Price Competition in Two-Sided Markets with Heterogeneous Consumers and Network Effects

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Abstract

We model a two-sided market with heterogeneous customers and two heterogeneous network effects. In our model, customers on each market side care differently about both the number and the type of customers on the other side. Examples of two-sided markets are online platforms or daily newspapers. In the latter case, for instance, readership demand depends on the amount and the type of advertisements. Also, advertising demand depends on the number of readers and the distribution of readers across demographic groups. There are feedback loops because advertising demand depends on the numbers of readers, which again depends on the amount of advertising, and so on. Due to the difficulty in dealing with such feedback loops when publishers set prices on both sides of the market, most of the literature has avoided models with Bertrand competition on both sides or has resorted to simplifying assumptions such as linear demands or the presence of only one network effect. We address this issue by first presenting intuitive sufficient conditions for demand on each side to be unique given prices on both sides. We then derive sufficient conditions for the existence and uniqueness of an equilibrium in prices. For merger analysis, or any other policy simulation in the context of competition policy, it is important that equilibria exist and are unique. Otherwise, one cannot predict prices or welfare effects after a merger or a policy change. Our conditions are related to the own- and cross-price effects, as well as the strength of the own and cross network effects. We show that most functional forms used in empirical work, such as logit type demand functions, tend to satisfy these conditions. Finally, using data on the Dutch daily newspaper industry, we estimate a flexible model of demand which satisfies the above conditions and evaluate the effects of a hypothetical merger.

JEL Classification: L13, L40, L82.

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1 Introduction

Markets are called two-sided if a) firms act as platforms that sell two different products or services to two different groups of customers, b) demand of at least one group depends on demand of the other group c) firms take the inter-relatedness of demands (or indirect network effect) into account when setting prices d) customers on one side of the market cannot pass on to customers on the other side of the market increases in the price they are asked by the platform (Rochet and Tirole, 2003; Evans, 2003; Filistrucchi, Geradin, and van Damme, 2012).

Traditional media markets are a typical example of two-sided markets (Anderson and Gabszewicz, 2008). They sell content and advertising space. Advertisers’ demand for ads on a media outlet increases with the number of consumers of content (viewers, readers, listeners, etc.), while consumers of content may also be, positively or negatively, affected by the quantity of advertising. Media firms are well aware of this relationship between the two demands they face and set prices accordingly. For instance, they may lower the price on one side in order to boost demand on the other side. Free newspapers and free-to-air TV are an extreme example of such a pricing policy.

The two-sided business model is also the most common business model on the Internet. Online trading platforms, such as Amazon or eBay, or intermediaries in the advertising market, such as Google with AdSense and AdWords, sell their services to buyers and sellers that both value the popularity of the platform on the other side of the market. Two-sided is also the business model of Google, as a provider for instance of search or email services, and of Facebook as a social network. Attracting users with various free services and make advertisers pay the bill is, in fact, the same business model of free-to-air TV and free newspapers.

Not least because of the emergence of the internet, economists and policy makers have become increasingly interested in two-sided markets. In the last ten years the theoretical literature on two-sided markets has grown rapidly.\footnote{See Rochet and Tirole (2003), Rochet and Tirole (2006) and Armstrong (2006) for the seminal theoretical studies and Rysman (2009) for a recent survey} A key insight in this literature is that pricing decisions in two-sided markets may be very different from pricing decisions in one-sided markets. From this, it follows that analyzing a two-sided market as if it were a single-sided market may lead to mistakes and unintended consequences in the application of competition policy (Evans, 2003; Wright, 2004). For example, one may falsely predict prices to increase on both sides of the market after a merger in the absence of productive efficiency gains. On the contrary, (Chandra and Collard-Wexler, 2009) present an economic model of the newspaper
market and show that it is not necessarily the case that a monopolist will choose to set higher prices on both market sides as compared to competing duopolists. Rather, a monopolist may choose to raise prices on one side and lower them on the other side.

Despite the growth of the theoretical literature on two-sided markets, most of the theoretical papers have either not modeled firms as setting prices on both sides of the market (Anderson and Coate, 2005) or have assumed linear demand (Armstrong, 2006) or have assumed price on one side has to be zero (Jean J. Gabszewicz and Sonnac, 2001, 2002) or have assumed one of the network effects to be zero. Similarly, for the models presented in structural econometric papers: Rysman (2004) presents a model to analyse the market for phone directories in the US where users of the directories clearly do not pay.\(^2\) Kaiser and Wright (2006) limit their analysis of magazines in Germany to markets with two magazines in order to be able to apply Armstrong (2006) Hotelling duopoly model. Argentesi and Filistrucchi (2007) and Fan (forthcoming) analyse the market for daily newspapers, in Italy and in the US respectively, estimating an insignificant effect of advertising on circulation and hence being able to assume no effect of advertising on readers.

In fact, while Bertrand competition is one of the standard oligopoly models used in industrial organization, the state-of-the-art in the analysis of two-sided markets does not allow to model firms as competing à la Bertrand on both sides of the market, except under the restrictive assumptions of one network effect or linear demand. This appears to be due more to technical difficulties rather than to empirical evidence showing that platforms do not set prices on both sides. In particular, there are feedback loops in two-sided markets in the presence of two network effects. This is because, for instance, advertising demand depends on the numbers of readers, which depends on the amount of advertising, which again depends on the amount of advertising and so on. The existence of these feedback loops implies that a price increase on one side has a complex effect on both demand on that side and demand on the other side. In practice, on the one hand, it is not clear that such a loop is finite, on the other hand it may be the case that quantities on the two-sides are not unique given prices on the two-sides. As a result, a multiplicity of optimal choices in monopoly (Weyl, 2010) and a multiplicity of equilibria in oligopoly naturally arise (White and Weyl, 2012). Yet, for merger analysis, or any other policy simulation in the context of competition policy, it is important that equilibria exist and are unique. Otherwise, one cannot predict prices or welfare effects after a merger or a policy change.

\(^2\)Similarly, Jeziorski (2012) analyses the market for radio in the US, under the reasonable assumption that listeners cannot be asked to pay even after the merger.
In general, the existence of network effects may give rise to multiple equilibria in both the consumers’ coordination game and the firms’ pricing game. For example, given the prices set by firms, it might be the case that all consumers choose one platform or another platform or they split among the two platforms. In fact, when consumers on one side choose a platform, they choose not only based on price on that side but also based on the expected number of users on the other side and *vice versa*. In equilibrium, these expectations need to be true. This is the coordination game. Uniqueness of demand given prices is a necessary condition for the existence of a unique equilibrium in the pricing game. Yet, even when demands are unique given prices, it may be the case that more than one equilibrium exists in the firm’s pricing game. Clearly, if there are multiple equilibria in the coordination game, multiple equilibria in the pricing game will be more likely.

To address these difficulties, Weyl (2010) and White and Weyl (2012) propose to model firms as setting insulating tariffs, i.e. price schedules conditional on the quantities on the other market side, instead of setting prices. For example, an oligopolistic newspaper publisher would not set advertising prices, but advertising prices depending on circulations of its own newspaper and the rivals’ newspapers. White and Weyl (2012) show that there is a unique equilibrium in insulating tariffs. However, in general, Nash equilibria in pure strategies and insulating tariffs equilibria will not coincide. It is an open question to what extent conclusions regarding price or welfare effects differ qualitatively and quantitatively depending on whether firms chose prices or price schedules. More importantly, whereas there are many instances in which firms charge prices conditional on their own quantity on the other side (e.g. when a price per viewer is charged to advertisers on TV), it is instead unclear that firms actually charge prices conditional on rivals’ quantities on the other side (e.g. whether the price of TV advertising on a each station changes also with the number of viewers of competing TV stations).

In this paper, we show how one can account for the feedback loops that arise when there are two network effects between the two market sides. We first derive an intuitive sufficient condition for demand to be unique given prices. This solves the issue of multiplicity of equilibria in the consumers’ coordination game. We then derive sufficient conditions for the existence and uniqueness of an equilibrium in prices. Both sets of conditions are related to the strength of the own and cross network effects.

We present a general model of a two-sided market with two network effects and heterogeneous consumers. In our model, consumers on one market side care differently about the amount and the type of advertising, and advertising demand depends on both the number and the distribution of consumer demographics, such as socioeconomic status, age and gender.
Our contribution will allow competition authorities to improve their quantitative assessment of mergers in two-sided markets. In fact, when assessing a merger between two newspaper publishers, it is important to quantify the price changes on each market side, and characterize the welfare effects. This has to be done in realistic, albeit simplified, settings in which a number of newspaper publishers own more than one newspaper (Filistrucchi, Klein, and Michielsen, 2012a,b). For this, parameters of sufficiently rich demand systems need to be estimated (Berry, 1994; Berry, Levinsohn, and Pakes, 1995) and marginal costs need to be inferred from prices. Intuitively, to infer marginal costs one searches for those values of the marginal costs such that the observed prices are optimal for the firms given the demand parameter estimates (Rosse, 1970). Having done so, one can use the estimates of the marginal costs to calculate the new equilibrium prices after the merger. It is now state-of-the art among academics and practitioners to conduct such a study for one-sided markets (for example Nevo, 2000a, or Budzinski and Ruhmer, 2010). In two-sided markets, however, this has only been done for special cases of two-sided markets. For example, the econometric models of Argentesi and Filistrucchi (2007), Van Cayseele and Vanormelingen (2009) and Fan (forthcoming) only model one indirect network effect. Rysman (2004) and Jeziorski (2012) allow instead for two network effects, but deal with the case when one of the two prices is zero. Moreover, while Chandra (2009) shows that advertising demand in media markets is related to subscriber characteristics and that this feeds back into firm’s pricing decision, none of the models allows for such effects.

For merger analysis, or any other policy simulation, it is important that equilibria exist and are unique. Otherwise, one cannot predict prices or welfare effects after a merger or a policy change. In one-sided markets, both properties usually hold for the specifications that are commonly used (Vives, 2001; Mizuno, 2003). We fill a gap in the two-sided markets literature by discussing under which conditions, in a two-sided market, Nash equilibria exist and are unique.

Finally, we estimate the model using data on the Dutch daily newspaper industry and evaluate the effects of a hypothetical merger.

2 Demand

2.1 General model

There are \( J \) platforms \( j = 1, \ldots, J \) that each serve two groups of customers. Demand from one group of customers depends on demand from the other group and vice versa. These platforms could be online
platforms such as search engines or social networks, or off-line platforms such as daily newspapers or magazines. In line with our empirical application, we will henceforth think of the platforms as newspapers, but our results apply more generally to all two-sided markets in which the platform charges membership fees.

Each newspaper \( j \) is owned by a newspaper publisher \( f \) and sells advertising space to advertisers, at a price \( p_{a}^{j} \), and subscriptions to readers, at a price \( p_{r}^{j} \). In principle, these prices can be zero or even negative on one side, in which case membership on that side is subsidized by the platforms. This is obviously the case when newspapers are distributed for free and profits are earned solely on the advertising side. Newspapers are not able to price-discriminate among the different groups of readers or advertisers. This means that they charge the same subscription price to all readers and the same advertising rate to all advertisers. There are \( G_{r} \) demographic groups of readers. An example of a group are the high income readers between the age of 30 and 40 who live in a particular part of the country. Conversely, there are \( G_{a} \) groups of advertisers. Here, each group corresponds to a combination of type of advertised product (e.g. pasta or clothing) and of type of advertisement (e.g. funny or informative). Importantly, advertising demand depends not only on the total number of readers of each of the \( J \) newspapers, but also on the distribution of readers across different demographic groups. This is sensible because certain types of advertisers will be willing to pay more for advertising space if there are, say, more high income individuals who read newspaper \( j \). Similarly, readership demand will not only depend on the amount of advertising, but also on the type of advertisements. For example, high income individuals may appreciate informative advertisements more than funny ones. Figure 1 shows the effect of feedback loops if there are two newspapers with no groups on the advertising side, but two groups of readers, with high and low income. There are therefore two advertising demands, \( q_{a}^{1} \) and \( q_{a}^{2} \), and four readership demands, \( q_{r}^{1l} \), \( q_{r}^{1h} \), \( q_{r}^{2l} \) and \( q_{r}^{2h} \) (subscripts \( l \) and \( h \) denote low and high income, respectively). Now suppose—as indicated in the

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3 Notice that these prices do not depend on whether advertisers and readers actually interact later on, and hence the market is a so-called two-sided non-transaction market and the prices are effectively membership fees. See also Filistrucchi, Geradin, and van Damme (2012).

4 We here stick to the most common assumption in theoretical models of two-sided markets. For a model of price discrimination in two sided markets, see Serfes and Liu (forthcoming). In the market for daily newspapers, while the cover price is in general the same for all newspapers, also advertising list prices do not feature price discrimination. However, it may be the case that price discrimination takes place on the advertising side through the granting of personalized discounts. Since we do not observe individual discounts to advertisers, in the empirical application that follows, we maintain the assumption of no price discrimination also on the advertising side.

5 There could also be differences in taste within a group of readers. The essential assumption we make advertising demand will only depend on the distribution of readers between groups, but not on taste differences of those readers within each demographic group. That is, we assume that advertisers care about the number of readers in the high income group, but not the taste of every single member or the distribution of tastes within that group. Notice, however, that in principle one can choose a narrow definition of a group to make this assumption plausible. For empirical work, this is of course restricted by the availability of data.
upper-left corner—that the advertising price for newspaper 1 decreases. This will affect both demands on the advertising side and they will subsequently affect all four readership demands, and they will again affect all advertising demands, and so on. This process may, or may not, converge. Generally, it will converge if network effects are not too strong. We provide a sufficient condition for convergence below.

Denote the two $J \times 1$ vectors of advertising and subscription prices as $p^a$ and $p^r$, respectively. Moreover, for $g^a = 1, 2, \ldots, G^a$ denote the $J \times 1$ vector of advertising quantities of group $g^a$ by $q^a_{g^a}$ and, for $g^r = 1, 2, \ldots, G^r$, denote the $J \times 1$ vector of reader quantities of group $g^r$ by $q^r_{g^r}$. Stack them into the $G^a J \times 1$ vector of advertising quantities of all groups, $q^a = \left( q^a_1, q^a_2, \ldots, q^a_{G^a} \right)^\prime$, and the $G^r J \times 1$ vector of reader quantities $q^r = \left( q^r_1, q^r_2, \ldots, q^r_{G^r} \right)^\prime$. From the firms’ perspective, demands at the group level on both market sides are functions of prices on the same market side and quantities at the group level on the other market side. For instance, aggregate advertising demand by one group of advertisers in newspaper $j$ is a function of all advertising prices and the distribution of readers in demographic groups in each newspaper. These demand functions will be denoted by $q^a = q^a(p^a, q^r)$ and $q^r = q^r(p^r, q^a)$. We assume that they are continuously differentiable. It will be convenient to express demands as functions

Figure 1: Feedback loops
of prices only, or put differently, to work with reduced-form demand functions. We will denote these reduced-form demand functions by \( q^a = \hat{q}^a(p^a, p') \) and \( q^r = \hat{q}^r(p^a, p') \). In principle, quantities need not be unique for given prices (in which case these would not be functions, but correspondences). One reason for this could be a coordination problem—an issue that has received considerable attention in the theoretical literature (see, for example, Rochet and Tirole, 2003, and Armstrong, 2006). To see this, suppose that advertisers like readers and readers like advertisements. Then, it could be an equilibrium that, for given prices, all advertisers and all readers go to one newspaper. Another equilibrium could be that they all go to another newspaper. In Assumption 1 we provide a sufficient conditions for existence and uniqueness of the reduced-form demand functions given prices. Here and in the following we follow Magnus (2010) and denote derivatives of a \( K_a \times 1 \)-vector \( a \) with respect to a \( K_b \times 1 \)-vector \( b \) by \( \partial a / \partial b' \) and call \( \partial a / \partial b' \) the \( K_a \times K_b \) Jacobian matrix.

**Assumption 1** (network effects). Assume that the feedback effects are not too strong in the sense that

\[
\sum_{l\in j} \left| \sum_{k\in g} \frac{\partial q^r_{lg}}{\partial q^a_{kg}} \cdot \frac{\partial q^a_{kg}}{\partial q^r_{lk}} \right| < 1
\]

and

\[
\sum_{l\in j} \left| \sum_{k\in g} \frac{\partial q^a_{lg}}{\partial q^r_{kg}} \cdot \frac{\partial q^r_{kg}}{\partial q^a_{lk}} \right| < 1
\]

for all \( j, g, q^a, q^r \).

The assumption is that feedback effects are not too strong. This is the case if at least one of the two network effects is not too strong. To better understand it, assume for the moment that advertising demand is of the constant elasticity form used in Rysman (2004),

\[
\log (q^a_j) = \alpha^a + \beta^a \log(p^a_j) + \gamma^a \log(q^r_j) + \epsilon_j.
\]

Assume that readership demand is given by a standard multinomial logit model with products \( j = 1, \ldots, J \), outside good \( j = 0 \) and market size \( M' \). Consider the simplest case in which the mean utility of a reader when purchasing good \( j \) is \( \delta^r_j = \alpha^r + \beta^r p^r_j + \gamma^r q^a_j \), normalize \( \delta^r_0 = 0 \) and denote the market shares by \( s^r_j = q^r_j / M' \). Then, the indirect network effects are

\[
\frac{\partial q^r_j}{\partial q^a_k} = -M' s^r_j s^a_k \gamma^r
\]
for \( j \neq \ell \) and

\[
\frac{\partial q_j'}{\partial q_j} = M' s_j'(1 - s_j') \gamma'.
\]

The first inequality in Assumption 1 holds if the sum of the absolute values of the changes in quantity \( q_j' \) that originate in changes of all other quantities \( q'_{\ell} \) and affect \( q_j' \) through \( q_a'_{\ell} \) is less than one. For this to be the case we need that

\[
\sum_{\ell} \left| \sum_k \frac{\partial q_j'}{\partial q_k'} \frac{\partial q_k'}{\partial q_j'} \right| = \sum_{\ell} \left| \frac{\partial q_j'}{\partial q_j'} \cdot \frac{\partial q_j'}{\partial q_j'} \right|
\]

\[
= \left| M' s_j'(1 - s_j') \gamma' \cdot \frac{\gamma'}{q_j'} \right| + \sum_{\ell \neq j} \left| -M' s_j' s_{\ell} \gamma' \cdot \frac{\gamma'}{q_{\ell}} \right|
\]

\[
= \left( (1 - s_j') - \sum_{\ell \neq j} s_j' \right) \cdot |\gamma' \gamma'|
\]

\[
= (1 - J \cdot s_j') \cdot |\gamma' \gamma'|
\]

\[< 1.\]

We only have to consider \( k = \ell \) in the above double sum because a change in the number of readers in newspaper \( \ell \) will only affect advertising demand of that newspaper. This shows that the assumption restricts the absolute value of the network effect to be not too big. Observe that it always holds if one of the two network effects is zero. We will further develop the intuition underlying this restriction in a couple of linear examples below.

Next consider another example in which demand on the advertising side is also described by a simple logit model with parameters \( \beta' \) and \( \gamma' \). Denote the indicator function by \( 1 \{ \cdot \} \). Then, Assumption 1 is

\[
\sum_{\ell} \left| \sum_k \frac{\partial q_j'}{\partial q_k'} \cdot \frac{\partial q_k'}{\partial q_{\ell}} \right| = \sum_{\ell} \left| \sum_k M' (s_j' 1\{ j = k \} - s_j' s_k) \gamma' \cdot M_{\ell} (s_{\ell}' 1\{ k = \ell \} - s_{\ell}' s_k) \gamma \right|
\]

\[
= M_{\ell} M' \sum \left| s_j' \sum_k (1\{ j = k \} - s_k) \cdot (s_{\ell}' 1\{ k = \ell \} - s_{\ell}' s_k) \gamma' \gamma \right|
\]

\[< 1.\]

Also here, the condition has the interpretation that the network effects are not too big. That is, given market shares on the advertising and readership side, the absolute value of \( \gamma' \gamma' \) needs to be small enough.
Under Assumption 1 the reduced-form demand functions exist and are unique for given prices. To state this formally, stack prices into the $2J \times 1$ vector $p = (p^a, p^r)'$, quantities into the $(G^a + G^r)J \times 1$ vector $q = (q^a, q^r)'$, and denote the vector-valued function giving the reduced-form quantities by $\hat{q}(p)$.

Lemma 1 (existence and uniqueness of reduced-form demand functions). For any vector $p \in \mathbb{R}^{2J}$ there is a unique set of quantities $\hat{q}(p)$ if Assumption 1 holds. Moreover, for any $q_0 \in \mathbb{R}^{(G^a + G^r)J}$ the sequence of iterates $\hat{q}_0, q(p, \hat{q}_0), q(p, q(p, \hat{q}_0)), \ldots$ converges to $\hat{q}$.

Proof. See p. 33 in Appendix A.

Notice that this proposition does not say that the equilibrium is unique. Rather, it says that there exists a unique set of quantities for given prices that we can solve for.

2.2 A linear demand example with one platform

Let us consider one newspaper facing demand for advertising and readership that is, respectively, linear in price on the same side and quantity on the other side,

\begin{align*}
q^a(p^a, q^r) &= \alpha^a - \beta^a p^a + \gamma^a q^r \\
q^r(p^r, q^a) &= \alpha^r - \beta^r p^r + \gamma^r q^a,
\end{align*}

with $\beta^a, \beta^r > 0$. Solving for $q^a$ and $q^r$ gives

\begin{align*}
\hat{q}^a(p^a, p^r) &= \frac{1}{1 - \gamma^a \gamma^r} \cdot \{ (\alpha^a + \alpha^r \gamma^r) - \gamma^a \beta^a p^a - \gamma^r \beta^r p^r \} \\
\hat{q}^r(p^a, p^r) &= \frac{1}{1 - \gamma^a \gamma^r} \cdot \{ (\alpha^r + \alpha^a \gamma^a) - \gamma^r \beta^a p^a - \gamma^a \beta^r p^r \},
\end{align*}

provided that $\gamma^a \gamma^r \neq 1$, which is the necessary and sufficient condition for existence of the reduced-form quantities.

Alternatively, we can write (1) in matrix notation,

\[ q = \alpha + Bp + \Gamma q, \]
with

\[ q = \begin{pmatrix} q^a \\ q^r \end{pmatrix}, \quad p = \begin{pmatrix} p^a \\ p^r \end{pmatrix}, \quad \alpha = \begin{pmatrix} \alpha^a \\ \alpha^r \end{pmatrix}, \quad B = \begin{bmatrix} -\beta^a & 0 \\ 0 & -\beta^r \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0 & \gamma^a \\ \gamma^r & 0 \end{bmatrix}. \]

We can solve for

\[ q = (I - \Gamma)^{-1} \cdot (\alpha + Bp) \]

provided that \( \det(I - \Gamma) = 1 - \gamma^a \gamma^r \neq 0 \). This shows that the above condition is actually a condition on the determinant of the matrix of network effects.

Based on (2) we can re-interpret the reduced-form demand functions as demands for complementary (if \( \gamma^a, \gamma^r > 0 \)) or substitute (if \( \gamma^a, \gamma^r < 0 \)) products. However, a middle case is also possible in which demand on one side depends negatively on the price of the other side but demand on the other side depends positively on the first price (\( \gamma^a > 0, \gamma^r < 0 \) or \( \gamma^a < 0, \gamma^r > 0 \)). This is the case for instance if advertisers attach a higher value to newspapers with more readers but readers dislike advertising.

The condition \( \gamma^a \gamma^r \neq 1 \) for existence is implied by Assumption 1, which in this linear context holds whenever \( |\gamma^a \gamma^r| < 1 \). Formally, a solution also exists if \( |\gamma^a \gamma^r| > 1 \). However, this solution is not meaningful in the context of demand, because the reduced-form quantities depend positively on their own price.

This shows that the conditions that guarantee existence of the reduced form demands are related to the size of the indirect network effects \( \gamma^a \) and \( \gamma^r \). In fact, what matters is the product of the two, because this is the module which is repeated in the loop.\(^6\) To see this, re-write (1) as a geometric series. Define \( \bar{\alpha}^a = \alpha^a - \beta^a p^a \) and \( \bar{\alpha}^r = \alpha^r - \beta^r p^r \). Then,

\[
q^a = \bar{\alpha}^a + \gamma^a (\bar{\alpha}^r + \gamma^r (\bar{\alpha}^a + \gamma^a (\bar{\alpha}^r + \gamma^r (\ldots))))
\]

\[
= \bar{\alpha}^a + \gamma^r \gamma^a \bar{\alpha}^a + (\gamma^a \gamma^r)^2 \bar{\alpha}^a + \ldots + \gamma^a \gamma^r \gamma^a \bar{\alpha}^a + \gamma^a \gamma^r \gamma^a \bar{\alpha}^a + (\gamma^a \gamma^r)^2 \bar{\alpha}^a + \ldots
\]

\[
= (\bar{\alpha}^a + \gamma^a \bar{\alpha}^a) \cdot \left(1 + \gamma^r \gamma^a + (\gamma^a \gamma^r)^2 + \ldots\right)
\]

and

\[
q^r = (\bar{\alpha}^r + \gamma^r \bar{\alpha}^r) \cdot \left(1 + \gamma^a \gamma^r + (\gamma^a \gamma^r)^2 + \ldots\right).
\]

\(^6\) One loop consists of advertising demand affecting readership demand and thereby affecting again advertising demand; likewise for the loop originating on the readership side.
Both converge to the reduced form quantities (2) if the absolute value of the common ratio, $\gamma a \gamma r$, is less than one. This, again, is the condition in Assumption 1(ii).

Writing demands in terms of a geometric series also shows that if one of the two network effects is zero, the reduced-form demand functions always exist because in that case the product of the two network effects is automatically zero. In that case the multiplier is equal to one.\(^7\)

Next consider the case in $|\gamma a \gamma r| > 1$. We have shown above that there is a unique set of quantities in this case as well. To derive a series representation for this case re-write (1) as

$$\tilde{q}^a(p, q^a) = \frac{q^a - \beta^a p^a}{\gamma a}$$
$$\tilde{q}^a(p^a, q^a) = \frac{q^a - \beta^a p^a}{\gamma a}$$

Then, we get

$$q^a = \left( -\frac{\alpha a}{\gamma a} - \frac{\alpha r}{\gamma a \gamma r} \right) \cdot \left( 1 + \frac{1}{\gamma a \gamma r} + \left( \frac{1}{\gamma a \gamma r} \right)^2 + \ldots \right)$$

and

$$q^r = \left( -\frac{\alpha r}{\gamma r} - \frac{\alpha a}{\gamma a \gamma r} \right) \cdot \left( 1 + \frac{1}{\gamma a \gamma r} + \left( \frac{1}{\gamma a \gamma r} \right)^2 + \ldots \right) .$$

From these we see that indeed, quantities exist if $|\gamma a \gamma r| > 1$ because in that case $1/\gamma a \gamma r < 1$ and the series are in powers of $1/\gamma a \gamma r$. However, we have already argued above that the resulting quantities will depend positively on own prices and that therefore this case is not economically meaningful. Besides, while the functions $q^a(p^a, q^a)$ and $q^r(p^a, q^a)$ have a natural interpretation because they are primitives of the model, the functions $\tilde{q}^a(p^a, q^a)$ and $\tilde{q}^r(p^a, q^a)$ are not meaningful in the sense that, for instance, $\tilde{q}^a(p^a, q^a)$ means that $q^a$ is chosen so that, for given $p^a$, the resulting number of readers is equal to $q^r$. One interpretation of Assumption 1 is therefore, that it excludes such dynamics in which there exist unique sets of quantities, but they have properties that are not economically meaningful. Here, this is

\(^7\)If, for instance,

$$q^a(p^a, q^a) = \alpha a - \beta a p^a + \gamma a q^a$$
$$q^r(p^r) = \alpha r - \beta r p^r$$

so that readers are not affected by advertising, then the reduced form demand functions are

$$\tilde{q}^a(p^a, p^r) = (\alpha a + \alpha r) - \beta a p^a - \gamma a \beta r p^r$$
$$\tilde{q}^r(p^r) = \alpha r - \beta r p^r ,$$

where it appears evident that reduced form readership demand is not affected by the advertising price (because by assumption advertising quantity does not affect advertising demand), while advertising demand is affected by the cover price (since the number of readers affects demand from advertisers).
because the convergent series has elements that have no economic interpretation. In the following, we will therefore only consider dynamics that satisfy Assumption 1.

2.3 A linear demand example with two platforms

Consider two newspapers facing demand functions for advertising and readership that are linear in all prices on the same side and all quantities on the other side,

\[ q^a_1 = \alpha^a_1 - \beta^a_{11} p^a_1 + \beta^a_{12} p^a_2 + \gamma^a_{11} q^r_1 + \gamma^a_{12} q^r_2 \]
\[ q^a_2 = \alpha^a_2 + \beta^a_{21} p^a_1 - \beta^a_{22} p^a_2 + \gamma^a_{21} q^r_1 + \gamma^a_{22} q^r_2 \]
\[ q^r_1 = \alpha^r_1 - \beta^r_{11} p^r_1 + \beta^r_{21} p^r_2 + \gamma^r_{11} q^a_1 + \gamma^r_{12} q^a_2 \]
\[ q^r_2 = \alpha^r_2 + \beta^r_{21} p^r_1 - \beta^r_{22} p^r_2 + \gamma^r_{21} q^a_1 + \gamma^r_{22} q^a_2, \]

with positive price coefficients.

This system can be written in matrix notation as

\[ q = \alpha + Bp + \Gamma q, \]

with

\[
\begin{pmatrix}
q^a_1 \\
q^a_2 \\
q^r_1 \\
q^r_2
\end{pmatrix}
= \begin{pmatrix}
\alpha^a_1 \\
\alpha^a_2 \\
\alpha^r_1 \\
\alpha^r_2
\end{pmatrix},
B = \begin{pmatrix}
-\beta^a_{11} & \beta^a_{21} & 0 & 0 \\
\beta^a_{21} & -\beta^a_{22} & 0 & 0 \\
0 & 0 & -\beta^r_{11} & \beta^r_{21} \\
0 & 0 & \beta^r_{21} & -\beta^r_{22}
\end{pmatrix},
\begin{pmatrix}
p^a_1 \\
p^a_2 \\
p^r_1 \\
p^r_2
\end{pmatrix}
\]

and block-diagonal matrix

\[
\Gamma = \begin{pmatrix}
0 & \Gamma^a \\
\Gamma^r & 0
\end{pmatrix} = \begin{pmatrix}
0 & 0 & \gamma^a_{11} & \gamma^a_{12} \\
0 & 0 & \gamma^a_{21} & \gamma^a_{22} \\
\gamma^r_{11} & \gamma^r_{12} & 0 & 0 \\
\gamma^r_{21} & \gamma^r_{22} & 0 & 0
\end{pmatrix}.
\]

We can solve for the quantities

\[ q = (I - \Gamma)^{-1}(\alpha + Bp) \]
if, and only if,
\[ \det (I - \Gamma) \neq 0. \]

1 – Γ is a partitioned matrix and this is equal to \( \det(I) \cdot \det(I - \Gamma^a \Gamma^r) = \det(I - \Gamma^a \Gamma^r) = \det(I - \Gamma^r \Gamma^a) \). So, the condition is in fact that

\[
\det (1 - \Gamma^a \Gamma^r) = \det \left( I - \begin{pmatrix} \gamma^a_{11} \gamma^r_{11} + \gamma^a_{12} \gamma^r_{21} & \gamma^a_{11} \gamma^r_{12} + \gamma^a_{12} \gamma^r_{22} \\ \gamma^r_{21} \gamma^a_{11} + \gamma^r_{22} \gamma^a_{12} & \gamma^r_{21} \gamma^a_{12} + \gamma^r_{22} \gamma^a_{22} \end{pmatrix} \right) = \det \left( I - \begin{pmatrix} \gamma^r_{11} \gamma^a_{11} + \gamma^r_{12} \gamma^a_{21} & 1 - \gamma^r_{11} \gamma^a_{12} + \gamma^r_{12} \gamma^a_{22} \\ 1 - \gamma^r_{21} \gamma^a_{11} + \gamma^r_{22} \gamma^a_{12} & \gamma^r_{21} \gamma^a_{12} + \gamma^r_{22} \gamma^a_{22} \end{pmatrix} \right) (3)
\]

\neq 0.

Assumption 1 is that

\[
\sum_{k=1}^{2} \sum_{\ell=1}^{2} |\gamma^a_{k \ell} \gamma^r_{k \ell}| = |\gamma^a_{11} \gamma^r_{11} + \gamma^a_{12} \gamma^r_{21}| + |\gamma^a_{11} \gamma^r_{12} + \gamma^a_{12} \gamma^r_{22}| < 1
\]

\[
\sum_{k=1}^{2} \sum_{\ell=1}^{2} |\gamma^a_{k \ell} \gamma^r_{k \ell}| = |\gamma^r_{11} \gamma^a_{11} + \gamma^r_{12} \gamma^a_{21}| + |\gamma^r_{11} \gamma^a_{12} + \gamma^r_{12} \gamma^a_{22}| < 1,
\]

and implies

\[
|\gamma^a_{11} \gamma^r_{11} + \gamma^a_{12} \gamma^r_{21}| < 1 - |\gamma^a_{11} \gamma^r_{12} + \gamma^a_{12} \gamma^r_{22}|.
\]

\[
|\gamma^r_{21} \gamma^a_{11} + \gamma^r_{22} \gamma^a_{12}| < 1 - |\gamma^r_{21} \gamma^a_{12} + \gamma^r_{22} \gamma^a_{22}|.
\]

Hence, the first term in parentheses in (3) is bigger than the third and the second is bigger than the fourth, and therefore \( \det (I - \Gamma^a \Gamma^r) > 0 \) and we can solve for the reduced-form quantities as functions of prices only.

We can also, as in the previous example, write quantities as a geometric series,

\[
q = (I + \Gamma \Gamma + (\Gamma \Gamma)^2 + \ldots) \cdot (I + \Gamma) \cdot (\alpha + Bp).
\]

This series converges to

\[
(1 - \Gamma)^{-1} \cdot (I + \Gamma) \cdot (\alpha + Bp) = (1 - \Gamma)^{-1} \cdot (\alpha + Bp)
\]

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if the absolute value of all eigenvalues of $\Gamma$ is strictly less than 1. Because of the block-diagonality of this matrix, with blocks $\Gamma^a\Gamma^r$ and $\Gamma^r\Gamma^a$ on the diagonal and blocks of zeros on the off-diagonal, this is the case if the absolute value of the eigenvalues of $\Gamma^a\Gamma^r$ (which are also the eigenvalues of $\Gamma^r\Gamma^a$) are strictly less than 1. They are given by

$$\lambda_{1,2} = \frac{(\gamma'_1^a \gamma'_1^r + \gamma'_2^a \gamma'_2^r)}{2} \pm \frac{1}{2} \sqrt{((\gamma'_1^a \gamma''_1^r) + (\gamma'_2^a \gamma''_2^r))^2 - 4 \text{det}(\Gamma^a\Gamma^r)}.$$ 

The maximal absolute value of the eigenvalues is obtained for $\text{det}(\Gamma^a\Gamma^r) = 0$. In particular, it holds that

$$(\gamma'_1^a \gamma'_1^r + \gamma'_2^a \gamma'_2^r) = (\gamma'_1^r \gamma'_2^a + \gamma'_2^r \gamma'_2^a) = (\gamma'_1^a \gamma'_2^r + \gamma'_2^a \gamma'_2^r)$$

Then, we have that

$$\lambda_1 = (\gamma'_1^a \gamma'_1^r + \gamma'_2^a \gamma'_2^r) + (\gamma'_1^r \gamma'_2^a + \gamma'_2^r \gamma'_2^a)$$

and $\lambda_2 = 0$. Assumption 1 implies that

$$(\gamma'_1^a \gamma'_1^r + \gamma'_2^a \gamma'_2^r) = (\gamma'_1^r \gamma'_2^a + \gamma'_2^r \gamma'_2^a) = (\gamma'_1^a \gamma'_2^r + \gamma'_2^a \gamma'_2^r) < \frac{1}{2}$$

and therefore, $|\lambda_1| < 1$ and the series will converge. Gandolfo (1996, p. 117) shows that this holds more generally for a linear system with $J > 2$ products. Lemma 1 above can be seen as a generalization of this result to non-linear systems.

3 Competition in prices

We will analyze a market in which firms compete in prices. The equilibrium concept will be Nash in pure strategies. In such an equilibrium, a firm $f$ takes the prices of its competitors as given and maximize the sum of profits over newspapers $\ell$ in their portfolio $F_f$,

$$\pi_f = \sum_{\ell \in F_f} \left\{ (p^a_{\ell} - mc^a_{\ell}) \cdot \left( \sum_{g=1}^{G^a} q^a_{\ell g} \right) + (p^r_{\ell} - mc^r_{\ell}) \cdot \left( \sum_{g=1}^{G^r} q^r_{\ell g} \right) \right\}. \quad (4)$$

Here, quantities $q^a_{\ell g}$ and $q^r_{\ell g}$ are functions of prices on the same market side and quantities on the other market side. That is, $q^a_{\ell g} = q^a_{\ell g}(p^a, q^a)$ and $q^r_{\ell g} = q^r_{\ell g}(p^r, q^r)$. For example, advertising demand in news-
paper $j$ depends on the price of advertising in that newspaper, $p^a_j$, but also on the number of readers in each of the demographic groups that read that newspaper, $q^g_{jg}$ for all $g \in G^a$, and also on all the other prices and all the other quantities on the other market side.

To formally define a Nash equilibrium in the context of our model denote the prices set by firm $f$ by $p_f$. Denote the prices set by all other firms by $p_{-f}$ and explicitly write profits as depending on prices set by $f$ and all its competitors $-f$, $\pi(p_f,p_{-f})$.

**Definition 1** (Nash equilibrium). A strategy profile $p^* = (p^*_f, p^*_{-f})$ is a *Nash equilibrium* if no unilateral deviation in strategy by any single firm is profitable for that firm, that is

$$\forall f, p_f : \pi_f(p^*_f, p^*_{-f}) \geq \pi_f(p_f, p^*_{-f}).$$

We will give conditions so that equilibrium prices satisfy the first order conditions. Towards this, observe that it is not possible to find a closed form of the first order conditions by taking the derivative of (4) with respect to the prices. This is because of the presence of feedback loops, which means that quantities on one market side depend on prices on the same market side and on quantities on the other market side. But quantities on the other market side again depend on quantities on the one market side, which again depend on prices on that first market side, and so on. However, using the reduced-form demand functions \( \hat{q}_{jg}^a = \hat{q}_{jg}^a(p^a, p^r) \) and \( \hat{q}_{jg}^r = \hat{q}_{jg}^r(p^a, p^r) \) we can rewrite (4) as

$$\pi_f = \sum_{j \in J_f} \left\{ (p^a_j - mc^a_j) \cdot \left( \sum_{g \in G^a} \hat{q}^a_{jg}(p^a, p^r) \right) + (p^r_j - mc^r_j) \cdot \left( \sum_{g \in G^r} \hat{q}^r_{jg}(p^a, p^r) \right) \right\}$$

and the first order conditions are given by the derivative with respect to all prices,

$$\frac{\partial \pi_f}{\partial p^a_j} = q^a_j + \sum_{i \in J_f} \left\{ (p^a_i - mc^a_i) \cdot \left( \sum_{g \in G^a} \frac{\partial \hat{q}^a_{ig}}{\partial p^a_i} \right) + (p^r_i - mc^r_i) \cdot \left( \sum_{g \in G^r} \frac{\partial \hat{q}^r_{ig}}{\partial p^a_i} \right) \right\} = 0 \quad (5)$$

on the advertising side and a similar expression on the readership side.

Denote the vector indicating which products are owned by firm $f$ by $\omega_f$. Then, $\Omega = \sum_f \omega_f \omega_f'$ is the Nevo (2000a, 2001)-type ownership matrix where $\Omega_{21} = 1$ if product $i$ and $j$ are owned by the same company, and $\Omega_{21} = 0$ otherwise. Define the $2J \times 2J$ matrix

$$\hat{Q} \equiv \begin{pmatrix} \sum_{g=1}^{G^a} \frac{\partial \hat{q}^a_{jg}}{\partial p^a_j} & \sum_{g=1}^{G^r} \frac{\partial \hat{q}^a_{jg}}{\partial p^r_j} \\ \sum_{g=1}^{G^a} \frac{\partial \hat{q}^r_{jg}}{\partial p^a_j} & \sum_{g=1}^{G^r} \frac{\partial \hat{q}^r_{jg}}{\partial p^r_j} \end{pmatrix} \circ \begin{pmatrix} \Omega & \Omega \\ \Omega & \Omega \end{pmatrix}.$$
The first matrix on the right hand side is the matrix of derivatives of the reduced form quantities, summed over demographic groups, respectively, with respect to prices. It is multiplied, elementwise, by the appropriate elements of the ownership matrix. Here, $[x_j]_j$ denotes the column vector consisting of the elements $x_j$, stacked on top of one another in the usual way. This gives that, for example, $[\hat{q}^a_{[g]} / \partial p^a]'_j$ is the $J \times J$ matrix of derivatives of quantities for demographic group $g$ on the advertising side with respect to prices on the advertising side. The summation is then over demographic groups.

To make this approach useful in practice, Proposition 2 relates the derivatives of reduced form quantities with respect to prices to properties of the original demand functions. The latter can typically be estimated using data on quantities and prices.

**Lemma 2** (price effects). The Jacobian matrix that consists of the partial derivatives of the reduced-form demand functions $\hat{q}(p)$ with respect to the prices is given by

$$
\frac{\partial \hat{q}(p)}{\partial p'} = - \begin{pmatrix}
-I & \partial q^a / \partial q' \\
\partial q' / \partial q'^d & -I
\end{pmatrix}^{-1} \begin{pmatrix}
\partial q^a / \partial p^d & 0 \\
0 & \partial q' / \partial p'^d
\end{pmatrix}
$$

provided that Assumption 1 holds.

**Proof.** See p. 35 in Appendix A. \hfill \square

Observe that we can write the vector of profits earned by firm $f$ as

$$
\pi_f(p_f, p_{-f}) = \omega_f \{(p_a - mc_a) \circ q^a + (p_n - mc_n) \circ q'\}.
$$

and that (5) is the $j$th row of the system of equations

$$
\hat{q} + \hat{Q}'(p - mc) = 0.
$$

(6)

In total, there are $2J$ rows, one for each of the $J$ products and 2 market sides. The first order conditions for the subscription prices are in row $J + 1$ till $2J$. From this, we get the unique vector of marginal costs that solves the first order conditions and the second order conditions. Taking the derivative of (6) with respect to prices gives,

$$
R \equiv \frac{\partial \hat{q}}{\partial p'} + (p - mc) \otimes I_J \frac{\partial \text{vec} \hat{Q}'}{\partial p'} + \hat{Q}' = \hat{Q} + (p - mc) \otimes I_J \frac{\partial \text{vec} \hat{Q}'}{\partial p'} + \hat{Q}',
$$
where \( \text{vec} A \) is the vectorizing operator that stacks the columns of \( A \) on top of one another. The second order conditions are that the respective sub-matrices of \( R \), denotes by \( R_f = R(\omega_f \omega_f') \), where \( R(\omega_f \omega_f') \) consists of the rows and columns of \( R \) for which the entry in \( \omega_f \omega_f' \) is one, are negative definite.

We summarize these results in our first proposition.

**Proposition 1.** A Nash equilibrium exists if there are prices such that

\[
\bar{q} + \hat{Q}'(p - mc) = 0
\]

and \( R_f \) is negative definite for all \( f \).

**Proof.** Follows from the definition of a Nash equilibrium.

If a Nash equilibrium exists, one can solve for the vector of marginal costs following Rosse (1970). For this, we do not need to assume that the equilibrium is unique. However, a necessary condition for this is that \( \hat{Q} \) is invertible. Lemma 2 shows that this condition is testable provided that demand parameters are known. In practice, one would first estimate advertising demand as a function of prices and readers, and readership demand as a function of subscription rates and the amount of advertising. Then, provided that the conditions given in Lemma (1) hold, one would use the result in Lemma (2) to calculate \( \hat{Q} \). This then allows one to calculate the vector of marginal costs using the result in the following proposition.

**Proposition 2.** Assume that Assumption 1 holds, that prices \( p \) satisfy the Nash equilibrium condition and that the second order conditions are satisfied. Then, the vector of marginal costs is given by

\[
mc = \hat{Q}^{-1} \bar{q} + p.
\]

For this to be correct one finally has to assess whether the equilibrium exists. According to Proposition 1 a sufficient condition for this to be the case is that the above mentioned second order conditions hold.

In practice, an equilibrium (under a different policy) can be found by solving (6) numerically. This involves repeatedly calculating \( \bar{q} \), following the second part of Lemma (1). In general, there could be multiple equilibria and therefore it could be of value for a policy simulation to have a set of sufficient conditions for uniqueness in hand. We provide such conditions in the following proposition. They are
related to properties of the best reply correspondence given prices of the other firms

$$b(p_{-f}) = \arg\max_{p_f} \pi_f(p_f, p_{-f}).$$

Under the assumption that $\pi_f(p_f, p_{-f})$ is quasi-concave, that we will henceforth make, $b(p_{-f})$ is a vector-valued function mapping the prices set by the competitors $-f$ into a set of prices set by firm $f$.

**Proposition 3** (uniqueness of equilibrium). Assume that $\pi_f(p_f, p_{-f})$ is quasi-concave and differentiable in $p_f$ for given $p_{-f}$. The equilibrium is unique if at least one of the following three conditions holds:

(i) the diagonal elements of $\partial^2 \pi_f(p_f, p_{-f}) / \partial (p_f', p_{-f}') \partial (p_f, p_{-f})$ dominate the off-diagonal elements within each row

(ii) $\partial^2 \pi_f(p_f, p_{-f}) / \partial (p_f', p_{-f}') \partial (p_f, p_{-f})$ is negative quasi-definite for all $p_f, p_{-f}$

(iii) the determinant of $-\partial^2 \pi_f(p_f, p_{-f}) / \partial (p_f', p_{-f}') \partial (p_f, p_{-f})$ is positive whenever $\partial \pi_f(p_f, p_{-f}) / \partial (p_f', p_{-f}') = 0$.

**Proof.** See p. 36 in Appendix A.

### 3.1 Back to the linear demand example with one platform

Before studying the interaction between firms, it is instructive to go back to the linear demand example of Section 2.2 with one platform. Profits of that one platform can be written as a function of the reduced-form demands $\hat{q}^a(p^a, p^r)$ and $\hat{q}^r(p^a, p^r)$. Denoting marginal costs by $mc^a$ and $mc^r$ we then have

$$\pi = (p^a - mc^a) \cdot \hat{q}^a(p^a, p^r) + (p^r - mc^r) \cdot \hat{q}^r(p^a, p^r) = (p^a - mc^a) \cdot \frac{1}{1 - \gamma^a \gamma^r} \cdot \{(\alpha^a + \alpha^r \gamma^a) - \beta^a p^a - \gamma^r \beta^r p^r \} + (p^r - mc^r) \cdot \frac{1}{1 - \gamma^a \gamma^r} \cdot \{(\alpha^r + \alpha^a \gamma^r) - \gamma^a \beta^a p^a - \beta^r p^r \}.$$

$(p^a - mc^a)$ and $(p^r - mc^r)$ are the margins on the advertising and readership side, respectively.

The first order conditions are

$$\frac{\partial \pi}{\partial p^a} = \hat{q}^a(p^a, p^r) + (p^a - mc^a) \cdot \frac{1}{1 - \gamma^a \gamma^r} \cdot (-\beta^a) + (p^r - mc^r) \cdot \frac{1}{1 - \gamma^a \gamma^r} \cdot (-\gamma^a \beta^a) = 0$$
\[
\frac{\partial \pi}{\partial p'} = \hat{q}'(p', p) + (p' - mc') \cdot \frac{1}{1 - \gamma' \gamma'} \cdot (-\gamma' \beta') + (p' - mc') \cdot \frac{1}{1 - \gamma' \gamma'} \cdot (-\beta') = 0.
\]

Looking at the second one, we see that this is the standard condition if \( \gamma' = 0 \). This is the case in which advertising demand does not depend on the number of readers. As a consequence, the platform will set subscription prices in the usual, one-sided way. Now consider the case in which \( \gamma' > 0 \) and \( \gamma = 0 \). This is the case in which advertising demand depends positively on the number of readers, but readership demand does not depend on the amount of advertising in the newspapers. Then, the first order condition for advertising prices is the standard one-sided one and the first order condition for readership is,

\[
\frac{\partial \pi}{\partial p'} = \hat{q}'(p', p) + (p' - mc') \cdot (-\gamma' \beta') + (p' - mc') \cdot (-\beta') = 0.
\]

Now, there is no feedback loop anymore, since one of the two network effects is zero. The first and the last term together give the effect of a marginal price increase on profits earned on the readership side, and the second term is the effect of readership orices on advertising profits. This effect arises because higher prices on the readership side decrease readership demand and, since advertising demand depends positively on readership demand via the indirect network effect \( \gamma' \), thereby advertising demand and advertising profits.

The second order conditions involve second derivatives with respect to prices and cross-derivatives,

\[
\frac{\partial^2 \pi}{(\partial p')^2} = -\frac{2\beta'}{1 - \gamma' \gamma'},
\]

\[
\frac{\partial^2 \pi}{(\partial p'^2)} = -\frac{2\beta'}{1 - \gamma' \gamma'},
\]

\[
\frac{\partial^2 \pi}{\partial p' \partial p'^2} = -\frac{\gamma' \beta' + \gamma' \beta'}{1 - \gamma' \gamma'}.
\]

Strict quasi-concavity requires that the first two are negative, which holds if, and only if, \( \gamma' \gamma' < 1 \); and that the squared cross-derivative is smaller than the product of the first two second derivatives,

\[
\left( \frac{\partial^2 \pi}{\partial p' \partial p'^2} \right)^2 < \frac{\partial^2 \pi}{(\partial p')^2} \cdot \frac{\partial^2 \pi}{(\partial p'^2)}.
\]

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This is equivalent to requiring
\[-(\gamma^a \beta^r + \gamma^r \beta^a) < 4\beta^a \beta^r.\]

The right hand side is always positive, and \(\beta^a\) and \(\beta^r\) on the left hand side are positive by definition. Therefore, strict quasi-concavity requires in addition to \(\gamma^r \gamma' < 1\) that neither \(\gamma^a\) nor \(\gamma'\) are too negative.

3.2 Back to the linear demand example with two platforms

We now go back to the linear demand example of Section 2.2 with two platforms. One can distinguish two cases: a) both platforms are owned by a monopolist b) each platform is owned by a different duopolist.

Let us start from the monopoly case. Its profits can be written as a function of the reduced-form demands \(\hat{q}^a(p^a, p^r)\) and \(\hat{q}^r(p^a, p^r)\). These can be stack into a \(\hat{q}(p)\). Denoting marginal the K X 1 vectors of marginal costs by \(mc^a\) and \(mc^r\) and stacking them into the 2K*1 mc vector we then have

\[\pi = (p - mc) \cdot \hat{q}(p)\]

which, substituting \(\hat{q} = (I - \Gamma)^{-1}(\alpha + Bp)\), becomes

\[\pi = (p - mc) \cdot (I - \Gamma)^{-1}(\alpha + Bp)\]

The first order conditions are therefore

\[
\frac{\partial \pi}{\partial p} = \hat{q}(p) + (p - mc) \cdot \frac{\partial \hat{q}(p)}{\partial p}
\]

or equivalently

\[
\frac{\partial \pi}{\partial p} = (I - \Gamma)^{-1}(\alpha - Bmc(p)) + 2(I - \Gamma)^{-1}Bp
\]

Rewriting it in terms of block-diagonal matrixes

\[
\frac{\partial \pi}{\partial p} = \begin{pmatrix}
I & -\Gamma^a \\
-\Gamma^r & I
\end{pmatrix}^{-1} \begin{pmatrix}
\alpha^a \\
\alpha^r
\end{pmatrix} + \begin{pmatrix}
B^a & 0 \\
0 & B^r
\end{pmatrix} \begin{pmatrix}
mc^a(p) \\
mc^r(p)
\end{pmatrix} + 2 \begin{pmatrix}
I & -\Gamma^a \\
-\Gamma^r & I
\end{pmatrix}^{-1} \begin{pmatrix}
B^a & 0 \\
0 & B^r
\end{pmatrix} \begin{pmatrix}
p^a \\
p^r
\end{pmatrix}
\]
where \((p^a - mc^a)\) and \((p^r - mc^r)\) are, respectively, the \(K \times 1\) vectors of margins on the advertising and readership side, we see that if for instance readers do not care about advertising, \(\Gamma^a = 0\), the price to advertisers is chosen as in a one-sided market. This is the case for instance in Argentesi and Filistrucchi (2007).

The second order conditions are instead that the matrix

\[
\frac{\partial \pi}{\partial p} = 2(I - \Gamma)^{-1} B - (I - \Gamma)^{-1} B \frac{\partial mc(p)}{\partial p}
\]

or

\[
\frac{\partial \pi}{\partial p} = (I - \Gamma)^{-1} B \left( 2I - \frac{\partial mc(p)}{\partial p} \right)
\]

is negative semi-definite.

The second order conditions are instead that the matrix

\[
(I - \Gamma)^{-1} B = \begin{pmatrix}
I & -\Gamma^a \\
-\Gamma^r & I
\end{pmatrix}^{-1}
\begin{pmatrix}
B^a & 0 \\
0 & B^r
\end{pmatrix}
\]

is negative semi-definite.

4 A hypothetical merger in the Dutch market for daily newspapers

We now study the effects of a hypothetical merger in the Dutch daily newspaper market. The market for daily newspapers in the Netherlands is described in Abbring and Van Ours (1994) and Filistrucchi, Klein, and Michielsen (2012a). The hypothetical merger we investigate is between publisher 1, De Persgroep, owning the Algemeen Dagblad (AD1), NRC Handelsblad (NRC), nrc.next (NRN), Het Parool (PAR), Trouw (TRO) and de Volkskrant (VOL), and publisher 2, the Telegraaf group, owning De Gooi- en Eemlander (GOO), Haarlems Dagblad (HAR), Leidsch Dagblad (LEI), Noordhollands Dag-
blad (NOR) and De Telegraaf (TEL). AD1 is a national-level newspaper with regional editions, NRC is a business-oriented national level newspaper, NRN is the corresponding evening edition, and PAR, TRO and VOL are other national level newspapers. The other group of newspapers consists of the regional level newspapers GOO, HAR, LEI and NOR, and the tabloid TEL.

The Netherlands are a small country that is extremely densely populated. Also within the country, there is considerable heterogeneity between more urban municipalities and more rural ones, which also affects the level of competition between newspapers. Figure 2 shows a map of The Netherlands at the municipality level, in which shades of blue depict levels of the Herfindahl-Hirschman-Index (HHI) on the readership side. The map shows that the level of concentration is high in the area around Amsterdam, Rotterdam and The Hague, which is in the west, and in the south. However, it is not clear whether these newspapers all operate in the same market—an implicit assumption that one has to make in order

\footnote{The HHI is defined as the sum of the squared market shares. Hence, 0 means infinitely many small firms, whereas 1 means that one firm serves the whole market. Here, we use market shares by firms and multiply multiply the obtained HHI it by 10,000.}

Figure 2: Readership concentration
to think of the HHIs as measuring competition. This is also relevant for the hypothetical merger we study here because the newspapers owned by publisher 1 are mainly higher quality national level newspapers, while the newspapers owned by publisher 2 are regional level newspapers and one tabloid national level newspaper. Ultimately, this question can only be answered once we have estimated a model for readership demand, which will allow us to characterize substitution patterns, for example by means of diversion ratios below.

For advertising demand, we use a constant elasticity specification,

\[
\log(q_a^t) = \alpha_j + \beta a \log(p_a^t) + \gamma a \log(q_r^t) + \epsilon_j,
\]

in which advertising demand depends only on the (own) advertising price and (own) circulation, and calibrate it so that the implied margins match the pattern documented in *Nederlands Uitgeversverbond* (2009).\(^9\)

The model for readership demand is a logit model that is estimated at the municipality level (Berry, 1994). We use data on market shares at the municipality level and data on market shares by demographic group at the national level to construct additional Petrin (2002)-type moments. Throughout, the market size is given by the population over 13 years of age. Furthermore, we control for region-paper fixed effects (5 regions) to capture different regional focus, also for national level newspapers, and for flexible time trend (year dummies) to capture the increased importance of outside options such as the evolution of the internet and the availability of free newspapers).\(^{10}\)

A straightforward, market-based measure for how valuable readers are to advertisers is given by the dependence of the advertising price per reader and socio-demographic characteristics of the subscribers of a newspaper. Figure 3 shows how the advertising price is related to the percentage of the readers in the highest wealth category. Blue dots are for national level newspapers and red dots for regional level newspapers. We see that controlling for wealth of the readers, regional newspapers charge higher advertising prices. This is confirmed in Table 1. We also include the percentage male readers as an additional explanatory variable. In our data, wealth and age are strongly correlated. Therefore, we do not include both, age and wealth measures at the same time. However, in the second column, we include

---

\(^9\)The parameter values are the same as in Affeldt, Filistrucchi, and Klein (forthcoming). Filistrucchi, Klein, and Michielsen (2012a,b) use a more restrictive specification where \(\beta a = -\gamma a\) so that advertising demand is of constant elasticity with respect to the price per reader.

\(^{10}\)The parameter values are the same as in Filistrucchi, Klein, and Michielsen (2012a,b) and Affeldt, Filistrucchi, and Klein (forthcoming).
age indicators instead of wealth indicators as explanatory variables. The finding is largely the same: newspapers charge higher prices if many of their readers are between 50 and 64 years old, are male,

Table 2 summarizes the demand elasticities, prices and marginal costs that we will use in the following. We report averages within groups of newspapers owned by the two publishers. The first part of the top panel of the table shows elasticities of advertising demand with respect to the advertising price, holding the number of readers constant, and with respect to the number of readers, holding the advertising prices constant. The second part of the top panel shows elasticities of advertising demand with respect to the advertising price and the subscription price, holding the respective other price fixed. They are obtained using the result in Lemma 2. The elasticity of advertising demand with respect to the price is $-1.02$, and $0.30$ with respect to the circulation of the newspaper.

On the subscription side the own-price elasticity is about $-1.8$ on average. Advertising is estimated to have a small but positive effect on circulation, with an elasticity of about $0.05$ on average, so that the market is found to be characterized by two indirect positive network effects between the demand for advertising and the demand for readership.

The bottom panel shows prices and inferred marginal costs. Advertising prices are per column mil-
Table 1: *Hedonic regressions for advertising price*

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log circulation</td>
<td>0.838</td>
<td>0.782</td>
</tr>
<tr>
<td></td>
<td>(0.096)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>regional newspaper</td>
<td>0.657</td>
<td>0.278</td>
</tr>
<tr>
<td></td>
<td>(0.178)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>linear time trend</td>
<td>0.017</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>percentage highest wealth category</td>
<td>2.172</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.640)</td>
<td></td>
</tr>
<tr>
<td>percentage middle wealth category</td>
<td>0.541</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.626)</td>
<td></td>
</tr>
<tr>
<td>percentage male readers</td>
<td>4.628</td>
<td>5.172</td>
</tr>
<tr>
<td></td>
<td>(0.820)</td>
<td>(0.618)</td>
</tr>
<tr>
<td>percentage age 35 to 49</td>
<td>1.261</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.221)</td>
<td></td>
</tr>
<tr>
<td>percentage age 50 to 64</td>
<td>6.667</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.894)</td>
<td></td>
</tr>
<tr>
<td>percentage age 65 and older</td>
<td>-0.092</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.769)</td>
<td></td>
</tr>
<tr>
<td>obs.</td>
<td>858</td>
<td>858</td>
</tr>
</tbody>
</table>

Hedonic regression with the log price per column millimeter of advertising as the dependent variable.

limeter and reflect the acquisition and typesetting costs for an additional column millimeter of advertising. For simplicity, we assume that there are no additional printing costs. All prices are in year-2002 euros. The initial situation is the one at the end of 2009.

In Table 3, we present summary statistics of the implied diversion ratios. The diversion ratio defined as the marginal effect of a price increase by one unit, divided by the own price effect. So, for example,

\[
D_{jk}^{X_{A}} = \frac{\partial \hat{q}_{jk}^{a}}{\partial p_{j}} / \frac{\partial \hat{q}_{j}^{a}}{\partial p_{j}}
\]

is the effect of losing one reader in newspaper \(j\) on advertising demand in newspaper \(k\). Each row is for a particular product and we present the sum of the diversion ratios across competing products. The table contains estimates that do not and do, respectively, take indirect network effects into account. The entries in the top part of the first of the three columns are zero because the advertising demand model assumes that direct cross-effects are zero on the advertising market.\(^{11}\) Then, one-sided diversion ratios on that side

\(^{11}\)This is an assumption that is commonly made in this context, see e.g. Rysman (2004), Van Cayseele and Vanormelingen (2009) and Fan (forthcoming). It means that, holding the number of subscribers constant, advertising demand in newspaper \(i\) depends only on the price of advertising in that newspaper, and not in others. Rysman (2004) argues that this is a reasonable assumption once readers single-home.
Table 2: Market characteristics

<table>
<thead>
<tr>
<th></th>
<th>firm 1</th>
<th>firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>average elasticities for advertising demand</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>advertising price $(\partial Q^A_j / \partial P^A_j) / (Q^A_j / P^A_j)$</td>
<td>-1.02</td>
<td>-1.02</td>
</tr>
<tr>
<td>circulation $(\partial Q^A_j / \partial Q^R_j) / (Q^A_j / Q^R_j)$</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td><strong>average elasticities for advertising demand incorporating feedback</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>advertising price $(\hat{\partial} Q^A_j / \partial P^A_j) / (Q^A_j / P^A_j)$</td>
<td>-1.04</td>
<td>-1.05</td>
</tr>
<tr>
<td>subscription price $(\hat{\partial} Q^R_j / \partial P^R_j) / (Q^R_j / P^R_j)$</td>
<td>-0.60</td>
<td>-0.51</td>
</tr>
<tr>
<td><strong>average elasticities for subscription demand</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>subscription price $(\partial Q^R_j / \partial P^R_j) / (Q^R_j / P^R_j)$</td>
<td>-1.96</td>
<td>-1.65</td>
</tr>
<tr>
<td>amount advertising $(\partial Q^R_j / \partial Q^A_j) / (Q^R_j / Q^A_j)$</td>
<td>0.04</td>
<td>0.07</td>
</tr>
<tr>
<td><strong>average elasticities for subscription demand incorporating feedback</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>advertising price $(\hat{\partial} Q^R_j / \partial P^R_j) / (Q^R_j / P^R_j)$</td>
<td>-0.38</td>
<td>-0.57</td>
</tr>
<tr>
<td>subscription price $(\hat{\partial} Q^R_j / \partial P^A_j) / (Q^R_j / P^A_j)$</td>
<td>-1.98</td>
<td>-1.69</td>
</tr>
<tr>
<td><strong>prices and marginal costs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>advertising price per column millimeter $P^A_j$</td>
<td>7.10</td>
<td>3.95</td>
</tr>
<tr>
<td>marginal cost advertising $C^A_j$</td>
<td>0.40</td>
<td>0.30</td>
</tr>
<tr>
<td>subscription price per year $P^R_j$</td>
<td>263.82</td>
<td>241.84</td>
</tr>
<tr>
<td>marginal cost subscription $C^R_j$</td>
<td>164.49</td>
<td>163.02</td>
</tr>
</tbody>
</table>

of the market are automatically zero once the quantity of readers is held constant. However, two-sided diversion ratios are positive. This is due to the fact that a drop in advertising demand negatively affects the subscription sales of that newspaper and increases the subscription sales of the other newspapers and thereby positively affects also their sales of advertisements. This is summarized in the top part of the second and third column. But since readers value advertising only very little, two-sided diversion ratios are still small and hardly different from the one-sided ones. A similar effect is at play in the lower part of the table, and also here the difference in the diversion ratios between column one and column three is small because one of the two network effects is small.

Finally, Table 4 shows the results of the merger simulation. The columns contain, respectively, prices before the merger, predicted prices after the merger, the percentage change, and a measure for the “upward pricing pressure” related to the merger, $UPp^*$. Affeldt, Filistrucchi, and Klein (forthcoming) show how to calculate this measure. In brief, it is the percentage efficiency gain that is necessary, for each newspaper and each market side at a time, so that the first order condition for setting the prices holds after the merger when evaluated at the pre-merger prices. In that sense, it is a measure for whether it is
Table 3: Diversion ratios

<table>
<thead>
<tr>
<th></th>
<th>without network effects</th>
<th>with network effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>advertising: first firm with newspapers . . .</td>
<td>advertising: first firm with newspapers . . .</td>
</tr>
<tr>
<td></td>
<td>same side advertising: first firm with newspapers . . .</td>
<td>subscriptions: first firm with newspapers . . .</td>
</tr>
<tr>
<td>AD1</td>
<td>0.0000</td>
<td>0.0022</td>
</tr>
<tr>
<td>NRC</td>
<td>0.0000</td>
<td>0.0030</td>
</tr>
<tr>
<td>NRN</td>
<td>0.0000</td>
<td>0.0010</td>
</tr>
<tr>
<td>PAR</td>
<td>0.0000</td>
<td>0.0010</td>
</tr>
<tr>
<td>TRO</td>
<td>0.0000</td>
<td>0.0012</td>
</tr>
<tr>
<td>VOL</td>
<td>0.0000</td>
<td>0.0035</td>
</tr>
</tbody>
</table>

...merging with the second firm with newspapers . . .

<table>
<thead>
<tr>
<th></th>
<th>with network effects</th>
<th>with network effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>advertising: first firm with newspapers . . .</td>
<td>subscriptions: first firm with newspapers . . .</td>
</tr>
<tr>
<td>AD1</td>
<td>0.1292</td>
<td>0.1305</td>
</tr>
<tr>
<td>NRC</td>
<td>0.1129</td>
<td>0.1140</td>
</tr>
<tr>
<td>NRN</td>
<td>0.1091</td>
<td>0.1103</td>
</tr>
<tr>
<td>PAR</td>
<td>0.0659</td>
<td>0.0664</td>
</tr>
<tr>
<td>TRO</td>
<td>0.0998</td>
<td>0.1009</td>
</tr>
<tr>
<td>VOL</td>
<td>0.0546</td>
<td>0.0227</td>
</tr>
</tbody>
</table>

...merging with the second firm with newspapers . . .

<table>
<thead>
<tr>
<th></th>
<th>with network effects</th>
<th>with network effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>advertising: first firm with newspapers . . .</td>
<td>subscriptions: first firm with newspapers . . .</td>
</tr>
<tr>
<td>AD1</td>
<td>0.0769</td>
<td>0.0780</td>
</tr>
<tr>
<td>NRC</td>
<td>0.0674</td>
<td>0.0441</td>
</tr>
<tr>
<td>NRN</td>
<td>0.0964</td>
<td>0.2402</td>
</tr>
<tr>
<td>PAR</td>
<td>0.0744</td>
<td>0.0870</td>
</tr>
<tr>
<td>TRO</td>
<td>0.0805</td>
<td>0.0732</td>
</tr>
<tr>
<td>VOL</td>
<td>0.0805</td>
<td>0.0732</td>
</tr>
</tbody>
</table>

Each row \( i \) shows the sum of the diversion ratios, over products \( j \) of the other firm, on the advertising side of firm 1 and 2, respectively, in the top panel, and on the readership side in the bottom panel. The columns correspond to the effect on either advertising or readership demand. That is, the cells contain values of \( \sum_j D_{ij}^{AA} \), \( \sum_j D_{ij}^{RA} \) and \( \sum_j D_{ij}^{RR} \) in the top panel and values of \( \sum_j D_{ij}^{RR} \), \( \sum_j D_{ij}^{RA} \) and \( \sum_j D_{ij}^{RR} \) in the bottom panel.

likely that prices will increase after the merger. In general, efficiency gains go along with lower prices, so the higher the efficiency gain that is necessary the more likely it is that prices will increase after a merger.

The table shows that prices are predicted to increase more on the advertising side. The economic reason for this is that readers care less about advertising than advertisers care about readers, and that this is internalized by the firms when setting prices. That means that they will be more reluctant to increase subscription prices, because this will also have a negative effect on their profits on the advertising side, while increasing advertising prices will mostly have an impact on advertising demand as the elasticity.
### Table 4: Merger simulation

<table>
<thead>
<tr>
<th></th>
<th>price pre</th>
<th>price post</th>
<th>perc. change</th>
<th>UPP*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>advertising:</strong> first firm with newspapers . . .</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AD1</td>
<td>17.43</td>
<td>18.01</td>
<td>3.30</td>
<td>0.12</td>
</tr>
<tr>
<td>NRC</td>
<td>0.68</td>
<td>0.77</td>
<td>13.70</td>
<td>0.07</td>
</tr>
<tr>
<td>NRN</td>
<td>1.41</td>
<td>1.51</td>
<td>7.48</td>
<td>0.10</td>
</tr>
<tr>
<td>PAR</td>
<td>1.82</td>
<td>1.90</td>
<td>4.23</td>
<td>-0.01</td>
</tr>
<tr>
<td>TRO</td>
<td>2.72</td>
<td>2.95</td>
<td>8.58</td>
<td>0.02</td>
</tr>
<tr>
<td>VOL</td>
<td>7.32</td>
<td>7.75</td>
<td>5.77</td>
<td>0.19</td>
</tr>
<tr>
<td>. . . merging with the second firm with newspapers . . .</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOO</td>
<td>3.86</td>
<td>4.01</td>
<td>4.02</td>
<td>-0.01</td>
</tr>
<tr>
<td>HAR</td>
<td>2.90</td>
<td>3.11</td>
<td>7.45</td>
<td>-0.02</td>
</tr>
<tr>
<td>LEI</td>
<td>13.11</td>
<td>15.03</td>
<td>14.67</td>
<td>-0.04</td>
</tr>
<tr>
<td>NOR</td>
<td>3.43</td>
<td>3.65</td>
<td>6.37</td>
<td>-0.02</td>
</tr>
<tr>
<td>TEL</td>
<td>7.68</td>
<td>8.26</td>
<td>7.47</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>subscriptions:</strong> first firm with newspapers . . .</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AD1</td>
<td>251.48</td>
<td>255.69</td>
<td>1.68</td>
<td>-3.65</td>
</tr>
<tr>
<td>NRC</td>
<td>246.12</td>
<td>256.21</td>
<td>4.10</td>
<td>-4.33</td>
</tr>
<tr>
<td>NRN</td>
<td>242.55</td>
<td>250.71</td>
<td>3.37</td>
<td>2.28</td>
</tr>
<tr>
<td>PAR</td>
<td>235.41</td>
<td>242.80</td>
<td>3.14</td>
<td>3.23</td>
</tr>
<tr>
<td>TRO</td>
<td>246.12</td>
<td>251.45</td>
<td>2.16</td>
<td>-1.75</td>
</tr>
<tr>
<td>VOL</td>
<td>316.98</td>
<td>323.12</td>
<td>1.94</td>
<td>-1.12</td>
</tr>
<tr>
<td>. . . merging with the second firm with newspapers . . .</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOO</td>
<td>197.22</td>
<td>203.16</td>
<td>3.01</td>
<td>7.69</td>
</tr>
<tr>
<td>HAR</td>
<td>249.87</td>
<td>257.55</td>
<td>3.07</td>
<td>3.88</td>
</tr>
<tr>
<td>LEI</td>
<td>238.98</td>
<td>245.87</td>
<td>2.88</td>
<td>2.11</td>
</tr>
<tr>
<td>NOR</td>
<td>294.31</td>
<td>300.63</td>
<td>2.15</td>
<td>1.02</td>
</tr>
<tr>
<td>TEL</td>
<td>273.07</td>
<td>279.57</td>
<td>2.38</td>
<td>2.79</td>
</tr>
</tbody>
</table>

This table shows prices in the initial situation, simulated prices after the merger, the percentage price change, as well as \( UPP^* \).

of readership demand with respect to the amount of advertising is very small. This is a remarkable and relevant finding for competition policy, especially when a consumer—that is, reader—surplus standard is adopted. In such a case, one may argue that in a market environment in which revenues are falling the merger may be beneficial in the sense that it ensures that now merged newspaper will stay in the market, which is a benefit if “diversity of opinion” is seen as a goal, while the price for this is not paid by the readers, but mostly by the advertisers.

### 5 Concluding remarks

We propose a tractable empirical model of a two-sided market in which consumers on one market side care about the amount and the type of advertising, e.g. on an online platform or in a daily newspaper, and
advertising demand depends on the distribution of consumer type such as socioeconomic status, age and gender. We show how one can account for the feedback loops that are typically present in such markets when recovering marginal costs from the first order conditions, having demand estimates in hand. Then, we derive sufficient conditions for the existence and uniqueness of an equilibrium. These conditions are related to the own- and cross-price effects, as well as the strength of the network effects. Finally, we estimate the model using data on the Dutch daily newspaper industry and evaluate the effects of a hypothetical merger.

Our results show that the conclusions may change dramatically when the two-sidedness of the market is taken into account. Our model entails the typical specification of advertising demand that is based on the idea that newspapers have a monopoly towards the advertisers when it comes to reaching their readers. Hence, the prediction of a merger simulation that ignores the two-sidedness will always be that prices will remain unaffected when firms merge. By the same token, the main and only price effects will be expected on the readership side, because there products are differentiated and firms compete in prices. However, we show that when a merger in this market is properly analyzed, which means that feedback effects between the two market sides are taken into account, then one will actually predict that prices will increase more on the advertising side, as compared to the readership side. This shows that one could make mistakes when analyzing a two-sided market as if it was a one-sided one. In this paper, we have developed general theoretical results that are directly useful to advance such structural analyzes of two sided markets with two network effects and heterogeneous consumers.

References


A Proofs

**Proof of Lemma 1.** Define the metric space \( \mathbb{R}^{(G^a + G^r)J} \times d \) with \( d(x,y) = \|x - y\| \) being the sup-norm. This metric space is complete. Recall that we have stacked prices on both market sides into the \( 2J \times 1 \)-vector \( p \equiv (p^a, p^r)' \) and demands on both market sides and by all groups of consumers into the \( (G^a + G^r)J \times 1 \)-vector \( q \equiv (q^a, q^r)' \). We now introduce some extra notation for this proof. In particular, denote the length of the vector of quantities by \( K \) and the demand function that gives demands for given prices and demands on the respective other market side by \( \tilde{q}(p,q) \equiv (\tilde{q}^a(p,q), \tilde{q}(p',q^a))' \). In this proof, we show that under Assumption 1 \( f(q) \equiv \tilde{q}(p,\tilde{q}(p,q)) \), which maps quantities into quantities for given
prices, is a contraction. For this, we show that there is a $\beta < 1$ such that for all $q = x, y$ in that space, $\|f(x) - f(y)\| \leq \beta \|x - y\|$.  

The derivative of the $j$th element of this vector $q$ with respect to the $k$th element is either zero—if $j$ and $k$ are on the same market side, or given by the indirect network effect. Define the block-diagonal matrix

$$
\Gamma(q) \equiv \begin{pmatrix} 0 & \frac{\partial \tilde{q}^a(p', q')}{\partial q'} \\ \frac{\partial \tilde{q}(p', q')}{\partial q''} & 0 \end{pmatrix}.
$$

Its elements are functions of the whole vector of prices and quantities, respectively. Assumption 1 is related to the matrix

$$
\Gamma(q)\Gamma(q) = \begin{pmatrix} 0 & \frac{\partial \tilde{q}^a(p', q')}{\partial q'} \\ \frac{\partial \tilde{q}(p', q')}{\partial q''} & 0 \end{pmatrix} \begin{pmatrix} 0 & \frac{\partial \tilde{q}(p', q')}{\partial q'} \\ \frac{\partial \tilde{q}^a(p', q')}{\partial q''} & 0 \end{pmatrix} = \begin{pmatrix} \frac{\partial \tilde{q}^a(p', q')}{\partial q''} & \frac{\partial \tilde{q}(p', q')}{\partial q'} \\ 0 & \frac{\partial \tilde{q}^a(p', q')}{\partial q''} & \frac{\partial \tilde{q}(p', q')}{\partial q'} \end{pmatrix}.
$$

Its $j\ell$th element is

$$
-\frac{\partial \tilde{q}^a_j(p', q')}{\partial q''} \frac{\partial \tilde{q}(p', q')}{\partial q'_\ell} = -\sum_k \frac{\partial \tilde{q}^a_j(p', q')}{\partial q'_k} \frac{\partial \tilde{q}^a_k(p', q')}{\partial q'_\ell}
$$

if $j$ and $\ell$ are both on the advertising side and

$$
-\frac{\partial \tilde{q}^a_j(p', q')}{\partial q''} \frac{\partial \tilde{q}^a_j(p', q')}{\partial q'_\ell} = -\sum_k \frac{\partial \tilde{q}^a_j(p', q')}{\partial q'_k} \frac{\partial \tilde{q}^a_k(p', q')}{\partial q'_\ell}
$$

if they are both on the readership side. Otherwise, they are zero.

Assumption 1 is that the sum of the absolute values of every row of $\Gamma(q)\Gamma(q)$ is less than 1, or

$$
\sum_\ell \left| \sum_k \frac{\partial \tilde{q}^a_j(p', q')}{\partial q'_k} \frac{\partial \tilde{q}^a_k(p', q')}{\partial q'_\ell} \right| < 1
$$

for every $j$.  

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By the gradient theorem of calculus we have
\[ f(x) - f(y) = \int_x^y \frac{\partial \tilde{q}(p,q)}{\partial q} \, dq = \int_x^y \Gamma(q) \Gamma(q) \, dq, \]
where \( x \) and \( y \) are two vectors of quantities.

Define \( \lambda \equiv \|x - y\| \). The \( j \)th row of \( \|f(x) - f(y)\| \) is therefore, if the \( j \)th element of \( q \) is an advertising quantity,
\[ |f_j(x) - f_j(y)| = \left| \int_y^x \Gamma(q) \Gamma(q) \, dq \right| \\
= \left| \int_y^x \sum_k \frac{\partial \tilde{q}_j^a(p^a,q^a)}{\partial q_k^r} \frac{\partial \tilde{q}_k^r(p^r,q^r)}{\partial q^a} \, dq \right| \\
= \left| \int_y^x \sum_k \frac{\partial \tilde{q}_j^a(p^a,q^a)}{\partial q_k^r} \frac{\partial \tilde{q}_k^r(p^r,q^r)}{\partial q^a} \, dq \right| \\
\leq \int_y^x \sum_k \frac{\partial \tilde{q}_j^a(p^a,q^a)}{\partial q_k^r} \frac{\partial \tilde{q}_k^r(p^r,q^r)}{\partial q^a} |dq| \\
\leq \int_{y_1}^{x_1} \cdots \int_{y_{JGa}}^{x_{JGa}} \max_{q^r} \left\{ \sum_{\ell} \left| \sum_k \frac{\partial \tilde{q}_j^a(p^a,q^a)}{\partial q_k^r} \frac{\partial \tilde{q}_k^r(p^r,q^r)}{\partial q^a} \right| \right\} |dq|_\ell \\
= \max_{q^r} \left\{ \sum_{\ell} \left| \sum_k \frac{\partial \tilde{q}_j^a(p^a,q^a)}{\partial q_k^r} \frac{\partial \tilde{q}_k^r(p^r,q^r)}{\partial q^a} \right| \right\} \int_{y_1}^{x_1} \cdots \int_{y_{JGa}}^{x_{JGa}} |dq|_\ell \\
\leq \max_{q^r} \left\{ \sum_{\ell} \left| \sum_k \frac{\partial \tilde{q}_j^a(p^a,q^a)}{\partial q_k^r} \frac{\partial \tilde{q}_k^r(p^r,q^r)}{\partial q^a} \right| \right\} \int_{y_1}^{x_1+\lambda} \cdots \int_{y_{JGa}+\lambda}^{x_{JGa}+\lambda} |dq|_\ell \\
= \beta \lambda \\
with \quad \beta = \max_{q^r} \left\{ \sum_{\ell} \left| \sum_k \frac{\partial \tilde{q}_j^a(p^a,q^a)}{\partial q_k^r} \frac{\partial \tilde{q}_k^r(p^r,q^r)}{\partial q^a} \right| \right\} < 1.

Proof of Lemma 2. We apply the implicit function theorem. Quantities are a function of prices and quantities on the other market side,
\[ q = q(p,q) \]
and the total derivative of
\[ q(p,q) - q = 0 \]
\[
\frac{\partial (q(p,q) - q)}{\partial p} dp + \frac{\partial (q(p,q) - q)}{\partial q} dq = 0. \tag{7}
\]

Here, the dimension of the first Jacobian matrix is \((G^a + G^r)J \times 2J\), the dimension of \(dp\) is \(2J \times 1\), the one of the second Jacobian matrix is \((G^a + G^r)J \times (G^a + G^r)J\), and the one of \(dq\) is \((G^a + G^r)J \times 1\).

Recalling that quantities depend on prices on the same market side and quantities on the other market side we get

\[
\frac{\partial (q(p,q) - q)}{\partial p} = \begin{pmatrix}
\frac{\partial q^a / \partial p^a}{\partial q^a / \partial p^a} & 0 \\
0 & \frac{\partial q^r / \partial p^r}{\partial q^r / \partial p^r}
\end{pmatrix} = \begin{pmatrix}
\frac{\partial q^a / \partial p^a}{\partial q^a / \partial p^a} & 0 \\
0 & \frac{\partial q^r / \partial p^r}{\partial q^r / \partial p^r}
\end{pmatrix}
\]

and

\[
\frac{\partial (q(p,q) - q)}{\partial q} = \begin{pmatrix}
\frac{\partial q^a / \partial q^a}{\partial q^a / \partial q^a} & \frac{\partial q^a / \partial q^a}{\partial q^a / \partial q^a} \\
\frac{\partial q^a / \partial q^a}{\partial q^a / \partial q^a} & \frac{\partial q^a / \partial q^a}{\partial q^a / \partial q^a}
\end{pmatrix} - \begin{pmatrix}
I & 0 \\
0 & I
\end{pmatrix} = \begin{pmatrix}
-I & \frac{\partial q^a / \partial q^a}{\partial q^a / \partial q^a} \\
\frac{\partial q^a / \partial q^a}{\partial q^a / \partial q^a} & -I
\end{pmatrix}.
\]

Together with (7) this gives

\[
\begin{pmatrix}
\frac{\partial q^a / \partial p^a}{\partial q^a / \partial p^a} & 0 \\
0 & \frac{\partial q^r / \partial p^r}{\partial q^r / \partial p^r}
\end{pmatrix} dp + \begin{pmatrix}
-I & \frac{\partial q^a / \partial q^a}{\partial q^a / \partial p^r} \\
\frac{\partial q^a / \partial q^a}{\partial q^a / \partial p^r} & -I
\end{pmatrix} dq = 0.
\]

Assumption 1 implies that

\[
\begin{pmatrix}
-I & \frac{\partial q^a / \partial q^a}{\partial q^a / \partial q^a} \\
\frac{\partial q^a / \partial q^a}{\partial q^a / \partial q^a} & -I
\end{pmatrix}
\]

is invertible (see Gandolfo, 1996, p. 117). Hence, the derivative of the implicit function of \(q\) as a function of \(p\) only is

\[
\frac{\partial \hat{q}(p)}{\partial p} = -\begin{pmatrix}
-I & \frac{\partial q^a / \partial q^a}{\partial q^a / \partial p^r} \\
\frac{\partial q^a / \partial q^a}{\partial q^a / \partial p^r} & -I
\end{pmatrix}^{-1} \begin{pmatrix}
\frac{\partial q^a / \partial p^a}{\partial q^a / \partial p^a} & 0 \\
0 & \frac{\partial q^a / \partial p^r}{\partial q^a / \partial p^r}
\end{pmatrix}.
\]

Proof of Lemma 3. We follow Vives (2001, p. 47f). There are three sufficient conditions for a unique equilibrium.

The first one is a dominant diagonal property. Under this condition, one can apply the contraction
mapping theorem that then implies that there is a unique fixed point of the best replies, which then is the Nash equilibrium.

The second condition is a property of the first order conditions. If the Jacobian matrix is negative quasi-definite, then the Gale-Nikaido theorem implies that the map from prices to values of the first order conditions is one-to-one, which proves that there is a unique vector of prices that solves the first order conditions. Hence, the equilibrium is unique.

The third condition allows one to apply the Poincaré-Hopf index theorem.

**B Empirical implementation**

In the estimation step, we estimate parameters of advertising demand using quarterly data at the national level and parameters of readership demand using both, yearly national level data by demographic groups and yearly municipality level data that are not at the level of demographic groups. In the following, unlike in the theoretical discussion in the paper, we indicate that we use panel data by adding a subscript \( t \).

Estimation of advertising demand parameters follows a standard linear instrumental variables procedure.

For readership demand estimation, there are two types of data and therefore we have two sets of moment conditions. The moments at the municipality level are the expectation of

\[
g_{mjt}(\theta_2) \equiv \xi_{mjt}(\theta_2)z_{mjt},
\]

where \( \xi_{mjt}(\theta_2) \) is the residual from an instrumental variables regression of mean utilities \( \delta_{mjt}(\theta_2) \) on characteristics \( x_{1mjt} \) using instruments \( z_{mjt} \) and thereby implicitly depends on the coefficients on those \( x_{1mjt} \), denoted by \( \theta_1 \). Here, we follow Berry, Levinsohn, and Pakes (1995) and obtain mean utilities for any given \( \theta_2 \) in the inner loop by means of a contraction mapping, also integrating over the distribution of demographics, and then estimate \( \theta_1 \) using the two-stage least squares estimator. So, minimizing a sample analogue of \( \mathbb{E}[g_{mjt}(\theta_2)] \) over \( \theta_2 \) implicitly involves a choice of \( \theta_1 \).

The aforementioned moments for readership demand are at the municipality-newspaper-time level. The additional moments we add at the national level, for different demographic groups, resemble the
additional moments used by Petrin (2002). They are expectations of

\[ g_{gjt}(\theta_2) \equiv (s_{gjt} - \hat{s}_{gjt}(\theta_2))z_{gjt}, \]

where subscript \( g \) is a demographic group, \( s_{gjt} \) is the market share of that group at the national level, and \( \hat{s}_{gjt}(\theta_2) \) is the predicted market share for that group at the national level, which we can simulate for a given \( \theta_2 \) by adding up implied market shares for different demographic groups. Similar to before, \( z_{gjt} \) is a vector of instruments.

Formally, the condition for identification of \( \theta_1 \) and \( \theta_2 \) is that

\[ g(\theta_2) \equiv \begin{pmatrix} E_mjt[g_{mjt}(\theta_2)] \\ E_gjt[g_{gjt}(\theta_2)] \end{pmatrix} = 0 \]

if, and only if, \( \theta_2 \) is equal to its population value. We henceforth assume that this is the case. Then, \( \theta_2 \) can be estimated by the generalized method of moments (GMM). In particular, our estimator is given by

\[ \hat{\theta}_2 = \arg\min_{\theta_2} \begin{pmatrix} E_mjt[g_{mjt}(\theta_2)] \\ E_gjt[g_{gjt}(\theta_2)] \end{pmatrix}' W \begin{pmatrix} E_mjt[g_{mjt}(\theta_2)] \\ E_gjt[g_{gjt}(\theta_2)] \end{pmatrix}, \]

where the hats denote averages and \( W \) is a positive semi-definite weighting matrix. The efficient estimator uses the inverse of the variance-covariance matrix of the moment conditions for this. In Petrin’s (2002) case, this weighting matrix is block-diagonal because the two sets of moments come from two independent sampling processes and the two blocks are the respective variance-covariance matrices of the moments. In our case, however, block-diagonality does not hold. This is because \( \xi_{mj} \), which enters the first set of moments, and \( s_{gjt} \), which enters the second, are correlated.

Hansen (1982) shows that the resulting estimator is consistent and normally distributed with variance-covariance matrix given by

\[ (G'WG)^{-1} G'WVWG (G'WG)^{-1}, \]

where \( G \) is the matrix of derivatives of the moments with respect to the estimated parameters, now
including both \(\theta_1\) and \(\theta_2\), and \(V\) is the variance-covariance matrix of the moment conditions,

\[
\begin{align*}
V \equiv & \mathbb{E}_{mjt} \left[ \begin{pmatrix}
g_{mj}(\hat{\theta}_2) \\
g_{gj}(\hat{\theta}_2)
\end{pmatrix}' \begin{pmatrix}
g_{mj}(\hat{\theta}_2) \\
g_{gj}(\hat{\theta}_2)
\end{pmatrix} \right].
\end{align*}
\]

Given consistent initial estimates \(\hat{\theta}_2\) it can be estimated by

\[
\hat{V} = \frac{1}{N} \sum_m \sum_j \sum_t \begin{pmatrix}
g_{mj}(\hat{\theta}_2) \\
g_{gj}(\hat{\theta}_2)
\end{pmatrix}' \begin{pmatrix}
g_{mj}(\hat{\theta}_2) \\
g_{gj}(\hat{\theta}_2)
\end{pmatrix},
\]

\[
= \frac{1}{N} \sum_m \sum_j \sum_t \begin{pmatrix}
g_{mj}(\hat{\theta}_2)'g_{mj}(\hat{\theta}_2) & g_{mj}(\hat{\theta}_2)'g_{gj}(\hat{\theta}_2) \\
g_{gj}(\hat{\theta}_2)'g_{mj}(\hat{\theta}_2) & g_{gj}(\hat{\theta}_2)'g_{gj}(\hat{\theta}_2)
\end{pmatrix}
\]

where \(N\) is the number of observations across \(m, j\) and \(t\).

Since our weighting matrix is the efficient one, we have that the variance-covariance matrix of our estimates is given by \((G'WG)^{-1}\). See also Newey and McFadden (1994) and Nevo (2000b) with the corresponding web appendix for additional details on the implementation. In brief, we first obtain a set of initial estimates of \(\theta_2\) as well as the implied estimates of \(\theta_1\), then estimate the variance-covariance matrix of the moment conditions by its sample analog, calculate \(W\), solve numerically for \(\hat{\theta}_2\), then estimate \(G\) and \(V\) by their sample analogs, and finally calculate the variance-covariance matrix of the estimates. In the simulations step, we use Halton sequences of length 80, treating the number of draws as going to infinity for the asymptotics. This implies that the simulation error is negligible. See McFadden (1989), Pakes and Pollard (1989) and Train (2003) for details.