Pre-Grant Patent Publication and Cumulative Innovation

Technical appendix

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Abstract

This appendix contains a proof that was omitted from the paper

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Proving that the equilibria in the filing and in the no-filing subgames are unique and the equilibrium investment levels are between 0 and 1: We will consider the filing subgame under the PP system. The proofs in the case of the CF system and in the case where firm 1 does not file for a patent are analogous.

When firm 1 files for a patent under the PP system, the best-response functions, $R^1(q^2|F)$ and $R^2(q^1|F)$ are determined implicitly by the equations

$$\frac{\partial \pi^1(q^1, q^2|F)}{\partial q^1} = q^2(1-\gamma\theta)(\pi_{yy} - \pi_{ny}) + \left(1 - q^2(1-\gamma\theta)\right)(\pi_{yn} - \pi_{nn}) - C'(q^1) = 0, \tag{1}$$

and

$$\frac{\partial \pi^2(q^1, q^2|F)}{\partial q^2} = (1-\gamma\theta)\left[q^1(\pi_{yy} - \pi_{ny}) + (1-q^1)(\pi_{yn} - \pi_{nn})\right] - \beta_L C'(q^2) = 0. \tag{2}$$

To show that $R^1(q^2|F)$ and $R^2(q^1|F)$ intersect only once inside the unit square, rewrite (1) and (2) as follows:

$$q^2 = H_1(q^1) = \frac{(\pi_{yn} - \pi_{nn}) - C'(q^1)}{(1-\gamma\theta)\Pi},$$

and

$$q^1 = H_2(q^2) = \frac{(1-\gamma\theta)(\pi_{yn} - \pi_{nn}) - \beta_L C'(q^2)}{(1-\gamma\theta)\Pi}.$$

When $\Pi > 0$ ($R^1(q^2|F)$ and $R^2(q^1|F)$ are downward sloping), $H_1(q^2)$ and $H_2(q^1)$ intersect in the (q^1, q^2) space inside the unit square provided that (i) $H_1(0) > 1$, (ii) $H_1(1) < 0$, (iii) $H_2(1) < 0$, (iv) $H_2(0) > 1$. Recalling that $C'(0) = 0$, conditions (i) and (iv) are both satisfied because Assumption A1 ensures that $\pi_{yn} - \pi_{nn} > \Pi$. Conditions (ii) and (iii) are satisfied because Assumption A2 ensures that $C'(1) > \pi_{yn} - \pi_{nn}$, and because $\beta_L > 1 > 1 - \gamma\theta$.

Next, suppose that $\Pi < 0$ ($R^1(q^2|F)$ and $R^2(q^1|F)$ are upward sloping). Now, $H_1(q^2)$ and $H_2(q^1)$ intersect in the (q^1, q^2) space inside the unit square provided that (i) $H_1(0) < 0$, (ii) $H_1(1) > 1$, (iii) $H_2(1) > 1$, (iv) $H_2(0) < 0$. Recalling that $C'(0) = 0$, conditions (i) and (iv) are both satisfied because $\Pi < 0$. Condition (ii) is satisfied if $(\pi_{yn} - \pi_{nn}) - (1-\gamma\theta)\Pi < C'(1)$. Since $\Pi < 0$, $(\pi_{yn} - \pi_{nn}) - (1-\gamma\theta)\Pi < (\pi_{yn} - \pi_{nn}) - \Pi = \pi_{yy} - \pi_{ny} < C'(1)$, where the equality follows because $\Pi \equiv \pi_{yn} + \pi_{ny} - \pi_{yy} - \pi_{nn}$ and the last inequality is implied by Assumption A2. Likewise, condition (iii) is satisfied if $\beta_L C'(1) > (1-\gamma\theta)(\pi_{yn} - \pi_{nn} - \Pi) = (1-\gamma\theta)(\pi_{yy} - \pi_{ny})$, which is ensured by Assumption A2.

To prove uniqueness, note that the slopes of $R^1(q^2|F)$ and $R^2(q^1|F)$ are given by $-\frac{C''(q^1)}{(1-\gamma\theta)\Pi}$ and $\frac{(1-\gamma\theta)\Pi}{\beta_L C''(q^2)}$. Assumption A2 ensures that $\left|-\frac{C''(q^1)}{(1-\gamma\theta)\Pi}\right| > 1 > \left|\frac{(1-\gamma\theta)\Pi}{\beta_L C''(q^2)}\right|$, which in turn implies that $R^1(q^2|F)$ and $R^2(q^1|F)$ intersect only once both when $\Pi > 0$ and when $\Pi > 0$. ■