The choice of technology and capital structure under rate regulation

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Abstract

This paper examines a regulated firm’s choice of technology. It presents a model in which regulatory opportunism induces the firm to adopt a technology that gives rise to cost functions with higher variable costs and lower fixed costs than is socially optimal. This distortion arises because the regulated price is positively correlated with marginal costs. Consequently, a technology with low marginal costs implies a low regulated price and hence is unattractive to the firm. Debt financing is shown to alleviate this distortion because it induces regulators to increase the regulated price to prevent the firm from financial distress, thereby reducing the cost to the firm of adopting technologies with low marginal costs. When regulators restrict the firm’s ability to issue debt, the firm may have an incentive to goldplate (i.e. waste resources). This incentive disappears when the firm can use its most preferred mode of financing.

Keywords: Rate regulation; Investment; Capital structure; Goldplating

JEL classification: G32; G38; L51

1. Introduction

The public utilities sector in the U.S. is subject to rate regulation by state regulatory commissions as well as federal agencies. Similarly, in Britain,
new regulatory bodies were established to apply price controls to the newly privatized public utilities in electricity, natural gas, telecommunications, and water. Given the importance of public utilities and the magnitude of investments they require, it seems natural to ask what is the effect of rate regulation on investment decisions of regulated firms. Indeed, this question was a main focus of the rate regulation literature in the last three decades. Traditionally, the maintained assumption in this literature is that the firm is endowed with a specific technology that is represented by a production function that depends on investment in physical capital and labor. Given this assumption, the discussion on the effects of rate regulation on investment, beginning with Averch and Johnson (1962), has centered around the question of whether regulated firms invest too much or too little in physical capital.

In practice, however, firms have a variety of technologies to choose from. These technologies may differ from one another not only in their technical properties, but also in their cost structures. An electric utility, for example, can choose between different mixes of base-load and peak-load generating units, with the former having a higher capital cost, but lower operating cost than the latter (Fuss and McFadden, 1978). Similarly, a telephone company can build redundancy into its network, thereby increasing its capital costs while reducing the cost of maintaining the network. The availability of different competing technologies suggests that models that take the firm's technology as given and simply examine the magnitude of investment in this technology provide an incomplete picture of the effects of rate regulation on investment.

This paper examines one aspect of a regulated firm's choice of technology, namely the choice between technologies with different mixes of variable and fixed costs. It is shown that rate regulation may induce regulated firms to choose technologies with higher variable costs and lower fixed costs than in the first-best (i.e. the technologies that a benevolent social planner would choose) and that the extent of this distortion depends crucially on the way the firm finances its investment in the capital market.

The interaction between the firm, the regulator, and the capital market is modeled as a three-stage game. The firm selects its technology in the first stage of the game, and this selection determines its cost structure. In the second stage, the firm issues a mix of equity and debt in the capital market to finance the cost of investing in the selected technology. Finally, the regulator establishes the regulated price in the third stage, taking the firm's

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1 For example, in 1990, investment in new plant and equipment in the U.S. public utilities sector totalled $65.91 billion and accounted for approximately 12.34% of total business expenditure for new plant and equipment (source: Department of Commerce, Bureau of the Census).
technology and capital structure as given. A key feature of this game is that the regulator does not precommit to particular regulated prices before the firm makes an irreversible investment decision. Consequently, the regulator may have an incentive to behave opportunistically by lowering the regulated price after investment has been made. This feature reflects the regulatory framework in both the U.S. and in Britain where regulators cannot make explicit precommitments to prices.\(^2\)

It should be pointed out, however, that the assumption that regulators lack the ability to precommit to prices represents an extreme case of regulatory opportunism, and that in practice, regulators may have some ability to precommit to prices, at least implicitly. First, rate cases are not held as soon as firms invest as assumed in this paper, but within a lag. This regulatory lag protects firms against opportunism at least in the short run. Second, regulators may intentionally introduce regulatory bureaucracy to make it unduly costly for them to acquire information about the firm's technology and lower prices accordingly (Sappington, 1986). Third, governments may alleviate the problem of regulatory opportunism by appointing regulators whose interests are closely aligned with those of firms (Spulber and Besanko, 1992). Finally, due to the repeated nature of the regulatory process, regulators may develop a reputation for not being hostile to firms to foster new investments (Salant and Woroch, 1992). Nevertheless, none of these mechanisms is perfect in the sense that typically regulatory opportunism remains a concern. The primary motivation for adopting an extreme version of regulatory opportunism is to highlight its impact on the choice of technology by regulated firms.

When investment involves sunk cost, regulatory opportunism has been shown in the literature to induce firms to underinvest, e.g. Spulber (1989, ch. 20) and Besanko and Spulber (1992).\(^3\) This paper shows that besides leading to underinvestment, regulatory opportunism may also distort the firm's choice of technology. Unlike the underinvestment problem, this bias

\(^2\) In the U.S., the inability of regulators to make explicit precommitments to prices stems from the fact that historically, courts gave regulatory commissions a great deal of leeway in choosing rates. According to the Supreme Court in the landmark Hope Natural Gas case of 1944, a regulatory agency is “not bound to the use of any single formula or combination of formulae in determining rates” (Federal Power Comm. v. Hope Natural Gas Co., 320 U.S. 591, 603, 1944). Moreover, in the United Railways case of 1930, the Supreme Court stated that “What will formulate a fair return in a given case is not capable of exact mathematical demonstration.” (United Railways & Elec. Co. v. West, 280 U.S. 234, 249, 251 1930). In Britain, the agencies that were established to regulated the newly privatized public utilities were given wide discretion in setting rates. For example, the telecommunication act of 1984 allows the Director General of Telecommunications to act “in a manner he considers best calculated”.

\(^3\) The absence of regulatory commitment to rates is also explored by Banks (1992) in the context of regulatory auditing and Spiegel and Spulber (1993, 1994) and Spiegel (1994) in the context of optimal capital structure of a regulated firm.
may arise even if all costs are avoidable. Assuming that the regulator wishes to maximize welfare, the regulated price is set as close as possible to marginal cost. Thus, the benefits from having a technology with low marginal costs are effectively expropriated by the regulator, rendering such a technology unattractive to the firm. Consequently, the firm has the incentive to choose a technology with higher marginal costs and lower fixed costs than is socially optimal. In the context of the electric power industry, for example, this suggests that rate regulation may induce electric utilities to invest in technologies which have relatively low capacity costs but high fuel costs. Moreover, it suggests that electric utilities may have little incentive to enhance their fuel efficiency. 4

The distortion in the choice of technology depends in this paper on the way the firm finances its investment. In particular, the distortion is alleviated when the firm uses debt financing. The reason for this is that debt serves as a substitute (albeit imperfect) for regulatory commitment to prices in the following way: by issuing debt, the firm increases the likelihood that it will become financially distressed. Since this event creates a deadweight loss, the regulator will try to ensure that the firm maintains a positive cash flow by setting the regulated price sufficiently above marginal cost. As a result, technologies with low marginal costs will no longer be associated with as low a regulated price as before, so the firm will choose a technology with lower marginal costs and higher fixed costs than under all-equity financing, closer to the first-best. Since debt alleviates the distortion in the choice of technology, it may be welfare-enhancing.

Regulated firms are sometimes accused of making unnecessary expenditures (goldplating) with the sole intention of inducing regulators to approve higher rates. 5 In the context of this paper, goldplating may be attractive to the firm because it induces the regulator to increase the regulated price in order to reduce the likelihood that the firm will become financially distressed. This price increase may more than compensate the firm for the waste of resources. In this paper, however, an optimally leveraged firm will never goldplate because it can issue debt instead. From the firm's perspective, this strategy has the advantage of leading to an increase in the regulated price without being wasteful.

4 Consistent with this prediction, Rose and Joskow (1990) report that in a sample of 144 electric utilities, the 1962 fuels costs of regulated investors-owned utilities were on average 14.7% higher than those of cooperative-owned utilities and 4.5% higher than those of government-owned utilities. The corresponding figures for 1972 were 27.5% and 9.5% respectively.

5 For a discussion and analysis of goldplating under rate-of-return regulation, see Westfield (1965), Zajac (1972), and Bailey (1973).
Although a vast literature exists on investment under rate regulation, surprisingly little theoretical research has been devoted to the issue of the choice of technology by regulated firms. Laffont and Tirole (1986) develop an optimal regulatory mechanism under asymmetric information. As in the current paper, this mechanism also leads to a bias toward too high marginal costs and too low fixed costs, but for a different reason. While here the bias arises because of regulatory opportunism and it exists for all output levels, in their model it arises because under asymmetric information the firm produces too little output and hence has an incentive to keep its fixed costs low even at the expense of high marginal costs. An important implication of this is that in Laffont and Tirole, given its output, the firm produces efficiently, whereas here, the firm produces inefficiently at all output levels. Sappington (1983) also develops an optimal regulatory mechanism under asymmetric information, but finds the reverse bias, i.e. the optimal regulatory strategy, induces the firm to adopt a technology with too much fixed costs and too little variable costs. This bias arises because, by raising fixed costs above their first-best level, the regulator is able to limit the information rents that firms command from their private information about the trade-off between fixed and variable costs. Crew and Kleindorfer (1986, ch. 8) examine the choice of technology in a traditional rate of return regulation model and find that tightening regulation leads to an increase in fixed cost and a decrease in variable costs. The rationale for this shift in costs is the Averch and Johnson rationale, stemming from the fact that high fixed cost and low variable cost are associated with capital intensive technologies.

The current paper is closely related to Spiegel and Spulber (1993, 1994) and Spiegel (1994). These papers also examine the strategic interaction between the capital structure of regulated firms, regulated prices, and investment. The focus of these papers, however, differs from the focus of the current paper. Spiegel and Spulber (1994) show that a regulated firm's capital structure has a significant effect on the regulated price and suggest that this may affect the firm's incentives to reduce its costs. Spiegel (1994) analyzes a more specific model which yields testable hypotheses concerning

6 For an earlier treatment of the capital structure of regulated firms see Sherman (1977) and Taggart (1981, 1985). Similarly to the current paper, Sherman finds that debt can have a beneficial effect on the firm's choice of technology. The intuition for his result is that in his model the regulator uses a rate-of-return regulation with the allowed rate of return being equal to a weighted average of the cost of equity and the cost of debt, plus some excess return. Since the cost of debt is lower than the cost of equity, issuing more debt is comparable to setting the allowed rate of return closer to the true cost of capital, thus moderating the Averch and Johnson effect.
the effects of changes in cost parameters and in the regulatory climate on
the equilibrium capital structure, regulated price, and investment the quality
of its output. Spiegel and Spulber (1993) examine the consequences of
asymmetric information regarding the firm's costs for its choice of capital
structure. None of these papers, however, examines the implications of
regulatory opportunism for the firm's choice of technology.

The remainder of this paper is organized as follows. The basic model is
presented in Section 2, and the regulatory process is considered in Section 3.
In Section 4 the effect of rate regulation on the firm's choice of technology is
studied under the assumption that the firm uses all-equity financing. Optimal
financing, involving a positive debt level, is examined in Section 5 and its
implications for the choice of technology are explored. In Section 6, the
basic model is used to examine goldplating. A summary of the main results
and concluding remarks are in Section 7. All proofs are in the Appendix.

2. The model

Consider a regulated firm that produces a single product or service. The
demand for the firm's output is given by \( q(p, z) = zQ(p) \), where \( p \) is the
regulated price set by the regulator and \( z \) is a random demand shock. The
demand shock, \( z \), is distributed on the interval \([z^-, z^+]\), where \( z^- > 0 \),
according to a differentiable distribution function \( f(z) \) and cumulative
distribution function \( F(z) \). To produce its output, the firm needs to invest \$k
in a production facility. This facility can be designed in a variety of ways.
Each design corresponds to a different technology and is associated with a
total operating cost function \( C(q, \gamma) = c(\gamma)q(p, z) + \gamma \). The firm's operating
cost, then, consists of a variable cost which is linear in output (constant
marginal cost) and a fixed cost. The latter can be thought of as the cost of
capacity and may include maintenance costs and the opportunity cost of
capital. The assumed properties of the demand and the cost functions are:

Assumption 1. \( Q'(p) < 0, Q''(p) \leq 0 \).

Assumption 2. For all \( \gamma \): \( c(\gamma) > 0, c'(\gamma) < 0 \).

Assumption 3. \( \lim_{\gamma \to 0} c'(\gamma) = -\infty, \lim_{\gamma \to \infty} c'(\gamma) = 0 \).

Assumption 1 is a standard assumption. Assumption 2 states that the
variable cost is positive and decreasing in $\gamma$. 7 Finally, Assumption 3 ensures
the existence of an interior solution for $\gamma$.

Typically, investment decisions are made in two steps. First, the firm has
to select a specific technology that it wishes to employ. Second, the firm has
to decide how much to invest in the selected technology. This paper,
however, is concerned only with the first step. 8 Therefore, the second step of
the investment decision is assumed to be of a 0–1 type: the size of
investment, $k$, is fixed and the firm can only decide whether or not to
undertake it. To simplify the analysis further, $k$ is assumed to be small
enough to ensure that investment is profitable. This leaves the design of the
production facility, i.e. the choice of $\gamma$, as the only meaningful investment
decision that the firm has to make.

The interaction between the regulated firm, the regulator, and outside
investors is modeled as a three-stage game (see Fig. 1). In stage 1, the firm
chooses its technology by selecting the cost parameter $\gamma$. This selection
determines the firm’s cost structure. In stage 2, the firm chooses a mix of
equity and debt needed to finance the cost of investment, $k$, by issuing new
shares and bonds to outsiders. Given this mix, the value of the firm’s
securities is determined in a competitive capital market. In stage 3, the
regulator establishes the regulated price, taking the firm’s technology and

7 Using Stigler’s (1939) terminology, the parameter $\gamma$ can be viewed as the degree of
flexibility that the firm’s technology exhibits: as $\gamma$ increases, the firm’s costs become less
responsive to fluctuations in output and hence more flexible (for alternative definitions of
flexibility and a literature survey, see Carlsson, 1989). Note, however, that unlike in the
literature on flexibility (e.g. Marschak and Nelson, 1962; Mills, 1984, 1986; Vives, 1989) where
the average cost function is U-shaped, here it is everywhere decreasing. Thus, while the
commonly used measure of flexibility is taken to be the inverse of $C_{eq}$ here it is the inverse of
$C_{\gamma}$. Despite this difference, the main feature of flexibility is nevertheless retained in that
increased flexibility makes production costs less responsive to fluctuations in output.

8 For an analysis of the second step of the investment decision in the absence of regulatory
commitment to rates, see Spulber (1989), Besanko and Spulber (1992), Spiegel (1994), and
Spiegel and Spulber (1994).
capital structure as given. Finally, the random demand shock, $z$, is realized, output is produced, and payments are made.

Two important assumptions underlie the sequential structure of the game. First, the regulated firm can choose its investment and capital structure at its own discretion. This assumption reflects the fact that for the most part, regulatory commissions in the U.S. do not intervene with investment and financial decisions of firms, while the philosophy in Britain is of "regulation with a light hand" (Vickers and Yarrow, 1988). Second, the regulated price is set after the firm has made its investment and financial decisions. This assumption reflects the fact that regulated decisions on rates are typically made on a much more frequent basis than firms' investment decisions. It also captures the lack of regulatory precommitment to prices that characterizes the regulatory framework in both the U.S. and Britain (see footnote 2).

Initially, the firm is all-equity and has no liquid assets. To finance $k$ the firm issues equity and debt to outsiders. Let $E(\alpha)$ be the market value of new shares representing a fraction $\alpha \in [0, 1]$ of the firm's equity, and let $B(D)$ be the market value of debt with face value $D$. Since the firm has no outstanding debt to begin with, $D$ represents the total debt obligation of the firm. Since $E$ and $B$ should cover the cost of the project, $k \leq E(\alpha) + B(D)$.

There is evidence, however, to suggest that regulatory commissions do not allow regulated firms to raise external funds in excess of the costs of investment in physical assets, see, for example, Phillips (1988, p. 220). Thus, the firm's budget constraint is

$$k = E(\alpha) + B(D).$$

The operating income of the firm is $zR(p, \gamma) - \gamma$, where $R(p, \gamma) = Q(p)(p - c(\gamma))$. Given a regulated price, $p$, a cost parameter, $\gamma$, and the

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9 This is despite the fact that most regulatory commissions in the U.S. have the authority to approve or reject investment and financial decisions of regulated firms. With regard to investment decisions. Brigham and Tapley (1986) argue that "utility managements have traditionally regarded choosing the composition and the construction program as a management prerogative, hence have not actively solicited commission inputs into the process, while commissions have not sought active involvement or responsibility" (pp. 16-23). As for securities issues, the Colorado Supreme court argues that "a guiding principle of utility regulation is that management is to be left free to exercise its judgment regarding the most appropriate ratio between debt and equity" (in Re Mountain States Teleph. & Teleg. Co. 39 PUR 4th 222, 247-248). Even when a deviation from this guiding principle is possible, "few commissions are willing to substitute their judgments for those of the management except in reorganization cases" (Phillips, 1988, p. 226). Moreover, U.S. courts in many states (e.g. Michigan, Oklahoma, Kansas, Delaware) restrict state commissions' scope of inquiry in security issue proceedings by directing commissions to inquire only whether the proposed projects are within the scope of the utility's corporate activity and not whether they are "reasonable" or "necessary" (for details see Howe, 1982).
firm’s debt obligation, \( D \), there exists a critical state of nature, \( z^* \), at which the firm is just able to break even. This state of nature is defined by

\[
\begin{align*}
  z^*(p, \gamma D) &= \begin{cases} 
    z^- & \text{if } z^- R(p, \gamma) \geq D + \gamma, \\
    \frac{D + \gamma}{R(p, \gamma)} & \text{if } z^- R(p, \gamma) < D + \gamma < z^+ R(p, \gamma), \\
    z^+ & \text{if } z^+ R(p, \gamma) \leq D + \gamma.
  \end{cases}
\end{align*}
\]

Since \( z R(p, \gamma) \) increases in \( z \), the probability that the firm fails to break even is \( F(z^*) \).

The critical state of nature, \( z^* \), is illustrated in Fig. 2. For states of nature above \( z^* \), \( z R(p, \gamma) > D + \gamma \), so the firm earns a positive profit.\(^{10}\) For states of nature below \( z^* \), \( z R(p, \gamma) < D + \gamma \), so the firm is unable to pay claimholders such as debtholders, input suppliers, and workers out of its earnings. Since the firm has no initial liquid assets, it becomes financially distressed. Nevertheless, it is assumed that even in this case all claimholders

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\(^{10}\) One may wonder why, if the regulator is truly opportunistic, would he not attempt to expropriate firm profits when \( z > z^* \), say by reducing future rates. But, anticipating such regulatory incentive, the firm will either pay out its profits as dividends or invest them before the next rate case. Indeed, Brigham and Tapley (1986) report that regulated firms maintain very high dividend pay out ratios (65% to 75% compared with 40% on average for non-regulated industrial companies).
are eventually paid in full and equityholders remain the residual claimants. To fulfill its financial obligations, the firm therefore needs to either borrow money from external sources, sell some of its assets, or ask the government to cover its deficit.\textsuperscript{11} From a social point of view, all three options are costly: given that regulators behave opportunistically, the capital market will require the firm to pay a very high interest rate on loans if it decides to borrow money from external sources. Selling assets can also be costly as the firm may fail to recover their full value. This is especially so when assets are firm-specific in the sense that their value in alternative uses is lower than their value to the firm. Finally, if the government is willing to bail out the firm, it may have to raise the necessary funds by imposing distorting taxes. Thus, financial distress creates a deadweight loss. This deadweight loss may in fact be exacerbated if normal production is interrupted.

In the light of its different potential sources, the deadweight loss is assumed to be proportional to the size of the firm's loss, so whenever $z \leq z^*$, the cost of financial distress is given by $t[D + \gamma - zR(p, \gamma)]$, where $t \in [0, 1]$. Note that the sunk cost of investment, $k$, has no direct effect on either the probability or the cost of financial distress, while fixed cost, $\gamma$, has a direct effect on both. Thus, in this model, the two types of cost have a very different impact on the regulatory process and on the firm's payoff, despite the fact that neither varies with output.

Let $m \in [0, 1]$ denote the firm's share in the cost of financial distress. At one extreme, if this cost is due to a high interest rate that the firm is required to pay on loans that it takes to finance its losses, $m = 1$. At the other extreme, if the cost of financial distress represents the shadow cost of public funds, $m = 0$. When the cost of financial distress is due to a sale of assets (which may affect the quality of the firm's output), or interrupted production, $m$ is between 0 and 1. Thus, the expected ex post profit of the firm, net of the firm's share in the expected cost of financial distress is

$$
\pi(p, \gamma, D) = \pi^*(p, \gamma, D) - \gamma - mt \int_{z^*}^{\infty} dF(z) \int_{z^*}^{\infty} [D + \gamma - zR(p, \gamma)]dF(z).
$$

The function, $\pi(p, \gamma, D)$, is the combined ex post expected return to equityholders (both old and new) and debtholders and is divided between

\textsuperscript{11} The option to liquidate the firm in order to pay claimholders is not considered here because it is hard to imagine that the government would let a natural monopoly be liquidated. Ruling out the possibility of liquidation, however, is inessential for the analysis. All the results remain unchanged even if the firm is liquidated once it becomes financially distressed, as long as liquidation creates a deadweight loss.
them according to their respective claims. Expected consumer surplus, net of consumer's share in the cost of financial distress, is

\[
CS(p, \gamma, D) = \hat{z} \int_p^{\infty} Q(\xi) d\xi - \left(1 - m\right) t \int_{z^{-}}^{z^{*}(p, \gamma, D)} [D + \gamma - zR(p, \gamma)] dF(z).
\]

(3)

3. The regulatory process

The solution concept used in this paper is subgame perfect equilibrium. Consequently, the three-stage game is solved backwards by assuming that at each stage, strategies are chosen optimally given the history of the game and the (correct) anticipation of the outcomes of subsequent stages. To this end, the regulatory process that takes place in stage 3 of the game is considered first. Given the firm's cost parameter, \(\gamma\), and the way the firm financed its investment, the regulator chooses the regulated price, \(p\), with the objective of maximizing the expected sum of consumer surplus and ex post firm profits, given by

\[
W(p, \gamma, D) = CS(p, \gamma, D) + \pi(p, \gamma, D).
\]

This characterization of the regulator's objective is consistent with Peltzman's (1976) political model of regulation and also reflects the landmark Supreme Court decision in the Hope Natural Gas case, according to which, "The fixing of 'just and reasonable' rates involves a balancing of the investor's and the consumers' interests", which should result in rates which are "within a range of reasonableness". This characterization also reflects the regulatory framework that was established in Britain in the electricity, natural gas, telecommunications, and water industries following their privatization in the 80s (e.g. Vickers and Yarrow, 1988).

Note that since both \(CS(p, \gamma, D)\) and \(\pi(p, \gamma, D)\) are net of the expected cost of financial distress, the regulator explicitly takes into account these costs when choosing \(p\). This concern about financial distress is consistent with Owen and Braeutigam (1978) who argue that, "One of the worst fears of a regulatory agency is the bankruptcy of the firm it supervises, resulting in 'instability' of services to the public or wildly fluctuating prices." It is also consistent with Vickers and Yarrow (1991) who report that "Regulators of

\[\text{As explained by the Pennsylvania commission, the range of reasonableness "is bounded at one level by investor interest against confiscation and the need for averting any threat to the security for the capital embarked upon the enterprise. At the other level it is bounded by consumer interest against excessive and unreasonable charges for service." (Pennsylvania Pub. Utility Comm. v. Bell Teleph. Co. of Pennsylvania, 43 PUR3d 241, 246 (Pa., 1962)).}\]
privatized utility companies in Britain are effectively required to ensure that they do not go bankrupt."

Let \( p^* = p^*(\gamma, D) \) be the regulated price that the regulator sets in stage 3 of the game, given the cost parameter that the firm chose in stage 1, and the debt level that was issued in stage 2. Then, \( p^* \) is characterized by the following first order condition,

\[
\frac{\partial W(p^*, \gamma, D)}{\partial p} = -\hat{z}Q(p^*) + R_p(p^*, \gamma) \left[ \hat{z}t + \int_{z^-} zdF(z) \right] = 0, \tag{4}
\]

where \( \hat{z} \) is the mean of \( z \), \( Z^*(\gamma, D) = z^*(p^*, \gamma, D) \), and \( R_p(p, \gamma) = Q(p) + Q'(p)(p - c(\gamma)) \). After manipulation, Eq. (4) can be rewritten as

\[
\frac{p^* - c(\gamma)}{p^*} = \frac{1}{\eta(p^*)} \left[ \frac{Z^*(\gamma, D)}{\hat{z} + t \int_{z^-} zdF(z)} \right] \left[ \frac{\hat{z}}{Z^*(\gamma, D)} \int_{z^-} zdF(z) \right], \tag{5}
\]

where, \( \eta(p) = -Q'(p)p/Q(p) \) is the elasticity of demand. Written in this way, the optimal regulated price can be interpreted as a modified Ramsey price: the markup of the regulated price above marginal cost is proportional to the inverse of the elasticity of demand. However, unlike the traditional Ramsey price which is derived by ensuring that the firm never incurs losses, here the regulator allows the firm to become financially distressed in some states and takes the cost of this event into account. Using the definition of \( z^* \), Assumption 1, and the fact that \( p^* > c(\gamma) \), it is straightforward to show that the second order condition for \( p^* \) holds, i.e. \( \frac{\partial^2 W(p^*, \gamma, D)}{\partial p^2} W_{pp}(p^*, \gamma, D) < 0 \).

From Eq. (5) it is easy to see that the deviation from marginal cost pricing decreases with the elasticity of demand and increases with the cost of financial distress and with the probability that it occurs. The effect of a change in \( \gamma \) on the regulated price, however, is ambiguous. To see this, fix \( D \) and differentiate Eq. (4) with respect to \( p^* \) and \( \gamma \) to obtain

\[
\frac{\partial p^*}{\partial \gamma} = -\frac{1}{W_{pp}(p^*, \gamma, D)} \left\{ -Q'(p^*)c'(\gamma) \left[ \hat{z}t + \int_{z^-} zdF(z) \right] + tZ^*(\gamma, D)f(Z^*(\gamma, D))R_p(p^*, \gamma) \frac{\partial Z^*(\gamma, D)}{\partial \gamma} \right\}, \tag{6}
\]
where from the definition of $z^*$, it follows that $\frac{\partial Z^*}{\partial \gamma} = 0$ if $Z^* = z^+$ or $Z^* = z^-$, and otherwise,

$$
\frac{\partial Z^*(\gamma, D)}{\partial \gamma} = \frac{R(p^*, \gamma) + Q(p^*)c'(\gamma)[D + \gamma]}{R(p^*, \gamma)^2}
$$

or

$$
R(p^*, \gamma) = \frac{1 + Z^*(\gamma, D)Q(p^*)c'(\gamma)}{R(p^*, \gamma)}.
$$

As Eq. (6) demonstrates, an increase in $\gamma$ has two effects on the regulated price. First, it lowers the marginal cost of production, which in turn provides the regulator, who moves after the firm, with an incentive to pass part of the resulting benefits to consumers by lowering the regulated price. This incentive is represented by the first term on the right-hand side of (6). Second, an increase in $\gamma$ affects the probability of financial distress. The regulator, in turn, adjusts the regulated price in response to this effect. But, as Eq. (7) indicates, this second effect can be either positive or negative, so in general one cannot determine the sign of $\frac{\partial p^*}{\partial \gamma}$ unambiguously.

This section is concluded by establishing the first-best solution as a benchmark. To this end, suppose that the regulated price, the cost parameter, $\gamma$, and the mode of financing are all chosen by a benevolent social planner whose objective is to maximize $W(p, \gamma, D)$. Clearly, since $D$ affects social welfare only through its impact on the probability and cost of financial distress, both of which are increasing in $D$, $W(p, \gamma, D)$ is maximized when $D = 0$. Thus, the first-best regulated price is $p^*_F = p^*(\gamma^*_F, 0)$, i.e. the modified Ramsey price evaluated at $D = 0$ and at the first-best cost parameter, $\gamma^*_F$. The latter can be found by maximizing $W(p^*_F, \gamma, 0)$ with respect to $\gamma$. Using the definition of $z^*$, the first order condition for $\gamma^*_F$, is:

$$
\frac{\partial W(p^*_F, \gamma^*_F, 0)}{\partial \gamma} = -Q(p^*_F)c'(\gamma^*_F) \left[ z^*_F(\gamma^*_F, 0) \right] + Q(p^*_F)c'(\gamma^*_F) \int_{z^-}^{z^*_F} zf(z) \, dz - \left[ 1 + tF(Z^*(\gamma^*_F, 0)) \right] = 0.
$$

The existence of an interior solution for $\gamma^*_F$ is ensured by Assumption 3. Eq. (8) shows that $\gamma^*_F$ is chosen by trading off variable cost (evaluated at $Q(p^*_F)$) and fixed cost.

4. The choice of technology under all-equity financing

In this section the firm is assumed to finance $k$ entirely with equity. Recalling that the firm has no outstanding debt to begin with, this assumption implies that $D = 0$ so that the firm is all-equity. This case is a
natural starting point for the analysis because the assumption that the firm is all-equity is implicitly made in virtually all the literature on rate regulation. Optimal financing and its implications for the firm's technology are examined in Section 5.

Let \( p^E \) denote the regulated price when the firm is all-equity. \( p^E \) is the modified Ramsey price evaluated at \( D = 0 \) and \( \gamma^E \), i.e. \( p^E = p^*(\gamma^E, 0) \). Now, consider stage 2 of the game, at which the firm issues equity to outsiders to raise \$k. Assuming that the capital market is competitive, new equityholders earn an expected return equal to the risk-free interest rate, which without loss of generality is normalized to 0. Thus, \( E^*(\alpha) = \alpha \pi(p^E, \gamma, 0) \). But, in equilibrium, the firm's budget constraint, given by Eq. (1), must be satisfied, so \( k = E^*(\alpha) \). Thus, in order to raise \$k, the firm has to give outsiders an equity participation of

\[
\alpha^* = \frac{k}{\pi(p^E, \gamma, 0)}. \tag{9}
\]

Anticipating the outcome of the regulatory process and the equilibrium in the capital market, the original owners of the firm choose in stage 1 of the game a cost parameter, \( \gamma^E \), to maximize their expected payoff, given by \( Y(\gamma, \alpha^*, 0) = (1 - \alpha^*)\pi(p^E, \gamma, 0) \). Using Eqs. (2) and (9), this expected payoff can be written as:

\[
Y(\gamma, 0) = zR(p^E, \gamma) - \gamma - mt \int_{z^-} [\gamma - zR(p^E, \gamma)]dF(z). \tag{10}
\]

Using the definition of \( z^* \), the first order condition for \( \gamma^E \) is

\[
\frac{\partial Y(\gamma^E, 0)}{\partial \gamma} = \left[ R_r(p^E, \gamma^E)\frac{\partial p^E}{\partial \gamma} - Q(p^E)c'(\gamma^E) \right] \\
\times \left[ z^*(\gamma^E, 0) \right] \\
\times \left[ \hat{z} + mt \int_{z^-} zdF(z) \right] - \left[ 1 + mtF(Z^*(\gamma^E, 0)) \right] \leq 0; \tag{11a}
\]

\[
\gamma^E \frac{\partial Y(\gamma^E, 0)}{\partial \gamma} = 0, \tag{11b}
\]

where \( \partial p^E/\partial \gamma = \partial p^*(\gamma, 0)/\partial \gamma \) is given by (6) (evaluated at \( D = 0 \)). Note that a marginal change in \( \gamma \) has two effects on \( Y(\gamma, 0) \). The first is an indirect effect due to an adjustment in the regulated price induced by a change in \( \gamma \). The second is a direct effect due to a change in the firm's cost structure and it has two components. The first component, given by the \( Q(p^E)c'(\gamma^E) \)
term, represents the decrease in variable cost. The second component, given by \(1 + mtF(\cdot)\), represents the increase in fixed cost. The solution for \(\gamma^E\) is characterized by the following proposition.

**Proposition 1.** Assume that the firm is all-equity and \(Y(\gamma, 0)\) is strictly concave in \(\gamma\). Then, the firm selects a technology with no fixed costs, i.e. \(\gamma^E = 0\). Since \(\gamma^E > 0\), the firm’s technology is inefficient.

**Proof.** See the Appendix.

Proposition 1 shows that regulatory opportunism induces the firm, when it is all-equity, to select a technology that has no fixed costs and higher than optimal marginal costs. The reason for this distortion is the following. When the firm is all-equity, the regulator has a strong incentive to pass the benefits from a reduction in marginal costs to consumers by lowering the regulated price. Since a reduction in fixed costs does not create a similar incentive, the firm prefers to lower its fixed costs as much as possible, so it selects a technology with zero fixed costs.\(^{13}\)

Since the equilibrium regulated price is always larger or equal to marginal cost and since the firm has no debt and no fixed costs, the firm always generates a non-negative cash flow. Thus,

**Proposition 2.** An all-equity regulated firm never becomes financially distressed, i.e. \(Z^E(0, 0) = z^-\).

Given the result of Proposition 2, Eq. (5) shows that when the firm is all-equity, the regulator uses marginal cost pricing, i.e. \(p^E = c(0)\). Hence, the payoff of the original equityholders is \(Y(0, 0) = -k\). Consequently, the firm will not invest at all, unless the government is willing to subsidize investment, or finds a way to commit to a regulated price which exceeds

\(^{13}\)To illustrate Proposition 1, consider the case where \(Q = 100 - p\), \(c = 1/\gamma\), and \(z\) is distributed uniformly over the interval \([0,2]\). Substituting in (4), \(p^E\) is defined implicitly by

\[
p^E = \frac{1}{\gamma} + \frac{\gamma^3(A - \gamma Q(p^E))}{4A^2Q(p^E)^2},
\]

where \(A = \gamma p^E - 1\). Differentiating this expression with respect to \(p^E\) and \(\gamma\) yields,

\[
\frac{dp^E}{d\gamma} = -\frac{1}{2\gamma^2} \left[ \frac{4A^3Q(p^E)^3 - \gamma^4(3(A - 1) - 2\gamma Ap^E - 2\gamma^3(A - 1)Q(p^E)^2)}{2A^3Q(p^E)^3 - \gamma^4AQ(p^E)^2 + \gamma^3A^2} \right].
\]

As \(\gamma \to 0\), the expression in the square brackets approaches 2, so \(\frac{dp^E}{d\gamma} \to -1/\gamma^2 = c'\). Thus, the benefit from lowering marginal costs is completely passed on to consumers, so the firm is not compensated for the associated increase in fixed costs. Hence, the firm will select a technology with \(\gamma = 0\).
marginal cost. In the next section, it will be shown that by allowing the firm to issue debt, regulators are able to implicitly make such a commitment.

5. The choice of technology under optimal financing

This section characterizes the optimal financial strategy of the firm and examines its implication for the choice of technology. It is shown that at the optimum, contrary to the assumption of Section 4 (and the implicit assumption in most of the rate regulation literature), the firm finances its investment, at least partially, with debt and the resulting capital structure has a positive effect on the firm's choice of technology.

5.1. Optimal financial strategy

Let \( (\alpha^L, D^L) \) be the optimal financial strategy of the firm and let \( \gamma^L \) be the cost parameter that the firm chooses under such a strategy. The pair \( \gamma^L \) and \( D^L \) induces the regulator to set in stage 3 of the game a regulated price \( p^L = p^*(\gamma^L, D^L) \). Anticipating this price, the firm issues in stage 2 new equity and debt to outsiders to raise \$k. Since by assumption the firm always fulfills its financial obligations, debt is completely riskless, so \( B(D) = D \).

This however does not imply that debt is costless from the firm's perspective: since debt adds to the financial obligations of the firm, it increases the likelihood of financial distress and therefore raises the expected cost of this event. Since the capital market is assumed to be competitive, both new equityholders and debtholders earn a zero net expected return on their investment. Using the firm's budget constraint given by Eq. (1), the equilibrium in the capital market is characterized by

\[
k = E^*(\alpha) + B^*(D) = \alpha \pi(p^L, \gamma^L, D^L) + D.
\] (12)

The left-hand side of (12) is the equilibrium market value of the firm's securities, which due to the firm's budget constraint, exactly covers the cost of the project. The first term on the right-hand side of (12) represents the share of new equityholders in the firm's expected ex post profits. The second term on the right-hand side of the equation is the payoff of debtholders that equals the face value of their claim. The financial strategy of the firm is fully characterized by a pair \( (\alpha, D) \) that satisfies (12).

Anticipating the outcome of the regulatory process and the equilibrium in the capital market and given \( \gamma \), the original owners of the firm choose a pair \( (\alpha, D) \) to maximize their expected payoff given by \( Y(\gamma, \alpha, D) = (1 - \alpha) \pi(\gamma, \alpha, D) \). Substituting for \( \alpha \) from Eq. (12), the original owner's expected payoff becomes
Thus, the optimal financial strategy of the firm becomes one of choosing an optimal debt level, $D^L$. Using the definition of $z^*$, the first order condition for an interior solution for $D^L$ is

$$
\frac{\partial Y(\gamma, D^L)}{\partial D} = R_p(p^L, \gamma) \left[ \hat{z} + mt \int_{z^*}^{\infty} z dF(z) \right] 
\times \frac{\partial p^L}{\partial D} - mtF(Z^*(\gamma, D^L)) = 0,
$$

(14)

where differentiating Eq. (4) with respect to $p^L$ and $D$ and using the definition of $z^*$ reveals that

$$
\frac{\partial p^L}{\partial D} = tZ^*_{(\gamma, D^L)} \frac{R_p(p^L, \gamma) f(Z^*(\gamma, D^L))}{-R(p^L, \gamma) W_{pp}(p^L, \gamma, D)} > 0.
$$

(15)

The reason why the last expression is positive is that $R_p(p^L, \gamma) > 0$ by Eq. (4), $W_{pp}(p^L, \gamma, D^L) < 0$ and $Z^* > z^* > 0$. The first term on the right-hand side of (14) represents the marginal benefit of debt from the firm's perspective. This benefit arises because an increase in debt leads to an increase in the regulated price, which in turn, since $R_p(p^L, \gamma) > 0$, leads to an increase in the operating income of the firm. The second term on the right-hand side of (14) represents the marginal cost of debt and is due to the increase in the expected cost of financial distress. At an interior optimum, the two terms must be equal. The following proposition shows that (14) indeed has an interior solution.

**Proposition 3.** The optimal level of debt is such that $0 < D^L < z^* R(p^L, \gamma) - \gamma$. Consequently, in equilibrium, $Z^* < z^*$, so the equilibrium probability of financial distress is strictly less than one. Moreover, $D^L$ decreases with the firm's share in the cost of financial distress, i.e. $\partial D^L/\partial m < 0$.

**Proof.** See the Appendix.

Proposition 3 shows that financing investment with some debt (but not too much) always improves on the payoff of the original owners of the firm. In equilibrium, the firm issues debt with face value $\min\{D^L, k\}$. In order to simplify the analysis it is assumed henceforth that $k$ is large enough so that
$D^L \leq k$ (but not too large to render the entire project unprofitable). Equity, then, is issued by the firm to finance $k - D^L$, which is the difference between the cost of investment and the amount raised by issuing debt. The firm issues a positive amount of debt in this model because, at least for small amounts of debt, the benefits associated with the increase in the regulated price exceed the firm's share in the cost of financial distress. Of course, in reality, the firm is likely to issue debt for many additional reasons, such as its effect on taxes, its ability to signal private information, its effect on agency costs and for corporate control reasons. Proposition 3 shows that even when all these reasons are absent, rate regulation is sufficient to induce the firm to issue debt.

Another implication of Proposition 3 is that for a given cost parameter, $p^{fb} < p^L$. The reason for this is that from Eq. (15) it follows that the regulated price increases with the firm debt. Since at the first-best $D = 0$, while in equilibrium $D^L > 0$, the result follows.

5.2. The choice of technology

Given its optimal financial strategy, the firm chooses in stage 1 of the game a cost parameter, $\gamma^L$, to maximize $Y(\gamma, D^L)$. Using the envelope theorem and the definition of $z^*$, the first order condition for $\gamma^L$ is

$$\frac{\partial Y(\gamma^L, D^L)}{\partial \gamma} = \left[ R_p(p^L, \gamma^L) \frac{\partial p^L}{\partial \gamma} - Q(p^L)c'(\gamma^L) \right]$$

$$\times \left[ \frac{\partial^2 p^L}{\partial \gamma^L} - Q(p^L)c'(\gamma^L) \right] - \left[ 1 + mtF(z^*(\gamma^L, D^L))\right] = 0,$$

(16)

where the expression $\frac{\partial p^L}{\partial \gamma} = \frac{\partial p^*(\gamma, D^L)}{\partial \gamma}$ is given by (6). The existence of an interior solution for $\gamma$ is shown in the Appendix. Eq. (16) has a similar interpretation to Eq. (11): a change in $\gamma$ affects both the regulated price and the firm's cost structure. The former effect is represented by the argument $R_p(p^L, \gamma^L)\frac{\partial p^L}{\partial \gamma}$, while the rest of the expression represents the latter effect. The next proposition offers a comparison between $\gamma^L$ and $\gamma^E$ and $\gamma^{fb}$.

**Proposition 4.** Assume that $Y(\gamma, D^L)$ is strictly concave in $\gamma$. Then, under optimal financing, the firm selects a technology with lower marginal costs and higher fixed costs than under all-equity financing. Assuming that at the

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14 For an excellent survey of the literature on capital structure, see Harris and Raviv (1991).

15 Dasgupta and Nanda (1993) prove a similar result. In their model, debt improves the bargaining position of the firm vis-à-vis consumers. In choosing an optimal debt level, the firm trades off this benefit against the associated increase in the expected cost of bankruptcy.
first-best, the impact of the cost parameter, \( \gamma \), on the reduction of the firm's variable costs, evaluated at the mean output, is less than one, i.e. 

\[-\hat{Q} (p^{fb}) c'(\gamma^{fb}) < 1\]

the equilibrium level of fixed costs is still below the first-best level.

**Proof.** See the Appendix.

The intuition behind Proposition 4 is straightforward. A leveraged regulated firm is allowed to charge a price in excess of its marginal cost, and as a result, it extracts some (but not all) of the benefits from reducing its marginal costs. In contrast, an all-equity firm does not extract any benefits from having such a reduction. Consequently, a leveraged firm chooses a technology with lower marginal costs than an all-equity firm. At the same time, given the assumption in the proposition, the firm does not extract all the social benefits from reducing its marginal costs while still bearing the entire costs of this reduction, so as a result, it chooses a technology with too high marginal costs. Since debt alleviates the distortion in the choice of technology it may be welfare-improving provided that the social benefits from the reduction in marginal costs outweigh the cost of the increase in the probability of financial distress and the loss in consumers' surplus resulting from the increase in the regulated price. Moreover, since \( \partial Y(\gamma, D)/\partial D > 0 \) (for small enough \( D \)), equityholders' payoff is larger than in the case of all-equity financing, so the firm may take investments that an all-equity firm would forgo.

6. Goldplating

Thus far, the regulated firm's choice of technology and its cost structure were examined by looking at the trade-off between fixed and marginal costs. This section examines another aspect of this choice, namely goldplating. This practice arises when a regulated firm inflates its costs deliberately, i.e. goldplates, by wasting resources or even by colluding with equipment suppliers, with the intention of inducing regulators to increase the regulated prices.

To examine the possibility of goldplating in the current model, suppose that \( c(\gamma) = c \) for all \( \gamma \). Given this assumption, \( \gamma \) can be viewed as goldplating: it increases fixed costs without lowering marginal costs. For example, \( \gamma \) may represent the cost of renting and maintaining luxurious offices, the cost of hiring too many employees, or excessive expenditure on R&D. Obviously, in the first-best solution, \( \gamma = 0 \). The reason why the firm may wish to choose a positive \( \gamma \) is that it makes it more susceptible to financial distress, thereby inducing the regulator to increase the regulated prices.
price (note from (6) that \( c'(\gamma) = 0 \) implies \( \partial p^*/\partial \gamma > 0 \)). Such an increase in \( p^* \) may more than compensate the firm for expending \( \gamma \).

Let \( \gamma^* \) be the equilibrium choice of goldplating. Since \( c \) is a constant, \( R(p, \gamma) \) is independent of \( \gamma \), i.e. \( R(p, \gamma) = R(p) \). Hence, \( \gamma \) affects the regulated price only through \( y^* \), exactly like debt. But, from (2) it follows that \( \partial z^*/\partial D = \partial z^*/\partial y = 1/R(p) \). Thus, it is clear that \( \partial p^L/\partial D = \partial p^L/\partial \gamma \).

Using this equality, it is easy to show that the first order condition for \( \gamma^* \) is

\[
\frac{\partial Y(\gamma^*, D)}{\partial \gamma} = \frac{\partial Y(\gamma^*, D)}{\partial D} - 1 \leq 0; \quad \gamma^* \frac{\partial Y(\gamma^*, D)}{\partial \gamma} = 0.
\]  

(17)

Now, consider an optimally leveraged firm. For such a firm, \( \partial Y(\gamma^*, D^L)/\partial D = 0 \) for all \( \gamma \), so \( \partial Y(\gamma^*, D^L)/\partial \gamma = -1 \), implying that \( \gamma^* = 0 \). Thus, an optimally leveraged firm never goldplates. Next, consider a regulated firm with a less than optimal debt level. From (17) it follows that a sufficient condition for such a firm to goldplate is \( \partial Y(0, D)/\partial D > 1 \). Thus,

**Proposition 5.** A regulated firm with an optimal capital structure never goldplates. In contrast, a firm with a suboptimal debt level may goldplate if \( \partial Y(0, D)/\partial D > 1 \).

The intuition behind Proposition 5 is straightforward. In order to induce the regulator to increase the regulated price, a regulated firm can either issue debt or goldplate. Debt, however, is preferable to goldplating because it is not wasteful: debtholders are buying the firm’s debt for \( B(D) \) which is part of the equityholders’ payoff. Thus, a regulated firm with an optimal capital structure does not need to goldplate. A regulated firm with too little debt in contrast may goldplate, provided that the increase in the regulated price outweighs the loss from wasting resources.

Finally, consider an all-equity firm. If \( \gamma = 0 \), the firm has no fixed obligations, so it never becomes financially distressed. Consequently, the regulator sets \( p^E = c \), so \( R(p^E) = 0 \). Substituting in (15), this yields \( \partial p^L/\partial D = \infty \). But, since \( \partial p^L/\partial \gamma = \partial p^L/\partial D \), this implies that an all-equity firm always finds it profitable to use some goldplating.

7. Conclusion

The choice of technology by regulated firms has been examined using a sequential game between a regulated firm and a regulator. The main insight of the paper is that the inability of the regulator to precommit to a particular regulated price before the firm makes an irreversible investment decision may induce the firm to select a technology with higher marginal cost and lower fixed cost than is socially optimal. This distortion arises because the
regulated price is chosen by the regulator to maximize welfare and is therefore decreasing in marginal cost and is unaffected by the level of fixed cost.

The distortion in the choice of technology is alleviated in the current model when the firm is leveraged. In this case, the firm is more likely to become financially distressed, so the regulated price no longer decreases with marginal cost by as much as in the case of an all-equity firm. Consequently, a technology with a low marginal cost becomes more attractive to the firm. Although the distortion is alleviated, it is not solved completely: given that the marginal costs are not decreasing too rapidly, the firm will still select a technology with higher than optimal marginal costs. Since debt leads to a higher regulated price, it may induce the firm to invest even if investment involves sunk cost. Thus, debt may be welfare-improving.

Finally, this paper examines the issue of goldplating, i.e. the possibility that regulated firms may waste resources in order to induce regulators to increase prices. It is shown that although regulated firms with low debt levels may be tempted to goldplate, they never wish to do so in the current model when their capital structures are optimal. This is because issuing debt induces regulators to increase prices just like goldplating, but has the advantage from the regulated firm's perspective of not being wasteful.

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Appendix

A.1. Proof of Proposition 1

Evaluate $\frac{\partial Y(\gamma,0)}{\partial \gamma}$ at $\gamma = 0$. Then, since $D = 0$, it follows from the definition of $z^*$ that $z^* = z^-$ (i.e. the firm never becomes financially distressed). Therefore,

$$\frac{\partial Y(0,0)}{\partial \gamma} = \ddot{z} \left[ R_p(p^E,0)\frac{\partial p^E}{\partial \gamma} - Q(p^E)c'(0) \right] - 1.$$  \quad (A1)

But, as (5) shows, when $z^* = z^-$, $p^E = c(\gamma)$. Hence, $R_p(p^E,0) = Q(p^E)$. Substituting in (A1) yields
\[
\frac{\partial Y(0,0)}{\partial \gamma} = \dot{z}Q(p^E)\left[\frac{\partial p^E}{\partial \gamma} - c'(0)\right] - 1. \tag{A2}
\]

It now remains to evaluate \(\partial p^E / \partial \gamma\) at \(\gamma = 0\). Substituting for \(z^* = z^-\) and 
\(p^E = c(0)\) in (6), and recalling that \(\partial z^- / \partial \gamma = 0\), the numerator of the expression equals \(\dot{z}Q(p^E)c'(0)\). As for the denominator, differentiating (4) with respect to \(p\), evaluating at \(D = \gamma = 0\) and \(p = p^E\), and using the fact that \(z^* = z^-\) and \(p^E = c(0)\) yields

\[
W_{pp}(p^E,0,0) = \dot{z}Q(p^E) + [2Q'(p^E) + Q''(p^E)(p^E - c(0))]
\times [\dot{z} + t \int z^* \frac{z}{z^*} f(z) + tZ^* f(Z^*)R_p(p^E,0) \frac{\partial Z^*}{\partial \gamma} = \dot{z}Q(p^E). \tag{A3}
\]

Thus, at \(\gamma = 0\), the denominator of (6) equals \(-\dot{z}Q(p^E)\), implying that at \(\gamma = 0\), \(\partial p^E / \partial \gamma = c'(0)\). Now, (A2) shows that \(\partial Y(0,0) / \partial \gamma = -1\), so by the strict concavity of \(Y(y,0)\) in \(\gamma\), \(\gamma^E = 0\). \(\square\)

A.2. Proof of Proposition 3

To prove that \(D^L > 0\), assume by way of negation that the firm uses all-equity financing. Then, by Proposition 2, \(F(Z^*) = 0\), so the second term on the left-hand side of (14) vanishes. Since the first term is positive for all \(D\), then \(\partial Y(\gamma,0) / \partial D = 0\), a contradiction to the optimality of \(D = 0\). To prove that \(D^L < z^+\), notice that the definition of \(Z^*\) implies that otherwise, \(Z^* = z^+\). In this case, \(\partial p^L / \partial D = 0\), so \(\partial Y(\gamma, D^L) / \partial D = -mt < 0\). Therefore, the firm never issues debt to the point where \(D^L \leq z^+R(p^L, \gamma) - \gamma\). As a result, in equilibrium, \(Z^* < z^+\), so \(F(Z^*) < 1\). Finally, note that the cost of debt, \(mtF(Z^*)\), increases with \(m\), while the benefit of debt is independent of \(m\). Consequently, \(\partial D^L / \partial m < 0\). \(\square\)

A.3. Proof that \(0 < \gamma^L < \infty\)

Substituting from (6) and (7) into \(\partial Y(\gamma, D^L) / \partial \gamma\) yields,

\[
\frac{\partial Y(\gamma, D^L)}{\partial \gamma} = \left\{ -R_p(p^L, \gamma)Q'(p^L)c'(\gamma)\left[\dot{z} + t \int z^* \frac{z}{z^*} f(z) \right] + tZ^* f(Z^*)R_p^2(p^L, \gamma) \frac{1 + Z^*Q(p^L)c'(\gamma)}{R(p^L, \gamma)} \right\} H - [1 + mtF(Z^*)], \tag{A4}
\]
where \( Z^* = Z^*(\gamma, D^L) \), and

\[
\begin{align*}
\dot{Z}^* + mt \int_{z^-} z F(z) \\
H = \frac{-W_{pp}(p^L, \gamma)}{R(p^L, \gamma)} > 0 \quad \text{(A5)}
\end{align*}
\]

Differentiating (4) with respect to \( p \) and evaluating at \( p^L \) and \( D^L \),

\[
W_{pp}(p^L, \gamma, D^L) = -\dot{Z} Q'(p^L) + [2Q'(p^L) + Q''(p^L)(p^L - c(\gamma))]
\times \left[ \dot{Z} + t \int_{z^-} z F(z) \right] - \frac{t f(Z^*)(Z^* R_p(p^L, \gamma))^2}{R(p^L, \gamma)} . \quad \text{(A6)}
\]

Substituting from (A6) into (A4) and simplifying terms,

\[
\frac{\partial Y(\gamma, D^L)}{\partial \gamma} = \left\{ -c'(\gamma)\left[(Q'(p^L))^2 - Q'(p^L)Q''(p^L)\right](p^L - c(\gamma)) \times \int_{z^-} z F(z) + c'(\gamma)Q(p^L)Q'(p^L)t \times \int_{z^-} z F(z) + \frac{t Z^* f(Z^*)R_p^2(p^L, \gamma)}{R(p^L, \gamma)} H - [1 + mt F(Z^*)] \right\} . \quad \text{(A7)}
\]

Now, from (14) it follows that

\[
mt F(Z^*) = \frac{t Z^* f(Z^*)R_p^2(p^L, \gamma)}{R(p^L, \gamma)} . \quad \text{(A8)}
\]

Substituting in (A7) and simplifying

\[
\frac{\partial Y(\gamma, D^L)}{\partial \gamma} = -c'(\gamma)\left\{ [(Q'(p^L))^2 - Q'(p^L)Q''(p^L)\right](p^L - c(\gamma)) \times \int_{z^-} z F(z) - Q(p^L)Q'(p^L)t \int_{z^-} z F(z) \right\} H - 1 . \quad \text{(A9)}
\]

Assumption 1 ensures that the coefficient of \( c'(\gamma) \) is positive. Hence, from Assumption 3 it follows that \( \lim_{\gamma \to 0} \frac{\partial Y(\gamma, D^L)}{\partial \gamma} = \infty \), implying that \( \gamma^L > 0 \). Assumption 3 also implies that \( \lim_{\gamma \to \infty} \frac{\partial Y(\gamma, D^L)}{\partial \gamma} = -1 \), implying that \( \gamma^L < \infty \). \( \square \)
A.4. Proof of Proposition 4

To prove the first part of the proposition, note that $\gamma^L > 0$, while Proposition 1 shows that $\gamma^E = 0$. To prove the second part of the proposition, substitute for $W_{pp}(p^L, \gamma, D^L)$ from the first order condition for $D^L$ in (14) into (6), substitute back into (16) and rearrange terms to obtain

$$
\frac{\partial Y(\gamma^L, D^L)}{\partial \gamma} = \frac{-Q'(p^L)c'(\gamma^L) \left[ \dot{z} + t \int z dF(z) R(p^L, \gamma^L)mF(Z^*(\gamma^L, D^L)) \right]}{R_p(p^L, \gamma^L)Z^*(\gamma^L, D^L)f(Z^*(\gamma^L, D^L))} - Q(p^L)c'(\gamma^L) \left[ \dot{z} + mt \int (z - Z^*(\gamma^L, D^L)) dF(z) \right] - 1
$$

$$
= 0. \quad (A10)
$$

Now, evaluate $\frac{\partial Y(\gamma, D^L)}{\partial \gamma}$ at $\gamma = \gamma^b$. Using (8),

$$
\frac{\partial Y(\gamma^b, D^L)}{\partial \gamma} = \frac{-Q'(p^L)c'(\gamma^b) \left[ \dot{z} + t \int z dF(z) R(p^L, \gamma^b)mF(Z^*(\gamma^b, D^L)) \right]}{R_p(p^L, \gamma^b)Z^*(\gamma^b, D^L)f(Z^*(\gamma^b, D^L))} - Q(p^L)c'(\gamma^b) \left[ \dot{z} + mt \int (z - Z^*(\gamma^b, D^L)) dF(z) \right]
$$

$$
+ t \left[ Q(p^L)c'(\gamma^b) \int z dF(z) + F(Z^*(\gamma^b, 0)) \right] + \dot{z}c'(\gamma^b)[Q(p^b) - Q(p^L)]. \quad (A11)
$$

The first two terms on the right-hand side of (A11) are negative because $c'(\gamma) < 0$ and because (4) implies that $R_p(p^L, \gamma) > 0$. The third term is also negative because by (8) it equals $-Q(p^b)c'(\gamma^b) + 1$, which given the assumption in the proposition, is negative. Finally, as argued in the last section, $p^b < p^L$ for all $\gamma$, so that $Q(p^b) > Q(p^L)$, implying that the last term is also negative. Since $Y(\gamma, D^L)$ is strictly concave in $\gamma$, $\gamma^b > \gamma^L$. □
References