The capital structure of a regulated firm

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We examine the equilibrium price, investment, and capital structure of a regulated firm using a sequential model of regulation. We show that the firm's capital structure has a significant effect on the regulated price. Consequently, the firm chooses its equity and debt strategically to affect the outcome of the regulatory process. In equilibrium, the firm issues a positive amount of debt and the likelihood of bankruptcy is positive. Debt raises the regulated price, thus mitigating regulatory opportunism. However, underinvestment due to lack of regulatory commitment to prices persists in equilibrium.

1. Introduction

Capital structure plays an important role in rate regulation due to the interaction between the investment and financial decisions of a regulated firm and the pricing choices of regulators. First, regulatory commissions set rates that depend on the firm's level of investment and capital structure, thus reflecting not only ratepayer interests, but also those of investors. The capital market, in turn, values the equity and debt of the regulated firm on the basis of its investment and capital structure, as well as on present and future regulatory policies. Second, the regulated firm makes its investment and financial decisions in anticipation of regulatory policies and the capital market's reactions. The purpose of this article is to explain these interactions and examine their implications for the regulatory process.

Rate regulation of public utilities in electricity, natural gas, telecommunications, cable TV, water services, and other industries is currently practiced by 50 state regulatory commissions as well as federal regulatory agencies. In 1989 the public utilities sector in the U.S. accounted for approximately 5.97% of the GNP and over 18.8% of total business expenditures for new plant and equipment.1 Given the significance of this sector, it is useful to understand the interaction between rate regulation, capital structure, and

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1 The output of the public utilities sector, including communications, equalled $246 billion in constant (1982) dollars in 1989. (Source: Department of Commerce, Bureau of Economic Analysis.) Investment in new plants and equipment totalled $106.11 billion in current dollars in 1989. (Source: Bureau of the Census.)
investment. To this end, we examine a sequential model of rate setting that explicitly accounts for regulatory policy, capital market equilibrium, and the firm’s financial strategy. Our main finding is that lack of regulatory commitment to rates provides the firm with an incentive to issue debt because debt mitigates the regulator’s incentive to act opportunistically. Thus, debt reduces the regulator’s incentive to lower rates as a response to the firm’s investment in cost reduction.

Our findings are consistent with the empirical evidence. A number of studies suggest that rate regulation creates an incentive for regulated firms to increase their debt levels, and others show that debt has a positive effect on regulated prices and on the allowed rate of return on equity. Bradley, Jarrell, and Kim (1984), in a study of 25 industries over the period 1962–1981, find that regulated firms such as telephone, electric and gas utilities, and airlines are consistently among the most highly levered firms. Taggart (1985) studies state electricity and natural gas regulation in the period 1912–1922, and concludes that the establishment of regulation increases the utility’s debt-equity ratio. Taggart attributes this in part to the reduction in the firm’s risk due to regulation, but cannot reject a “price influence” effect of debt on regulatory decisions. Besley and Bolton (1990), in a survey of 27 regulatory agencies and 65 utilities, find that approximately 60% of the regulators and utilities surveyed believe that an increase in debt relative to equity increases regulated prices. Hagerman and Ratchford (1978) show that, for a sample of 79 electric utilities in 33 states, the allowed rate-of-return on equity is increasing in the debt-equity ratio. Dasgupta and Nanda (1991a, 1991b), in a cross-section of U.S. electric utilities for the years 1980–1983, show that increased debt is taken on to cope with a regulatory environment that is harsher to shareholders. They find support for the view that debt precommitment can raise rates by causing the regulator to avoid bankruptcy costs.

The role of investment and capital structure in the strategic interaction between the regulator and the firm has not been addressed in the literature. Taggart (1981) identifies a “price-influence effect” of debt due to price increases by regulators seeking to reduce the risk of bankruptcy, but he does not examine the implications for equilibrium strategies. Capital structure theories have focused on tax considerations, agency costs, asymmetric information, and corporate control as the forces driving capital structure. Although these theories may also provide an explanation for the capital structure of regulated firms, they are not entirely satisfactory because none of them addresses the important interrelations between a regulated firm’s capital structure, its investment, and regulated rates.

We model the regulatory process as a three-stage game in which the players are a firm, a regulator, and outside investors. In the first stage of this game, a regulated firm chooses capital investment and capital structure. The market value of the firm’s debt and equity are established in a competitive capital market in the second stage. Finally, in the third stage, the firm’s price is established by the regulator. This structure reflects the dynamic nature of the regulatory process in which regulators can observe the investment and capital structure decisions of firms as well as the capital market equilibrium. The framework recognizes the greater flexibility of regulated rates in comparison with the

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2 Hyman, Toole, and Avellis (1987) compare the Bell Regional holding companies (BHCs), as a proxy for the telephone industry, to 104 industry groups and find that these companies remain highly leveraged even when risk is taken into account. Specifically, they find that the average debt ratio of the BHCs in 1986 was 40%, although their beta was .63. In comparison, the average debt ratio in 27 industries in which the average beta is .60–.99 was 28%, the average debt ratio in 50 industries in which the average beta is 1–1.39 was 32%, and the average debt ratio in 19 industries in which the average beta exceeds 1.40 was 30%.


4 See Myers (1984) for a discussion of tax-based theories, and Harris and Raviv (1991) for an extensive survey of theories based on agency costs, asymmetric information, and corporate control.
capital investment and capital structure commitment of the regulated firms. This implies limited commitment by regulators. Moreover, our structure reflects the fact that regulated firms are allowed to exercise discretion in choosing their capital structure and investment level. Howe (1982) finds that in many states (e.g., Michigan, Oklahoma, Kansas, Delaware) courts restrict the scope of state commissions' inquiry in security issue proceedings by directing the commissions to inquire only whether the proposed projects are within the range of the utility's corporate activity, and not whether they are "reasonable" or "necessary."

We show that in equilibrium the firm issues a positive amount of debt as a consequence of regulation. The regulator responds to this debt level by raising the regulated price to reduce the probability that the firm will become bankrupt. Nevertheless, in equilibrium the firm becomes bankrupt with a positive probability because the regulator does not increase the regulated price to the point where the firm is completely immune to bankruptcy. Because debt has a positive effect on the regulated price, it mitigates regulatory opportunism. This suggests that regulators permit debt financing as a means of making implicit binding commitments. However, despite the positive effect of debt on the regulated price, underinvestment persists in equilibrium, reflecting the lack of regulatory commitment to specific prices.

The consequences of limited regulatory commitment are examined by Banks (1992) in the context of regulatory auditing, and by Besanko and Spulber (1992) in a model of investment. This article goes beyond those studies by focusing on the crucial financial issues. The use of debt as a commitment device was examined in an oligopoly setting by Brander and Lewis (1986, 1988). The equilibrium implications of capital structure in our setting are, of course, different.

The article is organized as follows. We give the basic framework of the strategic regulation process in Section 2. The three-stage model of the regulatory process is set out in Section 3. The regulator's strategy for setting the optimal regulated price is examined in Section 4. In Section 5 we consider the capital market equilibrium. Equilibrium investment and the capital structure of the regulated firm are characterized in Section 6. Conclusions are given in Section 7. All proofs appear in the Appendix.

2. The basic framework

Regulators have considerable discretion in setting rates and in determining what is a "fair" rate of return. The Supreme Court, in its decision In re Permian Basin Rate Case, stated that there is a "broad zone of reasonableness" and that rates intended to balance investors' and consumers' interests are constitutionally permissible. Rate regulation functions as follows. The regulated firm files a tariff with the regulatory commission, which in turn holds a rate hearing that generally results in an adjustment of the firm's proposed tariff. Thus, the firm's prices are ultimately set by the regulatory process.

Formally, under rate regulation, the prices that the firm is allowed to charge are set such that the firm's expected revenues equal its estimated revenue requirement. The latter is based on an estimate of the firm's variable costs such as operating expenses, taxes, and depreciation, plus an allowed rate of return multiplied by the capital stock (rate base). The allowed rate of return is generally an average of the costs of debt and equity weighted by the relative proportions of debt and equity, usually measured at book value. The cost of

5 This procedure is followed not just under rate-of-return regulation but also under price-cap regulation, because regulatory commissions set price caps on the basis of the firm's cost of capital. For example, the FCC sets price caps on interstate access rates so as to ensure local exchange carriers a 11.25% rate of return on their investment. Similarly, the FCC has tentatively concluded that it will establish price caps on cable TV services to ensure cable operators a rate of return on their investment of approximately 10%--14%.
debt is usually taken to equal total interest payments per unit of the book value of debt. The estimated cost of equity is perhaps the most troublesome and is arrived at in various ways, including the discounted cash flow method and the earnings/price ratio method (see Phillips, 1988). Estimates of the cost of equity generally depend on regulatory assessment of investor expectations regarding the future performance of the firm and thus depend on future regulatory policies. (See, for example, Pettway, 1978; Myers, 1972; and Radford, 1988.) Alternative approaches based on comparable earnings require the regulator to identify firms with comparable risks.

In practice, because negotiations take place between the firm, consumers, and the regulator concerning each step in the calculation, regulators can exercise considerable discretion in the rate-setting process (see Spulber, 1989). In particular, regulators have some latitude in determining the underlying rate of return. A “fair” rate of return covers the cost of capital, but often exceeds the risk-free interest rate. It is important to emphasize that the regulator’s pricing policy affects the firm’s expected earnings, which in turn affect the firm’s cost of capital. The circularity of this process suggests that the regulated firm, the capital market, and the regulators all take into account the interrelated determination of the cost of capital and regulated prices.

To capture these institutional features, we consider the following three-stage game. In the first stage, the firm chooses the level of investment and a mix of equity and debt to finance this investment by issuing new shares and bonds to outsiders. In the second stage, the market value of the firm’s securities is determined in the capital market. Finally, in the third stage, the regulator establishes the regulated price by maximizing a welfare function, taking the firm’s investment and capital structure as given. The balancing of consumers’ and investors’ interests is made explicit by assuming that the welfare function is a weighted sum of consumers’ surplus and firm profits. Then, the regulated firm produces its output and the regulated market clears. The focus of this article is on characterizing the subgame equilibrium of the regulatory game.

The regulated firm is a monopolist producing output \( q \) at a regulated price \( p \). The demand for the firm’s output is given by \( q = Q(p) \), which is a twice differentiable, downward sloping, concave function, i.e., \( Q'(p) < 0 \), \( Q''(p) < 0 \). The firm’s cost function is \( C(q, z, k) \), where \( k \) is the firm’s investment and \( z \) is an efficiency parameter representing cost and technology shocks. The revenues of the firm are represented by a function \( R(p, z, k) \), where

\[
R(p, z, k) = pQ(p) - C(Q(p), z, k).
\]

The firm’s cost function is twice differentiable in \( q, z, \) and \( k \). Marginal costs are positive and nondecreasing, \( C_q(q, z, k) > 0, C_{qq}(q, z, k) \geq 0 \), where subscripts denote partial derivatives. Investment reduces total and marginal costs, \( C_k(q, z, k) < 0, C_{kk}(q, z, k) < 0 \), and the reduction in total costs is at a decreasing rate, \( C_{kk}(q, z, k) > 0 \). Let \( \lim_{k \to 0} C_k(q, z, k) = -\infty \), so that some investment is always profitable. Note that the assumption that marginal costs are nondecreasing, together with the concavity of the demand function, ensures that \( R_{pp}(p, z, k) \geq 0 \), provided that the regulated price exceeds the marginal costs.

The efficiency parameter \( z \) is a random variable distributed over the unit interval according to a positive density function \( f(z) \), with a cumulative distribution function \( F(z) \). Total and marginal costs are assumed to be decreasing in \( z \), \( C_z(q, z, k) < 0, C_{zz}(q, z, k) < 0 \) (i.e., higher values of \( z \) represent better states of nature). Finally, average cost at the worst
state of nature, when \( z = 0 \), is larger than the expected marginal costs for all output levels. That is, for all \( q \), \( C(q, 0, k)/q > \int_{0}^{1} C_{q}(q, z, k)dF(z) \). This assumption holds, for example, in the simple case in which a cost function consists of a fixed cost plus a marginal cost, \( c(z, k) \), that is decreasing in \( z \).

Initially, the firm is owned by a set of equityholders and is assumed to have no outstanding debt. Suppose that the initial equityholders decide to finance the cost of investment, \( k \), from external sources. The firm then issues new equity representing a fraction \( \alpha \) of the firm’s equity and bonds which promise to pay \( D \) (i.e., this is its face value). We allow for \( \alpha < 0 \), in which case the firm repurchases some of its existing equity. Let \( E \) denote the market value of the new equity, and let \( B \) denote the market value of bonds. Because \( E \) and \( B \) must cover the cost of investment, the firm’s budget constraint is given by \( k \leq E + B \). There is evidence, however, to suggest that regulatory commissions do not allow regulated firms to raise external funds in excess of the costs of investment in physical assets. (See, e.g., Phillips, 1988.) Hence, the firm budget constraint must hold with equality, i.e.,

\[
 k = E + B .
\]  

(2)

We assume that the regulated firm exercises discretion in its choice of a capital structure, which accords with general practice by regulated utilities.\(^9\)

For each debt obligation \( D \), regulated price \( p \), and investment level, \( k \), there is a critical value of the efficiency parameter, above which the firm is able to pay its debt. This critical value is defined by

\[
 z^{*} = \min\{z \geq 0: R(p, z, k) \geq D\}. \quad (3)
\]

Note that if \( R(p, 0, k) \geq D \), then \( z^{*} = 0 \), whereas for \( z^{*} > 0 \), \( R(p, z^{*}, k) = D \). For states of nature \( z \geq z^{*} \), the firm remains solvent: it pays \( D \) to bondholders and both old and new equityholders are the residual claimants. For states of nature \( z < z^{*} \), limited liability applies, the firm declares bankruptcy, and bondholders become the residual claimants. Thus, \( F(z^{*}) \) represents the probability of bankruptcy.

Bankruptcy imposes extra costs on the bondholders due to, among other things, legal fees and the transaction costs associated with reorganizing the firm and transferring ownership to bondholders. Bankruptcy costs depend on the size of the shortfall in the firm’s earnings from its debt obligation. They are represented by the function \( H(D - R(p, z, k)) \), which is assumed to be twice differentiable, increasing, and convex, with \( H(D - R(p, z, k)) = 0 \) for all \( D \leq R(p, z, k) \) (i.e., for all \( z \geq z^{*} \)). Let \( h(\cdot) > 0 \) be the marginal cost of bankruptcy. Because bondholders are also protected by limited liability, we assume that whenever \( H(D - R(p, z, k)) > R(p, z, k) \), the firm is liquidated.\(^10\) In this case, the cost of bankruptcy to bondholders is \( R(p, z, k) \), so their net payoff is zero. Otherwise, the firm is reorganized under the ownership of bondholders, who

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\(^9\) See Phillips (1988) and Dobesh (1985). The Colorado Supreme Court in Re Mountain States Teleph. & Teleg. Co. (39 PUR 4th 222, 247–248) stated that “a guiding principle of utility regulation is that management is to be left free to exercise its judgment regarding the most appropriate ratio between debt and equity.” However, see Taggart (1985) for information on early efforts by regulators to control utility company debt.

\(^10\) In practice, regulatory commissions have little power to prevent liquidation, because public utilities are not obligated to provide services at a loss. In the Texas Railroad Comm. v. Eastern Texas R.R. Co. case of 1924 the Supreme Court argued that “If at any time it develops with reasonable certainty that future operations must be at a loss, the company may discontinue operation and get what it can out of the property. . . To compel it to go on at a loss, or to give up the salvage value, would be to take its property without just compensation which is a part of due process of law.” (264 U.S. 79, 85 (1924)).
receive a net payoff of $R(p, z, k) - H(D - R(p, z, k))$. Let $z^{**}$ be the critical value of the efficiency parameter below which the firm is liquidated. Then, expected bankruptcy costs are given by

$$T(p, D, k) = \int_{0}^{z^{**}} R(p, z, k) dF(z) + \int_{z^{**}}^{z^*} H(D - R(p, z, k)) dF(z).$$

(4)

Given the regulated price, $p$, and the firm’s debt obligation, $D$, the expected profits of the firm are equal to the expected revenues net of expected bankruptcy costs,

$$\Pi(p, D, k) = \int_{0}^{1} R(p, z, k) dF(z) - T(p, D, k).$$

(5)

Expected profits represent the combined ex post expected return to equityholders (both old and new) and bondholders and are divided between them according to their respective claims.\(^{11}\) Note that the definition of profits excludes the cost of investment, because the regulatory authority treats these costs as sunk costs, and therefore excludes them from its objective function.

3. Definition of strategies and equilibrium

In this section we describe the strategies of each player and define the equilibrium of the three-stage regulatory game. Because we use subgame perfect equilibrium as our solution concept, we describe the strategies and solve the game backward. That is, we first consider the third stage of the game in which the regulated price is set. Then we specify the capital market equilibrium. Finally, we set out the firm’s optimization problem that takes place at the first stage of the game.

In the third stage of the game, the regulator chooses an optimal regulated price by maximizing a utilitarian welfare function given by

$$W(p, k, D) = CS(p) + b\Pi(p, k, D),$$

(6)

where $CS(p) = \int_{p}^{\pi} Q(p) dp$ is consumers’ surplus, and $b$ is a welfare weight satisfying $0 < b < 1$. The regulator’s optimal pricing strategy as a function of investment, $k$, and debt, $D$, is $p^*(k, D)$. Note that a regulated price, $p^*(k, D)$, corresponds to an allowed rate of return $\Pi(p^*(k, D), k, D)/k - 1$.

In the second stage of the game, the capital market clears. In equilibrium, the capital market correctly anticipates the regulator’s pricing strategy. Because the capital market is competitive, the firm’s securities are fairly priced. That is, the equilibrium market values of new equity, $E^*(k, \alpha, D)$, and bonds, $B^*(k, \alpha, D)$, adjust such that given the firm’s choice of investment, equity, and debt, $(k, \alpha, D)$, and the regulator’s pricing strategy, $p^*(k, D)$, investors earn an expected rate of return that is equal to the risk-free rate of return. Note that, because the capital market clears in anticipation of the regulator’s decision, the firm’s expected rate of return is determined in the capital market before the regulator actually sets the regulated price. It is important to emphasize that we do not assume that the regulator is able to make credible commitments to specific rates of return, so rates cannot be established through prior announcements.

\(^{11}\) Taxes are not included in our model so that we may focus on the incentive effects of bankruptcy costs. A tax advantage for debt relative to equity can imply an optimal capital structure. See for example, Kraus and Litzenberger (1973), Scott (1976), and Flath and Knoeber (1980).
In the first stage of the regulatory game, the firm chooses its investment, $k$, equity participation of outsiders, $\alpha$, and face value of its bonds, $D$. The firm’s objective is to maximize the expected payoff of its original shareholders, given by

$$ V(k, \alpha, D, p) = (1 - \alpha) \int_0^1 [R(p, z, k) - D]dF(z). \quad (7) $$

This payoff represents the original shareholders’ share of the net expected return to the firm over all states of nature in which the firm remains solvent. In equilibrium, the firm correctly anticipates the reaction of the capital market and the regulator’s pricing strategy. Thus, the equilibrium strategy of the firm, $(k^*, \alpha^*, D^*)$, is chosen to maximize the expression $V(k, \alpha, D, p^*(k, D))$, subject to the equilibrium conditions in the capital market and the firm’s budget constraint, $k = E^*(k, \alpha, D) + B^*(k, \alpha, D)$. The strategies $(k^*, \alpha^*, D^*, E^*(k, \alpha, D), B^*(k, \alpha, D), p^*(k, D))$ constitute a subgame perfect equilibrium of the three-stage regulation game.

4. The regulator’s pricing strategy

In this section we characterize the regulator’s optimal pricing strategy and examine how this strategy is influenced by the firm’s investment and capital structure. The regulator is assumed to place a high value on keeping the firm solvent. To represent this value, assume that the regulator sets a regulated price that is high enough to ensure that the firm is never liquidated for all debt levels. This imposes a no-liquidation constraint on the regulator’s problem to ensure that the firm’s revenues exceed the cost of bankruptcy even when the efficiency parameter is equal to zero, i.e., $R(p, 0, k) \geq H(D - R(p, 0, k))$. Given the no-liquidation constraint, $z^{**} = 0$.

Given $k$ and $D$, the regulator chooses an optimal pricing strategy, $p^* = p^*(k, D)$, with the objective of maximizing $W(p, k, D)$, subject to $R(p, 0, k) \geq H(D - R(p, 0, k))$. The first-order conditions for maximization are

$$ Q(p^*) = b\Pi_p(p^*, k, D), \quad \text{if} \quad R(p^*, 0, k) > H(D - R(p^*, 0, k)), $$

$$ R(p^*, 0, k) = H(D - R(p^*, 0, k)), \quad \text{otherwise}. \quad (8) $$

To interpret these conditions, note that the regulator’s objective function can be written as $W(p, k, D) = S(p, k) - bT(p, k, D)$, where $S(p, k) = CS(p) + b\int_0^1 R(p, z, k)dF(z)$ is the expected weighted surplus generated by the firm. Using this expression, it follows that if the no-liquidation constraint is nonbinding, the first-order condition for $p^*$ is $S_p(p^*, k) = bT_p(p^*, k, D)$. This equation indicates that the regulator sets the marginal expected weighted surplus equal to the weighted effect of the regulated price on expected bankruptcy costs. Otherwise, the regulator sets the regulated price just high enough to ensure that the firm is never liquidated.

The tradeoff between expected bankruptcy costs and higher prices is the significant aspect of the regulator’s decision. The regulator wishes to avoid bankruptcy costs, but faces deadweight welfare losses from pricing above expected marginal costs. The following proposition establishes two important properties of the optimal regulated price.

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12 There are other ways to guarantee that the firm remains solvent. The firm’s losses may be made up after the value of the efficiency parameter is realized, as occurs in successive rate hearings. Alternatively, existing owners may absorb the losses in anticipation of future earnings or borrow additional funds from the capital market. In the present one-period model, however, our approach requiring positive operating income seems the most natural, because it assures that neither bondholders nor equityholders will carry operating losses.
Proposition 1. (i) The optimal regulated price always exceeds expected marginal cost; and (ii) at the optimal regulated price, given a positive level of debt, the probability of bankruptcy is positive, $F(z^*) > 0$.

Proposition 1 demonstrates that if the regulated firm issues debt, then the regulator will always allow some bankruptcies. Although utility bankruptcies have been rare in the post-World War II period, they appear more likely as a consequence of investments in nuclear power plants. For example, Public Service Company of New Hampshire, co-owner of the Seabrook nuclear power plant, filed for bankruptcy in 1988. El Paso Electric Co. filed to reorganize under Chapter 11 in January 1992 after failing to reach an agreement with its creditors over $400 million in debt obligations, partly due to disallowed costs at their Palo Verde nuclear facility. The Washington Public Power Supply System (popularly known as “WHOOPS”) defaulted on $2.25 billion of municipal bonds in 1983 after three nuclear units were cancelled.

We now turn to a characterization of the regulator’s optimal pricing strategy, $p^*(k, D)$. We first consider the effect of the welfare weight on the regulator’s optimal pricing policy.

Proposition 2. If the no-liquidation constraint is nonbinding, then the optimal regulated price is increasing in the regulator’s welfare weight on profits, i.e., $\frac{\partial p^*}{\partial b} > 0$. Otherwise, $\frac{\partial p^*}{\partial b} = 0$.

Note that the analysis can be generalized to allow uncertainty regarding the regulatory climate as reflected in the parameter $b$. This will introduce an additional source of randomness for the firm’s investors.

An interesting finding reported by Besley and Bolten (1990) is that about 80% of the regulators and 63% of the utilities they surveyed believe that rates increase when the quality of debt deteriorates. In order to examine this issue in our model, we represent the quality of debt both in terms of the risk and the costs of bankruptcy. To model an exogenous increase in the risk of debt, let $a$ be a shift parameter in the distribution of $z$, such that $F_a(z; a) > 0$. That is, when $a$ increases, the efficiency parameter, $z$, is more likely to be low, so this implies, ceteris paribus, a higher probability of bankruptcy. Similarly, to model an exogenous increase in the costs of bankruptcy, let $t$ be a shift parameter in the marginal bankruptcy costs function, such that $h_t(\cdot, t) > 0$. Given this formulation, we establish the following proposition.

Proposition 3. If the no-liquidation constraint is nonbinding, then the optimal regulated price increases when the quality of the firm’s debt deteriorates, due to an increase either in the shift parameter, $a$, or in the bankruptcy cost parameter, $t$, i.e., $\frac{\partial p^*}{\partial a} > 0$, and $\frac{\partial p^*}{\partial t} > 0$. Otherwise, $\frac{\partial p^*}{\partial a} = 0$, and $\frac{\partial p^*}{\partial t} = 0$.

The proof of Proposition 3 is similar to the proof of Proposition 2 and hence is omitted from the Appendix. Proposition 3 provides an explanation for Besley and Bolten’s observation. The regulator responds to the deterioration in the quality of debt by increasing the regulated price in order to lower the expected costs of bankruptcy. This response is consistent with Owen and Braeutigam (1978), who argue that “One of the worst fears of a regulatory agency is the bankruptcy of the firm it supervises, resulting in ‘instability’ of services to the public or wildly fluctuating prices.”

The optimal price function, $p^*(k, D)$, need not be monotonic for all values of investment and debt. To see why, note that higher debt raises the likelihood of bankruptcy, thereby raising expected bankruptcy cost. This suggests that greater revenues would be provided by the regulator to reduce the likelihood of bankruptcy, but this need not imply a higher price. With regard to investment, a higher level of investment raises expected surplus and lowers expected bankruptcy costs by lowering production cost. This would suggest that lower revenues are needed. Again, this need not translate into lower prices.
Moreover, the increased cost of investment will be reflected in the regulated price. An important consideration is the effect of regulated price on the probability of bankruptcy. We can state a sufficient condition for a higher price to lower the probability of bankruptcy. To this end, let $\eta(p) = -p Q'(p)/Q(p)$ be the elasticity of demand. Then, the sufficient condition requires that for all $k$ and $D$

$$\frac{p^* - C_q(Q(p^*), z^*, k)}{p^*} < \frac{1}{\eta(p^*)},$$

(9)

where $p^* = p^*(k, D)$. Condition (9) implies that the optimal regulated price is less than the monopoly price for a firm with costs evaluated at $z^*$. This is equivalent to the statement that marginal revenues at the critical level of the efficiency parameter are positive, i.e., $R_p(p^*, z^*, k) > 0$. This in turn implies that a price increase lowers the likelihood of bankruptcy, as $\frac{dz^*}{dp} = R_p(p, z^*, k)/C_q(z^*, k) < 0$.

Proposition 4. Assume that the optimal regulated price satisfies (9) for all $k$ and $D$. Then the following hold: (i) the optimal regulated price is increasing in the firm’s debt obligation, i.e., $\frac{dp^*}{dD} > 0$; and (ii) the optimal regulated price is decreasing in the firm’s investment, i.e., $\frac{dp^*}{dk} < 0$.

Proposition 4 establishes that the price-influence effect of debt is positive. This suggests that the regulated firm will issue debt to raise the regulated price. At the same time, the price-influence effect of investment is negative. This reflects regulatory opportunism and suggests that the regulated firm will reduce its investment level in order to raise the regulated price.

5. Capital market equilibrium

In this section we set out the capital market equilibrium and assume that the capital market is competitive. The market clears in stage two of the game after the firm has chosen its investment and capital structure. Then, the market values of the firm’s securities adjust in anticipation of the regulated price, such that the expected rate of return to investors equals the risk-free rate of return, $1 + i$. Given the firm’s actions, $(k, \alpha, D)$, the market values of its securities are uniquely determined. In equilibrium, investors perfectly forecast the regulator’s optimal pricing strategy and take it into account in determining the firm’s expected revenues. To make this clear, define $R^*(z, k, D) = R(p^*(k, D), z, k)$ as the reduced-form operating income of the firm, evaluated at the optimal regulated price. Similarly, define $Z^* = z^*(p^*(k, D), k, D)$ and $T^*(k, D) = T(p^*(k, D), k, D)$, respectively, as the reduced-form critical value of the efficiency parameter and the reduced-form expected bankruptcy costs, evaluated at the optimal regulated price.

Given $(k, \alpha, D)$, the firm’s equity is priced such that new equityholders earn the risk-free rate,

$$E^* = E^*(k, \alpha, D) = \frac{1}{1 + i} \alpha \int_{z^*}^{z} [R^*(z, k, D) - D]dF(z).$$

(10)

The right side of (10) represents the discounted value of expected revenues of the firm net of debt payment over states of nature in which the firm remains solvent, and $\alpha$ is the
new equityholders’ share in these profits. Similarly, given \((k, \alpha, D)\), the firm’s bonds are priced such that bondholders also earn the risk-free rate,

\[
B^* = B^*(k, \alpha, D) = \frac{1}{1 + i} \left[ D(1 - F(Z^*)) + \int_0^{Z^*} R^*(z, k, D)dF(z) - T^*(k, D) \right].
\] (11)

Note that \(\alpha\) has no direct effect on the market value of bonds. The first term on the right side of (11) represents the discounted value of the expected return to bondholders over states of nature in which they are paid in full. The last two terms represent the discounted value of the firm’s expected revenues over states of nature in which the firm goes bankrupt and bondholders become the residual claimants, net of bankruptcy costs.

Interestingly, investment and debt have ambiguous effects on the equilibrium market values of equity and debt. Investment affects the market values of the firm’s securities both directly by lowering the firm’s costs, and indirectly through its effect on the regulated price. Although the former effect is always positive, the latter effect can be negative if a higher investment level lowers the regulated price and this lower price translates into lower revenues. Debt also has both direct and indirect effects on the equilibrium market values of the firm’s securities. The direct effect is due to the increase in the face value of bonds. The indirect effect is due to the effect of debt on the optimal regulated price. When (9) holds for all \(k\) and \(D\), the indirect effect is always positive. However, because the direct effect is negative for equity and is ambiguous for bonds, the net effect of debt is in general ambiguous.

6. The regulated firm’s strategy

The regulated firm’s investment and capital structure are chosen in the first stage of the game, before the capital market clears and before the regulator sets the regulated price. In equilibrium, the firm correctly anticipates the effect of its decisions on the reactions of both the capital market and the regulator. Thus, in equilibrium, the firm chooses investment, equity, and debt by taking into account the equilibrium market values of its equity and bonds, \(E^*(k, \alpha, D)\) and \(B^*(k, \alpha, D)\), and the regulator’s pricing strategy, \(p^*(k, D)\). This implies that the equilibrium strategy of the firm must satisfy the capital market’s equilibrium conditions, given by (10) and (11). Moreover, in equilibrium, the firm’s budget constraint must hold with equality, i.e., \(k = E^* + B^*\). Adding (10) and (11) and using the firm’s budget constraint, we can solve for a unique value of the equity participation of outsiders for each pair of \(k\) and \(D\),

\[
\alpha^*(k, D) = \frac{(1 + i)k - D(1 - F(Z^*)) - \int_0^{Z^*} R^*(z, k, D)dF(z) + T^*(k, D)}{\int_{Z^*}^{1} [R^*(z, k, D) - D]dF(z)}.
\] (12)

This allows a convenient representation of the firm’s strategy, \((k, \alpha, D)\), which effectively reduces to a choice of an investment level, \(k\), and a debt level, \(D\), with \(\alpha\) being determined by (12).

Substituting \(p^*(k, D)\) and \(\alpha^*(k, D)\) into (7), rearranging terms, and using (5), the objective function of the firm’s owners can be expressed as

\[
V(k, D) = V(k, \alpha^*(k, D), D, p^*(k, D)) = \Pi(p^*(k, D), k, D) - (1 + i)k.
\] (13)

Thus, the firm’s owners choose investment and debt to maximize expected revenues net of expected bankruptcy costs and the cost of capital.
The assumption that \( \lim_{k \to 0} C_k(q, z, k) = -\infty \) ensures that the equilibrium investment level, \( k^* \), is positive. In Proposition 5 below we prove that the equilibrium debt level, \( D^* \), is positive as well. Hence, the first-order conditions for the firm’s problem are

\[
\Pi_p(P^*, k^*, D^*) \frac{\partial P^*}{\partial k} = \int_0^1 C_k(Q(P^*), z, k^*)dF(z)
\]

\[
+ \int_0^{Z^*} h(D^* - R^*(z, k^*, D^*))C_k(Q(P^*), z, k^*)dF(z) + (1 + i) \tag{14}
\]

\[
\Pi_p(P^*, k^*, D^*) \frac{\partial P^*}{\partial D} = \int_0^{Z^*} h(D^* - R^*(z, k^*, D^*))dF(z), \tag{15}
\]

where \( P^* = p^*(k^*, D^*) \). Equation (14) reveals that the regulated firm takes into account the price effects of investment, as well as the effects of investment on cost reduction and on the marginal expected cost of bankruptcy, and the direct cost of investment. Equation (15) shows that at the optimum, the regulated firm trades off the marginal increase in the regulated price due to debt against the marginal increase in the expected cost of bankruptcy.

**Capital structure.** Equation (15) indicates that the firm may benefit from issuing debt through an increase in the regulated price. At the same time, issuing debt leaves the firm susceptible to a costly bankruptcy. This tradeoff implies that, in the present full-information framework, an unregulated firm would issue no debt because debt would only serve to create expected bankruptcy costs. In particular, this implies that the socially optimal level of debt is zero. We now show that when a firm is regulated, the benefit of debt exceeds its cost, at least for small amounts of debt, thereby inducing the firm to issue a positive amount of debt.

**Proposition 5.** In equilibrium, the regulated firm issues a positive amount of debt, i.e., \( D^* > 0 \). Consequently, the equilibrium probability of bankruptcy is positive, i.e., \( F(Z^*) > 0 \).

By issuing debt, the firm influences the regulator to raise the regulated price in order to avoid bankruptcy costs. This strategy, however, is costly to the firm, because the increase in the regulated price is not sufficiently large to make the firm completely immune from bankruptcy. However, for a small debt level, the increase in the regulated price has a first-order effect on the initial equityholders’ payoff, whereas the increase in expected bankruptcy costs has only a second-order effect. Hence, in equilibrium, the firm issues a positive amount of debt.

An important implication of Proposition 5 is that, in equilibrium, the price-influence of debt is locally positive, provided that the no-liquidation constraint is nonbinding. Because \( D^* > 0 \), (15) holds in equilibrium. But, if the no-liquidation constraint is nonbinding, then \( P^* \) is given by the first line of (8), evaluated at \( k^* \) and \( D^* \), so

\[
\Pi_p(P^*, k^*, D^*) = Q(P^*)/b > 0.
\]

Together with the fact that the right side of (15) is always positive, this implies that \( \partial P^*/\partial D > 0 \). That is, an increase in the equilibrium level of debt leads to an increase in the optimal regulated price. When the no-liquidation constraint is binding in equilibrium, this may no longer be true, because in general, \( \Pi_p(P^*, k^*, D^*) \) may be negative, in which case \( \partial P^*/\partial D < 0 \).

**Investment.** Thus far we have shown that, in equilibrium, the firm finances its investment, at least partially, with debt, and that consequently it goes bankrupt with positive
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Next, we consider the investment decision of the firm. To simplify the analysis, we assume in what follows that the no-liquidation constraint is nonbinding at the optimum. This assumption implies that the optimal regulated price is implicitly defined by the first line in (8).

Given $D^*$, the equilibrium investment level of the firm is given by (14), which shows that the firm chooses investment strategically because of the price-influence effect of investment, $\frac{\partial P^*}{\partial k}$. This effect is due to the fact that in our model the regulator sets the regulated price after the firm has already made an irreversible commitment to invest. At the optimum, the firm trades off the price-influence effect of investment against the net marginal return to investment, which is represented by the right side of (14). Because the first line in (8) indicates that $\Pi_r(P^*, k^*, D^*) = \frac{Q(P^*)}{b} > 0$, the regulated firm’s net marginal return to investment is either positive or negative depending on the sign of the price-influence effect of investment. In contrast, a competitive firm always sets the net marginal return to investment equal to $1 + i$.

Given our assumptions, the profit function is concave in investment.

$$
\Pi_{kk}(P^*, k^*, D^*) = -\int_0^1 C_{kk}(\cdot)dF(z) - \int_0^{Z^*} h(\cdot)C_{kk}(\cdot)dF(z)
- \int_0^{Z^*} h'(\cdot)C_k^{\prime}(\cdot)dF(z) - h(0)C_k(\cdot)f(Z^*) \frac{\partial Z^*}{\partial k} < 0,
$$

where $C_k(\cdot) < 0$, $C_{kk}(\cdot) > 0$, and $\frac{\partial Z^*}{\partial k} = -\frac{C_k(\cdot)}{C_k(\cdot)} < 0$. This implies that the regulated firm underinvests relative to the profit-maximizing level for the equilibrium output, $Q(p^*)$, and debt level, $D^*$, if $\frac{\partial P^*}{\partial k} > 0$, but overinvests otherwise.

In general, it is impossible to determine the sign of $\frac{\partial P^*}{\partial k}$. One special case is when condition (9) holds for all $k$ and $D$. Then, as Proposition 4 shows, the price-influence effect of investment is negative, leading to underinvestment. In the next proposition we consider a second special case in which the marginal cost of bankruptcy is constant. This assumption implies that, from the regulator’s perspective, the regulated price and investment, and the regulated price and debt, are strategic complements, i.e., $W_{pk}(P^*, k, D) > 0$, and $W_{pD}(P^*, k, D) > 0$.

**Proposition 6.** Assume that the marginal cost of bankruptcy is constant, i.e., $h(x) = h$ for all $x$. Then, the equilibrium price-influence effect of investment is negative, $\frac{\partial P^*}{\partial k} < 0$. Consequently, the regulated firm underinvests relative to the profit-maximizing level of output $Q(P^*)$ and debt $D^*$.

Proposition 6 establishes that, in equilibrium, the firm underinvests relative to the investment level that would be optimal for the equilibrium output and debt levels. This result is due to the opportunistic behavior of the regulator, who responds to the firm’s investment by cutting the regulated price, thereby limiting the ability of the firm’s owners to recover their investment. The extent of regulatory opportunism depends on the regulator’s welfare weight.

A more relevant comparison would be between the equilibrium investment level and the socially optimal investment level. As already noted, in our framework the socially optimal debt level is zero due to bankruptcy costs, so that the social optimum implicitly requires all-equity financing. To facilitate comparison with the social optimum, let consumers’ surplus and profit carry equal weights, i.e., $b = 1$. As the proof of part (ii) of Proposition 1 shows (see the Appendix), in the absence of debt the no-liquidation
constraint is nonbinding. Thus, the benchmark socially optimal investment, \( k^0 \), and price, \( p^0 \), are defined by the following two conditions:

\[ p^0 = \int_0^1 C_d(Q(p^0), z, k^0)dF(z), \]

\[ -\int_0^1 C_k(Q(p^0), z, k^0)dF(z) = 1 + i. \]

Condition (16) says that the socially optimal price equals expected marginal cost. Condition (17) says that the socially optimal marginal productivity of investment equals the risk-free rate of return. We now prove the following result.

**Proposition 7.** Assume that the marginal cost of bankruptcy is constant and that welfare weights are equal. Then, the following hold: (i) The regulated firm invests less than is socially optimal, \( k^* < k^0 \); and (ii) the regulated price is above the socially optimal price, \( P^* > p^0 \).

Proposition 7 is important because it shows that a regulated firm that chooses both investment and capital structure invests less than is socially optimal. This result is due to regulatory opportunism, which is reflected by the negative price-influence effect of investment. Earlier, Spulber (1989) derived a similar result, but in his model the firm is implicitly using all-equity financing. Proposition 7 shows that the underinvestment problem persists even when the firm is allowed to exercise discretion in deciding how to finance its investment.

Because debt entails the possibility of costly bankruptcy, the question arises as to why regulators permit firms to take on debt. The occurrence of underinvestment provides a clue. By permitting debt, the regulator makes an implicit commitment to the regulated firm, thereby restricting future opportunism. That is, the regulator makes it more difficult for himself to lower the regulated price after the firm invests in cost reduction. This suggests that the regulator will permit firms to take on debt only if debt increases the firm’s *ex ante* investment level such that the benefits from additional investment are sufficiently high to outweigh the expected costs of bankruptcy.\(^{13}\) Debt can therefore serve as an imperfect substitute for regulatory commitment to rates. This role is similar to the role that regulatory bureaucracy plays in Sappington (1986), in which a regulator commits himself to an inefficient regulatory process in order to protect the firm from *ex post* opportunism and thereby strengthen its incentives to reduce its costs *ex ante*.

**8. Conclusion**

The three-stage model of the regulatory process that we present shows that capital structure can play a role in the strategic interaction between regulators and firms. In equilibrium, the regulated firm issues a positive amount of debt as a consequence of regulation despite the presence of bankruptcy costs. Debt serves to raise the regulated rates as the regulator seeks to reduce expected bankruptcy costs, although the likelihood of bankruptcy remains positive at the equilibrium. This result is confirmed by previously cited empirical analyses of the effect of debt on regulated rates.

Our model allows regulators to set rates after the firm selects its investment and capital structure and after capital markets clear. The regulated firm is shown to invest less than

\(^{13}\) Another reason why regulators may allow firms to use debt financing is suggested in Spiegel (1992). There, debt financing is shown to have a positive effect on a regulated firm’s choice of technology and, in addition, it eliminates the firm’s incentive to engage in goldplating. As in the current article, these benefits may outweigh the cost of the associated increase in the probability of bankruptcy.
the socially optimal level, which in turn raises regulated rates above the optimal level. However, the issuance of debt mitigates the regulator’s incentive to act in an opportunistic manner, and may therefore provide the firm with an incentive to increase its level of investment above that of an all-equity firm.

The strategic issuance of debt by the firm may create incentives for regulators to place limits on debt as a means of controlling the risk of bankruptcy. However, as has been shown in financial market models, the firm’s capital structure can provide information regarding its costs and performance. This suggests the need for additional investigation of the informational aspects of capital structure in regulated industries.\(^{14}\)

Appendix

- **Proofs of Propositions 1–2 and 4–7 follow.**

**Proof of Proposition 1.** (i) From the constraint, it follows that \(R(p^*, 0, k) \geq H(D - R(p^*, 0, k)) \geq 0\). Using (1), this implies that \(p^* \geq C(q^*, 0, k)/q^*\), where \(q^* = Q(p^*)\). But, because by assumption, \(C(q, 0, k)/q > \int C_0(q, z, k)dF(z)\) for all \(q\), it follows that the optimal regulated price always exceeds expected marginal cost.

(ii) Assume by way of negation that \(z^* = 0\). Then, by the definition of \(z^*, R(p^*, 0, k) \geq D\). But, because by assumption \(H(D - R(p, z, k)) = 0\) for all \(D \leq R(p, z, k),\) and because \(D > 0\), it follows that \(R(p^*, 0, k) \geq D > 0 = H(D - R(p^*, 0, k))\). Hence, the constraint \(R(p^*, 0, k) \geq H(D - R(p^*, 0, k))\) is nonbinding. Substituting \(z^* = 0\) in the first line of (8), it follows that \(\frac{ap^*}{D} = 0\).

**Proof of Proposition 2.** When the no-liquidation constraint is nonbinding, \(p^*\) is implicitly defined by \(W_0(p^*, k, D) = 0\). Because \(W(\cdot)\) is concave in \(p\), it follows that \(\frac{ap^*}{D} = \text{sign} W_0(p^*, k, D)\), where

\[
\frac{ap^*}{D} = \frac{1}{bQ(p^*)} < 0.
\]

To determine the sign of this expression, recall from (9) that \(R_0(p^*, z^*, k) > 0\). Because \(R_0(p^*, z, k) = -C_0(\cdot)Q'(p^*) < 0\), it follows that

\[
R_0(p^*, z, k) \geq R_0(p^*, z^*, k) > 0, \quad \forall z \in [0, z^*].
\]

Now, by assumption, \(h(0) > 0\) and \(h'(\cdot) > 0\). Together with (A3) and with the fact that \(\frac{dz^*}{D} = \frac{1}{C_0(\cdot)} > 0\), this is sufficient to guarantee that both terms on the right side of (A2) are positive. Hence, \(W_0(p^*, k, D) > 0\), implying that \(\frac{ap^*}{D} > 0\).

Second, if the no-liquidation constraint is binding, \(p^*\) is implicitly defined by \(R(p^*, 0, k) = H(D - R(p^*, 0, k))\). Differentiating this equality with respect to \(p^*\) and \(D\), yields

\[
\frac{ap^*}{D} = \frac{h(\cdot)}{(1 + h(\cdot))R_0(p^*, 0, k)}.
\]

\(^{14}\) This issue is addressed in Spiegel and Spulber (1993).
Because $h(\cdot) > 0$, it only remains to show that $R_0(p^*, 0, k) > 0$. To this end, note that the regulator’s maximization problem can be formulated in terms of the Lagrangian

$$\mathcal{L} = W(p, k, D) + \lambda[\mathcal{R}(p, 0, k) - H(D - R(p, 0, k))].$$

(A5)

The first-order condition for $p^*$ is therefore

$$\mathcal{L}_p = W_0(p^*, k, D) + \lambda[1 + h(\cdot)]R_0(p^*, 0, k) = 0.$$  

(A6)

Using the definition of $W(p, k, D)$, and the fact that because the constraint is binding, $\lambda > 0$, it follows that

$$\int D R_0(p^*, z, k) dF(z) + [1 + h(\cdot)]R_0(p^*, 0, k) = Q(p^*) > 0.$$  

Clearly, because

$$R_0(p^*, z, k) = -C_{01}(\cdot)Q'(p^*) < 0,$$

the left side of the equation can be positive only if $R_0(p^*, 0, k) > 0$. Hence, $\partial p^*/\partial D > 0$.

(ii) Again, there are two cases to consider. First, if the no-liquidation constraint is nonbinding, then (8) indicates that $p^*$ solves $W_0(p, k, D) = 0$. Because $W(\cdot)$ is concave in $p$, it follows that $dp^*/dk = \text{sign} W_{p,k}(p^*, k, D)$. Differentiating this equality with respect to $p^*$, yields

$$W_{p,k}(p^*, k, D) = b\int R_0(p^*, z, k) dF(z) + b\int h'(D - R(p^*, z, k))R_0(p^*, z, k) dF(z) - b\int h'(D - R(p^*, z, k))R_0(p^*, z, k) dF(z) + bh(0)R_0(p^*, z, k)f(z) \frac{dz^*}{dk}.$$  

(A7)

Now, because $R_0(p^*, z, k) = -C_{01}(\cdot)Q'(p^*) < 0$, the first two terms in (A7) are negative. Because $R_0(p^*, z, k) = -C_{01}(\cdot) > 0$, $R_0(p^*, z, k) > 0$ by hypothesis, and because (A3) applies, the third term is also negative. Finally, by hypothesis and because $\partial z^*/\partial k = -C_{10}(\cdot)/C_{11}(\cdot) < 0$, the fourth term in (A7) is negative as well. Therefore, $W_{p,k}(p^*, k, D) < 0$, so that $\partial p^*/\partial k < 0$.

Second, if the no-liquidation constraint is binding, $p^*$ is implicitly defined by

$$R(p^*, 0, k) = H(D - R(p^*, 0, k)).$$

Differentiating this equality with respect to $p^*$ and $k$, yields

$$\frac{\partial p^*}{\partial k} = \frac{-h(\cdot)R_0(p^*, 0, k)}{1 + h(\cdot)R_0(p^*, 0, k)}.$$  

(A8)

The proof is completed by observing that $h(\cdot) > 0$, $R_0(p^*, 0, k) = -C_{01}(\cdot) > 0$, and $R_0(p^*, 0, k) > 0$ (see part (i)). $Q.E.D.$

**Proof of Proposition 5.** Assume by way of negation that $D^* = 0$. Using (3) and recalling that the no-liquidation constraint ensures that $R(p^*, 0, k) = 0$, it follows that $Z^* = 0$. Consequently, the right side of (15) vanishes. We now show that the left side of (15) is strictly positive, thus contradicting our assumption that $D^* = 0$.

Now, as in the proof of part (ii) of Proposition 1, it can be shown that when $Z^* = 0$, the no-liquidation constraint is nonbinding. Thus, $p^*(k, 0)$ is defined by the first line of (8), which implies that

$$\Pi_0(p^*, k, D) = Q(p^*)/b > 0.\text{ Hence, the sign of the left side of (15) depends only on the sign of }\partial p^*/\partial D = \partial p^*(k^*, 0)/\partial D.\text{ Note that the first line in (8) can also be written as }W_0(p^*, k, D) = 0.\text{ Together with the fact that }W(\cdot)\text{ is concave in }p,\text{ this implies that sign }\partial p^*(k^*, 0)/\partial D = \text{ sign }W_{p,k}(p^*, k^*, D^*) = 0\text{ and }dP^*/dk < 0.\text{ Hence, the firm underinvests relative to the profit-maximizing investment at output level }Q(P^*)\text{ and debt level }D^*.\text{ Q.E.D.}$

**Proof of Proposition 6.** Because $D^* > 0$, (15) is satisfied. From (15) we have $\partial P^*/\partial D > 0$. But, because sign $\partial P^*/\partial D = \text{ sign }W_{p,k}(P^*, k^*, D^*) > 0$. Given constant marginal bankruptcy costs, $W_{p,k}(P^*, k^*, D^*) = -bhR_0(P^*, Z^*, k^*)f(Z^*)/C_{11}(\cdot)$. Therefore, $R_0(P^*, Z^*, k^*) > 0$, which using (A7) implies $W_{p,k}(P^*, k^*, D^*) > 0$ and $\partial P^*/\partial k < 0$. Hence, the firm underinvests relative to the profit-maximizing investment at output level $Q(P^*)$ and debt level $D^*$. $Q.E.D.$
Proof of Proposition 7. Because the no-liquidation constraint is assumed to be nonbinding, it follows from the first line of (8) that

$$\frac{\partial p^*}{\partial k} = \frac{W_{ps}(p^*, k, D)}{W_{p}(p^*, k, D)}.$$  \hfill (A10)

Similarly,

$$\frac{\partial p^*}{\partial D} = \frac{W_{ps}(p^*, k, D)}{W_{p}(p^*, k, D)}.$$  \hfill (A11)

Now, because \(D^* > 0\), (15) is satisfied. Evaluating (A11) at \(D = D^*\), substituting into (15), and rearranging terms,

$$-W_{ps}(p^*, k, D^*) = \frac{W_{ps}(p^*, k, D^*)}{hF(Z^*)} \Pi_{d}(p^*, k, D^*),$$  \hfill (A12)

where \(h\) is a constant by assumption. Evaluating (A10) at \(D = D^*\) and using (A12),

$$\frac{\partial p^*}{\partial k} = \frac{W_{ps}(p^*, k, D^*)}{W_{p}(p^*, k, D^*)} \frac{hF(Z^*)}{\Pi_{d}(p^*, k, D^*)}.$$  \hfill (A13)

Now, using (A2) and (A7) and the assumption that \(h(\cdot)\) is constant, it follows that

$$\frac{\partial p^*}{\partial k} \leq -C_{p}(p^*, z^*, k) \frac{hF(Z^*)}{\Pi_{d}(p^*, k, D^*)}.$$  \hfill (A14)

Evaluating \(V_i(k, D^*)\) at \(k = k^0\) and using (17) yields

$$V_i(k^0, D^*) = \Pi_{d}(p^*, k^0, D^*) \frac{\partial p^*}{\partial k} - h \int_{k^0}^{z^*} C_{d}(Q(p^*), z, k^0)dF(z)$$

$$- \int_{k^0}^{1} C_{d}(Q(p^*), z, k^0)dF(z) - (1 + \delta),$$

$$= \Pi_{d}(p^*, k^0, D^*) \frac{\partial p^*}{\partial k} - h \int_{k^0}^{z^*} C_{d}(Q(p^*), z, k^0)dF(z)$$

$$- \int_{k^0}^{1} [C_{d}(Q(p^*), z, k^0) - C_{d}(Q(p^0), z, k^0)]dF(z),$$  \hfill (A15)

where \(p^* = p^*(k^0, D^*)\). Evaluating (A14) at \(k = k^0\) and substituting into (A15), yields

$$V_i(k^0, D^*) < h \left[ C_{d}(Q(p^*), z^*, k^0)F(Z^*) - \int_{k^0}^{z^*} C_{d}(Q(p^*), z, k^0)dF(z) \right]$$

$$- \int_{k^0}^{1} [C_{d}(Q(p^*), z, k^0) - C_{d}(Q(p^0), z, k^0)]dF(z).$$  \hfill (A16)

Because \(C_{d}(\cdot) < 0\), the expression in brackets is negative. Moreover, because \(\partial p^*/\partial D > 0\), then \(p^*(k^0, D^*) > p^0\). Together with the assumptions that \(Q'(p) < 0\) and \(C_{d}(\cdot) < 0\), this implies that the integral term is also negative. Hence, \(V_i(k^0, D^*) < 0\), which in turn implies that \(k^0 > k^*\).

Finally, because \(h\) is constant, \(\partial p^*/\partial k < 0\), so \(p^* = p^*(k^*, D^*) > p(k^0, D^*) > p^0\). Q.E.D.

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