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Bertrand competition when firms hold passive ownership stakes in one another

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ABSTRACT

We show that the Bertrand oligopoly model with cost asymmetries may admit multiple Nash equilibria when firms hold passive ownership stakes in each other. The equilibrium price may be as high as the monopoly price of the most efficient firm.

1. Introduction

Many industries feature a complex web of partial cross ownerships (PCO) among rival firms. Examples include the Japanese and the US automobile industries (Alley, 1997), the global airline industry (Airline Business, 1998), the Dutch financial sector (Dietzenbacher et al., 2000), the Nordic power market (Amundsen and Bergman, 2002), and the global steel industry (Gilo et al., 2006). Many of these PCO stakes are passive and give the investing firm a share in the target's profit but not in the target's decision making.

The competitive effects of PCO stakes have been examined earlier by Bolle and Güth (1992), Flath (1992) and Dietzenbacher et al. (2000) in the context of the Cournot model. Flath (1991) and Reitman (1994) examine the incentive to acquire PCO stakes in rivals. Malueg (1992), Gilo et al. (2006) and Gilo et al. (2009) show that PCO arrangements can facilitate collusion in infinitely repeated oligopoly models. In this paper, we consider an n-firm static Bertrand oligopoly model in which firms have different levels of (constant) marginal costs. We show that whenever the second most efficient firm (firm 2) has a direct or indirect stake in the most efficient firm (firm 1), the model admits multiple Nash equilibria. In all equilibria, firm 1 serves the entire market, but the upper bound on its equilibrium price increases with firm 2's stake and can be as high as the monopoly price of firm 1.

2. The model

Consider a Bertrand oligopoly with $n \geq 2$ firms which produce a homogeneous product and face a downward sloping demand function $Q(p)$. Each firm i has a constant marginal cost, c_i, and firms are ranked such that $c_1 < c_2 < \cdots < c_n$. The n firms simultaneously choose prices and the lowest price firm serves the entire market. When more than one firm charges the lowest price, consumers buy from the most efficient among these firms.[1] Given this tie-breaking rule, the market is always served by a single firm. The operating profit of each firm i given its price p_i is

$$y_i = Q(p_i)(p_i - c_i),$$

if it serves the entire market and 0 otherwise. We assume that y_i has a unique global maximizer, p_i^m, where $p_1^m < p_2^m < \cdots < p_n^m$ (see Tirole, 1988, Ch. 1.1.1.1). We also assume that $p_1^m > c_n$, so all firms are effective competitors.

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[1] This tie-breaking rule is standard (see e.g., Deneckere and Kovenock, 1996).

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We assume that the n firms are linked through a web of PCO stakes. These stakes are passive: each firm chooses its price unilaterally, but takes into account the resulting effect on its share in the rivals' profits. Specifically, let α_{ij} be firm i's stake in firm j and define the following $n \times n$ PCO matrix:

$$A = \begin{pmatrix} 0 & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & 0 & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & 0 \end{pmatrix}.$$

In the matrix A, row i specifies the stakes that firm i has in the n rivals, while column j specifies the stakes that the n rivals hold in firm j. Since each firm is also held by "real shareholders" (its controller and possibly outside stakeholders), the sum of each column in A is strictly less than 1.

Given the PCO matrix A, the accounting profits of the n firms, including their stakes in the profits of rivals, are implicitly defined by the following system of n equations in n unknowns:

$$\pi = y + A\pi, \tag{1}$$

where $y \equiv (y_1, \ldots, y_n)'$ is the vector of operating profits and $\pi \equiv (\pi_1, \ldots, \pi_n)'$ is the vector of accounting profits.

Since A is nonnegative and the sum of each of its columns is strictly less than 1, (1) has a unique solution (see Berck and Sydsæter, 1993, Ch. 21.1–21.22, p. 111) defined by:

$$\pi(A) = By,$$

where $B \equiv (I - A)^{-1}$. The ij-th entry in the matrix B, denoted b_{ij}, represents the aggregate share that the real shareholders of firm i have in y_j. The accounting profit of each firm i is $\pi_i(A) = \sum_{j=1}^{n} b_{ij} y_j$.

Lemma 1 in Gilo et al. (2006) proves that (i) $0 \le b_{ij} < b_{ii}$ for all i and all $j \neq i$; (ii) $b_{ij} > 0$ if firm i has a direct or an indirect stake in firm j and $b_{ij} = 0$ otherwise;[2] and (iii) $b_{ii} \ge 1$ for all i, with strict inequality if and only if $b_{ij} > 0$ and $b_{ji} > 0$.

3. The equilibrium

Throughout, we will rule out weakly dominated strategies, so $p_i \ge c_i$ for all i. Absent PCO arrangements, the Nash equilibrium vector of prices is $(c_2, c_2, p_3, \ldots, p_n)$, where $p_j \ge c_j$ for all $j \ge 3$. Given our tie-breaking rule, firm 1 serves the entire market.[3]

To characterize the set of Nash equilibria under PCO arrangements, recall that due to our tie-breaking rule, the market is always served by a single firm. Assume that this firm is j and its price is p^*. Then, $\pi_i(A) = b_{ij} Q(p^*) \left(p^* - c_j\right)$ for all i.

Lemma 1. *Let p^* be the lowest price in the market. Then, in a Nash equilibrium, firm 1 and at least one other firm charge p^*, where $c_2 \le p^* \le p_1^m$, and firm 1 serves the entire market.*

Proof of Lemma 1. Suppose that firm $j \ge 2$ charges p^* and serves the entire market. If firm 1 matches p^*, it will serve the entire market itself and earn $b_{11} Q(p^*)(p^* - c_1)$. If it does not, its profit is $b_{1j} Q(p^*) \left(p^* - c_j\right)$. Since $b_{ii} > b_{ij}$ and since $c_1 < c_j$, then $b_{11} Q(p^*)(p^* - c_1) > b_{1j} Q(p^*) \left(p^* - c_j\right)$. Hence, in every Nash equilibrium, firm 1 will charge p^* and will serve the entire market. Obviously, $p^* \le p_1^m$, otherwise firm 1 will deviate to p_1^m and increase its profit. Likewise, $p^* \ge c_2$ since firm 1 can always serve the entire market by charging c_2. To ensure that firm 1 cannot profitably deviate upward from p^*, at least one more firm must charge p^*. Given our tie-breaking rule, all consumers buy from firm 1. □

The next step is to characterize p^*. To this end, note from Lemma 1 that in every Nash equilibrium, $y_1 = Q(p^*)(p^* - c_1)$ and $y_i = 0$ for all $i \ge 2$. Hence, $\pi_i(A) = b_{i1} Q(p^*)(p^* - c_1)$ for all i. Since $p^* \le p_1^m$, firm 1 has no incentive to cut p_1 below p^*, and since at least one other firm charges p^*, raising p_1 above p^* will decrease $\pi_1(A)$ from $b_{11} Q(p^*)(p^* - c_1)$ to $b_{1j} Q(p^*) \left(p^* - c_j\right)$. As for firm $i \ge 2$, then deviating upward will not change $\pi_i(A)$. The most profitable deviation downward is to undercut p^* slightly; such deviation makes $\pi_i(A)$ arbitrarily close to $b_{ii} Q(p^*)(p^* - c_i)$. To rule out such deviations, p^* has to be such that for each firm $i \ge 2$,

$$b_{i1} Q\left(p^*\right)\left(p^* - c_1\right) \ge b_{ii} Q\left(p^*\right)\left(p^* - c_i\right),$$

or,

$$p^* \le p_i^* \equiv \frac{b_{ii} c_i - b_{i1} c_1}{b_{ii} - b_{i1}} = c_i + \frac{b_{i1}(c_i - c_1)}{b_{ii} - b_{i1}}. \tag{2}$$

Note that for all $i \ge 2$, $p_i^* \ge c_i$ with equality holding only if $b_{i1} = 0$.

We are now ready to state our main result.

Proposition 1. *Let $\widehat{p}^* = \min\left\{p_2^*, \ldots, p_n^*\right\}$, where each p_i^* is defined by Eq. (2). Then, in any Nash equilibrium, firm 1 serves the entire market at a price $p_1 \in [c_2, \min\{\widehat{p}^*, p_1^m\}]$. At least one more firm also charges p_1, while all other firms j charge $p_j \ge \max\left\{p_1, c_j\right\}$.*

4. Discussion

Proposition 1 implies that any price in the interval $[c_2, \min\{\widehat{p}^*, p_1^m\}]$ can be supported as the equilibrium price of firm 1. The reason for this is that when $b_{i1} > 0$ and p_1 is not too high, firm i prefers to let firm 1 serve the entire market at marginal cost c_1 and then get a share in y_1, rather than undercut firm 1 and serve the entire market at a higher marginal cost c_i. Since potentially there is a whole interval of p_1 that has this property, we may get multiple equilibria. This situation differs from the traditional Bertrand model because absent PCO arrangements, firms get positive payoffs only when they make sales.

Proposition 1 has at least two interesting implications.

Corollary 1. *The model admits multiple equilibria if and only if (i) firm 2 has a direct or indirect stake in firm 1, i.e., $b_{21} > 0$, and (ii) $c_2 > c_1$.*

Proof of Corollary 1. If firm 2 does not have a direct or indirect stake in firm 1, then $b_{21} = 0$. By (2), if $b_{21} = 0$ or $c_2 = c_1$, then $p_2^* = c_2 < p_i^*$ for all $i \ge 3$; hence $\widehat{p}^* = c_2$, so in the unique equilibrium, firm 1 serves the entire market at a price c_2. When $b_{21} > 0$ and $c_2 > c_1$, $p_2^* > c_2$; since $p_i^* > c_2$ for all $i \ge 3$, then $\widehat{p}^* > c_2$, so the model admits multiple equilibria. □

Corollary 1 implies that if $b_{12} = 0$, then firm 1 charges c_2 in every Nash equilibrium, so the PCO stakes of other firms are irrelevant. This result is in contrast to Gilo et al. (2009) who show that in a repeated Bertrand oligopoly model with cost asymmetries, PCO stakes that firm 1 holds in rivals are sufficient to facilitate collusion. The corollary also implies that cost asymmetry is crucial for the multiplicity of equilibria and the potential anticompetitive effect of PCO.

Corollary 2. *In equilibrium, consumers may end up paying as much as the monopoly price of firm 1.*

To illustrate, suppose that $n = 2$, $\alpha_{12} = 0$, $\alpha_{21} > 0$, and $c_2 = \gamma c_1 + (1 - \gamma) p_1^m$, where $\gamma \in (0, 1)$. Then, $\widehat{p}^* = p_2^* = \frac{c_2 - \alpha_{21} c_1}{1 - \alpha_{21}} = \frac{\gamma c_1 + (1-\gamma) p_1^m - \alpha_{21} c_1}{1 - \alpha_{21}}$, which exceeds p_1^m whenever $\alpha_{21} > \gamma$, i.e.,

[2] We will say that firm i has an indirect stake in firm j if it has a stake in a firm that has a stake in firm j, or has a stake in a firm that has a stake in a firm that has a stake in firm j, and so on.

[3] Blume (2003) shows that under the more conventional tie-breaking rule where firms split the market equally when they tie for the lowest price, the model admits an equilibrium in which $p_1 = c_2$, and firm 2 randomizes uniformly on the interval $[c_2, c_2 + \eta]$, where $\eta > 0$ is "small".

whenever firm 2 has a large enough stake in firm 1 and c_1 is sufficiently below c_2. The upper bound on the equilibrium price of firm 1 is then p_1^m, implying that there exists an equilibrium in which firm 1 charges its monopoly price.

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