Booklet of Abstracts

2019 Asymptotic Geometric Analysis

honoring Vitali Milman's 80^{th} birthday

July 29 - August 2, 2019

Vitali, traces and geometry

Monday
9:30-10:20
Yaglom

Noga Alon

Princeton University

More than 30 years ago Milman showed that combinatorial results about the number of distinct projections (traces) of a family of binary vectors on a prescribed number of coordinates yield interesting applications in the asymptotic study of finite dimensional normed spaces.

Since then this approach has been extended and refined by Vitali and by others, suggesting the investigation of the largest number of projections onto k coordinates guaranteed in every family of m binary vectors of length n.

This fundamental combinatorial question is intimately connected to additional topics and results in discrete mathematics, theoretical computer science, geometry, machine learning and more, and is wide open for most settings of the parameters.

I will describe some of the history, motivation and known results, focusing on a recent asymptotic solution of the question, in joint work with Moshkovitz and Solomon, for linear k and sub-exponential m.

Hypoelliptic Dirac operators and the trace formula

Monday
10:50-
11:40
Yaglom

Jean Michel Bismut

Université Paris-Sud (Orsay)

The hypoelliptic Laplacian is a family of operators acting on the total space of the tangent bundle of a Riemannian manifold, that interpolates between the ordinary Laplacian and the geodesic flow. It is obtained via a deformation of the associated Dirac operator. This deformation preserves global spectral invariants. In the case of locally symmetric spaces, the deformation is essentially isospectral. In the talk, I will describe applications of the hypoelliptic deformation in real and complex geometry, and also in the evaluation of semisimple orbital integrals.

Covering Systems

Béla Bollobás
Cambridge and Memphis

Monday 11:50-12:40 Yaglom

Introduced by Erdös in 1950, a covering system of the integers is a finite collection of arithmetic progressions in the integers whose union is the set $\mathbb{Z}$. Many beautiful questions and conjectures about covering systems were posed over the following decades, but until recently progress with them was slow. In particular, in 2007, Filaseta, Ford, Konyagin, Pomerance and Yu proved the 1980 conjecture of Erdös and Graham that for every constant C, if x is sufficiently large and the moduli are distinct elements of the interval $[x, Cx]$, then the density of integers uncovered by the union is bounded below by some constant depending only on C. Also, in 2015, in a breakthrough paper, Hough proved the major conjecture of Erdös from 1950 that every covering system with distinct moduli has a modulus which is at most 10^{16}.

Recently, Balister, Morris, Sahasrabudhe, Tiba and I have proved several results about covering systems: in particular, we have proved Schinzels conjecture from the 1960s that, for moduli $d_1 < \cdots < d_k$ with $d_1 \geq 2$, there are $d_i < d_j$ such that $d_i | d_j$, and answered Erdöss 1952 question concerning the asymptotic number of minimal covering systems with n progressions. In my lecture I shall give an introduction to this field and describe some of the recent results.

Algebraic Techniques in Combinatorial Geometry

Micha Sharir
Tel Aviv University

Monday 14:30-15:20 Lev

For the past 11 years, combinatorial geometry (and to some extent, computational geometry too) has gone through a dramatic revolution, due to the infusion of techniques from algebraic geometry and algebra that have proven effective in solving a variety of hard problems that were thought to be unreachable with more traditional techniques. The new era has begun with two groundbreaking papers of Guth and Katz, in 2008 and 2010. Their second paper has (almost completely) solved the notoriously hard distinct distances problem of Erdős, open since 1946.

It is now high time to celebrate the decade of extensive achievements that have been made since 2008, and in this talk I will survey some of the progress that has been made, including a variety of problems on distinct and repeated distances and other configurations, on incidences between points and lines, curves, and surfaces in two, three, and higher dimensions, on polynomials vanishing on Cartesian products with applications, on depth cycle elimination for lines and triangles in three dimensions, on range searching with semialgebraic sets, and I will most certainly run out of time while doing so.

Poisson hyperplanes: interactions with convex bodies

Monday
15:50-
16:40
Lev

Rolf Schneider
Albert-Ludwigs-Universität Freiburg

A Poisson process in the space of hyperplanes in $\mathbb{R}^d$ with a translation invariant distribution is perhaps the most natural and tractable arrangement of infinitely many random hyperplanes in $\mathbb{R}^d$. We consider such a hyperplane process $\widehat{X}$ and the tessellation X of $\mathbb{R}^d$ into polytopes that it induces, and describe various old and new aspects under which both are related to the geometry of convex bodies. A first question is: which polytopes appear in the tessellation X (with probability one), and how often? Then, several classical results from convex geometry, such as Aleksandrov–Fenchel inequalities, the Blaschke–Santaló inequality, Minkowski's existence theorem, stability for isoperimetric inequalities, turn out to be inevitable tools if one wants to answer a series of geometrically natural questions about $\widehat{X}$ and X. Increasing the intensity of the hyperplane process suggests various problems about the approximation of convex bodies by random polytopes.

Persistence barcodes in analysis and geometry

Monday
16:50-
17:40
Lev

Leonid Polterovich
Tel Aviv University

While originated in data analysis, persistence barcodes provide an efficient way to bookkeep topological information contained in the calculus of variations. I shall describe applications of barcodes to function theory and symplectic geometry. Based on joint works with Iosif Polterovich, Egor Shelukhin and Vukasin Stojisavljevic.

On some random convex sets generated by isotropic log-concave random vectors

Tuesday
9:30-10:20
Lev

Apostolos Giannopoulos
National and Kapodistrian University of Athens

Let K and C be centrally symmetric convex bodies in $\mathbb{R}^n$. Using rearrangement inequalities, Gluskin and V. Milman showed that if $\mathrm{vol}_n(C) = \mathrm{vol}_n(K)$ then

$$\|\mathbf{t}\|_{C^s,K} = \int_C \cdots \int_C \Big\| \sum_{j=1}^{s} t_j x_j \Big\|_K \, dx_1 \cdots dx_s \geqslant c\, \|\mathbf{t}\|_2$$

for all $\mathbf{t} \in \mathbb{R}^s$, where $c > 0$ is an absolute constant. We discuss upper bounds for this multi-integral expression in the case where C is isotropic. Our approach also provides an alternative proof of the lower bound.

In the second part of the talk we discuss estimates on the expected volume of various random convex sets. For any $\mathbf{x} = (x_1, \ldots, x_N) \in \oplus_{i=1}^N \mathbb{R}^n$ we denote by $T_{\mathbf{x}} = [x_1 \cdots x_N]$ the $n \times N$ matrix whose columns are the vectors x_i. Paouris and Pivovarov showed that if $N \geqslant n$ and $f_1, \ldots, f_N$ are probability densities on $\mathbb{R}^n$ with $\|f_i\|_\infty \leqslant 1$ then, for any centrally symmetric convex body K in $\mathbb{R}^N$,

$$\mathcal{F}_K(f_1, \ldots, f_N) = \int_{\mathbb{R}^n} \cdots \int_{\mathbb{R}^n} \big(\mathrm{vol}_n(T_{\mathbf{x}}(K))\big) \prod_{i=1}^{N} f_i(x_i)\, dx_N \cdots dx_1$$

is minimized when each f_i is the indicator function $\mathbf{1}_{D_n}$ of the Euclidean ball D_n of volume 1 in $\mathbb{R}^n$. We discuss upper and lower bounds for $\mathcal{F}_K(f_1,\dots,f_N)$ in the case where f_i are isotropic log-concave densities. Finally, given $N, n \geqslant 1$ and $r > 0$, we discuss upper and lower bounds for the expected volume

$$\mathbb{E}\Big[\mathrm{vol}_n\Big(\bigcap_{i=1}^{N} B(x_i, r)\Big)\Big]$$

of random ball polyhedra defined by an N-tuple of i.i.d. random points $x_1,\dots,x_N$ in $\mathbb{R}^n$ whose density f satisfies $\|f\|_\infty \leqslant 1$.

The talk is based on joint works with G. Chasapis and N. Skarmogiannis.

Solving Macromolecular Puzzles.

Tuesday 10:50-11:40 Lev

Haim Wolfson

Tel Aviv University

We shall present algorithms for the solution of key computational tasks in the modeling of large macromolecular assemblies, which are the factories of the living cell. Some of these tasks closely resemble solution of 3D toy puzzles.

A probabilistic variant of Sperner's theorem

Tuesday 11:50-12:40 Lev

Shoni Gilboa

Open University

A family of sets is called r-cover free if it does not contain $2 \le \ell + 1 \le r + 1$ distinct sets $A_0, A_1, \dots, A_\ell$ such that $A_0 \subseteq A_1 \cup \dots \cup A_\ell$. A classic theorem of Sperner states that the maximal cardinality of a 1-cover free family of subsets of an n-element set is $\binom{n}{\lfloor n/2 \rfloor}$. Estimating the maximal cardinality of an r-cover free family of subsets of an n-element set for $r > 1$ was also studied.

We study the following probabilistic variant of this problem. Let $S_0, S_1, \dots, S_r$ be independent and identically distributed random subsets of an n-element set. Which distribution minimizes the probability that $S_0 \subseteq S_1 \cup \dots \cup S_r$? A natural candidate is the uniform distribution on an r-cover-free family of maximal cardinality. For $r = 1$ such distribution is indeed best possible. However, for every $r > 1$ and n large enough, such distribution can be beaten by an exponential factor.

Joint work with Shay Gueron.

Amazing life of cell

Elena Vladimirsky
Jerusalem

Tuesday
14:00-
14:50
Lev

Many phenomena which are well-known in biology and medicine have no explanation from the classical stance of modern biochemistry and molecular biology. Especially it is connected with the amazing life of cell. It is well known that the dose-response effect of drugs does not always follow a linear relationship in the therapeutic dose range. For instance, small doses of some drugs cause a stimulating effect while the large doses of the same drugs have a depressive effect. Many mechanisms of the initiation and transduction of intracellular signals do not lend themselves to interpretation from the point of view of molecular biology. It becomes obvious from some figures: from 10 to 100,000 molecule-receptors may be expressed at the cell surface; 4,000 protein molecules take part in signal transduction; signal molecules have to cover great distance during their movement inside a cell (molecule diameter is about 2-10 nm while cell diameter is about 10,000 nm). The following questions arise: 1. How do signal molecules find their targets? 2. How their movement is directed? Our main assumption is that interaction between inductor and target molecules in cells is based on laws of physics. An inductor molecule emits a specific monochromatic radiation which is captured by the appropriate target molecule according to the bioresonance absorption principle triggering the emission of its own radiation and thus turning it from target into inductor. This is a chain process that creates a signal path, along which the activated molecules move and interact with each other through contact as described by molecular biology. As part of this process, all impact (information) is mediated through electro-magnetic particles (biophotons) that interact with each other in the electromagnetic field according to laws of constructive and destructive interference. Increase or decrease in the targets response depends on type of interference predominance. Due to this effect, weak signals are sometimes able to produce stronger response than strong ones as the increase in their number leads to expansion of the area of destructive interference. This principle was confirmed in our study using 3 experimental cell models. Further development of the biophotonic paradigm of information transduction in cell systems may contribute to better understanding of many normal and pathological processes in human body as well to improvement in some types of drug therapies. .

TBA

Mikhail Gromov
I.H.E.S., Paris

Tuesday
15:20-
16:10
Lev

A few conjectures on intrinsic volumes on Riemannian manifolds and Alexandrov spaces

Tuesday
16:20-
17:10
Lev

Semyon Alesker
Tel Aviv University

Decomposition of measures on the Boolean hypercube into approximate product measures

Wednesday
9:30-10:20
Lev

Ronen Eldan
Weizmann Institute

When can a random variable, taking values in the Boolean hypercube be approximated by a mixture of variables with roughly independent coordinates? Namely, given a measure μ on $\{0,1\}^n$ we want to be able to write $\mu = \sum_{i \in N} \mu_i$ with N as small as possible and μ_i being close to a product measure for most i. We will review several results regarding such entropy-efficient decompositions. This type of question has applications in several fields, which we will discuss:
1. Random graphs and large deviations,
2. Statistical mechanics - interacting particle systems,
3. Rounding of Semidefinite programs in computer science.

In particular, we will present an approach which relies on embedding of the measure to the space of paths of a Brownian motion, and see how this can be useful for the problem, as well as other aspects of the analysis of Boolean measures.

Operator Equations in Function Spaces

Wednesday
10:50-
11:40
Lev

Hermann König
Kiel

We first determine the form of the solutions of the Leibniz rule and the Chain rule operator equations in spaces of k-times continuously differentiable functions. Motivated by operator characterizations of the Fourier transform, we then study the "Extended Leibniz rule equation"

$$T(f \cdot g) = Tf \cdot Ag + Af \cdot Tg \quad ; \quad f, g \in \mathcal{S} \tag{0.0.1}$$

in the space of rapidly decreasing functions $\mathcal{S}$ on $\mathbb{R}$, real- as well as complex-valued functions. Here $T, A : \mathcal{S} \to \mathcal{S}$ are operators which are non-degenerate and satisfy a weak continuity condition. We determine the general form of the operators T and A satisfying (0.0.1). In the proof the operators T and A are shown to be localized which allows (0.0.1) to be reduced to functional equations involving two unknown functions.
This is joint work with Vitali Milman.

On extensions of the Milman-Pajor inequality

Wednesday
11:50-
12:20
Lev

Boaz Slomka
Open University

The Weyl principle in pseudo-Riemannian geometry.

Wednesday
12:20-
12:50
Lev

Dmitry Faifman
Université de Montréal

According to a famous theorem of Weyl, the volume of an epsilon-tube around a submanifold M in Euclidean space is a polynomial in epsilon, whose coefficients are, remarkably, intrinsic invariants of M. In different contexts they are known as the Lipschitz-Killing curvatures, the quermassintegrals, or the intrinsic volumes of M. In recent years, Alesker's theory of valuations on manifolds sparked a renewed interest in Weyl's theorem, and gave rise to some exciting new results in differential and integral geometry. In the talk we will discuss an extension of the Weyl theorem to the class of pseudo-Riemannian manifolds. Furthermore, we will extend the theory to a generic class of manifolds with a metric of non-constant signature. As an application we will deduce Chern-Gauss-Bonnet type theorems for such manifolds, and an array of Crofton formulas for the de Sitter space. Based on a joint work in progress with A. Bernig and G. Solanes.

Wittgenstein's philosophy of mathematics

Wednesday
18:10-
19:00
Amichai

Roy Wagner
E.T.H. Zürich

Wittgenstein's philosophy of mathematics is unique in that it does not try to reduce mathematics to some ideal reality, mental construction, formal mechanism or empirical description. Instead it portrays mathematics as a practice that is motivated by natural, psychological and social circumstances, but instead of simply reflecting them, it serves as a standard against which we describe them. To use a statement as a mathematical statement means to apply it as a norm, rather than as a description. To endorse a statement as mathematically true is to accept it as fitting into the system of the surrounding norms - an endorsement that is never simply determined by these norms.

The Gaussian Double-Bubble and Multi-Bubble Conjectures

Thursday
9:30-10:20
Dead Sea
Institute

Emanuel Milman
Technion

The classical Gaussian isoperimetric inequality, established in the 70s independently by Sudakov-Tsirelson and Borell, states that the optimal way to decompose $\mathbb{R}^n$ into two sets of prescribed Gaussian measure, so that the (Gaussian) area of their interface is minimal, is by using two complementing half-planes. This is the Gaussian analogue of the classical Euclidean isoperimetric inequality, and is therefore referred to as the “single-bubble case.

A natural generalization is to decompose $\mathbb{R}^n$ into $q \geq 3$ sets of prescribed Gaussian measure. It is conjectured that when $q \leq n+1$, the configuration whose interface has minimal (Gaussian) area is given by the Voronoi cells of q equidistant points. For example, for $q = 3$ (the “double-bubble conjecture) in the plane ($n = 2$), the interface is conjectured to be a “tripod or “Y - three rays meeting at a single point in 120 degree angles. For $q = 4$ (the “triple-bubble conjecture) in $\mathbb{R}^3$, the interface is conjectured to be a tetrahedral cone.

We confirm the Gaussian double-bubble and, more generally, multi-bubble conjectures for all $3 \leq q \leq n+1$. The double-bubble case $q = 3$ is simpler, and we will explain why. None of the numerous methods discovered over the years for establishing the classical $q = 2$ case seem amenable to the $q \geq 3$ cases, and our method consists of establishing a Partial Differential Inequality satisfied by the isoperimetric profile. To treat $q > 3$, we first prove that locally minimal configurations must have flat interfaces, and thus convex polyhedral cells. Uniqueness of minimizers up to null-sets is also established.

This is joint work with Joe Neeman (UT Austin).

Singularity of random Bernoulli 0/1 matrices

Thursday
10:50-
11:40
Dead Sea
Institute

Alexander Litvak
University of Alberta

Let M be a random $n \times n$ matrix, whose entries are independent 0/1 random variables taking value 1 with probability $p = p_n$. We provide sharp bounds on the probability that M is singular. Roughly speaking, this probability is comparable with the probability that M has either zero row or zero column. This is a joint work with Konstantin Tikhomirov.

Equivalent norms for relatively dense subsets

Thursday 11:50-12:40 Dead Sea Institute

Omer Friedland

Jussieu, Paris

Some recent works have shown that the heat equation posed on the whole Euclidean space is null-controllable in any positive time if and only if the control subset is a relatively dense set. This necessary and sufficient condition for null-controllability is linked to some uncertainty principles as the Logvinenko-Sereda theorem which give limitations on the simultaneous concentration of a function and its Fourier transform. In talk we present certain generalizations of this theorem for different classes of functions.

Complex Legendre Duality

Thursday 14:30-15:20 Dead Sea Institute

Bo'az Klartag

Weizmann Institute

The Legendre transform appears in the theory of the real Monge-Ampere equation and optimal transport in Euclidean spaces, since it intertwines between the convex potential of the optimal transport map and the convex potential of the inverse map. The theory of complex interpolation of normed spaces suggests an interesting interplay in the complex setting: Namely, in even dimensions the Legendre transform can also be viewed as a local symmetry of the complex Monge-Ampere equation. We introduce complex generalizations of the classical Legendre transform, which are additional local symmetries of the complex Monge-Ampere equation, each with its own unique fixed point. These new Legendre-type transforms give explicit local isometric symmetries for the Mabuchi metric on the space of Kahler metrics around any real analytic Kahler metric, answering a question originating in Semmes' work. Joint work with B. Berndtsson, D. Cordero-Erausquin and Y.A. Rubinstein.

Moments of Scores

Thursday 15:50-16:40 Dead Sea Institute

Sergey Bobkov

University of Minnesota, Minneapolis

If a random variable X has an absolutely continuous density $f(x)$, its score is defined to be the random variable $\rho(X) = f'(X)/f(X)$, where $f'(x)$ is the derivative of f. We will discuss upper bounds on the moments of the scores, especially in the case when X represents the sum of independent random variables.

Asymptotic Trace Formulas of Szegö Type for Ergodic Operators and Related Topics of Quantum Informatics

Thursday
16:50-
17:40
Dead Sea
Institute

Leonid Pastur

Institute for Low Temperatures Physics and Engineering, Kharkiv, Ukraine

We present a set up generalizing that for Szegö's theorem on the Töplitz (or discrete convolution) operators. Viewing the theorem as an asymptotic trace formula determined by a certain basic operator and by two functions (the symbol and the test function), we replace the Töplitz operator by an ergodic operator (e.g. random or quasiperiodic), in particular, by the discrete Schrodinger operator with ergodic potential. In the framework of this set up we discuss a variety of asymptotic formulas different from those given by Szegö's theorem and including certain Central Limit Theorems in spectral context and large block asymptotic formulas for the entanglement entropy of free disordered fermions.

On the change of variables $\lambda \mapsto \sqrt{\lambda}$

Friday
9:00-9:50
Amichai

Sasha Sodin

Queen Mary University, London

We shall discuss several applications of the change of variables $\lambda \mapsto \sqrt{\lambda}$, conjugated by the Fourier transform.

Flowers in convex geometry

Friday
9:50-10:20
Amichai

Liran Rotem

Technion

We will discuss a class of star bodies called flowers. A flower is an arbitrary union of Euclidean balls, each containing the origin. While flowers are not necessary convex, we will see that the class of flowers is in a one to one correspondence with the class of convex bodies. Using flowers one can understand many operations on convex bodies, both old and new. For example, obtaining from K its polar body is a highly non-local operation. However, the polar body to K can be obtained from the flower of K using what is essentially a pointwise map. Moreover, the class of flowers is surprisingly stable, and is closed under many natural operations. Some of these operations translate to new and interesting operations on convex bodies. Finally we will discuss the question of when is a flower convex, which leads to an interesting class of convex bodies we call "reciprocal bodies".

Based on joint work with Emanuel Milman and Vitali Milman.

Functionalization of geometric inequalities and beyond I

Friday
10:50-
11:20
Amichai

Dan Florentin
Kent State University

Functionalization of geometric inequalities and beyond II

Friday
11:20-
11:50
Amichai

Alexander Segal
Afeka College

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