

Optimal state-contingent regulation under limited liability

Robert Gary-Bobo*

and

Yossi Spiegel**

We consider an optimal regulation model in which the regulated firm's production cost is subject to random, publicly observable shocks. The distribution of these shocks is correlated with the firm's cost type, which is private information. The regulator designs an incentive-compatible regulatory scheme, which adjusts itself automatically ex post given the realization of the cost shock. We derive the optimal scheme, assuming that there is an upper bound on the financial losses that the firm can sustain in any given state. We first consider a two-type, two-state case, and then extend the results to the case of a continuum of firm types and an arbitrary finite number of states. We show that the first-best allocation can be implemented if the state of nature conveys enough information about the firm's type and/or the maximal loss that the firm can sustain is sufficiently large. Otherwise, the solution is characterized by classical second-best features.

1. Introduction

■ The optimal regulation literature has seen major developments in the past twenty-five years. This literature has mainly focused on the issue of how to regulate a firm when it has private information about its demand and cost functions. In practice, however, regulated rates are set for an extended period, typically a few years. During this period, the demand and cost conditions may be subject to random shocks. Therefore, it is important to design flexible regulatory mechanisms that can respond to these shocks. In particular, when regulated rates are not appropriately adjusted following negative external shocks, the firm may incur a large deficit that, due to limited liability, it may be unable to sustain. In this article we consider the design of an optimal regulatory mechanism that responds to *ex post* cost shocks and takes explicit account of the firm's limited ability to sustain losses.

Our analysis differs from classical optimal regulation theory (e.g., Baron and Myerson, 1982; Laffont and Tirole, 1986; and Lewis and Sappington, 1988) in three respects. First, we explicitly consider limited liability constraints: the *ex post* profit of the firm cannot fall below some given

* Panthéon-Sorbonne Economie and Paris School of Economics, Université Paris 1; garybobo@univ-paris1.fr.

** Tel Aviv University; spiegel@post.tau.ac.il.

We thank F. Forges, C. Gollier, P. Picard, J. Pouyet, J.-C. Rochet, B. Salanié, Jim Hosek (the Editor), two anonymous referees, and audiences in Munich, Stockholm, Milan, Paris, and Evanston for their help and comments.

(possibly negative) level.¹ Second, a publicly observable signal conveys information about the firm's hidden cost type and is used by the regulator to set the regulated rate. Finally, the public signal is a real cost shock that affects the firm's cost directly; consequently, the optimal production level of the firm is signal dependent. In this setting, the regulator designs an incentive-compatible regulatory scheme that adjusts itself automatically *ex post* given the realization of the publicly observed cost shock, before the firm produces. This regulatory scheme can be thought of as an "indexed" or state-contingent incentive scheme: the regulator does not have to redesign it after the realization of each cost shock.

Examples for the type of public cost shocks that we have in mind include equipment failures and fluctuations in input prices (e.g., fuel prices in the case of electric utilities). A large number of equipment failures may indicate that the firm's technology, and hence its unobserved cost type, is inefficient. The correlation between observed input prices and the firm's unobservable cost type may reflect the fact that the firm chooses its technology *ex ante* on the basis of its input price forecasts. If *ex ante* technological choices are relatively inflexible, the *ex post* realization of input prices will be correlated with the firm's technology, although due to forecasting errors, the correlation is bound to be imperfect.²

The optimal regulatory mechanism depends in our setting on the degree of correlation between the public signal and the firm's hidden cost type, as well as on the maximal deficit that the regulated firm can sustain. We find that whenever the realization of the random cost shocks conveys enough information about the firm's type and/or the maximal deficit that the firm can sustain in any given state is sufficiently large, the optimal regulatory scheme implements the first-best allocation, despite the fact that the firm's type, which determines the distribution of its costs in the various states of nature, is private information. By contrast, if the first-best allocation cannot be implemented, the solution is characterized by classical second-best features, i.e., the production levels of inefficient types are distorted downward to reduce the expected cost of informational rents, there is no distortion "at the top," and there is no (expected) rent "at the bottom." We obtain these results first in a simple two-type, two-state case, and then extend them to the case of a continuum of firm types and an arbitrarily large, but finite, number of states of nature.

The idea that regulators can exploit the correlation between *ex post* public signals and the firm's type and design signal-dependent transfers that implement the first-best allocation was first explored by Riordan and Sappington (1988). Their results are analogous to those of Crémer and McLean (1985, 1988) in the context of auction theory—the main difference being that in the context of auctions, the reports of other bidders play the role that the *ex post* public signals play in the Riordan and Sappington model. Our article differs from Riordan and Sappington in that the *ex post* signals are purely informational in their model, whereas in our model they are real cost shocks that affect not only the firm's transfers but also its cost and, hence, its output. More important, Riordan and Sappington's methodology allows them to prove that the first-best solution can be implemented under certain conditions, but it does not provide a characterization of the optimal regulatory scheme. In contrast, we fully characterize the optimal regulatory scheme, both when the first-best solution can be implemented and when it cannot.

It has long been recognized that limited liability constraints are important to assess the robustness of the first-best implementation results of Crémer and McLean (1985, 1988) and Riordan and Sappington (1988). Robert (1991) considers an auction problem in which each bidder can have finitely many possible types, but the types of different bidders are correlated. He shows that if there are upper bounds on the payments that the bidders can make, then the auctioneer may not be able to extract the full surplus from each bidder, as in Crémer and McLean (1988). Kosmopoulou and Williams (1998) consider a related model of group decision making

¹ The limited liability constraints could reflect the fact that if the firm makes sufficiently large losses, then it becomes financially distressed. The regulator wishes to avoid this situation, both because it is very costly and may lead to a disruption of the firm's normal operations, and because this situation is embarrassing to the regulator. For further discussion on this issue, see Spiegel and Spulber (1994, 1997).

² See the Appendix for a detailed example that illustrates this point.

with a continuum of agents' types. They show that it is impossible to implement the first-best allocation if agents' types are approximately independent and either the monetary transfers among agents, or their *ex post* payoffs, are subject to limited liability constraints.³

Closer to our article, Demougin and Garvie (1991) were the first to study optimal regulation with a continuum of firm types, correlated information, and limited liability constraints. We extend their analysis in several ways. First, as in Riordan and Sappington (1988), the signals in Demougin and Garvie are purely informational. Hence, in their article the firm's output is independent of the signals, whereas in ours it is state-contingent. Second, Demougin and Garvie consider either the case where the firm must earn nonnegative profits in each state of nature, or the case in which the regulator must use nonnegative transfers in every state. By contrast, in our model, the maximal loss that the firm can sustain in each state of nature is a parameter. In particular, we characterize the solution to the regulator's problem for various levels of this parameter. Third, the signal in Demougin and Garvie is binary, whereas we consider an arbitrarily large (but finite) number of states of nature. One of our contributions is to show that in order to implement the first-best solution, the regulator should use transfers that reward the firm in exactly one state and impose the same (minimal) punishment on the firm in all other states. Finally, while Demougin and Garvie rely on constrained calculus of variations techniques, our approach has the advantage of building on the by-now familiar and relatively simple methodology of Baron and Myerson (1982), which we adapt to the case of *ex post* cost shocks and limited liability constraints.

The rest of the article is organized as follows. In Section 2 we present the simple case of two types and two states. In Section 3 we study the case of a continuum of types and multiple signals. Section 4 concludes. The proofs are relegated to the Appendix.

2. The two-type, two-state case

■ Consider a regulated firm that produces a single product. The consumers' utility is

$$U(q, t) = S(q) - t, \quad (1)$$

where q is the firm's output and t is the total transfer made to the firm. We assume that $S(\cdot)$ is twice continuously differentiable, strictly increasing, and concave. The function $S(\cdot)$ and the transfer t can have at least two interpretations. If the regulator procures a public good from the firm, then q is simply the size or the quality of the public good, $S(q)$ is the gross aggregate utility that consumers derive from the public good, and t is the amount paid to the firm out of the state's budget. If the firm is a regulated monopoly producing a private good, then $S(q) = \int_0^q P(\xi)d\xi$ is the gross consumers' surplus, and $P(\cdot)$ is the inverse-demand function for the good. In that case, $t = P(q)q + A$ is the regulated firm's revenue, where $P(q)q$ is the aggregate sum of the usage fees that consumers pay, and A is either a subsidy paid to the firm out of the state's budget, or the aggregate sum of fixed fees paid by consumers.

The firm's cost of production depends on two separate, but correlated, factors: (i) the realization of a random, publicly observed state of nature (i.e., cost shock), s , and (ii) the firm's type, θ , which is private information for the firm. For instance, s could represent the number of costly equipment failures that the firm experiences, and θ could represent the efficiency of the firm's technology. A large number of equipment failures may then indicate that the firm's technology is inefficient. Alternatively, s could represent the observable components of the cost function, while θ could represent the unobserved components. In the Appendix we develop a detailed example that illustrates how s and θ might be correlated in this case.

In this section we assume that both s and θ are binary. We then generalize the analysis to the case of a continuum of firm types and multiple signals in the next section. Specifically, in this

³ Strausz (forthcoming) shows that another important factor for first-best implementation is the verifiability of the interim information that the principal receives about the agent. Under nonverifiability, the principal cannot implement the first best, and the optimal use of the interim information depends on the signal's accuracy and timing.

section we assume that the state of nature is either good (g) or bad (b), and the firm's cost type is either high (h) or low (ℓ). The firm's cost in state $s = g, b$ when its type is $\theta = h, \ell$ is $C_{\theta s}(q)$, where $C_{hs}(q) > C_{\ell s}(q)$ and $C'_{hs}(q) > C'_{\ell s}(q)$ for $s = g, b$ and $C_{\theta b}(q) \geq C_{\theta g}(q)$ for $\theta = h, \ell$. That is, high-type firms have higher total and marginal costs than low-type firms in every state of nature and the total cost of each type of firm in the bad state is greater than or equal to the corresponding total cost in good states. We also assume that $C_{\theta s}(q)$ is increasing, weakly convex in q , and $C_{\theta s}(0) < S'(0)$ for $\theta = h, \ell$ and $s = g, b$. Recalling that t denotes the firm's revenue, the firm's profit is

$$\pi_{\theta s}(q, t) = t - C_{\theta s}(q). \tag{2}$$

Let ϕ_ℓ be the probability that the firm's type is low and $\phi_h = 1 - \phi_\ell$ the probability that its type is high. We denote by $p_{\theta s}$ the conditional probability of state s given that the firm's type is θ . Hence, $p_{hg} + p_{hb} = p_{\ell g} + p_{\ell b} = 1$. For future reference it is convenient to write the conditional probabilities in matrix form as

$$H = \begin{pmatrix} p_{\ell g} & p_{\ell b} \\ p_{hg} & p_{hb} \end{pmatrix}, \tag{3}$$

and denote its determinant by J . Since in general $p_{\ell g} \neq p_{hg}$, the two types of firm differ from one another in two ways: (i) their cost of production in each state, and (ii) the likelihood that each state occurs. In this respect, our model differs from most of the literature on adverse selection, where types differ from one another with respect to a single parameter (e.g., their marginal costs). Note that when $p_{\ell g} \neq p_{hg}$, the conditional probabilities matrix H has full rank, and thus $J \equiv p_{\ell g} p_{hb} - p_{hg} p_{\ell b} \neq 0$. In what follows we will assume that $J \geq 0$.⁴ This assumption implies that $p_{hb}/p_{hg} \geq p_{\ell b}/p_{\ell g}$, so that a high (cost) type firm is more likely to draw a bad state than a low-type firm. When s represents the observable components of the cost function and θ the unobserved components, this means that firms with higher observed costs are also likely to have higher unobserved costs. And when s represents the number of equipment failures and θ represents the efficiency of the firm's technology, then $J \geq 0$ means that firms with costly inefficient technologies are likely to experience a larger number of equipment failures. In general, J can be viewed as a measure of the correlation between the firm's type and the realization of the state of nature: the higher is J , the stronger is the correlation between the firm's type and the state of nature; consequently, the realization of the state s is more informative about the firm's type.

□ **The regulator's problem under full information.** The regulator chooses a vector $(q_{hg}, q_{\ell g}, q_{hb}, q_{\ell b}, t_{hg}, t_{\ell g}, t_{hb}, t_{\ell b})$ that specifies a production level and a transfer from consumers to the firm for each type of firm and each state of nature. This vector is chosen *before* the state of nature is realized. The regulator's objective is to maximize the expected welfare function

$$\sum_{\theta} \sum_s \phi_{\theta} p_{\theta s} [U(q_{\theta s}, t_{\theta s}) + \alpha \pi_{\theta s}(q_{\theta s}, t_{\theta s})], \quad 0 \leq \alpha < 1. \tag{4}$$

The parameter α captures the regulator's marginal rate of substitution between net consumers' surplus and firm's profits. Since $\alpha < 1$, the regulator will try to minimize the firm's profits.

As usual, we assume that the regulator is constrained to select a mechanism, which ensures that the firm must at least break even on average (otherwise, the regulatory scheme amounts to a confiscation of property). In addition, we also assume that the firm's profit in every state cannot fall below M , where $M \leq 0$. That is, M is the maximal loss that the firm can sustain. Given these assumptions, using (1) and (2), and writing $\pi_{\theta s} = \pi_{\theta s}(q_{\theta s}, t_{\theta s})$ in order to simplify notation, the regulator's problem under full information can be written as

$$\max_{\substack{q_{hg}, q_{\ell g}, q_{hb}, q_{\ell b}, \\ t_{hg}, t_{\ell g}, t_{hb}, t_{\ell b}}} \sum_{\theta} \sum_s \phi_{\theta} p_{\theta s} [S(q_{\theta s}) - C_{\theta s}(q_{\theta s}) - (1 - \alpha)\pi_{\theta s}], \tag{RP}$$

⁴ The analysis when $J < 0$ is completely analogous.

subject to *ex ante* individual-rationality constraints,

$$\sum_s p_{\theta s} \pi_{\theta s} \geq 0, \quad \theta = h, \ell, \tag{EIR_\theta}$$

and state-by-state *ex post* individual-rationality (or limited-deficit) constraints,

$$\pi_{\theta s} \geq M, \quad \theta = h, \ell, \quad s = g, b. \tag{IR_{\theta s}}$$

Since $\alpha < 1$, it is optimal to set the transfers such that the EIR_θ constraints will be just binding. Substituting from the EIR_θ constraints into the objective function and recalling that $S'(q) > 0$, that $C_{\theta s}(q)$ is weakly convex in q , and that $C'_{\theta s}(0) < S'(0)$ for $\theta = h, \ell$ and $s = g, b$, it follows that the first-best production levels, $q_{\theta s}^*$, are defined implicitly by the first-order conditions

$$S'(q_{\theta s}) = C'_{\theta s}(q_{\theta s}), \quad \theta = h, \ell, \quad s = g, b. \tag{5}$$

That is, the optimal regulatory scheme involves marginal cost pricing in each state of nature. Since, by assumption, $C'_{hs}(q) > C'_{\ell s}(q)$ for $s = g, b$, it follows that $q_{\ell g}^* > q_{hg}^*$ and $q_{\ell b}^* > q_{hb}^*$: the low-cost firm produces more than the high-type firm in each state of nature. Given the optimal production levels, the optimal transfers, $t_{\theta s}^*$, are set such that

$$\sum_s p_{\theta s} \pi_{\theta s}^* = 0, \quad \pi_{\theta s}^* \geq M, \quad \theta = h, \ell, \quad s = g, b, \tag{6}$$

where $\pi_{\theta s}^* \equiv \pi_{\theta s}(q_{\theta s}^*, t_{\theta s}^*)$. When $M = 0$, (6) implies that $\pi_{\theta s}^* = 0$ for $\theta = h, \ell$, and $s = g, b$, so $t_{\theta s}^*(\theta) = C_{\theta s}(q_{\theta s}^*)$. However, when $M < 0$, the regulator has many degrees of freedom in choosing transfers that will satisfy (6).

□ **The regulator’s problem under asymmetric information.** We now turn to the case where the firm’s type is not observed by the regulator. By the Revelation Principle, we can without loss of generality restrict attention to direct-revelation mechanisms in which the firm truthfully reports its type to the regulator and, given a report $\hat{\theta} = h, \ell$, the regulator requires the firm to produce $q_{\hat{\theta} s}$ units in state s and gives the firm a transfer $t_{\hat{\theta} s}$ in state s . The regulator’s problem in that case is given by RP subject to the EIR_θ constraints, the $IR_{\theta s}$ constraints, and the following incentive-compatibility constraints:

$$\sum_s p_{\theta s} \pi_{\theta s} \geq \sum_s p_{\theta s} [t_{\hat{\theta} s} - C_{\theta s}(q_{\hat{\theta} s})], \quad \theta = h, \ell, \quad \hat{\theta} \neq \theta. \tag{IC_\theta}$$

Substituting for t from equation (2) into IC_ℓ and IC_h and simplifying, the two incentive constraints can be rewritten as

$$\sum_s p_{\ell s} \pi_{\ell s} \geq \sum_s p_{\ell s} [\pi_{hs} + \Delta_s(q_{hs})], \tag{IC_\ell}$$

and

$$\sum_s p_{hs} \pi_{hs} \geq \sum_s p_{hs} [\pi_{\ell s} - \Delta_s(q_{hs})], \tag{IC_h}$$

where $\Delta_s(q) \equiv C_{hs}(q) - C_{\ell s}(q)$ is the cost difference between the high-cost and the low-cost firms in state $s = g, b$.

To characterize the solution to the regulator’s problem, we will first simplify it through a series of lemmas. It should be noted that the problem cannot be simplified with the usual techniques of the mechanism-design literature. For instance, since in general $p_{\ell g} \neq p_{hg}$ and $p_{\ell b} \neq p_{hb}$, it is not true that if EIR_h is binding, then EIR_ℓ must be slack (as we shall see below, it is possible that at the optimum, both constraints are binding).

Lemma 1. At the optimum, EIR_θ and IC_θ , $\theta = \ell, h$ cannot both be slack.

Lemma 1 implies that each type of the firm either breaks even in expectation, or has a binding incentive constraint (to prevent it from misreporting its type), or both.

Lemma 2. At the optimum, $t_{\ell b}$ and t_{hg} can be set such that $IR_{\ell b}$ and IR_{hg} , respectively, will be binding while $IR_{\ell g}$ and IR_{hb} are slack.

Lemma 2 is useful because it implies that at any optimal solution, the transfers can be set, without any loss of generality, such that $\pi_{\ell b} = \pi_{hg} = M$. Once the output levels are determined, this allows us to pin down the value of one transfer for each type of firm. What is then left is to pin down the values of the two remaining state-contingent transfers. In economic terms, note that M can be interpreted as the largest punishment that the regulator can impose on the firm. With this interpretation in mind, Lemma 2 implies that each type of firm receives the largest feasible punishment in the state of nature that it is less likely to draw. Given that $J \geq 0$, the high-type (low-type) firm is less likely to draw the good (bad) state and hence is punished in the good (bad) state.

Lemma 3. If the optimal production levels are strictly monotonic with respect to the firm's type in each state of nature, i.e., $q_{\ell s} > q_{hs}$ or $s = g, b$, then EIR_ℓ and EIR_h cannot both be slack.

Lemma 3 implies that the optimal mechanism does not give a positive expected rent to both firm types: at least one type must break even in expectation.

Lemma 4. If the optimal production levels are strictly monotonic with respect to the firm's type in each state of nature, i.e., $q_{\ell s} > q_{hs}$ for $s = g, b$, then EIR_h is binding.

Lemma 4 implies that if output is strictly monotonic with respect to the firm's type in every state of nature, then the optimal mechanism is such that the high-type firm breaks even in expectation. That is, in expectation, there is no rent "at the bottom."

To characterize the optimal mechanism, it will be useful to define the information rents of the low-type firm under the first-best production plan. These rents, denoted by R_ℓ^* , reflect the expected payoff of a low-type firm from reporting that its type is high when the regulator requires the high-type firm to produce the first-best output levels, q_{hg}^* and q_{hb}^* . Recalling from Lemma 2 that $\pi_{\ell b} = \pi_{hg} = M$, it follows that

$$\begin{aligned}
 R_\ell^* &\equiv p_{\ell g} \left[M + C_{hg}(q_{hg}^*) - C_{\ell g}(q_{hg}^*) \right] + p_{\ell b} \left[-\frac{p_{hg}M}{p_{hb}} + C_{hb}(q_{hb}^*) - C_{\ell b}(q_{hb}^*) \right] \\
 &= \frac{JM}{p_{hb}} + p_{\ell g} \Delta_g(q_{hg}^*) + p_{\ell b} \Delta_b(q_{hb}^*).
 \end{aligned}
 \tag{7}$$

The intuition behind R_ℓ^* is as follows. The high-type firm gets a payoff of M in the good state and $-p_{hg}M/p_{hb}$ in the bad state. The latter payoff arises since the output levels under the first-best production plan are strictly monotonic with respect to the firm's type; hence by Lemma 4, EIR_h is binding. Thus, if the low-type firm reports that its type is high, its payoff is M with probability $p_{\ell g}$, and $-p_{hg}M/p_{hb}$ with probability $p_{\ell b}$. In addition, the low-type firm enjoys a cost saving of $\Delta_g(q_{hg}^*)$ in the good state and $\Delta_b(q_{hb}^*)$ in the bad state, due to its cost advantage over the high-type firm.

We are now ready to characterize the optimal mechanism.

Proposition 1. Recall that at the first best, both types of firm break even in expectation. Then,

- (i) The regulator can implement the first-best solution if and only if $R_\ell^* \leq 0$. At the optimum, $\pi_{\ell b}^* = \pi_{hg}^* = M < 0$, $\pi_{\ell g}^* = -(p_{\ell b}/p_{\ell g})M > 0$, and $\pi_{hb}^* = -(p_{hg}/p_{hb})M > 0$.
- (ii) If $R_\ell^* > 0$, the regulator cannot implement the first-best solution. The optimal production levels are denoted $q_{\ell g}^{**}$, $q_{\ell b}^{**}$, q_{hg}^{**} , and q_{hb}^{**} in this case, and are defined implicitly by the

following first-order conditions:

$$S'(q_{\ell s}^{**}) = C'_{\ell s}(q_{\ell s}^{**}), \quad s = g, b, \quad (8)$$

and

$$S'(q_{hs}^{**}) = C'_{hs}(q_{hs}^{**}) + (1 - \alpha) \frac{\phi_{\ell} P_{\ell s}}{\phi_h P_{hs}} \Delta'_s(q_{hs}^{**}), \quad s = g, b. \quad (9)$$

Part (i) of Proposition 1 is closely related to Riordan and Sappington (1988), who show that the first-best solution can be implemented, provided that the regulator can condition the regulatory scheme on the realization of an *ex post* signal that is correlated with the firm's type, and provided that the conditional probabilities matrix that specifies the likelihood of the various states of nature conditional on the firm's type has full rank (in our case, this simply means that $J \neq 0$). There are two important differences, however. First, unlike the *ex post* signals in Riordan and Sappington, which are purely informational and only affect the firm's transfers, here the states of nature affect the firm's cost, and hence its output level, directly.

Second and more important, to induce truthful reporting, the regulator needs to "punish" the low-type firm in the bad state of nature, which, given that $J \geq 0$, is less likely to be associated with the low-type firm (the regulator does not "punish" the high-type firm similarly, since, given the first-best output levels, this firm does not wish to report that its type is low). But unlike in Riordan and Sappington, we assume that the firm needs to earn at least M in every state of nature. Hence, the regulator has only a limited ability to punish the low-type firm for misreporting its type. Part (i) of the proposition shows that given this limitation, the first-best can be implemented if and only if $|MJ|$ is sufficiently large, that is, the maximal loss that the firm can sustain and/or the correlation between the likelihood of each state and the firm's type are sufficiently large. When these conditions hold, the low-type firm cannot get positive information rents from misreporting its type.

When $|MJ|$ is small, the low-type firm can get information rents by producing the first-best output levels of the high-type firm. Since $\alpha < 1$, these rents lower the value of the regulator's objective function, so the first-best solution cannot be implemented. Part (ii) of Proposition 1 shows that the regulator deals with this problem by distorting the high-type firm's output downward in both states of nature. The optimal solution then has the familiar second-best features: no distortion and positive rents "at the top" (the low-type firm produces its first-best output level in both states and gets a positive expected profit), and a downward output distortion but full rent extraction "at the bottom" (the high-type firm produces less than in the first-best solution in both states of nature and its expected profit is zero).⁵ Moreover, the proposition shows that the distortion of the high-type firm's output becomes larger as (i) the regulator places a smaller weight on the firm's profit, (ii) the relative probability that the firm's type is low, and (iii) the difference between the marginal costs of the low-type and high-type firms is large.

3. The case of a continuum of types and finitely many states

■ In this section we extend the preceding analysis by assuming that the regulated firm's type is drawn from the interval $\Theta = [\underline{\theta}, \bar{\theta}]$ and the set of states of nature is $\{1, \dots, n\}$, with higher states representing higher cost shocks (i.e., "worse" states of nature). In the next subsection we will present our basic assumptions about the cost functions and the distributions of types and states of nature. After that, we explore the conditions under which the regulator can implement

⁵ Kessler, Lulfesmann, and Schmitz (2005) show, in the context of a similar two-type, two-state principal-agent model with adverse selection, that at the optimum, the principal may distort the action of the inefficient agent (the output level of the high-cost firm in our context) upward rather than downward (Proposition 2 in their article). Their result differs from ours because they impose an upper bound on the transfers from the agent to the principal, whereas we impose a bound on the agent's payoff. We believe that in the context of a regulation model, bounds on the firm's profit are more natural than bounds on the size of the transfers (which are only one component of the firm's profit).

the first-best solution. Finally, we will characterize the second-best solution to the regulator’s problem, when the first-best solution cannot be implemented.

□ **Basic assumptions and notation.** We assume that the firm’s total cost function is linear and given by

$$C(q, s, \theta) = c_s(\theta)q,$$

where $0 \leq c_1(\theta) \leq c_2(\theta) \leq \dots \leq c_n(\theta)$ for all $\theta \in \Theta$. That is, higher states are associated with (weakly) higher marginal costs and therefore represent worse states of nature. In addition, we assume that for all $s \in \{1, \dots, n\}$, the marginal cost, $c_s(\theta)$, is a strictly positive, twice continuously differentiable, strictly increasing, and convex function of θ .

Let F denote the cumulative distribution function of the firm’s type, θ , on the support Θ , and assume that the associated density function, f , is strictly positive and continuously differentiable. The conditional probability of state s , given type θ , is denoted

$$p_s(\theta) = \Pr(s \mid \theta).$$

A regulatory scheme is a state-contingent array, $(q_s(\theta), t_s(\theta))_{s=1, \dots, n}$, where $q_s(\theta)$ is the required production level of type θ in state s and $t_s(\theta)$ is the associated transfer from the regulator to the firm.⁶ The profit of a type- θ firm in state s is

$$\pi_s(\theta) = t_s(\theta) - c_s(\theta)q_s(\theta). \tag{10}$$

As before, we assume that the firm’s profit in each state cannot fall below M , where $M \leq 0$.

The regulator’s problem is the continuous analog of (RP):

$$\max_{(q_s(\theta), t_s(\theta))_{s=1, \dots, n}} \int_{\underline{\theta}}^{\bar{\theta}} \sum_s p_s(\theta) [S(q_s(\theta)) - c_s(\theta)q_s(\theta) - (1 - \alpha)\pi_s(\theta)] f(\theta) d\theta, \quad 0 \leq \alpha < 1, \tag{RP_a}$$

subject to the *ex ante* participation constraints,

$$\sum_s p_s(\theta)\pi_s(\theta) \geq 0, \quad \forall \theta \in \Theta, \tag{EIR_\theta}$$

state-by-state limited-deficit constraints,

$$\pi_s(\theta) \geq M, \quad \forall \theta \in \Theta, \forall s \in \{1, \dots, n\}, \tag{IR_{\theta,s}}$$

incentive-compatibility constraints,

$$\sum_s p_s(\theta)\pi_s(\theta) \geq \sum_s p_s(\hat{\theta}) [t_s(\hat{\theta}) - c_s(\theta)q_s(\hat{\theta})], \quad \forall \theta, \hat{\theta} \in \Theta, \tag{IC_{\theta, \hat{\theta}}}$$

and nonnegativity constraints,

$$q_s(\theta) \geq 0, \quad \forall \theta \in \Theta, \forall s \in \{1, \dots, n\}.$$

The first-best solution to the regulator’s problem is attained in the absence of private information, in which case the $IC_{\theta, \hat{\theta}}$ constraints can be ignored. Since $\alpha < 1$, it is optimal to set the transfers as low as possible, so that the EIR_θ constraints will be just binding. Substituting from

⁶ As in Section 2, the transfer function, $t_s(\theta)$, includes the aggregate usage fees, $P(q_s(\theta))q_s(\theta)$, and the aggregate fixed fees (or subsidy), A .

the EIR_θ constraints into the objective function and maximizing with respect to $q_s(\theta)$ reveals that the first-best production level, $q_s^*(\theta)$, is implicitly defined by

$$S'(q_s^*(\theta)) = c_s(\theta), \quad \forall \theta \in \Theta, \forall s \in \{1, \dots, n\}. \tag{11}$$

Since $S(\cdot)$ is increasing, concave, and continuously differentiable, and since $c_s(\theta) > 0$, the solution $q_s^*(\theta)$ is positive and unique. By EIR_θ , the first-best transfers and expected profit of the firm must therefore be such that

$$\sum_s p_s(\theta)\pi_s^*(\theta) \equiv \sum_s p_s(\theta) [t_s^*(\theta) - c_s(\theta)q_s^*(\theta)] = 0, \quad \forall \theta \in \Theta. \tag{12}$$

When $M = 0$, (12) implies that the transfers are such that $t_s^*(\theta) = c_s(\theta)q_s^*(\theta)$ for all $s \in \{1, \dots, n\}$. That is, the firm just breaks even in every state. However, when $M < 0$, the regulator has many degrees of freedom in setting the transfers such that the $IR_{\theta,s}$ constraints will be satisfied; for instance, it is possible to set the transfers such that the firm will earn positive profits in some states and will incur losses (smaller than $|M|$) in other states.

In the next subsection we show that the first-best solution can be implemented even if the firm's type is private information, provided that the maximum deficit $|M|$ is sufficiently large. Then we will consider the second-best solution when it is impossible to achieve the first-best solution.

□ **Implementation of the first-best solution.** To establish conditions under which the first-best solution can be implemented, we first replace the $IC_{\theta,\hat{\theta}}$ constraints with the first-order necessary conditions for truthful revelation, assuming that the state-contingent regulatory scheme, $(q_s(\theta), t_s(\theta))_{s=1,\dots,n}$, is differentiable. We will then verify that the resulting solution is differentiable and will provide conditions ensuring that it is globally incentive compatible.⁷

Let $\hat{\theta}$ be the report of a firm whose true type is θ . If all firms report their types truthfully, then local incentive compatibility requires that

$$\begin{aligned} 0 &= \frac{d}{d\hat{\theta}} \left(\sum_s p_s(\theta) [t_s(\hat{\theta}) - c_s(\theta)q_s(\hat{\theta})] \right) \Big|_{\hat{\theta}=\theta} \\ &= \sum_s p_s(\theta) (t'_s(\theta) - c_s(\theta)q'_s(\theta)), \quad \forall \theta \in \Theta. \end{aligned} \tag{13}$$

Differentiating (12) and using (13), we get

$$\sum_s p'_s(\theta)\pi_s^*(\theta) = \sum_s p_s(\theta)c'_s(\theta)q_s^*(\theta) \equiv B^*(\theta), \quad \forall \theta \in \Theta. \tag{14}$$

Ignoring global incentive compatibility for the moment, the first-best solution can be implemented, provided that we can find transfers such that (12) and (14) hold simultaneously and the firm's profit, $\pi_s^*(\theta)$, is at least M in each state of nature.

To establish conditions under which we can find such transfers for the most stringent limited-deficit constraints, we will solve the following constrained optimization problem: for each θ ,

$$\max \min \{ \pi_1^*(\theta), \dots, \pi_n^*(\theta) \}, \tag{15}$$

subject to (12) and (14). If the solution to this maxmin problem is above M for all $\theta \in \Theta$, then it is possible to find transfers that implement the first-best solution. Otherwise, any system of transfers that induces truthful reports and leaves zero expected rents (i.e., satisfies equations (12)

⁷ On the differential approach to mechanism design, see Laffont and Maskin (1980).

and (14)) will necessarily be such that the firm would lose more than $|M|$ in at least one state of nature. Such a system of transfers would then violate at least one of the $IR_{\theta,s}$ constraints.

Lemma 5. The solution to the above maxmin problem must be such that the firm earns a profit in exactly one state of nature and incurs the same loss in all other states.

Lemma 5 is a key step in characterizing the least restrictive set of conditions under which it is possible to implement the first-best solution to the regulator’s problem, because it says that the first best can be implemented with a regulatory scheme that requires only two profit levels for the firm: a positive profit in one state and a loss in all other states. Moreover, this regulatory scheme involves minimal punishments in any given state of nature and hence has the “best shot” at satisfying the state-by-state limited-deficit constraints. Intuitively, to induce truth telling, the regulator needs to “punish” the firm whenever it misreports its type. Since the firm does not know in advance which state of nature will be realized, it takes into account only the expected level of the punishments. Hence, in order to satisfy the state-by-state limited-deficit constraints, it is optimal for the regulator to spread the punishments over as many states as possible. The regulator must then “reward” the firm in the remaining state of nature to ensure that it breaks even on average (otherwise its EIR_{θ} constraint is violated). From this intuition it is clear that we may also be able to implement the first-best solution to the regulator’s problem with other types of regulatory schemes, for instance, schemes that involve more than just one level of reward, or more than just one level of punishment, or a scheme that rewards the firm in more than one state of nature. However, such regulatory schemes will implement the first-best solution to the regulator’s problem under higher deficit bounds (in absolute value) than the maxmin scheme.

Given Lemma 5, we can now characterize the reward and punishment that the regulator uses in the maxmin scheme. Let i be the state in which the firm is rewarded and let j be any state in which the firm is punished. Since $\pi_s^*(\theta) = \pi_j^*(\theta)$ for all $s \neq \{i, j\}$, and using the fact that $\sum_s p_s(\theta) = 1$ and $\sum_s p'_s(\theta) = 0$, equations (12) and (14) can easily be solved to provide the profit values,

$$\pi_i^*(\theta) = \frac{(1 - p_i(\theta)) B^*(\theta)}{p'_i(\theta)}, \quad \pi_j^*(\theta) = -\frac{p_i(\theta) B^*(\theta)}{p'_i(\theta)}, \quad \forall j \neq i. \tag{16}$$

Since $M \leq \pi_j^*(\theta) \leq 0 \leq \pi_i^*(\theta)$, it follows from (16) that the reward state must be such that $p'_i(\theta) > 0$; this also ensures that the optimal profit levels are well defined. Moreover, it follows that $p_i(\theta) B^*(\theta) / p'_i(\theta) \leq -M$ for all $\theta \in \Theta$. Since we are interested in a regulatory scheme that implements the first-best solution to the regulator’s problem under the most stringent limited-deficit constraints, it is also obvious that the reward state must have the highest $p'_i(\theta) / p_i(\theta)$ ratio among all states.

Given (16), the maximum transfers that implement the first-best solution for the widest set of values of M are such that

$$t_s^*(\theta) = \begin{cases} c_i(\theta) q_i^*(\theta) + \frac{(1 - p_i(\theta)) B^*(\theta)}{p'_i(\theta)}, & s = i \\ c_s(\theta) q_s^*(\theta) - \frac{p_i(\theta) B^*(\theta)}{p'_i(\theta)}, & \forall s \neq i. \end{cases} \tag{17}$$

These transfers are clearly differentiable (recall that $p'_i(\theta) > 0$ for all $\theta \in \Theta$). We must now check that these transfers satisfy the $IC_{\theta, \hat{\theta}}$ constraints not only locally but also globally, i.e., for all $\theta, \hat{\theta} \in \Theta$. To this end, we first impose the following restrictions on the conditional probability system.

Assumption 1. The conditional probability $p_s(\theta)$ is a twice continuously differentiable function of θ with $p_s(\theta) \geq \varepsilon > 0$ for all $s \in \{1, \dots, n\}$ and all $\theta \in \Theta$. Moreover, $p_n(\theta)$ is an increasing and concave function of θ for all $\theta \in \Theta$.

Assumption 2. The conditional probability distribution $(p_s(\theta))_{s=1,\dots,n}$ has the first-order stochastic dominance (FOSD) property: $(p_s(\hat{\theta}))_{s=1,\dots,n}$ dominates $p_s(\theta)_{s=1,\dots,n}$ in the sense of strict first-order stochastic dominance if and only if $\hat{\theta} > \theta$.

Assumption 3. The likelihood ratio, $r_s(\theta) \equiv p_s(\theta)/p_n(\theta)$, is decreasing with θ for all $s \neq n$.

The first part of Assumption 1 ensures that all states of nature can be realized no matter what the firm’s type is. Otherwise, if $p_s(\theta) = 0$ for some s and some θ , the regulator would be able to rule out the possibility that the firm’s type is θ after observing state s . The second part of Assumption 1 says that the probability of drawing the worst state, n , increases with the firm’s type but at a decreasing rate. This implies in turn that less efficient types are more likely to draw state n than more efficient types. Riordan and Sappington (1988) show that the concavity of the likelihood function for the signal (our Assumption 1 in essence), together with the convexity of the cost function with respect to θ (which we also assume), ensure the existence of transfers that implement the first best. It is therefore not surprising that these properties will play a similar role in our setting. Assumption 2 implies that more efficient types have a higher probability of having good states (i.e., states with “small” index) than less efficient types. That is, $\sum_{s < t} p_s(\hat{\theta}) < \sum_{s < t} p_s(\theta)$ for $\hat{\theta} > \theta$ and $t < n$. Assumption 3 implies that $p'_s(\theta)/p_s(\theta) < p'_n(\theta)/p_n(\theta)$ for all $\theta \in \Theta$. It is therefore a form of the monotone likelihood ratio property (Milgrom, 1981) that is common in the mechanism-design literature.

Proposition 2. Suppose that Assumptions 1–3 hold and $q_s^*(\theta)c'_s(\theta)$ is nondecreasing with s for all θ , where $q_s^*(\theta)$ is defined by (11). Then, the maxmin transfers defined by (17) implement the first-best output levels if and only if

$$[p_n(\theta)/p'_n(\theta)]B^*(\theta) < -M, \quad \forall \theta \in \Theta. \tag{18}$$

Proposition 2 generalizes the first-best implementation theorem of Riordan and Sappington (1988). The only somewhat unusual assumption in the statement of Proposition 2 is “ $q_s^*(\theta)c'_s(\theta)$ is nondecreasing with s .” This assumption is only a sufficient condition for first-best implementation and only involves fundamental data of the model; in particular, using (11), we can reformulate it as

$$(S')^{-1}(c_s(\theta))c'_s(\theta) \leq (S')^{-1}(c_{s+1}(\theta))c'_{s+1}(\theta), \quad \forall s < n, \forall \theta \in \Theta.$$

The following simple example will help illustrate this assumption. Suppose that $S(q) = q^{1-\varepsilon} / (1 - \varepsilon)$ (the inverse demand for the regulated good, P , has constant elasticity, ε) and $c_s(\theta) = \theta s^\beta$, with $\beta > 0$. Then, $q_s^*(\theta)c'_s(\theta) = \theta^{-1/\varepsilon} s^{\beta(1-(1/\varepsilon))}$ is increasing with s for all $\theta \in \Theta$ if and only if $\varepsilon > 1$ (i.e., if the elasticity of the inverse-demand function exceeds one). Clearly, the assumption is not very restrictive and holds if the inverse demand for the regulated firm’s good is elastic; it holds trivially if c doesn’t depend on s .

□ **The second-best solution under *ex post* limited-deficit constraints.** In this subsection we assume that condition (18) fails, so that it is impossible to construct transfers that implement the first-best production level. We therefore characterize the second-best regulatory scheme that solves RP_a subject to the $IC_{\theta, \hat{\theta}}$, EIR_θ , and $IR_{\theta, s}$ constraints. Our strategy for solving this constrained optimization problem will be to substitute the necessary condition for local incentive compatibility (13) in RP_a and to ignore the *ex ante* participation constraints EIR_θ in a first step. We will then check that the resulting solution is differentiable, satisfies the EIR_θ constraints, and is globally incentive compatible.

Using (13) and the definition $r_s(\theta) \equiv p_s(\theta)/p_n(\theta)$, we establish the following result.

Lemma 6. The firm’s transfer in state n is given by

$$t_n(\theta) = \sum_s r_s(\bar{\theta})\pi_s(\bar{\theta}) + \sum_s r_s(\theta)c_s(\theta)q_s(\theta) - \sum_{s \neq n} r_s(\theta)t_s(\theta) + \int_\theta^{\bar{\theta}} \sum_s [r_s(x)c'_s(x)q_s(x) - r'_s(x)\pi_s(x)] dx. \tag{19}$$

Given (19), using the fact that $r_s(\theta) \equiv p_s(\theta)/p_n(\theta)$ and simplifying, the expected transfer of the firm, where the expectation is taken with respect to the firm's type and with respect to the state of nature, is given by

$$\begin{aligned} & \int_{\underline{\theta}}^{\bar{\theta}} \left[\sum_{s \neq n} p_s(\theta) t_s(\theta) + p_n(\theta) t_n(\theta) \right] f(\theta) d\theta \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \left[\sum_s r_s(\bar{\theta}) \pi_s(\bar{\theta}) + \sum_s r_s(\theta) c_s(\theta) q_s(\theta) \right] p_n(\theta) f(\theta) d\theta \\ &+ \int_{\underline{\theta}}^{\bar{\theta}} \left[\int_{\underline{\theta}}^{\bar{\theta}} \sum_s [r_s(x) c'_s(x) q_s(x) - r'_s(x) \pi_s(x)] dx \right] p_n(\theta) f(\theta) d\theta. \end{aligned} \tag{20}$$

After integration by parts, the expression in the last line of (20) becomes

$$\begin{aligned} & \int_{\underline{\theta}}^{\bar{\theta}} \sum_s [r_s(x) c'_s(x) q_s(x) - r'_s(x) \pi_s(x)] dx \varphi_n(\theta) \Big|_{\underline{\theta}}^{\bar{\theta}} \\ &+ \int_{\underline{\theta}}^{\bar{\theta}} \left[\sum_s [r_s(\theta) c'_s(\theta) q_s(\theta) - r'_s(\theta) \pi_s(\theta)] \varphi_n(\theta) \right] d\theta \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \left[\sum_s [r_s(\theta) c'_s(\theta) q_s(\theta) - r'_s(\theta) \pi_s(\theta)] \varphi_n(\theta) \right] d\theta, \end{aligned} \tag{21}$$

where $\varphi_n(\theta) \equiv \int_{\underline{\theta}}^{\theta} p_n(x) f(x) dx$. Substituting from (20) and (21) into (RP_a) and rearranging, the regulator's problem, given local incentive compatibility, becomes

$$\begin{aligned} & \max_{\substack{(q_s(\theta))_{s=1, \dots, n} \\ (t_s(\theta))_{s=1, \dots, n-1} \\ t_n(\bar{\theta})}} \int_{\underline{\theta}}^{\bar{\theta}} \sum_s [S(q_s(\theta)) - \alpha c_s(\theta) q_s(\theta)] p_s(\theta) f(\theta) d\theta \\ &- (1 - \alpha) \int_{\underline{\theta}}^{\bar{\theta}} \left[\sum_s r_s(\bar{\theta}) \pi_s(\bar{\theta}) + \sum_s r_s(\theta) c_s(\theta) q_s(\theta) \right] p_n(\theta) f(\theta) d\theta \\ &- (1 - \alpha) \int_{\underline{\theta}}^{\bar{\theta}} \left[\sum_s [r_s(\theta) c'_s(\theta) q_s(\theta) - r'_s(\theta) \pi_s(\theta)] \varphi_n(\theta) \right] d\theta, \end{aligned} \tag{RP_b}$$

subject to the EIR_θ constraints, the IR_{θ,s} constraints, and the constraints that $q_s(\theta) \geq 0$ for all $s = \{1, \dots, n\}$ and all $\theta \in \Theta$.

To characterize the solution to the regulator's problem, we shall study a relaxed version of RP_b in which we ignore the EIR_θ constraints and maximize the regulator's objective function pointwise, subject to the IR_{θ,s} constraints and the nonnegativity constraint on $q_s(\theta)$. We will then verify that the solution to the relaxed problem is differentiable and will provide sufficient conditions for this solution to satisfy the EIR_θ constraints and to be globally incentive compatible.

Lemma 7. The regulator's relaxed problem, RP_b, has a unique solution such that

$$S'(q_s^{**}(\theta)) = c_s(\theta) + (1 - \alpha) c'_s(\theta) \frac{F(\theta | n)}{f(\theta | n)}, \tag{22}$$

$$t_s^{**}(\theta) = c_s(\theta) q_s^{**}(\theta) + M, \quad \forall s \neq n, \forall \theta \in \Theta, \tag{23}$$

$$t_n^{**}(\theta) = c_n(\theta) q_n^{**}(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} \sum_s r_s(x) c'_s(x) q_s^{**}(x) dx - \frac{M(1 - p_n(\bar{\theta}))}{p_n(\bar{\theta})}, \quad \forall \theta \in \Theta. \tag{24}$$

This solution is differentiable in θ , and if the conditional hazard rate $f(\theta | n)/F(\theta | n)$ is non-increasing with θ , then $q_s^{**}(\theta)$ is decreasing with θ for all s , implying that less efficient types produce less in every state of nature.

Equation (22) is the continuous analog of (8) and (9). Since $F(\underline{\theta} | n) = 0$, it follows that $S'(q_s^{**}(\underline{\theta})) = c_s(\underline{\theta})$: the regulator uses marginal cost pricing for the lowest possible cost type of the firm so there is no distortion “at the top.” This is exactly as in the two-type, two-state case (see equation (8)). For higher-cost types, equations (11) and (22) reveal that since $S(\cdot)$ is concave and $c'_s(\theta) > 0$, then $q_s^*(\theta) > q_s^{**}(\theta)$ for all $s = \{1, \dots, n\}$ and all $\theta > \underline{\theta}$. Hence, unless $\theta = \underline{\theta}$, the regulator distorts the firm’s output level downward in every state of nature.⁸ Just like in the two-type, two-state case, the regulator distorts the firm’s output to a larger extent when (i) he attaches a smaller weight to firm’s profits, (ii) there is a bigger difference between the costs of different types of the firm, and (iii) there is a relatively low likelihood that the firm’s cost is low. Moreover, if the right-hand side of (22) is increasing with s , the firm’s output level is smaller in worse states of nature.

Equation (22) generalizes the results of Baron and Myerson (1982) and Demougin and Garvie (1991). To see how, suppose first that there is only one state of nature, i.e., $n = 1$, and let the marginal cost be given by $c(\theta) = \theta$. Then (22) becomes

$$S'(q^{**}(\theta)) = \theta + (1 - \alpha) \frac{F(\theta)}{f(\theta)},$$

which is just Baron and Myerson’s (1982) classic formula for second-best optimality.

Next, suppose that the regulator’s objective is to maximize consumers’ surplus so that $\alpha = 0$, and suppose that the marginal cost is state-independent and given by $c(\theta) = \theta$. Then (22) becomes

$$S'(q_s^{**}(\theta)) = \theta + \frac{F(\theta | n)}{f(\theta | n)},$$

which is just equation (21) in Demougin and Garvie (1991). In particular, this equation shows that the optimal output level is state-independent.⁹

Given (23) and (24), the expected profit of a type- $\bar{\theta}$ firm is $\sum_s p_s(\bar{\theta})\pi_s(\bar{\theta}) = 0$. Hence, the second-best solution features the familiar no (expected) rent “at the bottom” property. In the next proposition, we establish sufficient conditions under which all types $\theta < \bar{\theta}$ earn positive expected rents due to their private information. This ensures in turn that all the EIR_θ constraints are satisfied. In addition, the conditions ensure that the solution to the regulator’s relaxed problem is globally incentive compatible.

Proposition 3. Suppose that Assumptions 1–3 hold and the conditional hazard rate $f(\theta | n)/F(\theta | n)$ is nonincreasing with θ . Then, for all M , there exists a $\delta > 0$ such that if $|p'_s(\theta)| < \delta$ for all s in $\{1, \dots, n\}$, the solution characterized by (22)–(24) satisfies the EIR_θ constraints and is globally incentive compatible.

⁸ Noting that $c'_s(\theta)$ is the continuous analog of $\Delta'_s(\cdot)$ and that $F(\theta | n)/f(\theta | n)$ is the continuous analog of $\phi_L p_{Ls}/(\phi_h p_{hs})$, it follows that the distortion term in equation (22) is the exact analog of the distortion term in equation (9).

⁹ It is also interesting to note that if we replace $c_s(\theta)$ in equation (22) with $\bar{c}(\theta) \equiv \sum_s p_s(\theta)c_s(\theta)$, which is the expected value of $c_s(\theta)$ over all states of nature, then we get equation (15b) in Baron and Besanko (1984). They also assume that the regulated firm is subject to cost shocks, but the regulator in their model cannot observe the states of nature as in our model and hence cannot condition the firm’s output or transfers on these states. Instead, the regulator audits the firm’s costs and penalizes the firm if its reported costs are above its realized costs. Since the penalties are imposed after the firm has already produced, it is clear that the firm’s output in their model depends on $\bar{c}(\theta)$ rather than on $c_s(\theta)$ as in our model.

4. Conclusion

■ We studied the design of optimal regulatory schemes when the regulated firm is subject to *ex post* cost shocks. We showed that the regulator can design a regulatory scheme that adjusts itself automatically following the realization of each shock without having to renegotiate the entire scheme. This flexibility is important given that in practice, regulated rates are often set in advance for an extended period of time.

We showed that, under certain conditions, the regulator can exploit the correlation between firms' types and the likelihood of the various cost shocks and design a regulatory scheme that implements the first-best solution, despite the fact that the firm's type is private information. To implement the first best, the regulator needs to punish the firm if the realized state of nature is relatively unlikely given the firm's reported type. However, the regulator's ability to punish the firm in such states is constrained by the fact that (i) the firm must break even on average and (ii) the firm cannot sustain unlimited losses in any given state. Our analysis reveals that the scheme that implements the first-best solution under the most stringent limited-deficit constraints (i.e., the scheme that imposes the minimal punishments on the firm in any given state) "rewards" the firm in exactly one state of nature, and imposes the same "punishment" on the firm in all other states. This scheme is feasible provided that the correlation between the firm's type and the cost shocks is sufficiently strong, and/or the firm can sustain sufficiently large deficits in any given state. When these conditions fail, we are able to fully characterize the solution to the regulator's problem and show that it has classical second-best features. Another benefit of our approach is that it allows an extension of the well-known Baron and Myerson methodology to the case of *ex post* cost shocks and limited liability constraints, and can characterize the optimal solution using straightforward calculus.

Appendix

■ Proofs of Lemmas 1–7 and Propositions 1 and 2 follow.

We begin the Appendix by fully developing an example that illustrates how the *ex post* realization of input prices could be correlated with the firm's hidden cost type.

Example. Suppose the regulated firm uses two inputs, x_1 and x_2 , and has a CES production function: $q = (x_1^\rho + x_2^\rho)^{1/\rho}$, with $\rho \leq 1$. Moreover, suppose that at the time the firm needs to select x_1 and x_2 , the per-unit cost of input 1, $s = w_1$, is yet unknown. The firm forms an unbiased forecast of w_1 given by $\widehat{w}_1 = w_1 e^u$, where u is the forecasting error. We assume that the firm's forecast, \widehat{w}_1 , is private information for the firm. Given \widehat{w}_1 and given the per-unit cost of input 2, w_2 , which for simplicity is assumed to be common knowledge, the firm chooses x_1 and x_2 to minimize its expected expenditure, $\widehat{w}_1 x_1 + w_2 x_2$, subject to its technological constraint. The resulting conditional demands for the two inputs are $\bar{x}_1(\widehat{w}_1, w_2, q) = q[\widehat{w}_1^\sigma / (\widehat{w}_1^\sigma + w_2^\sigma)]^{1/\rho}$, $\bar{x}_2(\widehat{w}_1, w_2, q) = q[w_2^\sigma / (\widehat{w}_1^\sigma + w_2^\sigma)]^{1/\rho}$, where $\sigma \equiv \rho / (\rho - 1)$.

We have in mind that, at least in the short run, the choice of production techniques is inflexible, so that based on (\widehat{w}_1, w_2) , the firm is committed to its technological choice. Once the real input price $s = w_1$ is realized, the *ex post* cost function of the firm is

$$C(q, s, \theta) = s\bar{x}_1(\widehat{w}_1, w_2, q) + w_2\bar{x}_2(\widehat{w}_1, w_2, q) = (s\theta + w_2(1 - \theta)^\rho)^{1/\rho} q,$$

where $\theta \equiv [\widehat{w}_1^\sigma / (\widehat{w}_1^\sigma + w_2^\sigma)]^{1/\rho}$ is the firm's type.

Recalling that $\widehat{w}_1 = s e^u$ is private information and u is random, it is obvious that θ is private information, and that s and θ are correlated. One can easily verify that $C(q, s, \theta)$ satisfies the assumptions that we make in the article: the per-unit cost is strictly positive, increases with s , and is a twice-continuously differentiable function of θ . Moreover, $C(q, s, \theta)$ increases with θ provided that s is sufficiently larger than w_2 , and it is a convex function of θ provided that $\rho > 0$. *Q.E.D.*

□ **Proofs.** We now provide sketches of the proofs of Lemmas 1–4 and the complete proofs of Lemmas 5, 6, and 7 and Propositions 1 and 2. Detailed proofs of Lemmas 1–4 and Proposition 3 appear in a technical Appendix that is available at www.rje.org/main/sup-mat.html, as well as at the authors' personal web pages.

Sketch of the Proof of Lemma 1. If EIR_ℓ and IC_ℓ are slack, then either $IR_{\ell g}$ or $IR_{\ell b}$ or both are also slack, so it is possible to slightly lower $t_{\ell g}$ or $t_{\ell b}$ or both. Since $\alpha < 1$, this enhances the value of the regulator's objective function, while relaxing IC_h and having no effect on IR_{hg} and IR_{hb} . Similar arguments apply in the case of EIR_h and IC_h . *Q.E.D.*

Sketch of the Proof of Lemma 2. If $IR_{\ell b}$ is slack, then lowering $t_{\ell b}$ by $\varepsilon_{\ell b} > 0$ until $IR_{\ell b}$ is just binding and increasing $t_{\ell g}$ by $p_{\ell b}\varepsilon_{\ell b}/p_{\ell g}$ (to ensure that EIR_{ℓ} and IC_{ℓ} remain intact) relaxes $IR_{\ell g}$, and since $J \geq 0$, it also relaxes IC_h . Since the change does not affect EIR_h , IR_{hg} , IR_{hb} , and the regulator's objective function, the new allocation also solves the regulator's problem. Similar arguments apply in the case of IR_{hg} . *Q.E.D.*

Sketch of the Proof of Lemma 3. If EIR_{ℓ} and EIR_h are both slack, then IC_{ℓ} and IC_h are both binding by Lemma 1, and $\pi_{\ell b} = \pi_{hg} = M$ by Lemma 2. Dividing IC_{ℓ} by $p_{\ell g}$ and IC_h by p_{hg} , adding the two equations, and using the fact that $\pi_{\ell b} = \pi_{hg} = M$ yields an equation that contradicts the assumptions that $J \geq 0$ and that $\Delta_s(q) \equiv C_{hs}(q) - C_{\ell s}(q)$ is strictly increasing with q . *Q.E.D.*

Sketch of the Proof of Lemma 4. If EIR_h is slack, then IC_h is binding by Lemma 1, EIR_{ℓ} is binding by Lemma 3, and $\pi_{\ell b} = M$ by Lemma 2. Rewriting IC_h as an equation, and using the facts that $M < 0$, $J \geq 0$, $\Delta_g(q_{\ell g}) > 0$, and $\Delta_b(q_{\ell b}) > 0$ leads to a contradiction of the assumption that EIR_h is slack. *Q.E.D.*

Proof of Proposition 1. (i) Suppose that IC_{ℓ} and IC_h are both slack. Then, by Lemma 1, EIR_{ℓ} and EIR_h are binding. Since EIR_{ℓ} and EIR_h are binding while IC_{ℓ} and IC_h are slack, the solution to the regulator's problem is the first-best solution. To show that at the first-best solution, IC_{ℓ} and IC_h are indeed slack, note that since EIR_{ℓ} and EIR_h are binding, the left-hand sides of IC_{ℓ} and IC_h vanish. Hence, we only need to show that the right-hand sides of IC_{ℓ} and IC_h are both negative. The right-hand side of IC_h can be rewritten $(JM/p_{\ell g}) - p_{hg}\Delta_g(q_{\ell g}^*) - p_{hb}\Delta_b(q_{\ell b}^*)$, which is negative, since $J \geq 0$ and $M \leq 0$. Next, since EIR_h is binding, $\pi_{hb} = -(p_{hg}/p_{hb})M$. Consequently, the right-hand side of IC_{ℓ} becomes $(JM/p_{hb}) + p_{\ell g}\Delta_g(q_{hg}^*) + p_{\ell b}\Delta_b(q_{hb}^*) = R_{\ell}^* < 0$, as required.

(ii) If $R_{\ell}^* > 0$, the first-best solution cannot be implemented, since it violates IC_{ℓ} . Consequently, IC_{ℓ} must be binding at the optimum. Given that EIR_h is binding as well, $\pi_{hb} = -(p_{hg}/p_{hb})M$. Hence IC_{ℓ} can be written as

$$p_{\ell g}\pi_{\ell g} + p_{\ell b}M = \frac{JM}{p_{hb}} + p_{\ell g}\Delta_g(q_{hg}) + p_{\ell b}\Delta_b(q_{hb}), \tag{A1}$$

where the left-hand side is just the expected profit of the low-type firm.

The regulator's problem is given by RP subject to equation (A1), $\pi_{\ell b} = \pi_{hg} = M$, $\pi_{hb} = -(p_{hg}/p_{hb})M$, and subject to monotonicity of output. To characterize the solution, we shall ignore the monotonicity conditions, obtain a solution, and then verify that at this solution, output is indeed monotonic in the firm's type. Since EIR_h is binding, the high-type firm gets a zero expected profit, while the expected profit of the low-type firm is given by the right-hand side of (A1). Hence, the regulator's problem can be rewritten as

$$\max_{(q_{\theta s})} \sum_{\theta} \sum_s \phi_{\theta} p_{\theta s} [S(q_{\theta s}) - C_{\theta s}(q_{\theta s})] - (1 - \alpha)\phi_{\ell} \left[\frac{JM}{p_{hb}} + p_{\ell g}\Delta_g(q_{hg}) + p_{\ell b}\Delta_b(q_{hb}) \right].$$

The properties of S and $C_{\theta s}$ ensure that the solution to the regulator's problem is defined implicitly by the first-order conditions (8) and (9). To verify that output is monotonic in the firm's type, note that the conditions in the proposition imply that $q_{\ell s}^{**} = q_{\ell s}^*$ for $s = g, b$. Since $\alpha < 1$ and $\Delta'_s(q) > 0$ for $s = g, b$, these conditions also imply that $q_{hs}^{**} < q_{hs}^*$ for $s = g, b$. Hence, $q_{\ell s}^{**} = q_{\ell s}^* > q_{hs}^* > q_{hs}^{**}$ for $s = g, b$, as required. *Q.E.D.*

Proof of Lemma 5. Clearly, $\pi_s^*(\theta) = 0$ for all $s \in \{1, \dots, n\}$ cannot be a solution because it violates (14). By (12), then, the firm necessarily earns a positive profit in at least one state of nature and incurs a loss in at least one other state of nature.

Now fix θ and consider any solution to the maxmin problem. Let i be the state in which the firm's profit is highest and j be the state in which its loss is highest. That is, $\pi_j^*(\theta) \leq \pi_s^*(\theta) \leq \pi_i^*(\theta)$ for all $s \in \{1, \dots, n\}$. To show that the firm earns a positive profit in exactly one state and makes the same loss in all other states, we prove that $\pi_s^*(\theta) = \pi_j^*(\theta)$ for all $s \neq i$. To this end, suppose by way of negation that at the solution to the maxmin problem, there exists a state $k \neq \{i, j\}$ such that $\pi_j^*(\theta) < \pi_k^*(\theta) < \pi_i^*(\theta)$. Solving equation (12) for $\pi_i^*(\theta)$, substituting in (14), and simplifying, we get

$$\sum_{s \neq i} p_s(\theta)\gamma_s(\theta)\pi_s^*(\theta) = B^*(\theta), \quad \forall \theta \in \Theta, \tag{A2}$$

where $\gamma_s(\theta) \equiv p'_s(\theta)/p_s(\theta) - p'_i(\theta)/p_i(\theta)$. If we raise $\pi_j^*(\theta)$ slightly by ε and adjust $\pi_k^*(\theta)$ by $-p_j(\theta)\gamma_j(\theta)\varepsilon/(p_k(\theta)\gamma_k(\theta))$ to ensure that (A2) continues to hold, then by (12), $\pi_i^*(\theta)$ changes by $-(p_j(\theta)/p_i(\theta))[1 - (\gamma_j(\theta)/\gamma_k(\theta))]\varepsilon$. The profit levels in all other states remain unchanged. From these expressions it is clear that we can always choose ε small enough such that after the change, we still have $\pi_j^*(\theta) < \pi_k^*(\theta) < \pi_i^*(\theta)$. The fact that we have managed to raise the minimal profit level, $\pi_j^*(\theta)$, contradicts the assumed optimality of the solution to the maxmin problem. Consequently, it must be the case that $\pi_s^*(\theta) = \pi_j^*(\theta)$ for all $s \neq \{i, j\}$. Since by definition $\pi_j^*(\theta) < 0 < \pi_i^*(\theta)$, it follows that for all $\theta \in \Theta$, the firm earns a profit only in state i and incurs the same loss in all other states. *Q.E.D.*

Proof of Proposition 2. By Assumption 3,

$$r'_s(\theta) \equiv \frac{p'_s(\theta)p_n(\theta) - p_s(\theta)p'_n(\theta)}{(p_n(\theta))^2} = \frac{p_s(\theta)}{p_n(\theta)} \left[\frac{p'_s(\theta)}{p_s(\theta)} - \frac{p'_n(\theta)}{p_n(\theta)} \right] < 0, \quad \forall s \neq n.$$

Hence $p'_s(\theta)/p_s(\theta) < p'_n(\theta)/p_n(\theta)$ for all $s \neq n$, implying that state n has the highest $p'_i(\theta)/p_i(\theta)$ ratio and hence should be the one in which the firm is rewarded.

Since the maxmin transfers defined by (17) were derived from (12), these transfers clearly satisfy the EIR_θ constraint for all $\theta \in \Theta$. We now check that these transfers ensure global incentive compatibility. To this end, note that if we substitute the maxmin transfers from (17) with $i = n$ into the $IC_{\theta, \hat{\theta}}$ constraints and simplify, we get

$$0 \geq \frac{p_n(\theta) - p_n(\hat{\theta})}{p'_n(\hat{\theta})} B^*(\hat{\theta}) + \sum_s p_s(\theta) q_s^*(\hat{\theta}) (c_s(\hat{\theta}) - c_s(\theta)), \quad \forall \theta, \hat{\theta} \in \Theta, \tag{A3}$$

where the left-hand side vanishes because the transfers were chosen such that $\sum_s p_s(\theta) \pi_s^*(\theta) = 0$. Assuming that $\hat{\theta} > \theta$ and dividing by $\hat{\theta} - \theta$, the right-hand side of (A3) becomes

$$\left(\frac{p_n(\theta) - p_n(\hat{\theta})}{\hat{\theta} - \theta} \right) \frac{B^*(\hat{\theta})}{p'_n(\hat{\theta})} + \sum_s p_s(\theta) q_s^*(\hat{\theta}) \left(\frac{c_s(\hat{\theta}) - c_s(\theta)}{\hat{\theta} - \theta} \right).$$

Since $c_s(\theta)$ is increasing and convex, while by Assumption 1, $p_n(\theta)$ is increasing and concave,

$$\frac{c_s(\hat{\theta}) - c_s(\theta)}{\hat{\theta} - \theta} \leq c'_s(\hat{\theta}), \quad \frac{p_n(\theta) - p_n(\hat{\theta})}{\hat{\theta} - \theta} \leq -p'_n(\hat{\theta}).$$

Using the definition of $B^*(\cdot)$ we get

$$\begin{aligned} \left(\frac{p_n(\theta) - p_n(\hat{\theta})}{\hat{\theta} - \theta} \right) \frac{B^*(\hat{\theta})}{p'_n(\hat{\theta})} + \sum_s p_s(\theta) q_s^*(\hat{\theta}) \left(\frac{c_s(\hat{\theta}) - c_s(\theta)}{\hat{\theta} - \theta} \right) &\leq -B^*(\hat{\theta}) + \sum_s p_s(\theta) q_s^*(\hat{\theta}) c'_s(\hat{\theta}) \\ &= \sum_s q_s^*(\hat{\theta}) c'_s(\hat{\theta}) [p_s(\theta) - p_s(\hat{\theta})]. \end{aligned} \tag{A4}$$

The expression in the last line of (A4) is nonpositive, since by assumption, $q_s^*(\cdot) c'_s(\cdot)$ is weakly increasing with s , and since by Assumption 2, $p_s(\cdot)$ satisfies FOSD. Hence, (A3) is satisfied. The proof in the case of $\hat{\theta} < \theta$ is analogous. Consequently, global incentive compatibility is ensured.

Finally, we need to verify that the first-best production levels can be implemented with the maxmin transfers if and only if (18) holds. Since the maxmin transfers satisfy the EIR_θ constraints by construction, and since we already verified that they ensure global incentive compatibility, we only need to verify that the maxmin transfers satisfy the $IR_{\theta, s}$ constraints. The “if” part of the statement follows directly from the fact that if (18) holds, then the maxmin transfers ensure that the firm’s profit in states $1, \dots, n - 1$ is equal to or exceeds M . To prove the “only if” part, note that since, by construction, the maxmin transfers ensure that the firm’s loss is minimal in every given state, and since Assumption 3 ensures that state n has the highest $p_i(\theta)/p'_i(\theta)$ ratio, it is obvious that if (18) is violated for some θ , then under any system of transfers that is locally incentive compatible and leaves the firm no expected rent (i.e., satisfies equation (12) and (14)), at least one type of firm would incur a loss greater than $|M|$ in at least one state. That is, at least one of the $IR_{\theta, s}$ constraints will be violated. *Q.E.D.*

Proof of Lemma 6. Using (13) and the definition $r_s(\theta) \equiv p_s(\theta)/p_n(\theta)$, we can express $t'_n(\theta)$ as a function of the other $n - 1$ transfer functions:

$$t'_n(\theta) = \sum_s r_s(\theta) c_s(\theta) q'_s(\theta) - \sum_{s \neq n} r_s(\theta) t'_s(\theta).$$

Integrating $t'_n(\theta)$ from $\bar{\theta}$ (the “worst” type) to θ yields

$$\begin{aligned} t_n(\theta) &= t_n(\bar{\theta}) - \int_{\bar{\theta}}^{\theta} \left[\sum_s r_s(x) c_s(x) q'_s(x) - \sum_{s \neq n} r_s(x) t'_s(x) \right] dx \\ &= t_n(\bar{\theta}) - \sum_s r_s(\bar{\theta}) c_s(\bar{\theta}) q_s(\bar{\theta}) + \sum_{s \neq n} r_s(\bar{\theta}) t_s(\bar{\theta}) + \sum_s r_s(\theta) c_s(\theta) q_s(\theta) \\ &\quad - \sum_{s \neq n} r_s(\theta) t_s(\theta) + \int_{\bar{\theta}}^{\theta} \left[\sum_s [r_s(x) c_s(x)]' q_s(x) - \sum_{s \neq n} r'_s(x) t_s(x) \right] dx, \end{aligned}$$

where the second equality follows from integration by parts. Since, by definition, $\pi_s(\theta) = t_s(\theta) - c_s(\theta) q_s(\theta)$ and $r_n(\theta) = 1$, the first three terms in the above expression equal $\sum_s r_s(\bar{\theta}) \pi_s(\bar{\theta})$. Moreover, since $r'_n(\theta) = 0$, $\sum_{s \neq n} r'_s(x) t_s(x) =$

$\sum_s r'_s(x)t_s(x)$. Hence, the square-bracketed expression equals

$$\sum_s r_s(x)c'_s(x)q_s(x) + \sum_s r'_s(x)c_s(x)q_s(x) - \sum_{s \neq n} r'_s(x)t_s(x) = \sum_s [r_s(x)c'_s(x)q_s(x) - r'_s(x)\pi_s(x)].$$

We therefore write the firm's transfer in state n as in equation (19). *Q.E.D.*

Proof of Lemma 7. By Assumption 3, the coefficient of $\pi_s(\theta)$ in RP_b is negative for all $s \neq n$ (since $r'_n(\theta) = 0$, the coefficient of $\pi_n(\theta)$ is 0). Hence, it is optimal for the regulator to set transfers such that $\pi_s(\theta)$ will be as low as possible; given the $IR_{\theta,s}$ constraints, it follows that at the solution to the relaxed problem, $\pi_s^{**}(\theta) \equiv t_s^{**}(\theta) - c_s(\theta)q_s^{**}(\theta) = M$, $\forall s \neq n$. Substituting this equality in RP_b and recalling that $r'_n(\theta) = 0$, the first-order condition for $q_s(\theta)$ is

$$[S'(q_s(\theta)) - \alpha c_s(\theta)] p_s(\theta) f(\theta) - (1 - \alpha) c_s(\theta) p_s(\theta) f(\theta) - (1 - \alpha) r_s(\theta) c'_s(\theta) \varphi_n(\theta) = 0, \tag{A5}$$

for all s , and all $\theta \in \Theta$. Now, note that the cumulative distribution of θ , conditional on state n being realized, is given by

$$F(\theta | n) = \frac{\int_{\underline{\theta}}^{\theta} p_n(x) f(x) dx}{\int_{\underline{\theta}}^{\bar{\theta}} p_n(x) f(x) dx} = \frac{\varphi_n(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} p_n(x) f(x) dx}.$$

Hence,

$$\varphi_n(\theta) = \frac{F(\theta | n)}{f(\theta | n)} p_n(\theta) f(\theta).$$

Substituting this equality in (A5) and simplifying yields equation (22). Since $S(\cdot)$ is increasing and concave, $q_s^{**}(\theta)$ is differentiable and unique. If $f(\theta | n)/F(\theta | n)$ is nonincreasing with θ (so $F(\theta | n)/f(\theta | n)$ is nondecreasing with θ), then, together with the assumption that $c_s(\theta)$ is increasing and convex, the right-hand side of (22) is increasing with θ , so $q_s^{**}(\theta)$ is decreasing with θ .

To characterize the transfers, note that (23) follows immediately from the fact that $\pi_s(\theta) = M$ for all $s \neq n$, and all $\theta \in \Theta$. As for (24), note that since by definition $r'_n(\theta) = 0$, $t_n(\bar{\theta})$ appears only in the second line of RP_b . Hence, it is clear that it is optimal to choose the transfers of type $\bar{\theta}$ such that $\sum_s r_s(\bar{\theta})\pi_s(\bar{\theta}) = 0$. Substituting this equality in (19), using (23) and the fact that $r_n(\theta) = 1$ and $r'_n(\theta) = 0$, yields

$$\begin{aligned} t_n^{**}(\theta) &= c_n(\theta)q_n^{**}(\theta) + \sum_{s \neq n} r_s(\theta) [c_s(\theta)q_s^{**}(\theta) - t_s^{**}(\theta)] + \int_{\theta}^{\bar{\theta}} \sum_s r_s(x)c'_s(x)q_s^{**}(x)dx - \int_{\theta}^{\bar{\theta}} \sum_{s \neq n} r'_s(x)Mdx \\ &= c_n(\theta)q_n^{**}(\theta) - \sum_{s \neq n} r_s(\theta)M + \int_{\theta}^{\bar{\theta}} \sum_s r_s(x)c'_s(x)q_s^{**}(x)dx - \sum_{s \neq n} r_n(\bar{\theta})M + \sum_{s \neq n} r_s(\bar{\theta})M. \end{aligned}$$

Noting that $\sum_{s \neq n} r_n(\bar{\theta}) = (\sum_{s \neq n} p_s(\bar{\theta})/p_n(\bar{\theta})) = (1 - p_n(\bar{\theta}))/p_n(\bar{\theta})$ yields (24). From (23) and (24) it is clear that $t_s^{**}(\theta)$ is unique and differentiable for all s . *Q.E.D.*

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