay it even if the cost shock occurs). The beliefs of equityholders, however, do matter, since $l$-types are more profitable than $h$-types; consequently, both types will try to convince equityholders that their type is $l$ in order to boost the market value of their equity. But since there are no countervailing incentives, signalling is costless, so $l$-types cannot separate themselves. Note that this result is independent of the specific refinement we use. The application of undefeated equilibrium, however, allows us to eliminate all Pareto-dominated pooling equilibria.

When $D(\theta) < k \leq \bar{D}(\theta) + (1 - \theta)c$, the situation is more complex. In this case, the only candidate for an undefeated separating equilibrium is the Pareto–dominant separating equilibrium from the firm’s perspective, because this equilibrium defeats all other separating equilibria. In this equilibrium (if it exists), either $l$-types or $h$-types reach their debt targets. However, the Pareto-dominant separating equilibrium is in turn defeated by the Pareto-dominant pooling equilibrium, in which $D^* = \min\{k, \bar{D}(\theta)\}$, since both types find the pooling equilibrium more attractive. Pooling dominates separation because the equity overpricing effect that $h$-types enjoy by pooling with $l$-types is sufficiently large to outweigh the negative cost effect that they face in equilibrium. For $l$-types the opposite is true: the positive cost effect that they enjoy under pooling is sufficiently large to outweigh the associated equity underpricing effect.

It should be noted that unlike in the case where $D(\theta) < k \leq \bar{D}(\theta)$, now the nonexistence of separating equilibria does depend on the specific refinement we use. Nevertheless, this result seems intuitive because all separating equilibria are Pareto-dominated by pooling at $D^* = \min\{k, \bar{D}(\theta)\}$. The proof uses the fact that the critical debt function $D(\theta)$ is concave to show that the Pareto-dominant pooling PBE defeats the Pareto-dominant separating PBE (i.e., both types are strictly better off in the former equilibrium). The concavity of the debt level follows from the form of the regulator’s objective function and the regulator’s choice of a pricing policy.

Finally, note that there is a substantial difference between the case where $k < \bar{D}(\theta)$ and the case where $k \geq \bar{D}(\theta)$. In the former, the firm uses debt financing on the margin, while in the latter it uses equity financing on the margin.

## 6. Properties of the equilibrium

- Having fully characterized the equilibrium, we are now ready to examine its properties. First, substituting for the equilibrium debt level and regulated price into (2)
reveals that the probability of bankruptcy in equilibrium is zero for both types of firms, regardless of the size of the project. Hence,

**Proposition 6.** In equilibrium, the regulated firm’s debt is completely riskless; hence, the firm does not face the possibility of bankruptcy.

It should be emphasized that the firm’s debt is riskless because of the leverage effect: the regulator responds to the firm’s debt by setting a regulated price such that the firm can repay it with probability one. The result of Proposition 6 is consistent with the observation that bankruptcies have been very rare in the U.S. utility sector since the mid-1930s. It is similar to Spiegel (1994) but stands in contrast with Spiegel and Spulber (1994) and Spiegel (1996), because here and in Spiegel (1994), the regulator maximizes the product of consumer surplus and profits, which leads to prices based on average costs, whereas in Spiegel and Spulber (1994) and Spiegel (1996), the regulator maximizes the sum of consumer surplus and profits, which leads to marginal cost pricing. The latter has the feature that an increase in the regulated price benefits the firm not only on the margin, but also on its inframarginal units; consequently, the firm is more than compensated for the increase in its expected cost of bankruptcy and is therefore willing to issue risky debt. With average cost pricing, there is no similar effect: once the firm is exposed to bankruptcy, it bears part of the associated costs, so debt levels beyond $D(\theta)$ are not profitable to the firm.

Second, we examine the implication of asymmetric information for the capital structure of the firm. To this end, recall that the firm’s debt target under full information is $\bar{D}(\theta)$. This debt target varies across firms based on their expected costs. Under asymmetric information, in contrast, the debt target of the firm depends on the prior beliefs of the regulator and outside investors rather than on the firm’s true cost parameter, (i.e., it depends on $\theta$ rather than on $\theta$). This implies that when the project is large, there should not be a significant empirical correlation between the capital structure of the firm and its expected value. If, however, the project’s size is small or medium, the model admits separating equilibria in which firms with high expected value (i.e., low expected costs) rely more heavily on debt financing (and may even use all-debt financing with medium-sized projects), whereas firms with low expected value (i.e., high expected costs) rely more heavily on equity financing.

When the project is smaller than the debt target under asymmetric information, $\bar{D}(\theta)$, the firm’s capital structure is decoupled from cost and demand parameters (see Propositions 3–5). On the other hand, when the project is larger than $\bar{D}(\theta)$, cost and demand parameters affect capital structure. Thus, the size of the project relative to the debt target alters the effects of cost and demand parameters on the regulated price. In the large-projects case, both firms use a combination of debt and equity financing. Accordingly, we consider comparative statics for the large-projects case where $k$ exceeds the debt target $\bar{D}(\theta)$, in which case $D^* = \bar{D}(\theta)$, $\alpha^* = (k - \bar{D}(\theta))/(1 - \theta)c$, and $p^* = \bar{D}(\theta) + c$.

**Proposition 7.** Suppose that $k > \bar{D}(\theta)$. Then, $D^*$ is increasing in the prior probability $\theta$, decreasing in the cost shock $c$, increasing in the bankruptcy cost $t$, increasing in the consumers’ willingness to pay $V$, and decreasing in the welfare weight $\gamma$; and $\alpha^*$ is increasing in the size of the project $k$, decreasing in $\theta$, increasing in $c$, decreasing in $t$, decreasing in $V$, and increasing in $\gamma$.

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19 Note however that debt is risky because the regulator is unwilling to raise the regulated price enough to ensure that bankruptcy never happens. Nonetheless, with marginal cost pricing, the firm finds it optimal to issue a high level of debt to induce the regulator to increase prices, despite the fact that the increase in prices is not sufficient to prevent bankruptcy in all states of nature.
Proposition 7 implies that when \( k \) is large, debt and equity are substitutes in the sense that an increase in debt financing due to a change in an exogenous parameter leads to a decrease in equity financing and vice versa. The only exception is that an increase in \( k \) raises \( a^* \) without affecting \( D^* \). This is because the debt target does not depend on the size of the project and since the firm uses equity financing on the margin.

Finally, we examine the impact of changes in exogenous parameters on the equilibrium regulated price, \( p^* \).

**Proposition 8.** Suppose that \( k > \bar{D}(\theta^*) \). Then, \( p^* \) is increasing in the prior probability, \( \theta^* \), increasing in the bankruptcy cost \( t \), increasing in the consumers’ willingness to pay \( V \), and decreasing in the welfare weight \( \gamma \). The markup \( p^* - c \) is decreasing in the cost shock.

### 7. Conclusion

- The sequential game with asymmetric information between the firm, a regulator, and outside investors shows that regulated firms can affect their rates by properly choosing their capital structure. The regulator is concerned with the possibility that the firm will go bankrupt and incur a deadweight loss. This creates a leverage effect: when the firm issues debt, the regulator responds by increasing rates in order to reduce the likelihood of bankruptcy, enabling the firm to capture a larger share in the surplus it generates. Anticipating the regulator’s response, the firm chooses its debt target by trading off higher rates induced by the leverage effect against the increase in expected bankruptcy costs.

Because the firm’s costs are private information, capital structure can be used as a signalling device. When the beliefs of the regulator and outside investors are consistent (in the sense of the refinement of undefeated equilibria), the model admits only Pareto-undominated equilibria. These equilibria can be either pooling or separating, depending on the size of the firm’s investment. When the size of the investment is relatively small, the model may admit both a continuum of separating equilibria and a pooling undefeated equilibrium, all of which are payoff-equivalent. In the separating equilibria, firms with low probability of a cost shock issue relatively little equity, while firms with a high probability of a cost shock rely more heavily on equity financing. In a pooling equilibrium, both types issue the same debt level that completely offsets the benefits and costs from separation for both types.

When the investment project is medium-sized, the model admits a unique separating undefeated equilibrium, in which firms with a low probability of a cost shock use all-debt financing, while firms with a high probability of a cost shock separate themselves by using enough equity financing. Finally, when investment is large, the model admits a unique undefeated equilibrium, which is pooling. This equilibrium is sustained by the fact that neither type of firm has an incentive to distinguish itself, as the potential gains for each type from revealing its identity to one receiver are outweighed by the loss associated with the negative response of the second receiver. Since the equilibrium financial strategy of the firm depends in this case on the prior beliefs of the regulator and outside investors rather than on the true cost parameter of the firm, the capital structure choice of the firm is decoupled from its private information about its value. Empirically, therefore, there need not be a correlation between capital structure and the expected value of the firm.

In addition to the regulatory leverage effect, firms have many reasons to issue debt, such as taxation, agency costs, and corporate control considerations.\(^{20}\) Controlling

\[^{20}\text{See Myers (1984) for a discussion of tax-based theories of capital structure and Harris and Raviv (1991) for a survey of theories based on agency costs and corporate control.}\]
for these other effects, our analysis implies that regulated firms would be more leveraged than unregulated firms. This helps to explain the empirical finding of Bradley, Jarrell, and Kim (1984) that firms in regulated industries are among the most highly leveraged. The absence of bankruptcy in equilibrium explains why despite being so highly leveraged, regulated firms have hardly ever gone bankrupt under traditional rate regulation. Our model suggests that as the process of deregulation proceeds in the utility industries, regulated utilities will either have to lower their debt-equity ratios or face serious financial difficulties.

Appendix

The beliefs of receivers (outside investors and the regulator in the current model) are said to be inconsistent with the set of types \( T \) if

\[
\begin{align*}
 b & \neq \frac{b^\theta m(\theta^\prime)}{b^\theta m(\theta^\prime) + (1 - b^\theta)m(\theta^\prime)} \quad \text{for any } m: \{\theta^\prime, \theta^\prime\} \to [0, 1] \text{ satisfying } \\
 m(\theta) & = 1 \ \forall \ \theta \in T, \ m(\theta) = 0 \ \forall \ \theta \notin T,
\end{align*}
\]

where \( T_1 \) is the set of types who strictly prefer the alternative equilibrium to the proposed one. For a definition of undefeated equilibrium in the general case, the reader is referred to Mailath, Okuno-Fujiwara, and Postlewaite (1993).

**Definition A1.** An undefeated equilibrium is a PBE such that the following consistency requirement on the beliefs of outside investors and the regulator off the equilibrium path is satisfied:

Consider a proposed equilibrium, \( \sigma \), and a capital structure \((\sigma', D')\), that is not chosen in \( \sigma \), but is chosen by at least one type of firm in an alternative equilibrium, \( \sigma' \). Let \( T \) be the set of firm’s types that choose \((\sigma', D')\) in \( \sigma' \). If each member of \( T \) prefers \( \sigma' \) to \( \sigma \), with a strict preference for at least one member of \( T \), then upon observing \((\sigma', D')\), the posterior beliefs of outside investors and the regulator must be consistent with the set \( T \) in the following sense:

\[
\theta^\prime = \theta^* = \begin{cases} 
\theta^\prime_h, & \text{if only type } h \text{ prefers } \sigma \text{ to } \sigma', \\
[\theta^\prime_h, \theta^\prime_l], & \text{if type } h \text{ strongly prefers } \sigma \text{ to } \sigma' \text{ and type } l \text{ weakly prefers it}, \\
\theta^\prime_h, & \text{if both types prefer } \sigma \text{ to } \sigma', \\
[\theta^\prime_h, \theta^\prime_l], & \text{if type } l \text{ strongly prefers } \sigma \text{ to } \sigma' \text{ and type } h \text{ weakly prefers it}, \\
\theta^\prime_l, & \text{if only type } l \text{ prefers } \sigma \text{ to } \sigma'.
\end{cases}
\]

If the posterior beliefs of outside investors and the regulator satisfy condition (A1), then the proposed equilibrium is said to be undefeated, that is, no alternative equilibrium defeats it.

**Derivation of the condition on \( t \) and \( c \) that ensures that the limited liability constraint on debtholders is never binding.** First recall that the firm goes bankrupt only if the cost shock occurs. Hence, the payoff of debtholders in the event of bankruptcy is \( Y_D = p^* - c - t(D - p^* + c) \), where \( p^* \) is given by the third line in equation (10). Differentiating this expression with respect to \( D \) reveals that \( \partial Y_D / \partial D < 0 \); consequently, it is sufficient to verify that \( Y_D \geq 0 \) at the highest debt level that the firm will ever issue. This debt level, denoted by \( D(\theta) \), is implicitly defined by the equation \( D = p^* \) (debt levels beyond this level lead to bankruptcy with probability one, and are therefore dominated strategies for the firm). Using the third line in (10) yields

\[
D(\theta) = \frac{(1 - \gamma)(V - \theta c) + c(1 + \theta t)}{1 + (1 - \gamma)\theta t} - c(1 - \theta).
\]

The debtholders’ payoff at this debt level as a function of \( t \) is

\[
\tilde{Y}_D(t) = D(\theta) - c(1 - \theta).
\]

Now, \( \partial^2 \tilde{Y}_D / \partial t^2 < 0 \), indicating that \( \tilde{Y}_D \) is concave in \( t \). Together with the fact that \( \tilde{Y}_D(0) = (1 - \gamma)V - c(1 - \gamma \theta) \geq 0 \), where the inequality follows since \( \gamma < \tilde{\gamma} \leq (V - c)(V - \theta c) \), this implies that \( \tilde{Y}_D \geq 0 \) for all \( t < \theta \), where \( \theta \) is the largest root of the equation \( \tilde{Y}_D(t) = 0 \). Specifically,
Hence, \( t < T \) is a sufficient condition for the payoff of debtholders to be nonnegative for all debt levels that are undominated strategies for the firm.

Proof of Proposition 3. Let \( Y(\theta) = Y(D^*(\theta), \theta, \theta|\theta) \) be the expected payoff of \( \theta \)-types in a separating equilibrium, and let \( Y(\theta'|\theta) = Y(D^*(\theta'), \theta, \theta'|\theta) \) be the expected payoff of \( \theta' \)-types when they mimic the financial strategy of \( \theta' \)-types. Incentive compatibility requires that in a separating equilibrium, \( Y(\theta) \succeq Y(\theta'|\theta) \), \( \theta, \theta' \in \{\theta, \theta'\} \). Using equation (17), this condition can be written as

\[
\frac{c(\theta - \theta')}{(1 - \gamma)(Y(\theta') + k - D^*(\theta'))} \left[ \frac{\gamma Y(\theta')}{1 - \gamma} + D^*(\theta') - k \right] \geq 0, \quad \theta, \theta' \in \{\theta', \theta\}.
\]  

(5)\( ^{(A5)} \)

where \( Y(\theta') + k - D^*(\theta') \geq 0 \), since \( Y(\theta') > 0 \) and \( D^*(\theta') \leq k \). Noting that \( \theta - \theta' > 0 \) if \( \theta = \theta' \) and \( \theta - \theta' < 0 \) otherwise, it follows that in a separating equilibrium it must be the case that

\[
\frac{\gamma Y(\theta')}{1 - \gamma} + D^*(\theta') > k > \frac{\gamma Y(\theta')}{1 - \gamma} + D^*(\theta').
\]  

(5)\( ^{(A6)} \)

Since \( Y(\theta') > 0 \), the left inequality holds for any \( D^*(\theta') \) sufficiently close to \( k \); the lower bound on \( D^*(\theta') \), defined in the proposition, is the lowest \( D^*(\theta') \) that satisfies the left inequality. A necessary condition for the right inequality to hold is \( k > \gamma Y(\theta')(1 - \gamma) \), because \( D^*(\theta') \leq 0 \). Hence, \( k > \gamma Y(\theta')(1 - \gamma) \) is also a necessary condition for a separating PBE (when this inequality fails, there is no positive \( D^*(\theta') \) that deters \( l \)-types from mimicking \( h \)-types). The upper bound on \( D^*(\theta') \), defined in the proposition, is the largest \( D^*(\theta') \) that satisfies the right side of (A6). Every nonnegative pair \( (D^*(\theta'), D^*(\theta')) \) that satisfies (A6) can be supported as the debt-level choices in a separating equilibrium. One belief function that supports these debt levels as equilibrium outcomes is such \( \theta' = \theta = \theta' \) if \( D < D^*(\theta') \) or \( D = D^*(\theta') \), and \( \theta' = \theta = \theta' \) otherwise (there are other belief functions that support these equilibria). The equilibrium equity participation of outsiders is then determined by substituting for \( D^*(\theta) \) in equation (16). Since \( k \leq \hat{D}(\theta) \), the expected payoff of a \( \theta \)-type firm is exactly as in the full-information case.

Next, let \( Y(\theta'|\theta) = Y(D^*, \theta, \theta'|\theta) \) be the expected payoff of \( \theta' \)-types in a pooling equilibrium (if it exists) in which both types issue a debt level \( D^* \). Then \( p^* = \hat{D}(\theta) + c \). A pooling PBE exists only if \( Y(\theta'|\theta) \geq Y(\theta), \theta \in \{\theta, \theta'\} \). Using the first line of equation (17), this condition can be written as

\[
\frac{c(\theta - \theta')}{(1 - \gamma)(1 - \gamma)(V - \theta'|c) - D^*} \left[ \gamma(V - \theta'|c) + D^* - \frac{k}{1 - \gamma} \right] \geq 0, \quad \theta \in \{\theta, \theta'\}.
\]  

(7)\( ^{(A7)} \)

Clearly, (A7) can hold for both \( \gamma \) if and only if the expression in the square brackets vanishes. Hence, in a pooling PBE, \( D^* = k(1 - \gamma) - \gamma (V - \theta'|c) \). Substituting for \( D^* \) in equation (16) reveals that \( a^0 = \gamma \). One belief function that supports \( D^* \) as the outcome of a pooling PBE is such that \( \theta' = \theta = \theta' \) if \( D < D^* \), \( \theta' = \theta = \theta' \) if \( D > D^* \), and \( \theta' = \theta = \theta' \) otherwise (again, this belief function is not unique). Since (A7) must hold with equality, both types receive their separating equilibrium expected payoffs. Moreover, since all equilibria (pooling and separating) are payoff-equivalent, definition (A1) implies that all of them are undefeated. Q.E.D.

Proof of Proposition 4. First, suppose by way of negation that there exists an undefeated separating equilibrium in which \( D^*(\theta') < k \). Incentive compatibility requires that in equilibrium, \( Y(\theta) \succeq Y(\theta'|\theta) \) and \( Y(\theta') \succeq Y(\theta'|\theta) \). Now consider a deviation by \( l \)-types to \( D(\theta') = k \). From equation (17) it is easy to see that the deviation increases \( Y(\theta) \) and leaves \( Y(\theta'|\theta) \) unaffected; hence, the first inequality continues to hold. Now, evaluated at \( D(\theta') = k \), \( Y(\theta'|\theta) = (1 - \theta)c \). Since by assumption \( k < \hat{D}(\theta') = (1 - \gamma)(V - \theta'|c) - (1 - \theta)c \) (see equation (11)), it follows that \( (1 - \theta)c < (1 - \gamma)(V - \theta'|c) - k \). But the last expression equals \( Y(\theta') \), so the second inequality continues to hold as well. Therefore, \( D^*(\theta') = k \) can also be the outcome of a separating PBE. In the new equilibrium, though, \( Y(\theta') \) is higher than before while \( Y(\theta) \) is unchanged (\( Y(\theta) \) does not depend on \( D \)), so the new equilibrium Pareto dominates the putative equilibrium. Hence, definition (A1) implies that in a separating undefeated equilibrium (if it exists), \( D^*(\theta') = k \); this implies in turn that \( a^0(\theta) = \theta \). The condition \( Y(\theta') \geq Y(\theta'|\theta) \) determines \( D^*(\theta') \); using equation (17), this condition implies that \( D^*(\theta') \leq (kY(\theta') - D(\theta'))/\gamma(Y(\theta') - Y(\theta')) \). Since \( Y(\theta') > Y(\theta) \), there exists a positive \( D^*(\theta') \) that satisfies this condition provided that \( kY(\theta') > D(\theta') \). Substituting for \( D^*(\theta') \) in equation (16) yields \( a^0(\theta') \). To show that a pair \( (k, D^*(\theta')) \), such that \( 0 < D^*(\theta') < (kY(\theta') - D(\theta'))/\gamma(Y(\theta') - Y(\theta')) \) can be supported as the debt-level choices in an undefeated equilibrium, we must find an appropriate belief function. One such belief function is such that \( \theta' = \theta = \theta' \) if \( D < k \), and \( \theta' = \theta = \theta' \) if \( D = k \). If \( D > k \) is not feasible, since debt is
riskless). Given this belief function, l-types can only lose by choosing $D < k$, while h-types lose if they choose $D = k$, and have nothing to gain by deviating to $D < k$ ($D \neq D^*(\theta)$).

Next we consider pooling equilibria. If $k < \bar{D}(\theta)$, then $D^* \leq \bar{D}(\theta)$ (recall that since debt is riskless, $D$ cannot exceed $k$), so (A7) is a necessary condition for a pooling equilibrium. However, (A7) can hold only if $D^* = k(1 - \gamma) - \gamma(V - \theta c)$, in which case $Y(\theta^*|\theta) = \bar{D}(\theta) + (1 - \theta)c - k$. But since by assumption $\tilde{D}(\theta) < k$, $Y(\theta^*|\theta) < c(1 - \theta)$, which is the expected payoff of an l-type firm in the Pareto-dominant separating PBE in which $D^*(\theta) = k$. Moreover, we know from Proposition 3 that h-types are indifferent between pooling at $D^* = k(1 - \gamma) - \gamma(V - \theta c)$ and separating. Hence, definition (A1) implies that the Pareto-dominant separating equilibrium defeats the putative pooling equilibrium.

If $k > \bar{D}(\theta)$, then $D^*$ may exceed $\bar{D}(\theta)$, and in fact must exceed it, otherwise the pooling PBE will be defeated by the Pareto-dominant separating PBE in which $D^*(\theta) = k$. Therefore, equation (17) implies that $Y(\theta^*|\theta) = (1 - \theta)(D^* + c(1 - \theta) - k)(1 - \theta)$, $\theta = (\theta^*, \theta^*)$. This expected payoff increases with $D^*$, so by definition (A1), the only candidate for an undefeated pooling equilibrium is such that $D^* = k$ (this equilibrium defeats all pooling PBE in which $D^* < k$). In equilibrium, the payoff of l-types remains $(1 - \theta)c$, while the payoff of h-types becomes $Y(\theta^*|\theta^*) = (1 - \theta)c - k$. But since by assumption $\tilde{D}(\theta) > k$, $Y(\theta^*|\theta^*) < \bar{D}(\theta) + (1 - \theta)c - k = Y(\theta)$, so by definition (A1), the Pareto-dominant separating equilibrium defeats the putative pooling equilibrium. Q.E.D.

**Proof of Proposition 5.** We prove the proposition through a series of four lemmas.

**Lemma 1.** Suppose that $k \leq \bar{D}(\theta)$. Then there exists a unique undefeated equilibrium which is pooling. In this equilibrium, $D^* = k$, $a^* = 0$, and $p^* = k + c$.

**Proof.** When $D^* = k$, the payoff of a 0-type firm is $Y(\theta^*|\theta) = (1 - \theta)c$. Since this is the highest payoff that each type can achieve, definition (A1) implies that this pooling equilibrium defeats all other PBE. One belief function that supports $D^* = k$ as an equilibrium outcome is such that $\theta' = O1' = (\theta^*, \theta^*)$ if $D = k$ and $\theta' = \theta^* = \theta^*$ if $D < k$ (since $D$ is riskless, $D > k$ is not feasible). Finally, the equilibrium price is determined by equation (10). Q.E.D.

**Lemma 2.** Suppose that $E(\theta^* < k \leq \bar{D}(\theta) + (1 - \theta)c$. Then the model admits a unique Pareto-dominant pooling PBE. In this equilibrium, $D^* = \bar{D}(\theta)$.

**Proof.** First we show that $D^* > \bar{D}(\theta)$. To this end, recall from Proposition 3 that the model can admit only one pooling PBE in which $D^* < \bar{D}(\theta)$, and assume that the necessary condition for the existence of this PBE is satisfied (otherwise we are done). Now we shall show that both types are better off pooling at $\bar{D}(\theta)$ than pooling at $D^* < \bar{D}(\theta)$. By Proposition 3, the expected payoff of an h-type firm in the unique pooling PBE in which $D^* < \bar{D}(\theta)$ is $Y(D^*, \theta, \theta^*|\theta^*) = \bar{D}(\theta) + (1 - \theta)c - k$. On the other hand, equation (17) implies that the expected payoff of h-types when the firms pool at $\bar{D}(\theta)$ is

$$Y(\bar{D}(\theta), \theta, \theta^*|\theta^*) = \frac{(1 - \theta)c}{(1 - \theta)}.$$

(As 8)

But since $\bar{D}(\theta) < \bar{D}(\theta) < k$,

$$Y(\bar{D}(\theta), \theta, \theta^*|\theta^*) > \bar{D}(\theta) + (1 - \theta)c - k$$

$$> \bar{D}(\theta) + (1 - \theta)c - k$$

(A9)

That is, h-types are better off pooling at $\bar{D}(\theta)$. As for l-types, equation (17) implies that their payoff when $D^* < \bar{D}(\theta)$ is

$$Y(D^*, \theta, \theta^*|\theta^*) = [D^* + (1 - \theta)c - k] \times \frac{\bar{D}(\theta) + (1 - \theta)c - D^*}{\bar{D}(\theta) + (1 - \theta)c - D^*}.$$  

(A10)

But since $D^* < \bar{D}(\theta) < \bar{D}(\theta)$ and $\theta < \theta^*$,

$$Y(D^*, \theta, \theta^*|\theta^*) < [D^* + (1 - \theta)c - k] \times \frac{1 - \theta^*}{1 - \theta}$$

$$< \bar{D}(\theta) + (1 - \theta)c - k \times \frac{1 - \theta^*}{1 - \theta}$$

(A11)

implying that l-types are also better off pooling at $\bar{D}(\theta)$.
Having shown that $D^* > \hat{D}(\theta)$, we next show that $D^* = \overline{D}(\theta)$. To this end, note from equation (17) that the expected payoff of a $\theta$-type firm in pooling equilibria such that $D^* > \hat{D}(\theta)$ is

$$
Y(D^*, \theta', \theta|\theta) = \begin{cases} 
[D^* + (1 - \theta)c - k] \times \frac{1 - \theta}{1 - \theta'} & \text{if } D^* \leq \hat{D}(\theta), \\
[D(\theta') + (1 - \theta)c - k - (1 - \gamma)\theta(V + c - D^*)] \times \frac{1 - \theta}{1 - \theta'} & \text{if } D^* > \hat{D}(\theta).
\end{cases}
$$

This expression attains a unique maximum at $\overline{D}(\theta)$, so definition (A1) implies that the pooling equilibrium in which $D^* = \overline{D}(\theta)$ Pareto dominates all other pooling equilibria. One belief function that supports this pooling equilibrium is such that $\theta' = \theta = \theta$ if $D \geq \hat{D}(\theta)$ and $\theta = \theta^k = \theta^p$ if $D < \hat{D}(\theta)$. Given these beliefs, the payoffs of both types are given by equation (A12), which is maximized at $\hat{D}(\theta)$. Hence no type will increase $D$ to above $\hat{D}(\theta)$. When $h$-types deviate to $D < \hat{D}(\theta)$, their expected payoff becomes $\hat{D}(\theta^h) + (1 - \theta)c - k$, which by (A9) is less than their equilibrium expected payoff. Hence, $h$-types will not deviate. As for $l$-types, given the above belief function, their expected payoff when they deviate to $D < \hat{D}(\theta)$ becomes

$$
Y(D, \theta', \theta|\theta) = \begin{cases} 
[\hat{D}(\theta^h) + (1 - \theta)c - k] \times \frac{\hat{D}(\theta^h) + (1 - \theta)c - D}{\hat{D}(\theta) + (1 - \theta)c - D} & \text{if } D \leq \hat{D}(\theta), \\
[D + (1 - \theta)c - k] \times \frac{1 - \theta'}{1 - \theta^h} & \text{if } D > \hat{D}(\theta).
\end{cases}
$$

This payoff increases with $D$, so whenever $D < \hat{D}(\theta)$,

$$
Y(D, \theta', \theta|\theta) < \overline{D}(\theta) + (1 - \theta)c - k \times \frac{1 - \theta'}{1 - \theta^h} < Y(\overline{D}(\theta), \theta, \theta|\theta),
$$

implying that $l$-types will not deviate as well.

To prove existence, it only remains to verify that both types indeed have an incentive to undertake the project. A sufficient condition for this is $k \leq \overline{D}(\theta) + (1 - \theta)c$, because this condition guarantees that the initial equityholders of both types of firm receive a nonnegative expected payoff when they undertake the project. When this condition holds, the model admits a unique undefeated pooling equilibrium in which $D^* = \overline{D}(\theta)$. Q.E.D.

**Lemma 3.** Suppose that $k > \overline{D}(\theta)$. Then there does not exist a separating undefeated equilibrium in which $D^*(\theta) < \overline{D}(\theta)$ and $D^*(\theta^p) < \overline{D}(\theta^p)$.

**Proof.** Recall that when $D^*(\theta) < \overline{D}(\theta)$ and $D^*(\theta^p) < \overline{D}(\theta^p)$, all PBE (separating and pooling) are payoff equivalent. Lemma 2 (in particular (A9) and (A11)) shows that these equilibria are Pareto dominated by the pooling equilibrium in which $D^* = \overline{D}(\theta)$, and hence by definition (A1) they are defeated. Q.E.D.

**Lemma 4.** Suppose that $k > \overline{D}(\theta)$. Then there does not exist a separating undefeated equilibrium.

**Proof.** First we show that when $k > \overline{D}(\theta)$, there exists a unique candidate for a separating undefeated equilibrium. Then we show that this equilibrium is defeated by the pooling equilibrium in which $D^* = \overline{D}(\theta)$. To find a candidate for a separating equilibrium, recall that incentive compatibility requires that in a separating equilibrium, $Y(\theta') \geq Y(\theta|\theta')$, $\theta \in (\theta^p, \theta^p)$. Since $Y(\theta)$ attains a unique maximum at $\overline{D}(\theta)$ (see Figure 2), we can restrict attention without a serious loss of generality to separating equilibria in which $D^*(\theta) \leq \overline{D}(\theta)$ and $D^*(\theta^p) \leq \overline{D}(\theta^p)$. Moreover, by Lemma 3, it must be that in an undefeated separating equilibrium (if it exists), $D^*(\theta) > \overline{D}(\theta)$ and $D^*(\theta^p) > \overline{D}(\theta)$. Using equation (17), the incentive-compatibility condition can therefore be written as

$$
Y(\theta') \geq Y(\theta) \frac{1 - \theta'}{1 - \theta}, \quad \theta, \theta' \in (\theta^p, \theta^p).
$$

This inequality implies that in a separating equilibrium,

$$
Y(\theta) = Y(\theta^p) \frac{1 - \theta^p}{1 - \theta^p}.
$$

(A16)
By equation (10), \( p^* = D + c \) for \( D \in [\underline{D}(\theta), \overline{D}(\theta)] \). Substituting for \( p^* \) in (17) reveals that 
\[
Y(\theta) = D^* + (1 - \theta)c - k, \quad \text{so (A16) becomes}
\]
\[
D^*(\theta') + (1 - \theta')c - k = \frac{(1 - \theta')(D^*(\theta') + (1 - \theta)c - k)}{1 - \theta'}.
\]
Equation (A17) implies that in a separating equilibrium (if it exists),
\[
D^*(\theta') = H(D^*(\theta')) = \frac{D^*(\theta')(1 - \theta') + k(\theta' - \theta)}{1 - \theta'}.
\]
Moreover, since \( Y(\theta') \) increases in \( D \) for all \( D \leq \overline{D}(\theta') \), and since \( Y(\theta') \) increases in \( D \) for all \( D \leq \overline{D}(\theta') \), it must be the case that in an undefeated separating equilibrium either \( D^*(\theta') = \overline{D}(\theta') \) or \( D^*(\theta') = \overline{D}(\theta') \), otherwise the equilibrium can be defeated by a separating equilibrium in which both types issue more debt such that (A18) still holds. A necessary condition for an undefeated separating equilibrium in which \( D^*(\theta') \leq \overline{D}(\theta') \) is \( H(\overline{D}(\theta')) \leq \overline{D}(\theta') \) (otherwise there does not exist a pair \( (D^*(\theta'), H(D^*(\theta')) \) that satisfies (A18)). Using (A18), this necessary condition can be written as
\[
k \leq \frac{\overline{D}(\theta')(1 - \theta') - \overline{D}(\theta)(1 - \theta)}{\theta' - \theta}.
\]
Similarly, a necessary condition for an undefeated separating equilibrium in which \( D^*(\theta') = H(D^*(\theta')) = \overline{D}(\theta') \) is \( H^{-1}(\overline{D}(\theta')) \leq \overline{D}(\theta') \). Using (A18), this necessary condition can be written as
\[
k \geq \frac{\overline{D}(\theta')(1 - \theta') - \overline{D}(\theta)(1 - \theta)}{\theta' - \theta}.
\]
Since either (A19) or (A20) must hold, the model admits a (unique) Pareto-dominant separating PBE.

Next, we show that the pooling PBE in which \( D^* = \overline{D}(\theta) \) defeats the Pareto-dominant separating equilibrium. The expected payoffs in the Pareto-dominant pooling PBE in which \( D^* = \overline{D}(\theta) \) are
\[
Y(\overline{D}(\theta), \theta', \theta| \theta) = \frac{(\overline{D}(\theta') + (1 - \theta')c - k(1 - \theta))}{1 - \theta'}, \quad \theta \in \{\theta', \theta\}.
\]
On the other hand, the expected payoffs in the Pareto-dominant separating PBE are given by
\[
Y(\theta) = D^*(\theta) + (1 - \theta)c - k, \quad \theta \in \{\theta', \theta\},
\]
where \( D^*(\theta') = H(D^*(\theta')). \) If \( D^*(\theta') = \overline{D}(\theta') \), then a comparison of (A21) with (A22) reveals that a sufficient condition for the Pareto-dominant pooling PBE to defeat the Pareto-dominant separating PBE (i.e., both types are strictly better off in the former equilibrium) is
\[
k < \frac{\overline{D}(\theta')(1 - \theta') - \overline{D}(\theta)(1 - \theta)}{\theta' - \theta}.
\]
but (A23) is implied by (A19) and \( \overline{D}(\theta) \) concave. Hence, the Pareto-dominant separating PBE is defeated. If on the other hand \( D^*(\theta') = H(D^*(\theta')) = \overline{D}(\theta') \), then a comparison of (A21) with (A22) reveals that the sufficient condition for the Pareto-dominant pooling PBE to defeat the Pareto-dominant separating PBE becomes
\[
k > \frac{\overline{D}(\theta')(1 - \theta') - \overline{D}(\theta)(1 - \theta)}{\theta' - \theta}.
\]
But (A24) is implied by (A20) and \( \overline{D}(\theta) \) concave. Hence, the Pareto-dominant separating PBE is defeated. Q.E.D.

Lemmas 2–4 imply that when \( \overline{D}(\theta) < k \leq \overline{D}(\theta) + (1 - \theta)c \), the model admits a unique undefeated equilibrium which is pooling. In this equilibrium, \( D^* = \overline{D}(\theta) \), so by equations (16) and (10), \( \alpha^* = \max\{0, (k - \overline{D}(\theta))/((1 - \theta)c)\} \), and \( p^* = D^* + c \). Q.E.D.
References


