Pre-Grant Patent Publication, R&D, and Welfare*

By

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Abstract: In Europe and in Japan, patent applications are automatically published at 18 months from the filing date regardless of whether a patent has been or will ever be granted. In the U.S., applicants who file for a patent only in the U.S. can choose to keep their applications confidential until a patent is actually granted. We examine the consequences of this difference in patent disclosure rules in a setting where patent protection is imperfect in the sense that patent applications may be rejected and patents are not always upheld in court. We show that pre-grant publication of patents leads to fewer applications and fewer inventions, but for a given number of inventions, it raises the probability that new technologies will reach the product market and thereby enhances consumers’ surplus and possibly total welfare as well.

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1. Introduction

The two main objectives of patent systems are to encourage inventors to engage in R&D by granting them a temporary monopoly over the use of their inventions and to facilitate the dissemination of new technologies for the benefit of society at large. Economists generally agree that the U.S. patent system puts a greater emphasis on the first objective, while the system of the European Patent Office (EPO) and the Japanese patent system emphasize more the second goal. One example that highlights this different emphasis is the fact that patent applications in Europe and Japan are automatically published at 18 months from the filing date (a pre-grant patent publication, or PG-Pub system), whereas applicants in the U.S. who do not file for a patent in foreign countries can choose to keep their patent applications confidential until a patent is actually granted.

The economic literature has already studied various aspects of patent laws, including the optimal length and breadth of patents (e.g., Nordhaus, 1969; Gilbert and Shapiro, 1990; Klemperer, 1990; Gallini, 1992; Chang 1995; Green and Scotchmer 1995; Matutes, Regibeau, and Rockett, 1996; Eswaran and Gallini, 1996; O’Donoghue, 1998; and O’Donoghue, Scotchmer, and Thisse, 1998), priority rules such as "first to file" versus "first to invent" (e.g., Scotchmer and Green, 1990), novelty requirements (e.g., Scotchmer and Green, 1990; Scotchmer 1996; Eswaran and Gallini, 1996; and O’Donoghue, 1998), and the optimal renewal of patents (Cornelli and Schankerman, 1999). However, pre-grant patent publication (PG-Pub) has received very little attention in the economic literature. Given the intense public debate

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1 For example, Ordover (1991) argues that "... current U.S. policy stance that advocates very strong intellectual property rights may have gone too far in protecting the interests of the innovator." (p. 58-59), and "The Japanese patent system subordinates the short-term interests of the innovator in the creation of exclusionary rights to the broader policy goal of diffusion of technology." (p. 48).

2 Most countries have a PG-Pub system (Ragusa 1992). Prior to the American Investors Protection Act (AIPA) of 1999, the U.S. had a confidential filing system (CF system) as all patent applications were kept confidential until a patent was granted. The act has modified this rule by requiring that patent applications will be published at 18 months from the earliest filing date provided that the applicant has also filed for a patent in a foreign country. Hence, the current system in the U.S. is a hybrid of the PG-Pub and the CF systems. Interestingly, the initial versions of the legislation applied the 18 months rule to all patent applications in the U.S. Strong opposition to this proposal, mainly from small and independent inventors, eventually led to a compromise whereby applicants who file only in the U.S. can keep their applications confidential until a patent is issued.

3 The only exceptions we are aware of are Aoki and Prusa (1996) and Johnson and Popp (2003). Aoki and Prusa (1996) show that PG-Pub credibly commits the first filer to a given technology choice and hence facilitates collusion in the product market. Johnson and Popp (2003) look at U.S. patent citation data and find that more significant inventions take longer to go through the application process and hence are more likely to be affected by the switch to PG-Pub. They then simulate the potential gains from PG-
in the U.S. about PG-Pub prior to the legislation of the American Investors Protection Act (AIPA) of 1999, it seems that a formal economic analysis of this issue is badly needed.

In this paper we posit a model that allows us to study the effect of PG-Pub on firms’ R&D and patenting behavior and evaluate the resulting implications for consumers’ surplus, social welfare, and the incentive to invent. Our main premise is that PG-Pub affects not only the incentives to invent in the first place but also subsequent R&D efforts. To capture this idea, we consider two firms that engage in an R&D process aimed at inventing a basic technology (that has no commercial value of its own) and then developing a commercial application of this basic technology. Most of our analysis focuses on the second phase of the R&D process that begins after one firm (firm 1) has already invented the basic technology (towards the end of the paper, we examine the impact of PG-Pub on the incentives of the two firms to invent the basic technology in the first phase). Having invented, firm 1 needs to decide whether to apply for a patent on its basic technology. The benefit from having a patent is that it allows firm 1 to sue firm 2 for patent infringement, if the latter manages to invent the basic technology and commercialize it. If firm 1 wins in court, firm 2 cannot use the new technology in the product market. However, applying for a patent is costly for firm 1 because the patent or the patent application may reveal technological information to firm 2 and thereby diminish firm 1’s technological advantage.

The difference between the PG-Pub and the confidential filing (CF) systems in our model is that under the PG-Pub system, the details of firm 1’s invention are revealed whenever it files for a patent (possibly with some delay) even if its patent application is eventually rejected, whereas under the CF system, the details of firm 1’s invention are revealed only if a patent is actually granted. We examine the effect of PG-Pub on the incentive of firm 1 to file for a patent on the basic technology and on the investments of the two firms in the second phase of the R&D process. We then examine the resulting implications for consumers’ surplus and social welfare.

We show that the implications of PG-Pub depend on the strength of patent protection, which depends in our model on two factors: (i) the likelihood that the patent office will grant firm 1 a patent, and (ii) the likelihood that the court will rule in favor of firm 1 in a patent infringement suit. Thus, patent protection in our model is imperfect and its strength depends on the actions of two separate branches of the government: the patent office and the court.\(^4\) PG-Pub matters however only if patent protection is

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\(^4\) The assumption that patent protection is imperfect has also been made elsewhere. Meurer (1989), Anton and Yao (2000, 2003), and Choi (1999) assume that patents can be challenged in court and may
strong or intermediate because firms do not file for patents under neither filing system when patent protection is weak.

When patent protection is strong, firm 1 files for a patent under both filing systems. Not surprisingly, since PG-Pub reveals information on firm 1’s basic technology to firm 2, it encourages firm 2 to invest more in the second phase of the R&D process while inducing firm 1 to invest less. What is less clear is the effect of PG-Pub on the aggregate level of investment. But since R&D investments in the second phase of the R&D process are strategic substitutes, we can show that PG-Pub raises firm 2’s investment by more than it lowers firm 1’s investment. When the cost of investment is quadratic, this implies in turn that there is a higher likelihood that the new technology will reach the product market, so consumers become better-off. When the marginal cost of investment is sufficiently large, social welfare (measured as the sum of consumers’ surplus and profits) increases as well because the gap between the investments of firm 1 and 2 shrinks, implying that the allocation of investments between the two firms becomes more efficient. But, since PG-Pub benefits firm 2 and hurts firm 1, it weakens the incentive to invest in the first phase of the R&D process and hence discourages basic research.

Things are much more subtle when patent protection is intermediate. Now firm 1 files for a patent under the CF system but not under the PG-Pub system. The resulting effect on the investments in the second phase of the R&D process depends now on the likelihood that a patent will be upheld in court. When this likelihood is large, firm 1 cuts its investment level while firm 2 invests more, although the effect on the aggregate level of investment is ambiguous. By contrast, when the likelihood that a patent will be upheld in court is small, the effect of PG-Pub on the investments of the two firms is ambiguous, although the aggregate level of investment falls unambiguously. Nonetheless, PG-Pub benefits consumers regardless of the likelihood that a patent will be upheld in court. One reason for this is that firm 1 does not file for a patent under the PG-Pub system and hence cannot block firm 2’s use of the new technology in the product market if firm 2 develops it. When the marginal cost of investment is sufficiently large, PG-Pub is socially desirable if patents are likely to be upheld in court, and socially undesirable otherwise. As in the strong protection case, this is due to the gap between the investments of the two firms which be ruled as invalid, but unlike in our model, the possibility that patent applications may be rejected plays no role in these papers. Kabla (1996) also assumes that patent applications may be rejected, but she does not consider the possibility that patents may not be upheld in court. Waterson (1990) and Crampes and Langinier (2002) assume that suing for patent infringement is costly so patent holders do not always sue imitators. By contrast, we assume that it always pays the patent holder to sue for patent infringement. Finally, Crampes and Langinier (1998) show that under certain conditions, firms may choose not renew their patents in order to conceal favorable market information from potential entrants.
is smaller under the PG-Pub system when patents are relatively likely to be upheld in court and larger when they are not. Finally, PG-Pub has an ambiguous effect on the incentives to invent in first phase of the R&D process as it hurts both firms. However, when the cost of investment is quadratic, it hurts firm 1 by more than it hurts firm 2, so overall it weakens the incentives to invent in the first phase just as in the strong protection case.

The rest of the paper is organized as follows: in Section 2 we describe the model and in Sections 3 and 4 we study the equilibrium under the PG-Pub and CF systems. In Section 5 we compare the two systems in terms of the equilibrium patenting and investment behavior of the two firms and use the results to examine the implications of PG-Pub for consumers’ surplus and social welfare. In Section 6 we examine the implications of PG-Pub for the incentives to invent. We conclude in Section 7. All proofs are in the Appendix.

2. The model
We consider two firms that engage in an R&D process that requires the invention of a basic technology (that has no commercial value of its own), and the development of a commercial application of this basic technology. We distinguish between two phases in the R&D process: in the first phase, both firms are trying to invent the basic technology. This phase ends when one firm, firm 1, has successfully invented the basic technology. In the second phase, firm 1 is trying to develop a commercial application of its basic technology, while its rival, firm 2, still needs to engage in basic research before it can turn to the development of a commercial application.

In Sections 3-5 we focus on the second phase of the R&D process. At the beginning of this phase, firm 1 needs to decide whether or not to apply for a patent on its basic technology. If it does, some of its technological advantage over firm 2 may be lost as firm 2 may learn details about firm 1’s invention either from the patent application if it is made public, or from the actual patent if firm 1’s application is approved. On the other hand, if firm 1 gets a patent, it can sue firm 2 if the latter manages to invent the basic technology and commercialize it, on the grounds that firm 2 infringes on firm 1’s

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5 We assume that the fact that firm 1 has invented the basic technology is common knowledge. This could be justified on the grounds that firm 1 has an incentive to publicly announce this fact, given that, as we shall see below, the R&D investments in the remainder of the R&D process are strategic substitutes.
patent; if firm 1 wins in court, firm 2 cannot use the new technology in the product market. Given firm 1’s patenting decision, both firms choose their R&D investment levels in the second phase of the R&D process. These investments determine the likelihood of each firm to bring the new technology to the product market.

Towards the end of the paper in Section 6, we ask how PG-Pub affects the incentives of the two firms to invest in the first phase of the R&D process.

The filing system: We consider a pre-grant patent publication system (PG-Pub system) and a confidential filing system (CF system). Under PG-Pub system the contents of patent applications are automatically published after a certain period of time from the application date (typically 18 months). Under the CF system, patent applications are kept confidential until a patent is granted, and if a patent is not granted then no information is revealed.

In practice, patent protection is imperfect both because patent applications are sometimes rejected by the patent office if they are not deemed sufficiently novel, useful, or nonobvious, and because actual patents are not always upheld in court. We capture these imperfections by assuming that if firm 1 applies for a patent on the basic technology, it gets a patent with probability \( \theta \in [0,1] \), and if firm 1 sues firm 2 for patent infringement, it wins in court with probability \( \gamma \in [0,1] \). As we shall see, although both \( \gamma \) and \( \theta \) affect the strength of patent protection, the distinction between them will play an important role.

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6 We abstract from the possibility that firm 1 will license its invention to firm 2. While this is done mainly to simplify the analysis, it is straightforward to formalize the idea that firm 1 always sues for patent infringement in order to develop reputation for vigorously protecting its intellectual property and thereby deters future infringements on its inventions, by considering a repeated model of R&D competition. We also assume that if firm 1 files for a patent, it does so without delay. This assumption is motivated by the fact that in practice, there are many advantages to file for a patent as early as possible even under the first-to-invent rule used in the US. (see Kelly, 1995).

7 The acceptance rates of patent applications in 1993 were 74% in Europe, 67.2% in Japan, and only 65.2% in the U.S (these numbers were constructed from Table II-8, p. 26, in Institute for Intellectual Property, 1995). Allison and Lemley (1998) find that out of the 300 final patent validity decisions by U.S. courts during the period 1989-1996, only 162 patents (54%) were held valid. In Japan, plaintiffs in intellectual property cases concluded in the lower courts in 1995 (including patents, utility models, industrial designs, and copyrights) won in 51 cases out of 478, while 298 cases settled out of court (Japanese Supreme Court General Secretariat, 1996).
in our analysis. Throughout we treat $\theta$ and $\gamma$ as exogenous parameters.\textsuperscript{8}

\begin{itemize}
  \item \textbf{The cost of R&D in the second phase of the R&D process:} To capture the uncertainty that is inherent in any R&D process we assume that the outcome of the second phase of the R&D process is binary: each firm either succeeds to develop the new technology or it fails. Therefore, at the end of the second phase, either both firms have the new technology, both do not have it, or one firm (either firm 1 or 2) has it while the other does not. We assume that given firm 1’s filing decision, but before the patent office decides whether to grant firm 1 a patent, firms 1 and 2 simultaneously choose how much to invest in the second phase of the R&D process.\textsuperscript{9} The investments of firms 1 and 2, $q_1$ and $q_2$, determine their respective probabilities of successfully developing the new technology.

  Since firm 1 has already invented the basic technology, it can focus on the development of an application technology right from the start of the second phase of the R&D process. By contrast, firm 2 still needs to engage in basic research before turning to the application technology. We capture this advantage of firm 1 by assuming that its cost of achieving a given $q$ is lower than that of firm 2. Specifically, we assume that firm 1’s cost of investment in the second phase is $C(q)$, where $C(q)$ is twice continuously differentiable, increasing, and strictly convex, with $C'(0) = 0$. Firm 2’s cost of investment depends on how fast it manages to invent the basic technology. We assume that the speed at which this happens is positively affected by firm 2’s ability to learn about firm 1’s invention from firm 1’s patent application or actual patent. Hence we assume that firm 2’s expected cost in the second phase is $\beta_h C(q)$ if it learns about firm 1’s invention from the patent application, $\beta_m C(q)$ if it learns about firm 1’s invention from firm 1’s patent when it is issued, and $\beta_n C(q)$ if it does not learn about firm 1’s invention at all, where $\beta_h > \beta_m > \beta_n > 1$. The assumption that $\beta_h > \beta_m > \beta_n$ is consistent with Mansfield, Schwartz and Wagner (1981) who estimate that the average ratio between the cost of imitating an existing technology ($\beta_h C(q)$ or $\beta_m C(q)$ in our model) and the cost of innovating it from scratch ($\beta_n C(q)$ in our model) is 0.65.

\end{itemize}

\textsuperscript{8} According to the enablement doctrine of patent law, "claims ought to be bounded to a significant degree by what the disclosure enables, over and beyond prior art" (Merges and Nelson, 1994, p. 10). Thus, in a more general model where firm 1 can choose the scope of its disclosure, the likelihood that a court will upheld firm 1’s patent would be an endogenous variable.

\textsuperscript{9} This timing reflects the fact that patent examination is typically a lengthy process: in 1997, the average pendency of patents in the U.S. was 26.5 months from the original filing date (see Table 4 in the 1997 U.S. Patent office annual report).
Competition in the product market: At the end of the second phase of the R&D process, the two firms compete in the product market. Instead of assuming a specific type of product market competition, we simply assume that if only one firm succeeds in developing the new technology (this firm can be either firm 1 or 2), the net present value of its profits is $\pi_{yn}$ and the net present value of its rival’s profits is $\pi_{ny}$. If both firms succeed in developing the new technology, the net present value of their profits is $\pi_{yy}$, and if neither firm succeeds, the net present value of their profits is $\pi_{nn}$. Throughout, we make the following assumptions:

A1 $\pi_{yn} > \pi_{yy} \geq \pi_{nn} \geq \pi_{ny}$

A2 $\pi_{yn} + \pi_{ny} > 2\pi_{yy}$

A3 $C'(1) > \pi_{yn} - \pi_{nn}$, and $C''(q) > \pi_{yn} + \pi_{ny} - \pi_{yy} - \pi_{nn}$ for all $q \in [0, 1]$.

Assumptions A1 and A2 are consistent with a broad class of duopoly models. For example, if the new technology is cost-reducing, then in a Bertrand model with homogeneous products and linear cost functions, $\pi_{yn} > 0 = \pi_{yy} = \pi_{nn} = \pi_{ny}$, while in a Cournot model with homogeneous products, $\pi_{yn} > \pi_{yy} > \pi_{nn} > \pi_{ny}$, and whenever the demand and cost functions are linear, $\pi_{yn} + \pi_{ny} > 2\pi_{yy}$, for a sufficiently large cost reduction. Assumption A3 ensures that the best-response functions of firms 1 and 2 are well-behaved.

The sequence of events and the solution concept: The sequence of events is summarized in Figure 1. In Sections 3-5 we analyze the second phase of the R&D process which begins after firm 1 has already invented the basic technology. For each filing system, we solve the model backwards to obtain a subgame perfect equilibrium. Given firm 1’s filing decision, there are two subgames: a filing subgame and a no-filing subgame. For each subgame, we solve for the Nash equilibrium levels of investment. We then compare the two subgames and solve for firm 1’s decision on whether or not to file for a patent on

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10 To illustrate, suppose that the inverse demand function is $P = 6-x_1-x_2$, where $x_i$ is the output of firm $i$, $i = 1, 2$, and assume that firm i’s marginal cost of production is 0 if it develops the new technology and $k < A/2$ otherwise. Then, $\pi_{yn} = (A+k)^2/9$, $\pi_{yy} = A^2/9$, $\pi_{ny} = (A-2k)^2/9$, implying that Assumption A2 holds whenever $A2/5 < k < A/2$.

11 Note that Assumption A2 and the assumption that $\pi_{yn} \geq \pi_{nn}$ ensure that $\pi_{yn} - \pi_{nn} \geq \pi_{yy} - \pi_{ny}$; hence, $C'(1) > \pi_{yn} - \pi_{nn}$ implies that it is too costly to invest up to the point where developing the new technology becomes a sure thing, regardless of whether the rival firm has the new technology.
Firm 1 invents a basic technology and needs to decide whether or not to file for a patent. If it does, it receives a patent on the basic technology with probability \( \theta \).

The two firms simultaneously choose \( q^1 \) and \( q^2 \) which determine their respective probabilities of developing a commercial application technology.

The outcome of the second phase of the R&D process is realized.

If firm 2 develops an application technology and firm 1 has a patent, then firm 1 sues firm 2 for patent infringement and wins in court with probability \( \gamma \).

The two firms compete in the product market.

Figure 1: The sequence of events
its invention, and we also compare the two filing systems in terms of their impact on consumers and on social welfare. In Section 6 we consider the initial phase of the R&D process and examine the effect of PG-Pub on the incentives of the two firms to invent the basic technology.

3. The Pre-Grant Patent Publication (PG-Pub) system

When firm 1 files for a patent under the PG-Pub system, it can prevent firm 2 from bringing the new technology to the product market (if firm 2 develops it) with probability $\gamma \theta$ (the probability that a patent is granted and is upheld in court). Recalling that the R&D investments of firms 1 and 2 in the second phase of the R&D process, $q^1$ and $q^2$, also represent their respective probabilities of developing the new technology, it follows that the probability that firm 2 will develop the new technology and will be able to bring it to the product market is $q^2(1-\gamma \theta)$. Therefore, when firm 1 files for a patent, the expected payoffs of the two firms are

$$\pi^1(q^1, q^2 | F) = q^1 [q^2(1-\gamma \theta) \pi_{yy} + (1-q^2(1-\gamma \theta)) \pi_{yn}]$$

(1)

$$+ (1-q^1) [q^2(1-\gamma \theta) \pi_{ny} + (1-q^2(1-\gamma \theta)) \pi_{ny}] - C(q^1),$$

and

$$\pi^2(q^1, q^2 | F) = q^1 [q^2(1-\gamma \theta) \pi_{yy} + (1-q^2(1-\gamma \theta)) \pi_{yy}]$$

(2)

$$+ (1-q^1) [q^2(1-\gamma \theta) \pi_{yn} + (1-q^2(1-\gamma \theta)) \pi_{yn}] - \beta \gamma C(q^2).$$

The first line on the right side of (1) describes firm 1’s payoff when it succeeds to develop the new technology. With probability $q^2(1-\gamma \theta)$, firm 2 also develops the new technology and can bring it to the product market, so firm 1’s payoff is $\pi_{yy}$, and with probability $1-q^2(1-\gamma \theta)$, firm 2 either fails to develop the new technology or is prevented from using it in the product market, so firm 1’s payoff is $\pi_{yn}$. The second line on the right side of (1) describes the corresponding expressions when firm 1 fails to develop the new technology. The interpretation of equation (2) is similar, except that firm 2’s cost of R&D is higher since unlike firm 1, it does not have access to the basic technology right from the start of the second phase of the R&D process.

Absent filing, firm 1 cannot prevent firm 2 from using the new technology if firm 2 develops it.\footnote{We assume however that firm 2 cannot patent its invention of the basic technology given that firm 1’s invention already serves as prior art.}
Hence, firm 1’s expected payoff under both systems is

\[ \pi^1(q^1, q^2 | \text{NF}) = q^1 \left[ q^2 \pi_{xx} + (1 - q^2) \pi_{yn} \right] + (1 - q^1) \left[ q^2 \pi_{yx} + (1 - q^2) \pi_{ny} \right] - C(q^1). \] (3)

Note that this expression coincides with \( \pi^1(q^1, q^2 | \text{F}) \) if either \( \gamma = 0 \) or \( \theta = 0 \) because then firm 1 gets no protection when it files for a patent. Similarly, the expected payoff of firm 2 is

\[ \pi^2(q^1, q^2 | \text{NF}) = q^1 \left[ q^2 \pi_{yy} + (1 - q^2) \pi_{ny} \right] + (1 - q^1) \left[ q^2 \pi_{yy} + (1 - q^2) \pi_{nn} \right] - \beta_H C(q^2). \] (4)

This expression differs from \( \pi^2(q^1, q^2 | \text{F}) \) in two ways: first, the probability that firm 2 uses the new technology in the product market is now \( q^2 \) instead of \( q^2(1-\gamma \theta) \). Second, absent filing, firm 2 has no access to technical information about firm 1’s basic technology, so its cost of investment in the second phase of the R&D process is \( \beta_n C(q^2) \) instead of \( \beta_L C(q^2) \).

**Lemma 1:** (The equilibrium in the PG-Pub system.) Assumptions A1-A3 ensure the existence of a unique Nash equilibrium in filing subgame in which \( q^1_F, q^2_F \in [0,1] \), and a unique Nash equilibrium in the no-filing subgame in which \( q^1_{\text{NF}}, q^2_{\text{NF}} \in [0,1] \). The equilibrium investment levels are such that \( q^2_{\text{NF}} < q^2_F < q^1_F < q^1_{\text{NF}} \) when \( \gamma \theta = 0 \) and \( q^2_F < q^2_{\text{NF}} < q^1_{\text{NF}} < q^1_F \) when \( \gamma \theta \geq 1 - \beta_L / \beta_H \).

Figure 2a shows the equilibrium points in the filing subgame, \( F_0 \), and the no-filing subgame, \( \text{NF} \), when \( \gamma \theta = 0 \). Then firm 1 gets no protection if it files for a patent, so \( R^1(q^2 | \text{F}) = R^1(q^2 | \text{NF}) \). Since \( \beta_H > \beta_L \), the marginal cost of \( q^2 \) is higher in the no-filing subgame, so \( R^2(q^1 | \text{F}) > R^2(q^1 | \text{NF}) \). As \( \gamma \theta \) increases, firm 1 gets more patent protection, and as Figure 2b shows, \( R^1(q^2 | \text{F}) \) shifts to the right whereas \( R^2(q^1 | \text{F}) \) shifts down. As a result, the equilibrium point moves southeast from \( F_0 \) to \( F \). Figure 2c shows that when \( \gamma \theta \geq 1 - \beta_L / \beta_H \), \( R^2(q^1 | \text{F}) \) drops below \( R^2(q^1 | \text{NF}) \), so \( F \) is attained southeast of \( \text{NF} \). Hence, \( q^1_F > q^1_{\text{NF}} \) and \( q^2_F < q^2_{\text{NF}} \). In all cases, the assumption that \( \beta_H > \beta_L \) > 1 ensures that \( F_0, \text{NF}, \) and \( F \) lie below the diagonal, so \( q^1 > q^2 \).

Now let \( \pi^1_F = \pi^1(q^1_F, q^2_F | \text{F}) \) and \( \pi^1_{\text{NF}} = \pi^1(q^1_{\text{NF}}, q^2_{\text{NF}} | \text{NF}) \) be the Nash equilibrium payoffs of firm 1 in the filing and in the no-filing subgames, and define \( \pi^2_F \) and \( \pi^2_{\text{NF}} \) similarly. Then,

**Proposition 1:** (Firm 1’s filing decision under the PG-Pub system.) There exists a critical value of \( \gamma \theta \), denoted \( \overline{\gamma} \theta \), where \( \overline{\gamma} \theta \in (0, 1 - \beta_L / \beta_H) \), such that \( \pi^1_F > \pi^1_{\text{NF}} \) as \( \gamma \theta \leq \overline{\gamma} \theta \).

Proposition 1 implies that firm 1 files for a patent under the PG-Pub system if and only if the
Figure 2a: The best-response functions under the PG-Pub system when $\gamma \theta = 0$

Figure 2b: The best-response functions under the PG-Pub system when $0 < \gamma \theta < 1 - \beta_H / \beta_L$

Figure 2c: The best-response functions under the PG-Pub system when $\gamma \theta > 1 - \beta_H / \beta_L$
effective protection of patents, $\gamma \theta$, exceeds a threshold level, $\overline{T}$). Intuitively, firm 1 does not file for a patent when $\gamma \theta$ is small because this reveals technological information to firm 2, while offering firm 1 only little protection against imitation. As $\gamma \theta$ increases, patents receive stronger protection so filing becomes more attractive to firm 1. When $\gamma \theta > \overline{T}$, firm 1’s benefit from raising its chance to block firm 2 from using the new technology exceeds the corresponding loss from revealing technological information to firm 2, so firm 1 files for a patent.

Proposition 1 also shows that the threshold $\overline{T}$ is bounded from above by $1-\beta_L/\beta_H$, where $\beta_L/\beta_H$ is the ratio of imitation to invention costs. This implies that we should expect more patent applications when (i) the cost of imitating firm 1’s patent is high (i.e., $\beta_L$ is high), and (ii) the cost of developing the new technology from scratch is low (i.e., $\beta_H$ is low). Implication (i) follows because the higher is $\beta_L$, the smaller is firm 1’s cost of filing for a patent. The intuition for implication (ii) is that as $\beta_H$ falls, firm 2 invests more in the no filing subgame and hence is more likely to develop the new technology. Hence, firm 1 has a stronger incentive to file for a patent in an attempt to block firm 2’s use of the new technology if firm 2 successfully develops it.

4. The Confidential Filing (CF) system

Absent filing, the expected payoffs of firms 1 and 2 do not depend on the filing system and hence are still given by equations (3) and (4). Consequently, the Nash equilibrium in the no-filing subgame continues to be $(q_{1NF}, q_{2NF})$ as in the case of the PG-Pub system. As for the filing subgame, firm 1’s expected payoff is also the same across the two filing systems, and hence is still given by equation (1).

Firm 2’s expected payoff in the filing subgame is given by

$$\pi^2(q_1, q_2 | F) = q_1 \left[ q_2^2 (1-\gamma \theta) \pi_{yy} + (1-q_2^2 (1-\gamma \theta)) \pi_{yn} \right]$$

$$+ (1-q_1^2) \left[ q_1^2 (1-\gamma \theta) \pi_{yn} + (1-q_1^2 (1-\gamma \theta)) \pi_{nn} \right] - \beta_g C(q_2^2),$$

(5)

where $\beta_g = \theta \beta_m + (1-\theta) \beta_H$. This expression differs from firm 2’s corresponding payoff under the PG-Pub system (equation (2)) because here firm 2 learns about firm 1’s invention only when a patent is granted, and this occurs with probability $\theta$. Since patents are typically issued more than 18 months after the filing date, we assume that firm 2’s cost under the CF system, conditional on firm 1 getting a patent, is $\beta_m C(q_2^2)$, where $\beta_L < \beta_m < \beta_H$. With probability $1-\theta$, firm 1’s patent application is rejected and firm 2 does not learn about firm 1’s invention, in which case its cost is $\beta_H C(q_2^2)$.

Equation (5) reveals that under the CF system, the likelihood of getting a patent affects the filing
subgame not only through the effective protection parameter, $\gamma \theta$, but also through firm 2’s cost function. Hence, unlike the PG-Pub system, now $\gamma$ and $\theta$ have potentially different impacts on the equilibrium.

**Lemma 2:** *(The equilibrium in the CF system.)* Assumptions A1-A3 ensure the existence of a unique Nash equilibrium in filing subgame in which $q^*_1, q^*_2 \in [0,1]$. The Nash equilibrium in the no-filing subgame, $(q^*_1^{NF}, q^*_2^{NF})$, coincides with that under the PG-Pub system. The equilibrium investment levels are such that $q^*_2^{NF} = q^*_F < q^*_2 = q^*_1$ if $\theta = 0$, $q^*_2 < q^*_2^{NF} < q^*_2^{NF} < q^*_1^{NF}$ if $\theta > 0$ and $\gamma \geq 1-\beta_L/\beta_H$, and either $q^*_2^{NF} < q^*_F$, or $q^*_1^{NF} < q^*_F$, or both, if $\theta > 0$ and $\gamma < 1-\beta_L/\beta_H$. Moreover, $q^*_1 + q^*_2 > q^*_1^{NF} + q^*_2^{NF}$ for all $\gamma < 1-\beta_L/\beta_H$.

Figure 3a shows that in the extreme case where $\theta = 0$ (no patents are granted), the equilibrium points in the filing subgame, $F$, and in the no-filing subgame, $NF$, coincide. As $\theta$ increases, patents are more likely to be granted, so $R(q^*_1 | F)$ shifts to the right. At the same time, firm 2 has a lower probability of using the new technology in the product market, so the marginal benefit from $q^2$ falls. Since firm 2 is also more likely to learn about firm 1’s invention, the marginal cost of $q^2$ falls as well. Whether $R^2(q^1 | F)$ shifts up or down, depends on the value of $\gamma$. When $\gamma \geq 1-\beta_L/\beta_H$, patents are likely to be upheld in court, so the marginal benefit from $q^2$ falls more than the marginal cost of $q^2$. Consequently, as Figure 3b shows, $R^2(q^1 | F)$ shifts down, so the equilibrium point in the filing subgame, $F$, lies southeast of NF. When $\gamma < 1-\beta_L/\beta_H$, a small increase in $\theta$ lowers the marginal cost of $q^2$ by more than it lowers the marginal benefit from $q^2$, so $R^2(q^1 | F)$ shifts up. Figure 3c shows that now, $F$, can lie either northwest, northeast, or southeast of NF, so the comparison between the two subgames is in general ambiguous. Nonetheless, since $F$ lies above a 45 degree line passing through NF, the aggregate level of investment is larger in the filing subgame.

Let $\pi^1_F = \pi^1(q^*_1, q^*_2 | F)$ and $\pi^2_F = \pi^2(q^*_1, q^*_2 | F)$ be the equilibrium payoffs of firms 1 and 2 in the filing subgame, and recall that the equilibrium payoffs in the no-filing subgame are $\pi^1_{NF}$ and $\pi^2_{NF}$, as in Section 3. Now,

**Proposition 2:** *(Firm 1’s filing decision under the CF system.)* For each $\theta > 0$, there exists a critical value of $\gamma$, denoted $\gamma^*$, where $\gamma \in (0, 1-\beta_L/\beta_H)$, such that $\pi^1_F \geq \pi^1_{NF}$ as $\gamma \leq \gamma^*$. Furthermore, $\pi^2_{NF} > \pi^2_F$ for all $\gamma > 1-\beta_L/\beta_H$.

Proposition 2 implies that given the likelihood of getting a patent, $\theta$, firm 1 files for a patent under
Figure 3a: The best-response functions under the CF system when $\theta = 0$.

Figure 3b: The best-response functions under the CF system when $\gamma > 1 - \beta_M/\beta_H$.

Figure 3c: The best-response functions under the CF system when $\gamma < 1 - \beta_M/\beta_H$. 
the CF system if and only if the likelihood that the patent will be upheld in court exceeds a threshold level, \( \bar{\gamma} \). This threshold is bounded from above by \( 1 - \beta_M / \beta_H \).^{13}

5. The implications of PG-Pub for R&D, patenting, and welfare

Having examined the two filing systems in isolation, we now compare them in order to determine the impact of PG-Pub on firm 1’s incentive to file for a patent on the basic technology, on firms 1 and 2’s investments in the second phase of the R&D process, and on consumer surplus and social welfare.

5.1 the effect of PG-Pub on patenting behavior and on the R&D investments

As a preliminary step, we begin by comparing the equilibrium investment levels and expected payoffs under the two filing systems assuming that firm 1 files for a patent. We do not need to have a similar comparison for the case where firm 1 does not file for a patent since then PG-Pub is irrelevant.

Lemma 3: (Comparing the equilibrium investment levels and expected payoffs in the filing subgame under the two filing systems.) Suppose that firm 1 files for a patent under both systems. Then,

(i) \( q^2_F < q^2_F < q^1_F < q^1_F \) and \( q^1_F + q^2_F > \bar{q}^1_F + \bar{q}^2_F \)

(ii) \( \pi^1_F < \pi^1_F \) and \( \pi^2_F > \pi^2_F \)

Lemma 3 is only a preliminary step towards examining the effect of PG-Pub on the R&D investments of the two firms in the second phase of the R&D process and their expected payoffs since firm 1 need not have the same incentives to file for a patent under the two systems. To see the intuition behind Lemma 3 it is useful to look at Figure 4 which illustrates the best response functions of the two firms. The expected marginal cost of firm 2 is higher under the CF system since then firm 2 has access to information on firm 1’s invention only when firm 1 gets a patent (even then the information is typically revealed more than 18 months from the filing date). Consequently, \( \bar{R}^2(q^1 | F) \) lies below \( R^2(q^1 | F) \). Since firm 1’s best-response function is the same under the two filing systems, the equilibrium point under the

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13 In a previous version of the paper we showed that the threshold, \( \bar{\gamma} \), above which firm 1 files for a patent under the CF system may either increase or decrease with \( \theta \), depending on the sensitivity of \( \bar{q}^2_F \) with respect to \( \theta \). Intuitively, an increase in \( \theta \) affects firm 1 directly by raising its chances to get a patent and indirectly by affecting \( \bar{q}^2_F \). When \( \bar{q}^2_F \) decreases, the indirect effect reinforces the direct effect so firm 1 has a stronger incentive to file. When \( \bar{q}^2_F \) increases, the indirect effect is negative (firm 2 is more likely to succeed in the second phase of the R&D process). Hence, the negative indirect effect may dominate if \( \bar{q}^2_F \) is sufficiently elastic with respect to \( \theta \).
PG-Pub system, $F$, is attained northwest of the equilibrium point under the a CF system, $\bar{F}$. Given Assumption A3, $\bar{F}$ lies under a 45 degrees line passing through $F$, so the aggregate level of investment is larger under the PG-Pub system. Part (ii) of Lemma 3 shows that in the filing subgame, PG-Pub benefits firm 2 and hurts firm 1.

Given Lemma 3 it is now possible to compare the incentives of firm 1 to file for a patent under the two systems. To this end, note that the result that $\pi^1_F > \pi^1_{NF}$, together with the fact that $\pi^1_F > \pi^1_{NF}$ if $\gamma > \frac{\bar{\gamma}}{\theta}$ (Proposition 1) and $\pi^1_F > \pi^1_{NF}$ if $\gamma > \bar{\gamma}$ (Proposition 2), implies that $\bar{\gamma}$ lies strictly below $\frac{\bar{\gamma}}{\theta}$ in the $(\gamma, \theta)$ space. Hence,

**Proposition 3:** (Firm 1’s filing decision under the PG-Pub and CF filing systems.) Firm 1 does not file for a patent under both filing systems if $\gamma < \bar{\gamma}$, files for a patent under both systems if $\gamma > \frac{\bar{\gamma}}{\theta}$, and files for a patent only under the PG-Pub system if $\bar{\gamma} \leq \gamma \leq \frac{\bar{\gamma}}{\theta}$.

Using Proposition 3, we can now distinguish between 3 possible cases that differ with respect to the strength of patent protection (see Figure 5). When $\gamma < \bar{\gamma}$, patents receive weak protection since they are relatively hard to defend in court. Consequently, firm 1 does not file for a patent under neither filing system so PG-Pub is irrelevant. Examples for industries where this might be the case include some mature industries like textile, food processing, and fabricated metal products (Arundel and Kabala 1998, Levin et. al., 1987). When $\gamma > \frac{\bar{\gamma}}{\theta}$, patents receive relatively strong protection since they are likely to be upheld in court; hence, firm 1 files for a patent under both filing systems. Yet, PG-Pub is not irrelevant because it affects the investment levels of the two firms. Examples for industries where patents are regarded as providing strong protection include pharmaceuticals, organic chemicals, and pesticides (Arundel and Kabala 1998, Levin et. al., 1987, Mansfield, 1986). Finally, when $\bar{\gamma} \leq \gamma \leq \frac{\bar{\gamma}}{\theta}$, patent protection is intermediate and firm 1 files for a patent only under the CF system. Industries where patents provide an intermediate protection (relative to other forms of protection such as, secrecy, securing a lead-time advantage over rivals, learning curve advantages, and investment in sales or service efforts), include chemical products, relatively uncomplicated mechanical equipment, electrical equipment, and Petroleum (Levin et al., 1987, Mansfield, 1986).

The analysis so far reveals that PG-Pub has at least three important implications which are corollaries of Propositions 1-3 and Lemmas 2-3.

**Corollary 1:** A switch from a CF system to a PG-Pub system induces firm 1 to file for a patent for a
Figure 4: The best-response functions in the filing subgame under the PG-Pub and the CF systems.

Figure 5: Firm 1’s equilibrium profits under the two patent systems. Firm 1 does not file for a patent under both systems when patent protection is weak, files under both systems when patent protection is strong, and files only under the CF system when patent protection is intermediate.
smaller set of parameters.

Corollary 1 follows immediately from Proposition 3. It suggests that PG-Pub may discourage the dissemination of technological information, contrary to what many proponents of this system argue.\textsuperscript{14} The reason of course is that proponents of the PG-Pub overlooked the fact that PG-Pub will have an adverse effect on the incentives of firms to file for patents. The adverse effect of PG-Pub on the incentives of firm 1 to patent its basic technology confirms Gilbert’s (1994) intuition that "There is at least a theoretical potential for the publication of applications prior to the patent grants to have adverse incentive effects because of the potential for appropriation of the intellectual property when no patents are ever issued. To avoid appropriation of intellectual property, some investors who otherwise would apply for patents might rely instead on trade secrets protection." However, Proposition 3 qualifies this argument by suggesting that this adverse effect of PG-Pub pertains only to industries in which patent protection is intermediate.

**Corollary 2:** When patent protection is strong, PG-Pub leads to a decrease in $q^1$, and an increase in $q^2$ and in the aggregate level of investment. When patent protection is intermediate, PG-Pub leads to a decrease in $q^1$, an increase in $q^2$, and has an ambiguous effect on the aggregate level of investment if $\gamma > 1 - \beta_M / \beta_H$. If $\gamma < 1 - \beta_M / \beta_H$, PG-Pub has an ambiguous effect on $q^1$ and $q^2$ but lowers the aggregate level of investment unambiguously.

Tepperman (2002) studies the effect of Canada’s 1989 Patent Act reform that led to a switch from a confidential filing system with a first-to-invent priority rule to a PG-Pub system with a first-to-file priority rule on the behavior of 84 Canadian firms from various industries. He finds that on average, firms have increased their R&D spendings following the reform. This finding is consistent with Corollary 2 when patent protection is strong but not when it is intermediate. Tepperman also finds that following the

\textsuperscript{14} For example, in a Congress hearing in February 1997, Rep. Howard Coble (then the chairman of the subcommittee on Courts and Intellectual Property) stated that PG-Pub "... will benefit American inventors, innovators, and society at large ... by furthering the constitutional incentive to disseminate information regarding new technologies more rapidly ..." Similarly, Rep. Sue W. Kelly, argued that "It’s also an imperative that we have an 18-month publication of patent applications for all inventors ... How can we say that our businesses do not need to know about technology until actually a patent issues? We cannot in good conscious make such judgments because we neither know which technological inventions may be industry-critical, nor from whom or from what source such inventions will arise." Both statements appear in http://commdocs.house.gov/committees/judiciary/hju40523.000/hju40523_0f.htm
reform, firms have increased their patenting intensity. This finding is inconsistent with Corollary 1. It should be noted though that the inconsistency between Corollaries 1 and 2 and Tepperman’s findings may be explained by the fact that firms have stronger incentives to invest in R&D and to patent their inventions under the first-to-file rule than under the first-to-patent rule (Green and Scotchmer, 1990).

**Corollary 3:** *PG-Pub hurts firm 1 and benefits firm 2 when patent protection is strong and when it is intermediate and* \[
\gamma > 1 - \frac{\beta_f}{\beta_H}.
\]

The result that PG-Pub hurts firm 1 follows by revealed preferences and the fact that firm 1 chooses to file for a patent under the CF system even though it could choose to keep the basic technology secret. Putnam (1997) estimates that PG-Pub is associated with a $479 decrease in the mean value of patents. In our model, firm 1’s loss is even larger since Putnam’s estimate is conditional on a patent being granted, while we examine the impact of PG-Pub on the unconditional expected profit of firm 1. The effect of PG-Pub on the payoff of firm 2 follows from part (ii) of Lemma 3 when patent protection is strong, and from Proposition 2 when patent protection is intermediate and \[
\gamma > 1 - \frac{\beta_f}{\beta_H}.
\]

In the context of our model it is natural to assume that small inventors will mainly play the role of firm 1 while large corporations may play the roles of both firms 1 and 2. This is because large corporations who are active in product development, are likely to have the capacity and resources needed to absorb the informational spillovers generated by PG-Pub, whereas small inventors typically do not have such resources and are mainly busy developing a small number of original inventions. With this interpretation in mind, Corollary 3 suggests that PG-Pub is likely to hurt small inventors but may benefit large corporations. This can explain perhaps why the main opposition for adopting a PG-Pub system in the U.S. came from small and independent inventors while the main support for PG-Pub came from large corporations.

We conclude this section by noting that since firm 1’s invention lowers the cost of R&D from \[
\beta_h C(q) \text{ to } C(q),
\]

it is natural to associate higher values of \(\beta_H\) with inventions of higher quality. That is, the higher is the quality of firm 1’s invention, the larger is the reduction in the cost of developing the new technology. Given this interpretation, we now ask how PG-Pub affects the average quality of inventions for which firm 1 files for a patent. Since our model is too general to provide a conclusive answer to this question, we shall make the following assumption:

\[
A4 \quad C(q) = rq^2/2, \text{ where } r > \pi_{yy} - \pi_{nn}.
\]
The restriction on r ensures that Assumption A3 is satisfied.

**Proposition 4:** (The effect of PG-Pub on the quality of inventions for which firm 1 files for a patent.)
Given Assumption A4, firm 1 files for a patent under the PG-Pub system if and only if \( \beta_H \leq \beta_L/(1-\gamma\theta)^2 \), whereas under the CF system, it files for a patent if and only if \( \beta_H \leq \beta_M/(1-2\gamma+\theta\gamma^2) \). Since \( \beta_L/(1-\gamma\theta)^2 < \beta_M/(1-2\gamma+\theta\gamma^2) \), the average quality of inventions for which firm 1 files for a patent is higher under the CF system than under the PG-Pub system.

Roughly speaking, holding \( \beta_L \) and \( \beta_M \) fixed, an increase in \( \beta_H \) (and hence in the quality of firm 1’s invention) raises the cost of patenting since it means that more information is revealed to firm 2 (firm 2’s cost drops from \( \beta_H C(q) \) to \( \beta_L C(q) \) under the PG-Pub system and to \( \beta_C(q) \) under the CF system). Therefore, firm 1 files for a patent if and only if the quality of its invention is not too high. But, given Assumption A4, the cost of patenting is higher under the PG-Pub system, so on average, PG-Pub leads to a reduction in the quality of patented inventions relative to the CF system.

### 5.2 The implications of PG-Pub for consumer surplus and social welfare

Let \( S_{yy} \) be the net present value of consumers’ surplus when both firms develop the new technology, and define \( S_{yn} \) and \( S_{nn} \) similarly for the cases where only one firm, and when neither firm develop it. Social welfare is given by the sum of consumers’ surplus and firms’ profits, so \( W_{yy} = S_{yy} + 2\pi_{yy}, W_{yn} = S_{yn} + \pi_{yn} + \pi_{ny}, \) and \( W_{nn} = S_{nn} + 2\pi_{nn}. \)

Since the comparison between consumers’ surplus and social welfare under the two systems is in general very complex, we shall impose Assumption A4. Recalling from Proposition 1 that \( \gamma\theta \) is implicitly defined by \( \pi_1 = \pi_{NF} \), and recalling from Proposition 2 that \( \gamma \) is implicitly defined by \( \pi_1 = \pi_{NF} \), it is straightforward to establish that given Assumption A4, \( \gamma\theta = 1 - \sqrt{\beta_L/\beta_M} \) and \( \gamma = (1 - \sqrt{\beta_C/\beta_H})/\theta. \)

Therefore, patent protection is strong if \( \gamma > (1 - \sqrt{\beta_L/\beta_M})/\theta, \) intermediate if \( (1 - \sqrt{\beta_C/\beta_H})/\theta \leq \gamma \leq (1 - \sqrt{\beta_L/\beta_M})/\theta, \) and weak if \( \gamma < (1 - \sqrt{\beta_C/\beta_H})/\theta. \) In addition to Assumption A4, we make the following assumptions:

A5 \[ S_{yy} \geq S_{yn} \geq S_{nn}, S_{yy} + S_{nn} \geq 2S_{yn}, \text{ and } S_{yn} - S_{nn} > \pi_{nn} - \pi_{ny} \]

A6 \[ W_{yy} \geq W_{yn} \geq W_{nn} \]
Assumption A5 implies that the net present value of consumers’ surplus is increasing with the number of firms that bring the new technology to the product market at an increasing rate. It also implies that if only one firm brings the new technology to the product market, the resulting benefit to consumers outweighs the loss to the firm that does not bring it to the product market. Assumption A6 implies that social welfare is increasing with the number of firms that bring the new technology to the product market. Both assumptions hold in a broad class of oligopoly models; for instance, when the new technology is cost-reducing, Assumptions A5 and A6 hold in the Cournot model with homogeneous products and a linear demand and in the Bertrand model with linear cost functions.

Expected Consumers’ surplus: Holding the number of inventions fixed across the two filing systems, the expected consumers’ surplus under both filing systems when firm 1 files for a patent is,

\[ S(q^1, q^2 | F) = q^1 q^2 (1 - \gamma \theta) S_{yy} + (1 - q^1)(1 - q^2(1 - \gamma \theta)) S_{nn} \]

\[ + [q^1 (1 - q^2(1 - \gamma \theta)) + (1 - q^1)q^2(1 - \gamma \theta)] S_{yn}. \]

Likewise, the expected consumers’ surplus under both systems absent filing is given by,

\[ S(q^1, q^2 | NF) = q^1 q^2 S_{yy} + (1 - q^1)(1 - q^2) S_{nn} + [q^1 (1 - q^2) + (1 - q^1)q^2] S_{yn}. \]

Let \( S_F \equiv S(q^1, q^2 | F) \) be the equilibrium expected value of consumers’ surplus under the PG-Pub system when there is filing, and define \( S_p \equiv S(q^1, q^2 | F) \) similarly for the CF system. When firm 1 does not file for a patent, PG-Pub plays no role, so the equilibrium expected value of consumers’ surplus, denoted \( S_{NF} \equiv S(q^1, q^2 | NF) \), is the same under both filing systems.

When patent protection is strong, firm 1 files for a patent under both systems. Hence, we need to compare \( S_F \) and \( S_p \). Given Assumption A4, the equilibrium levels of investment under the CF system are

\[ \frac{1}{\bar{q}_F^1} = \frac{(\pi_{yn} - \pi_{nn})(r \beta_\theta + (1 - \gamma \theta) \Pi)}{r^2 \beta_\theta - (1 - \gamma \theta)^2 \Pi^2}, \quad \frac{1}{\bar{q}_F^2} = \frac{(\pi_{yn} - \pi_{nn})(1 - \gamma \theta)(r + \Pi)}{r^2 \beta_\theta - (1 - \gamma \theta)^2 \Pi^2}, \]

where \( \Pi \equiv \pi_{yy} + \pi_{nn} - \pi_{yn} \pi_{ny} < 0 \) by Assumption A2. Assumptions A1 and A4 ensure that \( r > -\Pi \). Together with the assumption that \( \beta_{ii} > \beta_{ij} > 1 \geq 1 - \gamma \theta \), this implies in turn that \( \bar{q}_F^1 \) and \( \bar{q}_F^2 \) are strictly between 0 and 1. Under the PG-Pub system, the investment levels are also given by equation (8), except that now \( \beta_{L} \) replaces \( \beta_\theta \). Substituting for \( \bar{q}_F^1 \) and \( \bar{q}_F^2 \) into (6),
where $S \equiv S_{yy} + S_{nn} - 2S_{yn} > 0$ by Assumption A5. The expression for $S_F$ is identical to $\tilde{S}_F$, except that $\beta_L$ replaces $\beta_\theta$.

In the intermediate protection case, firm 1 files for a patent under the CF system but not under the PG-Pub system. Therefore, we need to compare $\tilde{S}_F$ and $S_{NF}$, where $S_{NF}$ is given by the right side of (9) evaluated at $\theta = 0$ ($\theta = 0$ means that no information is revealed to firm 2, exactly as if firm 1 did not file for a patent).

**Proposition 5:** (The effect of PG-Pub on consumers.) Suppose that Assumptions A4 and A5 hold and patent protection is intermediate or strong, i.e., $\gamma \geq \gamma^*$ (otherwise PG-Pub is irrelevant). Then PG-Pub enhances consumers’ surplus. Moreover, when patent protection is intermediate, the increase in consumers’ surplus due to PG-Pub is larger when $\gamma$ is larger.

Intuitively, in the strong protection case, firm 1 files for a patent under both filing systems; but as Lemma 3 shows, firm 1 invests less and firm 2 invests more under the PG-Pub system. Given Assumption A4, the latter effect dominates, so the new technology is more likely to reach the product market and this makes consumers better-off. Under intermediate protection, firm 1 files for a patent under the CF system but not under the PG-Pub system. To examine how this affects consumers, note that as $\gamma$ increases, patents are more likely to be upheld in court, so firm 1 is more likely to block firm 2 from bringing the new technology to the product market; hence, consumers’ surplus under the CF system, $\tilde{S}_F$, decreases with $\gamma$. On the other hand, under the PG-Pub system, firm 1 does not file for a patent so consumers’ surplus, $S_{NF}$, is independent of $\gamma$. Since $S_{NF} = \tilde{S}_F$ when $\gamma = (1 - \sqrt{\beta_\theta/\beta_H})/\theta$, consumers’ surplus is higher under a PG-Pub system and moreover, the gain of consumers from PG-Pub is larger, the larger is $\gamma$.

**Expected social welfare:** Holding the number of inventions fixed across the two filing systems, the expected social welfare when firm 1 files for a patent is $W_F = S_F + \pi_F + \pi^2_F$ under the PG-Pub system,
and \( \bar{W}_F = \bar{S}_F + \bar{\pi}_1 + \bar{\pi}_2 \) under the CF system. When firm 1 does not file for a patent, the expected social welfare is \( W_{NF} = S_{NF} + \pi_{1, NF} + \pi_{2, NF} \). When patents receive a strong protection, firm 1 files for a patent under both systems. Hence, the equilibrium expected social welfare is \( \bar{W}_F \) under the CF system and \( W_F \) under the PG-Pub system. Given Assumption A4 and using equations (1), (2), (8), and (9),

\[
\bar{W}_F = W_{nn} + \frac{(\pi_{yy} - \pi_{nn})^2 (1 - \gamma \theta)^2 (r + \Pi) \{ r \beta_\theta + (1 - \gamma \theta)^2 \Pi \} S}{(r^2 \beta_\theta - (1 - \gamma \theta)^2 \Pi^2)^2} \\
+ \frac{(\pi_{yy} - \pi_{nn}) \{ r \beta_\theta + (1 - \gamma \theta)^2 \Pi \} \{ S_{yy} - S_{nn} + \pi_{ny} - \pi_{nn} \}}{r^2 \beta_\theta - (1 - \gamma \theta)^2 \Pi^2} \\
+ \frac{(\pi_{yy} - \pi_{nn})^2 r \{ (r \beta_\theta + (1 - \gamma \theta)^2 \Pi)^2 - \beta_\theta (1 - \gamma \theta)^2 (r + \Pi)^2 \}}{2(r^2 \beta_\theta - (1 - \gamma \theta)^2 \Pi^2)^2}.
\]

The expression for \( W_F \) is identical to \( \bar{W}_F \), except that \( \beta_L \) replaces \( \beta_\theta \).

In the intermediate protection case, firm 1 files for a patent only under the CF system. Hence, the equilibrium expected social welfare is \( \bar{W}_F \) under the CF system and \( W_{NF} \) under the PG-Pub system, where \( W_{NF} \) is equal to \( \bar{W}_F \) when it is evaluated at \( \theta = 0 \), since under the CF system, the situation when \( \theta = 0 \) is the same as if firm 1 did not file for a patent.

Using the fact that \( \bar{W}_F \) and \( W_F \) differ only with respect to \( \beta \), and \( \bar{W}_F \) and \( W_{NF} \) differ only with respect to \( \theta \), we prove the following result:

**Proposition 6:** (The welfare implications of PG-Pub.) Suppose that Assumptions A4-A6 hold and let

\[
\tilde{r}(\beta) = -\Pi \{ Y + \sqrt{\beta} Y + \beta - (1 - \gamma \theta)^2 \} \left( \sqrt{\beta} + (1 - \gamma \theta) \right)^{\frac{2}{3}} Y, \quad Y = \left( \sqrt{\beta} - (1 - \gamma \theta) \right)^{\frac{2}{3}} \left( \sqrt{\beta} + (1 - \gamma \theta) \right)^{\frac{1}{3}}.
\]

Then,

(i) a sufficient condition for PG-Pub to enhance expected welfare when patent protection is strong is \( r \geq \tilde{r}(\beta_\theta) \);

(ii) a sufficient condition for PG-Pub to enhance (lower) expected welfare when patent protection is intermediate is \( r \geq \tilde{r}(\beta_\theta) \) and \( \gamma > (\prec) (\beta_H - \beta_M)/(\beta_H + \beta_M) \); moreover, if \( r \geq \tilde{r}(\beta_\theta) \) and \( \gamma > (\prec) (\beta_H - \beta_M)/(\beta_H + \beta_M) \), the welfare gain (loss) from to PG-Pub is larger (smaller) the larger is \( \gamma \).

Proposition 6 reveals that when \( r \), which measures the slope of the marginal cost of investment in the second phase of the R&D process is sufficiently large, PG-Pub is socially desirable if patent
protection is strong, but depending on the value of $\gamma$, it may or may not be socially desirable when patent protection is intermediate. Intuitively, Lemma 3 shows that when patent protection is strong, the gap between $q^1$ and $q^2$ is smaller under the PG-Pub system. Since the cost functions are convex, this implies that all else equal, the allocation of investments is more efficient under the PG-Pub system, and the resulting efficiency gain increases with $r$. Consequently, when patent protection is strong, PG-Pub is surely welfare enhancing when $r$ is sufficiently large. This result is reinforced by the fact that as $r$ increases, the aggregate levels of investment under the two filing systems converge, so the two systems differ mainly with respect to the allocation of investments between the two firms.

When patent protection is intermediate, things are more complex because the sufficient condition also depends on the likelihood that patents will be upheld in court, $\gamma$. The reason that $\gamma$ matters now is that Lemma 2 shows that when $\gamma > 1 - \beta_M / \beta_H$, the allocation of investments between the two firms is more even under the PG-Pub system, whereas when $\gamma < 1 - \beta_M / \beta_H$, the opposite holds. Given the convexity of the cost functions, the allocation of investments is more efficient under the PG-Pub system if $\gamma$ is large and more efficient under the CF system if $\gamma$ is small.

To get a better sense for the welfare implications of PG-Pub, we consider the following example.

**A Cournot example with a cost-reducing technology:** Suppose that the two firms are Cournot-competitors and face an inverse demand function $P = 6 - x_1 - x_2$, where $x_i$ is the output of firm $i$, $i = 1, 2$. In addition, assume that firm $i$’s marginal cost of production is 0 if it develops the new technology and 3 otherwise. Given these assumptions, $\pi_{yn} = 9$, $\pi_{yy} = 4$, $\pi_{mn} = 1$, $\pi_{ny} = 0$, $S_{yy} = 8$, $S_{yn} = 4.5$, and $S_{nn} = 2$; these values are consistent with Assumptions A1, A2, and A5. To ensure that $r > \pi_{yn} / \pi_{nn}$ as Assumption A4 requires, let $r > 8$. The example allows us to derive the precise conditions under which PG-Pub enhances or lowers social welfare (Proposition 6 only reports sufficient conditions).

In the strong protection case, PG-Pub is welfare-enhancing when $W_F - \bar{W}_F > 0$. In Figure 6, we set $\beta_L = \beta_M = 2$ and $\beta_H = 3$ (i.e., $\beta_L / \beta_H = 0.66$, similarly to the ratio of the cost of imitation to the cost of invention obtained by Mansfield, Schwartz, and Wagner (1981)) and present $W_F - \bar{W}_F$ as a function of $r$ for different combinations of $\gamma$ and $\theta$. The figure shows that PG-Pub is welfare-enhancing if and only if $r$ is sufficiently large. Moreover, the figure shows that when PG-Pub is socially desirable, it generates a larger welfare gain as $\theta$ is smaller ( patents are granted with a small probability) and as $\gamma$ is smaller ( patents are unlikely to be upheld in court). To see why, note from equation (12) that the difference between $\bar{q}_F - q^2_F$ and $q^1_F - q^2_F$ widens as $\gamma$ and $\theta$ decrease, so the efficiency gain from PG-Pub increases. Thus, PG-Pub is more likely to be socially desirable when the marginal cost of developing new products
Figure 6: The change in welfare due to PG-Pub in the strong protection case

Figure 6a: gamma = 0.7

Figure 6b: theta = 0.7
rises sufficiently fast, and the welfare gain (when there is one) is bigger when γ and θ are small.

When the protection of patents is intermediate, PG-Pub is welfare-enhancing if \( W_{NF} - W_{\bar{F}} > 0 \). In Figure 7, we set \( \beta_L = \beta_M = 2, \beta_H = 18, \) and \( \theta = 0.25 \) and present \( W_{NF} - W_{\bar{F}} \) as a function of \( r \) for five values of \( \gamma \) (we restrict \( \gamma \) to be between 0.118 and 0.667 since protection is intermediate and hence \( (1 - \sqrt{\beta_L/\beta_H})/\theta \leq \gamma \leq (1 - \sqrt{\beta_L/\beta_H})/\theta) \). Since \( \beta_L/\beta_H = 2/18 \), patents create a relatively large informational spillover. When \( \gamma \) is relatively large (\( \gamma = 0.5 \) and 0.6), PG-Pub is welfare enhancing if and only if \( r \) is sufficiently large (above 8.241 and 8.245, respectively), whereas when \( \gamma \) is relatively small (\( \gamma = 0.2, 0.3, \) and 0.4) the opposite is true (\( r \) is below 8.231, 8.234, and 8.238, respectively). Figure 7 also shows that when PG-Pub is socially desirable, it generates a larger welfare gain when \( \gamma \) is large, i.e., when patents are relatively likely to be upheld in court. As explained above, this is due to the effect of \( \gamma \) on the allocation of investments between the two firms, which in turn affects the efficiency of R&D.

**Foreign patent applications and domestic welfare:** At least in the U.S., many patent applications are made by foreign inventors whose payoffs should be ignored if we are only interested in domestic welfare. For instance, between 1993 and 1997, 42.2% of all patent applications in the U.S. were made by non-U.S. residents and 43.6% of all U.S. patents were issued to non-U.S. residents.\(^{15}\) To examine how the PG-Pub of patent applications filed by foreign inventors affects domestic welfare, suppose that firm 1 is a foreign firm. Then, domestic welfare in the strong protection case is \( S_F + \pi^2_F \) under the PG-Pub system and \( \bar{S}_F + \bar{\pi}^2_F \) under the CF system. Since Proposition 5 and Lemma 3, respectively, imply that \( S_F \geq \bar{S}_F \) and \( \pi^2_F \geq \bar{\pi}^2_F \), it is clear that PG-Pub enhances domestic welfare. In the intermediate protection case, domestic welfare is still \( \bar{S}_F + \bar{\pi}^2_F \) under the CF system, but under the PG-Pub system it becomes \( S_{NF} + \pi^2_{NF} \). Proposition 5 implies that now, \( S_{NF} \geq \bar{S}_F \), while Proposition 2 implies that \( \pi^2_{NF} > \pi^2_F \) whenever \( \gamma > 1 - \beta_L/\beta_H \). Hence, whenever \( \gamma \geq 1 - \beta_L/\beta_H \), the patent is sufficiently likely to be upheld in court, PG-Pub surely enhances domestic welfare. When \( \gamma < 1 - \beta_L/\beta_H \), the comparison between \( \pi^2_{NF} \) and \( \pi^2_F \) is ambiguous, so we cannot determine the impact on domestic welfare without imposing further structure on the model.

**Proposition 7:** *(The welfare implications of PG-Pub when firm 1 is a foreign firm.)* If firm 1 is a foreign

---

\(^{15}\) See Tables 2,6,9, and 10 in the 1997 U.S. Patent office annual report. In addition, in 1997, 17 organizations among the top 30 organizations receiving U.S. patents, and 7 out of the top 10, were foreign (see http://www.ipo.org/Top2001997.html).
Figure 7: The change in welfare due to PG-Pub in the intermediate protection case (theta = 0.25)
firm then PG-Pub always enhances domestic welfare when patent protection is strong. If patent protection is intermediate, a sufficient condition for PG-Pub to enhance domestic welfare is that \( \gamma \geq 1 - \beta_L / \beta_H \) (i.e., the patent is sufficiently likely to be upheld in court).

Comparing Propositions 6 and 7 reveals that PG-Pub is more likely to enhance domestic welfare if firm 1 is a foreign firm. This is because PG-Pub always hurts firm 1, so if we ignore firm 1’s payoff, we get a more positive picture of the welfare implications of PG-Pub. In addition, Proposition 7 shows that in the intermediate protection case, domestic welfare is more likely to increase when patents \( \gamma \) is relatively large. To understand why, note that as \( \gamma \) increases, firm 1 which is now a foreign firm, is more likely to file for a patent and block the domestic firm, firm 2, from using the new technology in the product market. But, in the intermediate protection case, PG-Pub induces firm 1 to stop filing for patents, so firm 2 is more likely to use the new technology in the product market if it develops it.

5.3 The timing of PG-Pub
In countries that have already adopted the PG-Pub system, patent applications are published at 18 months from the filing date (Ragusa 1992). We now examine the impact of the timing of publication on social welfare. To this end, we shall assume that as PG-Pub is done earlier, \( \beta_L \) becomes lower (i.e., the cost of imitation under the PG-Pub system falls). Then:

**Proposition 8:** (The effect of cutting the time between the filing date and the publication date.) Suppose that Assumptions A4-A6 hold. Then, as \( \beta_L \) falls (publication is made earlier), there are fewer patent applications under the PG-Pub system, but so long as \( r \geq \bar{r}(\beta_L) \), the welfare gain from PG-Pub when patent applications are made, grows.

Proposition 8 shows that cutting the time between the filing date and the publication date has mixed welfare effects: on the one hand, it increases the cost of patenting, so less technological information is disseminated. On the other hand, conditional on patents being filed, the welfare gain from PG-Pub increases at least when the cost of R&D is sufficiently convex (note that this is also the condition for PG-Pub to be socially desirable).

6. The incentives to invent
Thus far we have focused on the implications of PG-Pub on the second phase of the R&D process. We
now go back to the first phase and ask how PG-Pub affects the firms’ incentives to invent the basic technology. To this end, suppose that the investments in the first phase enhance the chance of each firm to be the first to invent the basic technology. Consequently, the benefit from investment in the first phase is $B = \pi_1 - \pi_2$, i.e., the difference between the expected profits of being firm 1 and being firm 2. We argue that the filing system that leads to a higher $B$, provides a stronger incentive to invent the basic technology. As before, we only study the cases of strong and intermediate protection since PG-Pub is completely irrelevant when patent protection is weak.

In the strong protection case, firm 1 files for a patent under both filing systems, so the benefit from investing in the first phase of the R&D process is $B_F = \pi_1 - \pi_2$ under the PG-Pub system, and $\bar{B}_F = \pi_1 - \pi_2$ under the CF system. Since part (ii) of Lemma 3 implies that PG-Pub hurts firm 1 and benefits firm 2, it is clear that $B_F - \bar{B}_F < 0$. Hence, PG-Pub weakens the incentive to invent the basic technology.

Things are more subtle when protection is intermediate. Now PG-Pub induces firm 1 to stop filing for a patent and this has an adverse effect on both firms. To study this case further, we impose Assumption A4. Then,

$$\bar{B}_F = \frac{(\pi_{yn} - \pi_{nn}) (\pi_{yn} + \pi_{nn} - 2 \pi_{ny}) r (\beta_\theta - (1 - \gamma \theta)^2)}{2 (r^2 \beta_\theta - (1 - \gamma \theta)^2 \Pi^2)}. \quad (11)$$

Under the PG-Pub system, firm 1 does not file for a patent, so the benefit from investing in the first phase is $B_{NF} = \pi_1 - \pi_2$, where $B_{NF}$ is identical to $\bar{B}_F$ when $\theta = 0$. The impact of PG-Pub, then, depends on the sign of $B_{NF} - \bar{B}_F$.

**Proposition 9:** *(The impact of PG-Pub on the incentives to invent the basic technology.)* PG-Pub weakens the incentive to invent the basic technology when patent protection is strong, and given Assumption A4, it also weakens the incentive to invent the basic technology when patent protection is intermediate. Moreover, given Assumption A4, the negative impact of PG-Pub on the incentive to invent the basic technology decreases with $\gamma$ when patent protection is strong but increases with $\gamma$ when patent protection is intermediate.

Proposition 9 supports the concern of opponents of PG-Pub that PG-Pub might discourage
inventors.\textsuperscript{16} Given the importance of inventions to the economy as a whole, this adverse effect of PG-Pub should be given a serious consideration. In addition, the proposition shows that as patents become more likely to be upheld in court, this drawback of PG-Pub becomes less significant if patent protection is strong, but more significant if patent protection is intermediate. The reason for this difference is that when protection is strong, firm 1 files for a patent under both filing systems. As patents become more likely to be upheld in court, PG-Pub is less detrimental to firm 1 and less beneficial to firm 2, so its negative effect on the incentive to invent diminishes. When patent protection is intermediate, firm 1 does not file for a patent under the PG-Pub system so $\gamma$ does not affect the incentive to invent. But, since an increase in $\gamma$ boosts the incentive to invent under the CF system, the detrimental effect of PG-Pub on the incentive to invent (i.e., the difference between $B_{NF}$ and $B_{F}$) increases.

7. Conclusion

This paper contributes to the recent public debate in the U.S. about the desirability of adopting a pre-grant patent publication system (PG-Pub). Our results suggest that PG-Pub discourages patent applications in industries in which patent protection is intermediate and may have an adverse effect on the quality of inventions that are patented and on the incentives to invent. At the same time, our results also suggest that, holding the number of inventions fixed, PG-Pub may raise the likelihood that new technologies will reach the product market by either raising the aggregate level of investment in the development of application technologies or by lowering the legal hurdles for introducing such technologies into the market by firms who do not own patents on the underlying basic technologies. This implies that once we fix the number of inventions, PG-Pub benefits consumers and may also enhance social welfare. When the inventions are made by foreign firms, the positive effect on domestic welfare is even larger.

Another advantage of PG-Pub is that it can help eliminate the so-called "submarine" patents which are patent applications that are intentionally delayed by applicants until a similar idea is commercialized by someone else (typically a large corporation), at which point the application is completed and entitles

\textsuperscript{16} For instance, a group of 26 American Nobel Laureates in economics, physics, chemistry, and medicine, led by Franco Modigliani, argued in an open letter sent to the U.S. Senate in September 1997, that the legislation that included a provision for PG-Pub "... will prove damaging to American small inventors and thereby discourage the flow of new invention... It will do so by curtailing the protection they obtain through patent relative to the large multi-national corporation." (see http://www.eagleforum.org/psr1998/mar98/psrmar98.html).
the patentholder to collect royalties.\textsuperscript{17} Although in our model it is common knowledge that firm 1 has already invented the basic technology, in reality this is not necessarily the case; a PG-Pub system has the advantage of eliminating the incentive to engage in submarine patents by giving other innovators a due warning that they should direct their efforts in a different direction.

Finally, although our model is quite general (we do not assume a particular type of competition in the product market, we do not need to distinguish between product and process inventions, and we derive many of the results without assuming a particular functional form for the R&D cost functions), there is no doubt a need for further investigation of PG-Pub. In particularly, throughout the paper we have assumed away the possibility that firm 1 may license its invention to firm 2 instead of suing firm 2 for patent infringement. In future research it would be interesting to examine how PG-Pub affects the incentive of firms to engage in licensing agreements and how it affects the terms of these agreements. Such investigation is particularly important for industries like pharmaceuticals, electronic components and accessories, and computers and office equipment where patent protection is either strong or intermediate (so that PG-Pub is highly relevant) and licensing agreements are prevalent (Anand and Khanna, 2000).

\textsuperscript{17} A case in point are the patents that were issued in the 1980’s and the 1990’s to Jerome Lemelson for bar code-scanning and “machine vision” technologies which he first filed for in 1954 and 1956. According to a story published in the \textit{American Lawyer} in May 1993, Lemelson collected $500 million in royalties from manufacturers who inadvertently infringed on his patents. It should be noted though that there is a disagreement on the significance of submarine patents. For more details, see for instance http://www.ipo.org/submarine.htm
Appendix

Following are the proofs of Lemmas 1-3, Propositions 1-2, 4-6, and 8-9, and Corollary 2.

**Proof of Lemma 1:** When firm 1 files for a patent, its best-response function, $R^1(q^2 | F)$, is determined implicitly by,

$$
\frac{\partial \pi^1(q^1, q^2 | F)}{\partial q^1} = q^2(1 - \gamma \theta)(\pi_{yy} - \pi_{ny}) + (1 - q^2(1 - \gamma \theta))(\pi_{yn} - \pi_{nn}) - C'(q^1) = 0. \quad (A-1)
$$

Similarly, firm 2’s best-response function, $R^2(q^1 | F)$, is determined implicitly by

$$
\frac{\partial \pi^2(q^1, q^2 | F)}{\partial q^2} = (1 - \gamma \theta)(\pi_{yy} - \pi_{ny}) + (1 - q^1(1 - \gamma \theta))(\pi_{yn} - \pi_{nn}) - \beta_L C'(q^2) = 0. \quad (A-2)
$$

Assumptions A1 and A3 ensure that $R^1(q^2 | F)$ and $R^2(q^1 | F)$ are well-defined, single-valued, and downward sloping in the $(q^1, q^2)$ space. Hence, $q^1$ and $q^2$ are strategic substitutes. A Nash equilibrium in the filing subgame, $(q^1_F, q^2_F)$, is determined by the intersection of $R^1(q^2 | F)$ and $R^2(q^1 | F)$. We now prove that Assumptions A1-A3 ensure that $R^1(q^2 | F)$ and $R^2(q^1 | F)$ intersect only once in the unit square.

To this end, let us rewrite (A-1) and (A-2) as follows:

$$
q^2 = H_1(q^1) = \frac{(\pi_{yn} - \pi_{nn}) - C'(q^1)}{(1 - \gamma \theta) \Pi}, \quad (A-3)
$$

and

$$
q^1 = H_2(q^2) = \frac{(1 - \gamma \theta)(\pi_{yy} - \pi_{ny}) - \beta_L C'(q^2)}{(1 - \gamma \theta) \Pi}, \quad (A-4)
$$

where $\Pi \equiv \pi_{yy} + \pi_{nn} - \pi_{yn} - \pi_{ny} \leq 2 \pi_{yy} - \pi_{yn} - \pi_{ny} < 0$ (the first inequality follows because $\pi_{yy} > \pi_{nn}$ by Assumption A1 and the second inequality follows from Assumption A2). $H_1(q^1)$ and $H_2(q^2)$ intersect in the $(q^1, q^2)$ space in the unit square (recall that $q^1$ and $q^2$ are probabilities and hence must be between 0 and 1) provided that (i) $H_1(0) > 1$ (ii) $H_1(1) < 0$, (iii) $H_2(1) < 0$, and (iv) $H_2(0) > 1$. Condition (ii) is satisfied if $C'(1) > \pi_{yn} - \pi_{nn}$, which is ensured by Assumption A3. Condition (iii) is satisfied if $C'(1) > (1 - \gamma \theta)(\pi_{yn} - \pi_{nn})/\beta_L$; since $\beta_L > 1 > 1 - \gamma \theta$, this inequality is implied by Assumption A3. Since $\Pi < 0$ and recalling from Assumption A3 that $C'(0) = 0$, conditions (i) and (iv) are both satisfied if $\pi_{yn} - \pi_{nn} > -(1 - \gamma \theta)\Pi$. It is now easy to verify that the last inequality holds since $\pi_{yy} > \pi_{ny}$.

To prove uniqueness, note that the slopes of $R^1(q^2 | F)$ and $R^2(q^1 | F)$ are given by $C''(q^1)/((1 - \gamma \theta)\Pi)$
and \((1-\gamma \theta)\Pi/\beta_L C''(q^2)\), respectively. Given Assumption A3, \(C''(q^1)/(1-\gamma \theta)\Pi < -1 < (1-\gamma \theta)\Pi/\beta_L C''(q^2)\), which in turn implies that \(R^1(q^2 \mid F)\) and \(R^2(q^1 \mid F)\) intersect only once.

When firm 1 does not file for a patent, the two best-response functions, \(R^1(q^2 \mid NF)\) and \(R^2(q^1 \mid NF)\), respectively, are implicitly defined by:

\[
\frac{\partial \pi^1(q^1, q^2 \mid NF)}{\partial q^1} - q^2(\pi_{yy} - \pi_{ny}) + (1-q^2)(\pi_{yn} - \pi_{nn}) - C'(q^1) = 0, \tag{A-5}
\]

and

\[
\frac{\partial \pi^2(q^1, q^2 \mid NF)}{\partial q^2} = q^1(\pi_{yy} - \pi_{ny}) + (1-q^1)(\pi_{yn} - \pi_{nn}) - \beta_H C'(q^2) = 0. \tag{A-6}
\]

Assumptions A1 and A3 ensure that \(R^1(q^2 \mid NF)\) and \(R^2(q^1 \mid NF)\) are well-defined, single-valued, and downward sloping. A Nash equilibrium in the no-filing subgame, \((q^1_{NF}, q^2_{NF})\), is determined by the intersection of \(R^1(q^2 \mid NF)\) and \(R^2(q^1 \mid NF)\). The proof that Assumptions A1-A3 ensure that \(R^1(q^2 \mid NF)\) and \(R^2(q^1 \mid NF)\) intersect only once in the unit square is similar to the corresponding proof in the filing subgame and is therefore omitted.

Next, we turn to the equilibrium levels of investment. Since \(\gamma\) and \(\theta\) do not appear in (A-5) and (A-6), \(q^1_{NF}\) and \(q^2_{NF}\) are independent of \(\gamma\) and \(\theta\). On the other hand, Assumptions A1 and A2 ensure that \(\pi_{yy} + \pi_{nn} - \pi_{ny} - \pi_{yn} < 2\pi_{yy} - \pi_{yn} - \pi_{ny} < 0\), so (A-1) and (A-2) imply that \(\partial R^1(q^2 \mid F)/\partial (\gamma \theta) > 0\) and \(\partial R^2(q^1 \mid F)/\partial (\gamma \theta) < 0\). Since \(q^1\) and \(q^2\) are strategic substitutes, it follows that \(q^1_F\) increases and \(q^2_F\) decreases with \(\gamma \theta\).

Now suppose that \(\gamma \theta = 0\). Then, by (A-1) and (A-5), \(R^1(q^2 \mid F) = R^1(q^2 \mid NF)\). Since \(\beta_L < \beta_H\), equations (A-2) and (A-6) imply that \(R^2(q^1 \mid F) > R^2(q^1 \mid NF)\) for all \(q^1\) and since \(q^1\) and \(q^2\) are strategic substitutes, it follows that \(q^1_F < q^1_{NF}\) and \(q^2_F > q^2_{NF}\). To prove that \(q^2_F < q^1_F\), note that if \(\gamma \theta = 0\) and \(\beta_L = 1\), equations (A-1) and (A-2) are symmetric, and hence \(q^2_F = q^1_F\). As \(\beta_L\) increases from 1, \(R^2(q^1 \mid F)\) shifts so since the best-response functions are downward sloping, it must be that \(q^2_F < q^1_F\). The proof that \(q^2_{NF} < q^1_{NF}\) is similar to the proof that \(q^2_F < q^1_F\).

Finally, suppose that \(\gamma \theta \geq 1-\beta_L/\beta_H\), and rewrite (A-2) as follows:

\[
\frac{\partial \pi^2(q^1, q^2 \mid F)}{\partial q^2} = q^1(\pi_{yy} - \pi_{ny}) + (1-q^1)(\pi_{yn} - \pi_{nn}) - \frac{\beta_L C'(q^2)}{1-\gamma \theta} = 0. \tag{A-7}
\]

Since \(\gamma \theta \geq 1-\beta_L/\beta_H\), the third term here exceeds the third term in (A-6), so \(R^2(q^1 \mid F) \leq R^2(q^1 \mid NF)\). Together with the fact that by (A-1) and (A-5), \(R^1(q^2 \mid F) > R^1(q^2 \mid NF)\) for all \(\gamma \theta > 0\), it follows that \(q^1_F\)
> q^1_{NF} and q^2_F < q^2_{NF}.  Q.E.D.

**Proof of Proposition 1**: By equations (3) and (4), \( \pi^1_{NF} \) and \( \pi^2_{NF} \) are independent of \( \gamma \) and \( \theta \). Using the envelope theorem, equation (1) implies that

\[
\frac{\partial \pi^1_F}{\partial (\gamma \theta)} = -q^2_F \left[ q^1_F (\pi_{yy} - \pi_{yr}) + (1 - q^1_F) (\pi_{ny} - \pi_{nr}) \right] + \frac{\partial \pi^1_F}{\partial q^2_F} \frac{\partial q^2_F}{\partial (\gamma \theta)}.  \tag{A-8}
\]

Assumption A1 ensures that the expression inside the square brackets and \( \partial \pi^1_F / \partial q^2 \) are negative. Since \( \partial q^2_F / \partial (\gamma \theta) < 0 \) by Lemma 1, it follows that \( \partial \pi^1_F / \partial \gamma \theta > 0 \). The proof that \( \partial \pi^2_F / \partial \gamma \theta < 0 \) is analogous.

To prove the existence of \( \gamma \theta \in (0, 1 - \beta_L / \beta_H) \), such that \( \pi^1_F = \pi^1_{NF} \) as \( \gamma \theta \rightarrow 0 \), note that \( \gamma \theta \) is defined implicitly by \( \pi^1_F = \pi^1_{NF} \). Since \( \pi^1_F \) increases with \( \gamma \theta \), whereas \( \pi^1_{NF} \) is independent of \( \gamma \theta \), it is sufficient to show that \( \pi^1_F < \pi^1_{NF} \) if \( \gamma \theta = 0 \) and conversely if \( \gamma \theta = 1 - \beta_L / \beta_H \). If \( \gamma \theta = 0 \), equations (1) and (3) imply that \( \pi^1(q^1_F, q^2_F) = \pi^1(q^1_{NF}, q^2_{NF}) \). Consequently,

\[
\pi^1_F < \pi^1(q^1_{NF}, q^2_{NF} | F) = \pi^1(q^1_{NF}, q^2_{NF} | NF) \leq \pi^1_{NF},  \tag{A-9}
\]

where the strict inequality follows because \( \partial \pi^1(q^1_F, q^2_F | F) / \partial q^2 < 0 \) and because by Lemma 1, \( q^2_F > q^2_{NF} \) when \( \gamma \theta = 0 \), and the weak inequality is implied by revealed preferences (i.e., the definition of \( q^1_{NF} \)).

Next suppose that \( \gamma \theta = 1 - \beta_L / \beta_H \). Then Lemma 1 indicates that \( q^2_F < q^2_{NF} \). Using equations (1) and (3) and Assumption 1, it is easy to show that \( \pi^1(q^1_F, q^2_F | F) > \pi^1(q^1_{NF}, q^2_{NF} | F) \) for all \( q^2 > 0 \) and all \( \gamma, \theta > 0 \), so

\[
\pi^1_F \geq \pi^1(q^1_{NF}, q^2_F | F) > \pi^1(q^1_{NF}, q^2_{NF} | F) = \pi^1_{NF}, \tag{A-10}
\]

where the left inequality is implied by revealed preferences and the right inequality follows because \( \partial \pi^1(q^1_F, q^2_F | F) / \partial q^2 < 0 \) and \( q^2_F < q^2_{NF} \).  Q.E.D.

**Proof of Lemma 2**: The expected payoff functions in the no-filing subgame do not depend on the filing system. Hence the Nash equilibrium in the no-filing is once again \( (q^1_{NF}, q^2_{NF}) \).

Moreover, since firm 1’s expected payoff does not depend on the filing system, its best-response function in the filing subgame, \( R^1(q^2 | F) \), continues to be defined implicitly by equation (A-1). The best-response function of firm 2, \( \overline{R}^2(q^1 | F) \), is now defined implicitly by
where $\theta$ stand for "equal in sign." Substituting for $C'(q^2)$ from equation (A-11) and rearranging terms,

$$
\frac{\partial \bar{R}^2(q^2 | F)}{\partial \theta} = \frac{\beta_M}{\beta_H} \left[ q^2_F (\pi_{yy} - \pi_{nm}) + (1 - q^2_F) (\pi_{yn} - \pi_{nn}) \right] - \gamma (1 - \gamma \theta) \Pi \beta_H C'(q^2),
$$  \hspace{1cm} (A-13)

where $\pm$ stand for "equal in sign." Substituting for $C'(q^2_F)$ from equation (A-11) and rearranging terms,

$$
\frac{\partial \bar{R}^2(q^2 | F)}{\partial \theta} = \frac{\beta_M - \beta_H}{\beta_H} C'(q^2_F),
$$  \hspace{1cm} (A-12)

Since the expression outside the square brackets is positive, it follows that $\partial \bar{R}^2(q^1 | F)/\partial \theta > 0$ as $\gamma > 1 - \beta_M/\beta_H$. Thus, when $\gamma > 1 - \beta_M/\beta_H$, $\partial \bar{R}^2(q^1 | F)/\partial \theta < 0$. Together with the fact that $\partial R^1(q^2 | F)/\partial \theta > 0$ and the fact that $q^1$ and $q^2$ are strategic substitutes and $R^1(q^2 | F)$ is steeper than $\bar{R}^2(q^1 | F)$, this implies that $\bar{q}^1_F$ increases and $\bar{q}^2_F$ decreases. When $\gamma < 1 - \beta_M/\beta_H$, $\partial \bar{R}^2(q^1 | F)/\partial \theta > 0$. Since $\partial R^1(q^2 | F)/\partial \theta > 0$ as well, it follows that either $\bar{q}^1_F$ increases, or $\bar{q}^2_F$ increases, or both.

When $\theta = 0$, (A-1) coincides with (A-5) and (A-11) coincides with (A-6), so $R^1(q^2 | F) = R^1(q^2 | NF)$ and $\bar{R}^2(q^1 | F) = R^2(q^1 | NF)$. The proof that $\bar{q}^1_F > \bar{q}^2_F$ when $\theta = 0$ and the proof for the case where $\theta > 0$ are similar to the corresponding proofs in Lemma 1.

Finally, suppose that $\gamma = 1 - \beta_M/\beta_H$. Then, $\bar{R}^2(q^1 | F) = R^2(q^1 | NF)$, so $(q^1_F, \bar{q}^2_F)$ and $(q^1_{NF}, q^2_{NF})$ lie on the same curve in the $(q^1, q^2)$ space, with $(q^1_F, \bar{q}^2_F)$ being southeast of $(q^1_{NF}, q^2_{NF})$. Using (A-11), the slope of this curve is $\partial \bar{R}^2(q^1 | F)/\partial q^1 = -(1 - \gamma \theta) \Pi / \beta_H C'(q^2_F)$. Given Assumption A3, $C''(q) > -\Pi$ for all $q \in [0, 1]$, so $\partial \bar{R}^2(q^1 | F)/\partial q^1 > -1$, implying that $(q^1_F, \bar{q}^2_F)$ lies above a 45 degrees line passing through $(q^1_{NF}, q^2_{NF})$. Consequently, $\bar{q}^1_F + \bar{q}^2_F > q^1_{NF} + q^2_{NF}$. When $\gamma < 1 - \beta_M/\beta_H$, $\bar{R}^2(q^1 | F)$ shifts upward thereby reinforcing the result. \hspace{1cm} Q.E.D.
Proof of Proposition 2: The proof that \( \gamma \) exists and decreases with \( \beta_L \) and increases with \( \beta_H \) is similar to the proof of Proposition 1 and is therefore omitted. To compare the \( \pi^2_F \) and \( \pi^2_{NF} \), suppose that \( \gamma > 1 - \beta_M / \beta_H \). Using equation (4) and recalling that \( \pi^2(q^1, q^2 | F) \) is given by (2) with \( \beta_0 \) instead of \( \beta_L \), we get,

\[
\pi^2(q^1_F, q^2_F | NF) - \pi^2_F = \theta \gamma q^2_F \left[ (\pi^1_F - \pi^1_M) - (1 - q^1_F)(\pi^1_M - \pi^1_M) \right] - \theta (\beta_H - \beta_M) C(q^2_F).
\]  

(A-14)

Substituting for the square bracketed term from equation (A-11) and recalling that \( C(q) \) is strictly convex,

\[
\pi^2(q^1_F, q^2_F | NF) - \pi^2_F = 0 \left[ \gamma \beta_0 q^2_F C'(q^2_F) - (\beta_H - \beta_M) C(q^2_F) \right] > \frac{\theta \beta_H C(q^2_F)}{1 - \gamma \theta} \left[ \gamma - \left( 1 - \frac{\beta_M}{\beta_H} \right) \right] > 0.
\]

(A-15)

Using this inequality,

\[
\pi^2_{NF} > \pi^2(q^1_F, q^2_F | NF) > \pi^2(q^1_F, q^2_F | NF) > \pi^2_F.
\]

(A-16)

where the first (weak) inequality follows by revealed preferences, and the second (strict) inequality follows because \( \partial \pi^2(q^1, q^2 | NF) / \partial q^1 < 0 \), and \( q^1 > q^1_{NF} \) for all \( \gamma > 1 - \beta_M / \beta_H \). Q.E.D.

Proof of Lemma 3: (i) Equations (A-2) and (A-11) and the assumption that \( \beta_0 > \beta_L \), imply that \( R^2(q^1 | F) < R^2(q^1 | F) \). Since firm 1’s best-response function is the same under the PG-Pub and CF systems and \( q^1 \) and \( q^2 \) are strategic substitutes, it follows that \( q^1_F < q^1_{NF} \) and \( q^2_F > q^2_{NF} \). To prove that \( q^2_F < q^1_{NF} \), note that if \( \gamma \theta = 0 \) and \( \beta_L = 1 \), equations (A-1) and (A-2) are symmetric and hence \( q^2_F = q^1_{NF} \). As \( \gamma \theta \) increases from 0 and \( \beta_L \) increases from 1, \( R^2(q^1 | F) \) shifts down while \( R^1(q^2 | F) \) shifts to the right; since the two best-response functions are downward sloping, it follows that \( q^2_F < q^1_{NF} \).

To examine the aggregate level of investment, note that since firm 1’s best-response function in the filing subgame is the same under both filing systems, \( (q^1_F, q^2_F) \) and \( (q^1_{NF}, q^2_{NF}) \) lie on the same curve in the \( (q^1, q^2) \) space, with \( (q^1_F, q^2_F) \) being southeast of \( (q^1_{NF}, q^2_{NF}) \). Using equation (3), the slope of this curve is \( \partial R^1(q^2 | F) / \partial q^2 = (1 - \gamma \theta) \Pi / C''(q^1_{NF}) \). Given Assumption A3, \( C''(q) > -\Pi \) for all \( q \in [0, 1] \), so \( \partial R^1(q^2 | F) / \partial q^2 > -1 \), implying that \( (q^1_F, q^2_F) \) lies below a 45 degrees line passing through \( (q^1_{NF}, q^2_{NF}) \). Consequently, \( q^1_F + q^2_F \leq q^1_{NF} + q^2_{NF} \).

(ii) First, note that
\[ \pi_F^1 < \pi^1(q_F^1, q_F^2 | F) \leq \bar{\pi}_F^1, \]  
where the left inequality follows since \( \partial \pi^1(q_F^1, q_F^2 | F)/\partial q^2 < 0 \) and since \( q_F^2 > q_F^1 \), and the right inequality follows by revealed preferences. Second, using equation (4) and the fact that \( \bar{\pi}^2(q_F^1, q_F^2 | F) \) is given by (2) with \( \beta_q \) instead of \( \beta_L \), it follows that \( \pi^2(q_F^1, q_F^2 | F) > \bar{\pi}^2(q_F^1, q_F^2 | F) \). Hence,

\[ \pi_F^2 > \pi^2(q_F^1, q_F^2 | F) > \bar{\pi}^2(q_F^1, q_F^2 | F) \geq \bar{\pi}_F^2, \]  
where the left inequality follows from revealed preferences and the middle inequality follows since \( \partial \bar{\pi}^2(q_F^1, q_F^2 | F)/\partial q^1 < 0 \) and \( q_F^1 > \bar{q}_F^1 \). \( Q.E.D. \)

Proof of Corollary 2: Proposition 3 implies that when patent protection is strong, the equilibrium investment levels are \( q_F^1 \) and \( q_F^2 \) under the PG-Pub system and \( \bar{q}_F^1 \) and \( \bar{q}_F^2 \) under the CF system. The effect of PG-Pub on the investment levels in this case follows immediately from part (i) of Lemma 3.

When patent protection is intermediate, Proposition 3 implies that the equilibrium investment levels are \( q_{NF}^1 \) and \( q_{NF}^2 \) under the PG-Pub system and \( \bar{q}_F^1 \) and \( \bar{q}_F^2 \) under the CF system. The effect of PG-Pub on the investment levels in this case follows immediately from part (ii) of Lemma 2. \( Q.E.D. \)

Proof of Proposition 4: Given Assumption A4, firm 1’s expected payoff if it files for a patent under the PG-Pub system is

\[ \pi_F^1 = \pi_{nn} + \frac{(\pi_{yn} - \pi_{nn})}{2 \beta_L} \left( r \beta_L + (1 - \gamma \theta)^2 \pi^2 \right) + \frac{(\pi_{yn} - \pi_{nn})(\pi_{nn} - \pi_{yn})(r + \pi^2)(1 - \gamma \theta)^2}{r^2 \beta_L^2 + (1 - \gamma \theta)^2 \pi^2}. \]  

If firm 1 does not file for a patent, its expected payoff, \( \pi_{NF}^1 \), is given by a similar expression with \( \beta_L \) replacing \( \beta_{Hi} \) and \( \theta = 0 \). Clearly, \( \pi_F^1 \) is independent of \( \beta_{Hi} \) while a straightforward (though tedious) differentiation reveals that given Assumption A2 and the assumptions that \( r > \pi_{yn} - \pi_{nn} \) and \( \beta_L > 1 \geq 1 - \gamma \theta \), \( \pi_{NF}^1 \) is increasing with \( \beta_{Hi} \). Now, setting \( \pi_F^1 = \pi_{NF}^1 \) and solving for \( \beta_{Hi} \), the largest \( \beta_{Hi} \) for which firm 1 still files for a patent under the PG-Pub system is \( \beta_{Hi}/(1 - \gamma \theta)^2 \).

Under the CF system, the expected payoff of firm 1 if it files for a patent, \( \bar{\pi}_F^1 \), is similar to \( \pi_F^1 \), except that \( \beta_q = \theta \beta_{Hi} + (1 - \theta) \beta_{Hi} \) replaces \( \beta_L \). Now things are more complex since both \( \pi_F^1 \) and \( \pi_{NF}^1 \) depend on \( \beta_{Hi} \). Setting \( \pi_F^1 = \pi_{NF}^1 \) and solving for \( \beta_{Hi} \) yields 3 solutions, but two of them are less than \( \beta_m \) and are
therefore irrelevant (by assumption, \( \beta_H > \beta_M \)). The third solution is equal to \( \beta_M/(1-2\gamma+\theta\gamma^2) \). Since the derivative of \( \pi_1^F - \pi_{1NF} \) is decreasing at \( \beta_H = \beta_M/(1-2\gamma+\theta\gamma^2) \) it follows that firm 1 files for a patent if and only if \( \beta_H < \beta_M/(1-2\gamma+\theta\gamma^2) \). \( Q.E.D. \)

**Proof of Proposition 5:** In the strong protection case, we need to compare \( \tilde{S}_F \) (consumers’ surplus under the CF system) and \( S_F \) (consumers’ surplus under the PG-Pub system). Now,

\[
S_F - \tilde{S}_F = \frac{(\pi_{yn} - \pi_{mn}) r (1-\gamma \theta)^2 (r + \Pi)^2 (\beta_\theta - \beta_L) (S_{yn} - S_{mn})}{(r^2 \beta_\theta - (1-\gamma \theta)^2 \Pi^2) (r^2 \beta_L - (1-\gamma \theta)^2 \Pi^2)} \tag{A-20}
\]

\[
+ (\pi_{yn} - \pi_{mn})^2 (r + \Pi) (1-\gamma \theta) \left[ \frac{r \beta_L - (1-\gamma \theta)^2 \Pi}{(r^2 \beta_L - (1-\gamma \theta)^2 \Pi^2)^2} - \frac{r \beta_\theta - (1-\gamma \theta)^2 \Pi}{(r^2 \beta_\theta - (1-\gamma \theta)^2 \Pi^2)^2} \right] S.
\]

Since \( \beta_\theta > \beta_L \), this expression is strictly positive, implying that PG-Pub makes consumers better-off.

In the intermediate protection case, we need to compare \( \tilde{S}_F \) (consumers’ surplus under the CF system) and \( S_{NF} \) (consumers’ surplus under the PG-Pub system). Now,

\[
S_{NF} - \tilde{S}_F = \frac{(\pi_{yn} - \pi_{mn}) r (r + \Pi)^2 (\beta_\theta - \beta_H(1-\gamma \theta)) (S_{yn} - S_{mn})}{(r^2 \beta_\theta - (1-\gamma \theta)^2 \Pi^2) (r^2 \beta_H - \Pi^2)} \tag{A-21}
\]

\[
+ (\pi_{yn} - \pi_{mn})^2 (r + \Pi) \left[ \frac{r \beta_H - \Pi}{(r^2 \beta_H - \Pi^2)^2} - \frac{(1-\gamma \theta)^2 (r \beta_\theta + (1-\gamma \theta)^2 \Pi)}{(r^2 \beta_\theta - (1-\gamma \theta)^2 \Pi^2)^2} \right] S.
\]

Recalling that in the intermediate protection case, \( \gamma \geq \left(1 - \sqrt{\beta_\theta/\beta_H}/\theta \right) \), we get \( \beta_\theta - \beta_H(1-\gamma \theta)^2 \geq 0 \), so the first line of (A-21) is positive. The square bracketed expression inside the second line, is increasing with \( \gamma \) and it vanishes at \( \gamma = \left(1 - \sqrt{\beta_\theta/\beta_H}/\theta \right) \); hence the second line is positive as well, so \( S_{NF} > \tilde{S}_F \) for all parameter values in the intermediate protection case. Finally, it is straightforward to establish that the first line of (A-21) is increasing with \( \gamma \). Since the second line is also increasing with \( \gamma \), it follows that the gain of consumers from PG-Pub is larger the larger is \( \gamma \). \( Q.E.D. \)

**Proof of Proposition 6:** (i) Since in the strong protection case, expected social welfare under the CF system, \( \bar{W}_F \), and under the PG-Pub system, \( W_F \), differ only with respect to \( \beta \), we can establish a sufficient condition for \( W_F > \bar{W}_F \) by replacing \( \beta_\theta \) with \( \beta \) in equation (10) and deriving a condition that ensures that \( \partial \bar{W}_F / \partial \beta < 0 \) for all \( \beta \in [\beta_L, \beta_\theta] \). From equation (10),
The expression outside the square brackets in (A-22) is negative and the last two expressions inside the square brackets are positive (the last term is positive by Assumption A5). Hence $Z(r, \beta) \geq 0$ is sufficient for $\partial \tilde{W}_F / \partial \beta < 0$ for all $\beta \in [\beta_L, \beta_U]$, which in turn ensures that $W_F > \tilde{W}_F$. Now, surely, $Z(r, \beta) > 0$ for all $\beta \in [\beta_L, \beta_U]$. Recalling that $r > -\Pi$ and noting that $Z(r, \beta)$ is a convex function of $r$ and that $Z'(r, \beta) < 0$ and $Z(-\Pi, \beta) < 0$, it follows that $Z(r, \beta) > 0$, provided that $r \geq \bar{r}(\beta)$, where $\bar{r}(\cdot)$ is defined in the proposition.

(ii) Recall that in the intermediate protection case, expected social welfare under the CF system is $\tilde{W}_F$, and under the PG-Pub system, it is $W_{NF}$, where $W_{NF}$ is equal to $\tilde{W}_F$ when it is evaluated at $\theta = 0$. Since $\tilde{W}_F$ and $W_{NF}$ differ only with respect to $\theta$, a sufficient condition for PG-Pub to enhance (lower) welfare is that $\partial \tilde{W}_F / \partial \theta > 0$ ($\partial W_{NF} / \partial \theta < 0$) for all $\theta \in [0, \theta_U / \gamma)$. Using equation (10), we get

$$
\frac{\partial \tilde{W}_F}{\partial \theta} = \frac{(\pi_{ym} - \pi_{mm}) r (1 - \gamma \theta)(r + \Pi)(\beta_H - \beta_M - \gamma(\beta_H + \beta_M))}{2(r^2 \beta - (1 - \gamma \theta)^2 \Pi)^3} \times [(\pi_{ym} - \pi_{mm}) Z(r, \beta)]
$$

$$
+ 2 (\pi_{ym} - \pi_{mm}) M(\beta) S + 2 (r + \Pi)(r^2 \beta - (1 - \gamma \theta)^2 \Pi^2)(S_{ym} - S_{mm} + \pi_{ny} - \pi_{mn}),
$$

where

$$
M(\beta) = (r + (1 - \gamma \theta) \Pi)^2 + r^2 (\beta - 1) + 2 r \gamma \theta (1 - \gamma \theta) \Pi > 0,
$$

and

$$
Z(r, \beta) = r^2 \beta (r + 3 \Pi) + (1 - \gamma \theta)^2 \Pi^2 (3 r + \Pi).
$$

To determine the sign of the derivative, note that the expression inside the square brackets is similar to the expression inside the square brackets in (A-22), and hence is positive when $r \geq \bar{r}(\beta)$. In that case, the sign of the derivative depends on the sign of $(\beta_H - \beta_M - \gamma(\beta_H + \beta_M))$. Finally, note that $W_{NF}$ is independent of $\gamma$, while using equation (10),
If $r \geq r^\bar{\theta} (\beta \theta)$, then
\[
\frac{\partial \tilde{W}_F}{\partial \gamma} < 0.
\]
Thus, if $W_{NF} > \tilde{W}_F$ (PG-Pub is welfare-enhancing), the welfare gain from PG-Pub increase as $\gamma$ increases. If on the other hand $W_{NF} < \tilde{W}_F$ (PG-Pub is welfare-reducing), the welfare loss from PG-Pub becomes smaller as $\gamma$ increases. \quad Q.E.D.

Proof of Proposition 8: Under the PG-Pub system, firm 1 files for a patent if and only if
\[
y < \frac{1 - \sqrt{\beta_L/\beta_H}}{\theta}.
\]
As $\beta_L$ falls, the right side of the inequality increases, so firm 1 files for a smaller set of parameters. If the inequality still holds, firm 1 files for a patent under both filing systems, so the impact of PG-Pub on expected social welfare is given by $W_F - \tilde{W}_F$ (i.e., the difference between expected welfare under PG-Pub and under CF). To examine how $\beta_L$ affects $W_F - \tilde{W}_F$, note that $\tilde{W}_F$ is independent of $\beta_L$, while equation (A-22) implies that if $r \geq r^\bar{\theta}(\beta_L)$, then $\partial \tilde{W}_F/\partial \beta_L < 0$. Hence, whenever $r \geq r^\bar{\theta}(\beta_L)$, lowering $\beta_L$ boosts the welfare gain from PG-Pub. \quad Q.E.D.

Proof of Proposition 9: Given Assumption A4, the benefit from innovating under the CF system when patent protection is strong or intermediate is
\[
\tilde{B}_F = \frac{(\pi_{yn} - \pi_{nm})(\pi_{yn} + \pi_{mn} - 2\pi_{ny})}{2\left(r^2 \beta_\theta - (1 - \gamma \theta^2)^2 \Pi^2 \right)}.
\]
Under the PG-Pub system, the benefit from innovating is $B_F$ if protection is strong, where $B_F$ is identical to $\tilde{B}_F$ except that $\beta_L$ replaces $\beta_\theta$. Thus, the impact of PG-Pub on the incentive to invent when patent protection is strong depends on the sign of the following expression:
\[
B_F - \tilde{B}_F = -\frac{(\pi_{yn} - \pi_{nm})(\pi_{yn} + \pi_{mn} - 2\pi_{ny})}{4\left(r^2 \beta_L - (1 - \gamma \theta^2)^2 \Pi^2 \right)} < 0. \quad (A-28)
\]
Straightforward calculation reveals that this expression increases with $\gamma$, hence PG-Pub weakens the incentive to invent, but less so as $\gamma$ increases.

In the intermediate protection case, the impact of PG-Pub on the incentive to invent depends on
the sign of the following expression:

\[
B_{NF} - \overline{B}_F = \frac{(\pi_{yn} - \pi_{nn}) (\pi_{yn} + \pi_{nn} - 2\pi_{ny}) r (r^2 - \Pi^2) (\beta_H (1 - \gamma \theta)^2 - \beta_\theta)}{2 \left( r^2 \beta_H - \Pi^2 \right) \left( r^2 \beta_\theta - (1 - \gamma \theta)^2 \Pi^2 \right)}, \tag{A-29}
\]

where \( \beta_H (1 - \gamma \theta)^2 - \beta_\theta \leq 0 \), since in the intermediate protection case, \( \gamma \geq \left( 1 - \sqrt{\beta_\theta / \beta_H} \right) / \theta \). Hence, PG-Pub weakens the incentives to invent in this case as well. However now, straightforward calculation reveals that \( B_{NF} - \overline{B}_F \) decreases with \( \gamma \) so the negative impact of PG-Pub increases when \( \gamma \) increases. \( Q.E.D. \)
References


