SAT-Based Invariant Inference and Its Relation to Concept Learning

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erc Supervised Verification of Infinite-State Systems

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Mooly Sagiv

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CERTORA

James R. Wilcox

W UNIVERSITY of WASHINGTON

SAT-Based Invariant Inference

- predicate abstraction
  [CAV'97, POPL'02]
- symbolic abstraction
  [VMCAI'04,'16]
- interpolation
  [CAV'03, TACAS'06]
- IC3/PDR
  [VMCAI'11, FMCAD'11]
- abduction
  [OOPSLA'13]
- SyGuS
  [FMCAD'13,...]
- ICE learning
  [CAV'14, POPL'15]
- ...

Why do they succeed?

Why do they fail?

(How can we make them better?)

Goal

Understand SAT-based invariant inference
from the perspective of exact learning with queries

[POPL'20] Complexity and information in invariant inference. Feldman, Immerman, Sagiv, Shoham
[POPL'21] Learning the boundary of inductive invariants. Feldman, Sagiv, Shoham, Wilcox
[POPL'22] Property-directed reachability as abstract interpretation in the monotone theory. Feldman, Sagiv, Shoham, Wilcox
[SAS'22] Invariant Inference With Provable Complexity From the Monotone Theory. Feldman, Shoham

Safety of Transition Systems

(Un)reachability problem: no bad state is reachable from the initial states

Init:

$(x_1, \ldots, x_n) \coloneqq 0 \ldots 0$

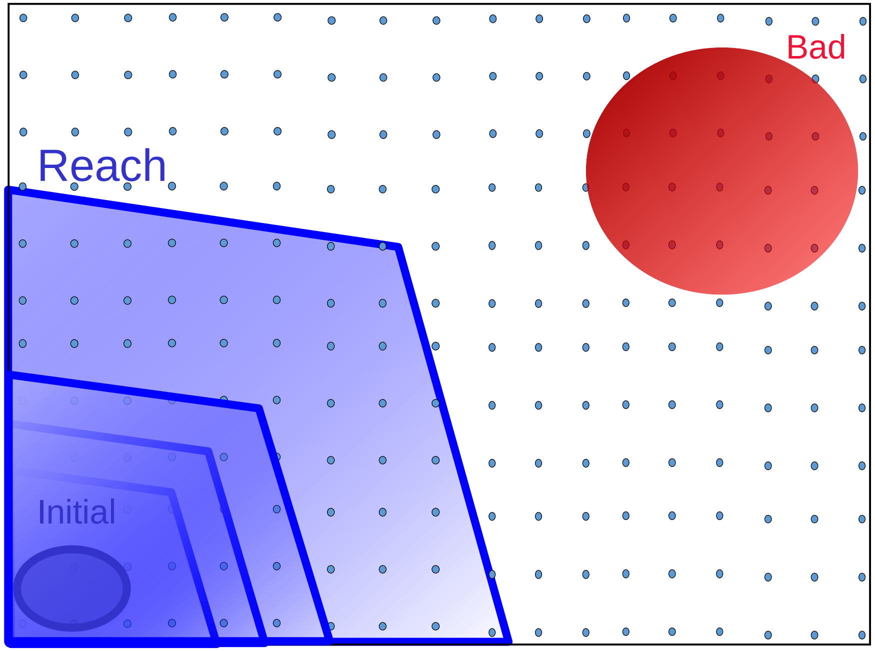
Bad:

$(x_1, \ldots, x_n) = 1 \ldots 1$

δ:

$y_1, \ldots, y_n \coloneqq *$

$x_1, \ldots, x_n \coloneqq (x_1, \ldots, x_n) + 2 \cdot (y_1, \ldots, y_n) \pmod{2^n}$



Inductive Invariants

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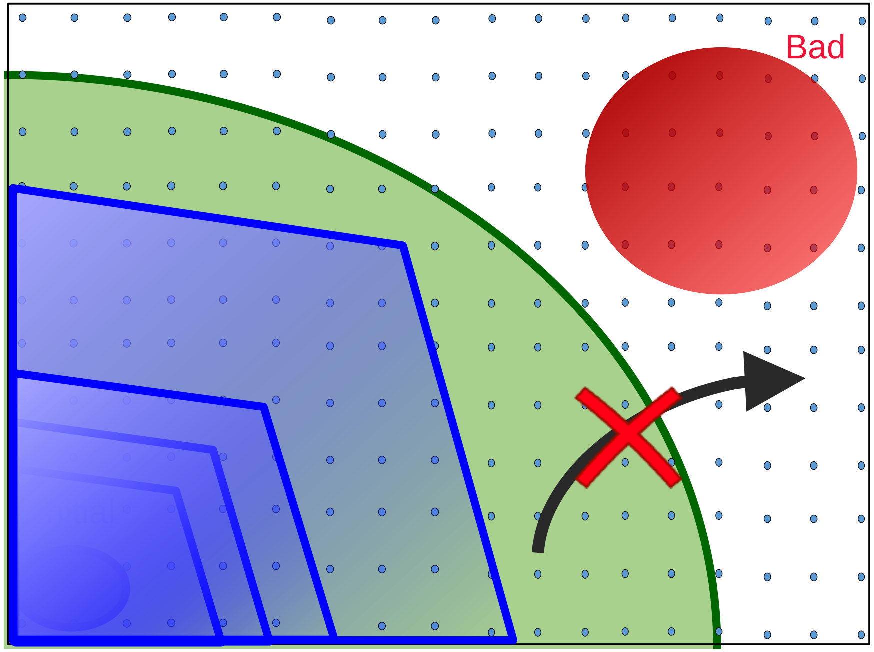

Initiation: $\text{Init} \subseteq I$

Safety: $I \cap \text{Bad} = \emptyset$

Consecution: $\{I\}\ \delta\ \{I\}$

Inductive Invariants

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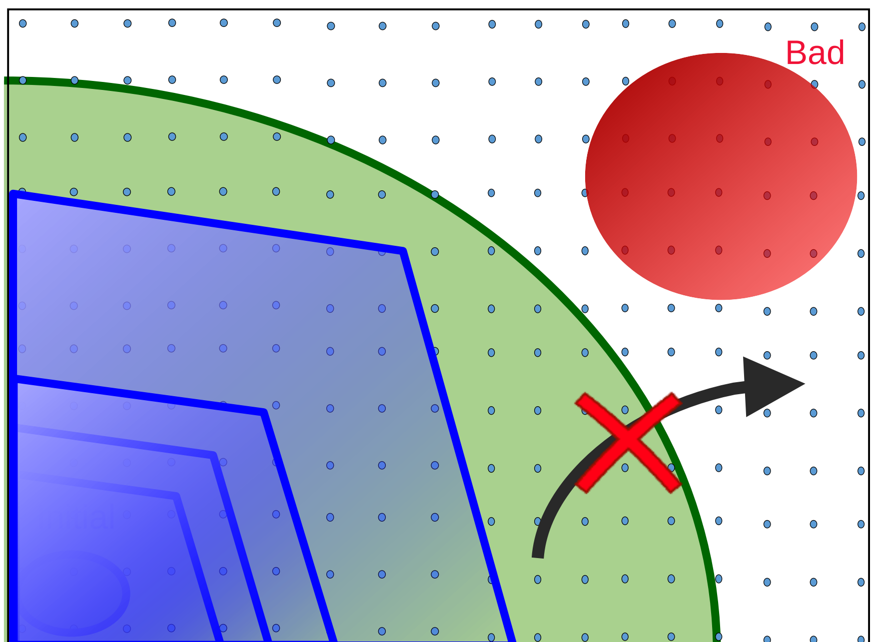
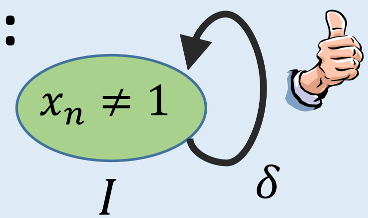


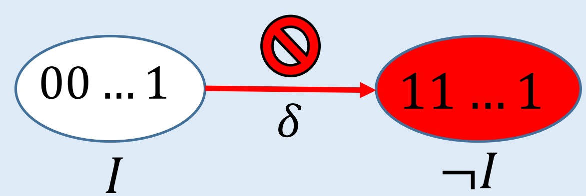
$I\colon\ x_n \neq 1$

Inductive:



$I\colon\ (x_1, \ldots, x_n) \neq 1 \ldots 1$

Not inductive:



Invariant Inference

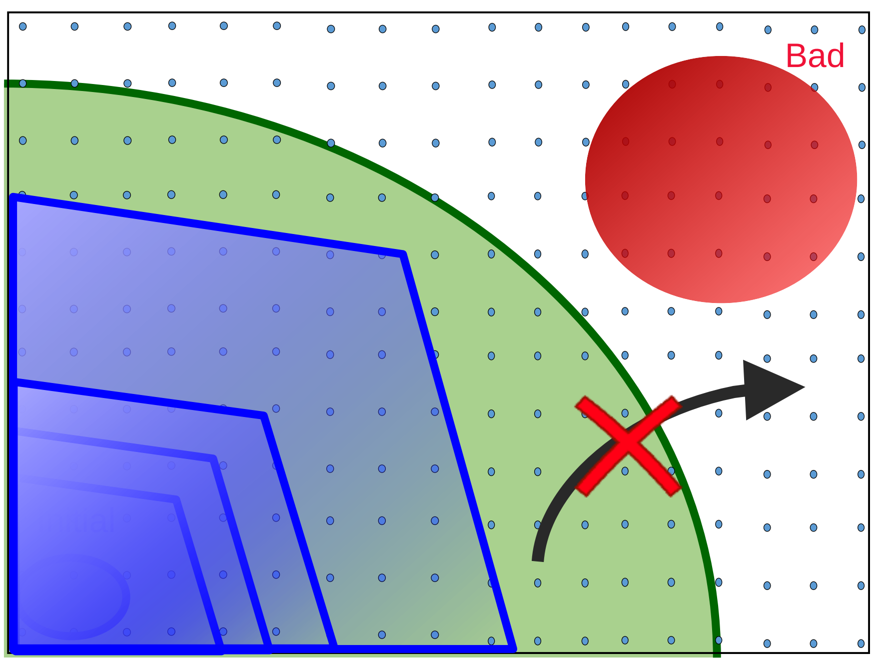
(Un)reachability problem: no bad state is reachable from the initial states

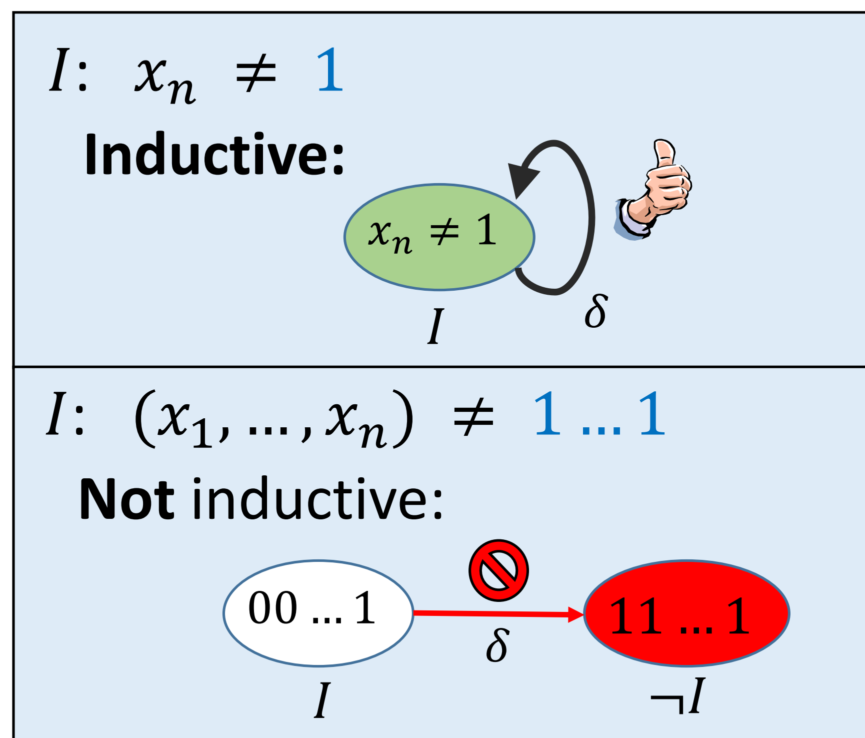
Goal: Find inductive invariants automatically

Bad:

$(x_1, \ldots, x_n) = 1 \ldots 1$

$2 \cdot (y_1, \ldots, y_n) \pmod{2^n}$

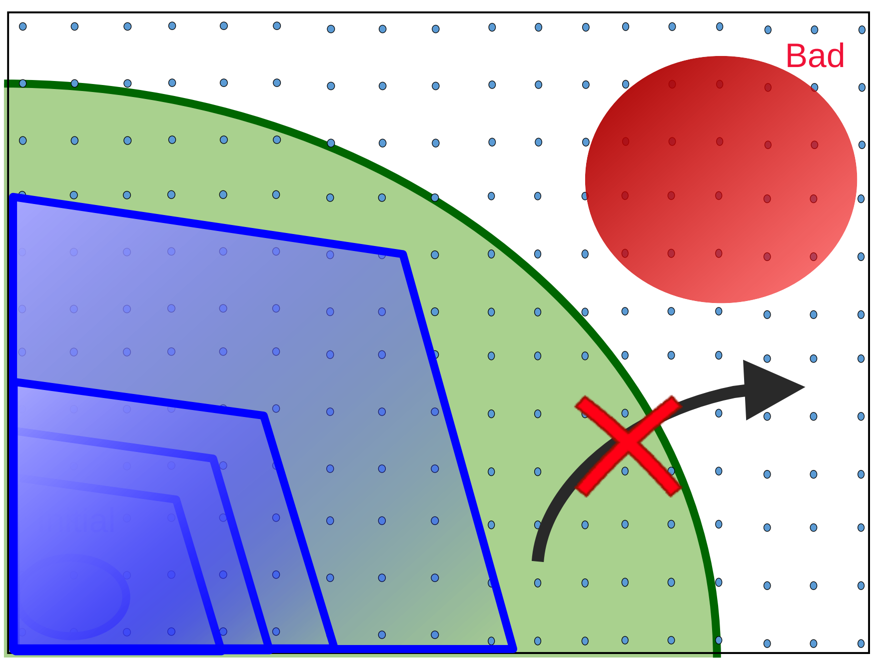


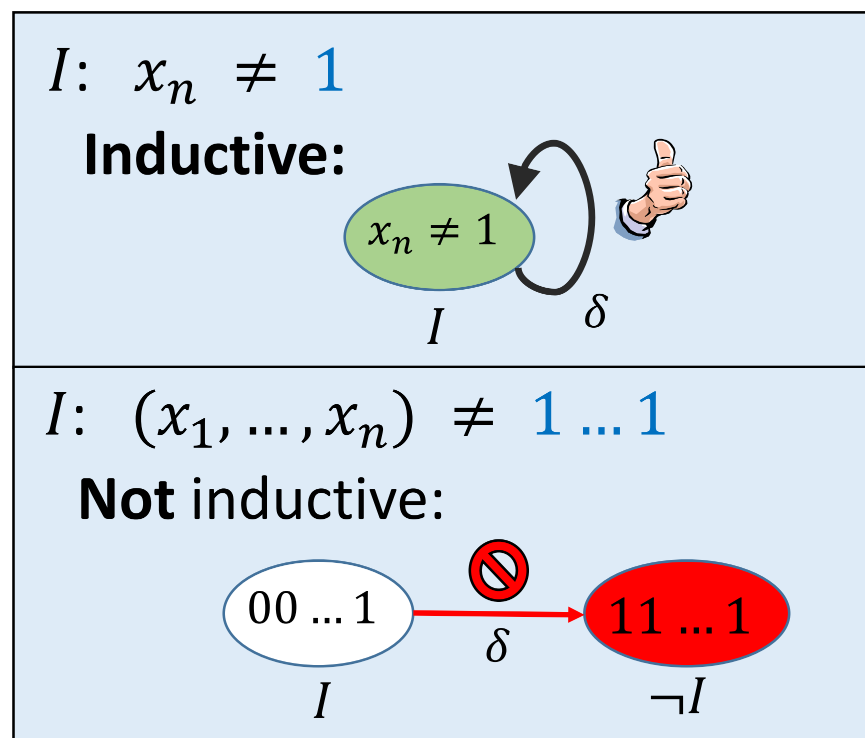


SAT-based Invariant Inference

Goal: Find inductive invariants automatically

Means: Employ a SAT solver





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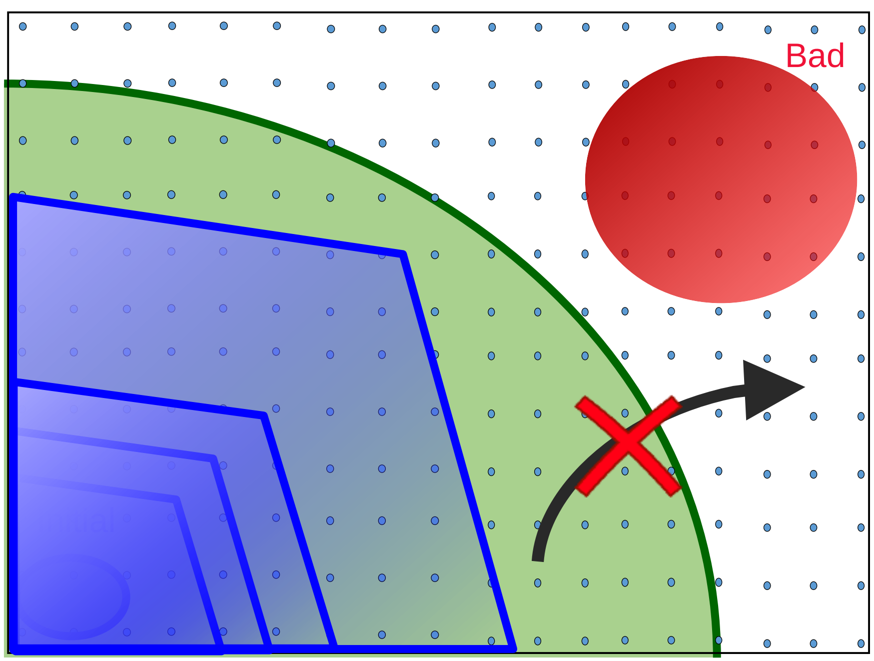


Init, Bad: formulas over V

$\boldsymbol{\delta}$: formula over V, V'

SAT query Examples:

Initiation: **Init** $\wedge \neg \boldsymbol{I}$ unsat?

Safety: $\boldsymbol{I} \wedge$ **Bad** unsat?

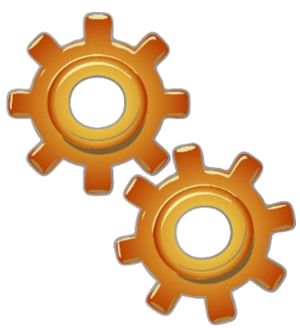
Cons.: $\boldsymbol{I} \wedge \boldsymbol{\delta} \wedge \neg \boldsymbol{I'}$ unsat?

* $I' = I[V \mapsto V']$

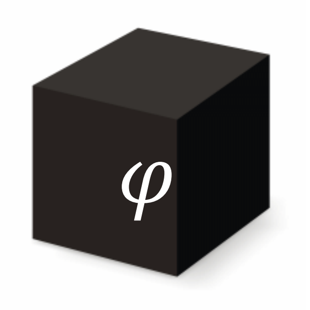
Exact Concept Learning with Equivalence & Membership Queries

Goal: learn an unknown concept φ

learning algorithm



oracle



is it ψ_1?

✓ / ✗+counterexample

is it ψ_2?

✓ / ✗+counterexample

does $\sigma_3 \vDash$?

✓ / ✗

…

Membership Equivalence

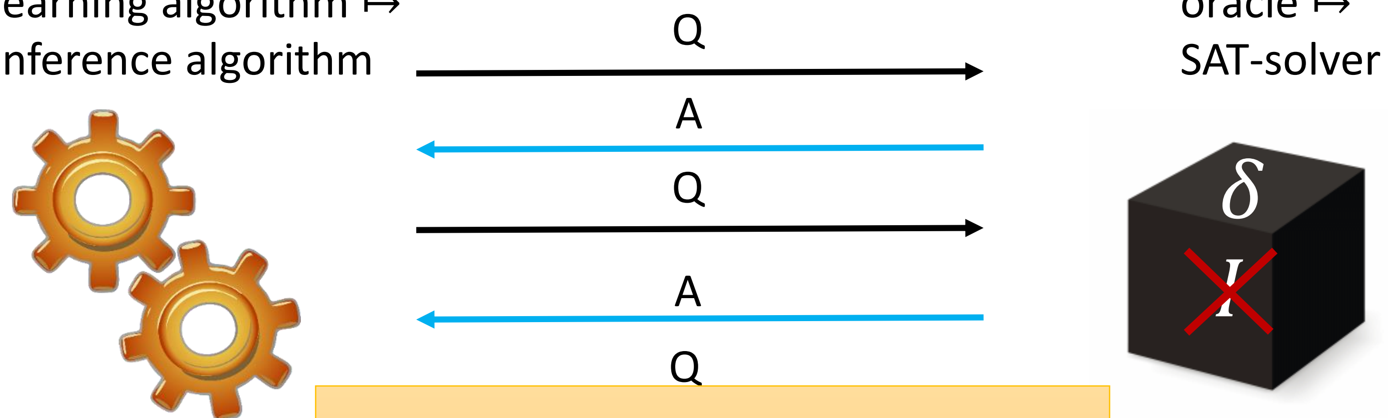
[ML'87] Queries and Concept Learning. Angluin

SAT-Based Invariant Inference as Inference with Queries

Goal: infer an unknown inductive invariant I

learning algorithm ↦ inference algorithm

oracle ↦ SAT-solver



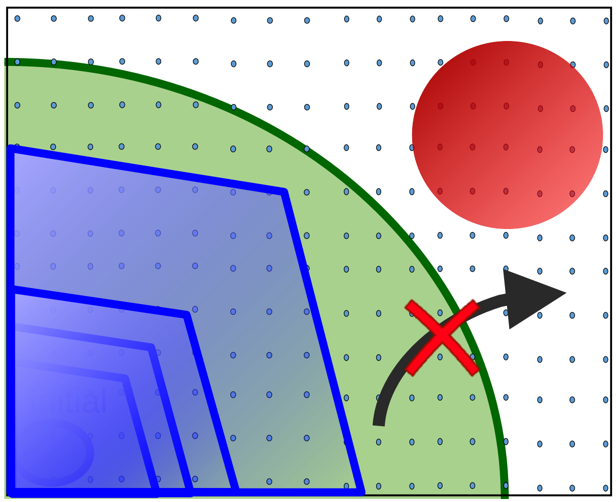
Which SAT queries?

Algorithms cannot access the transition relation directly, only through SAT queries

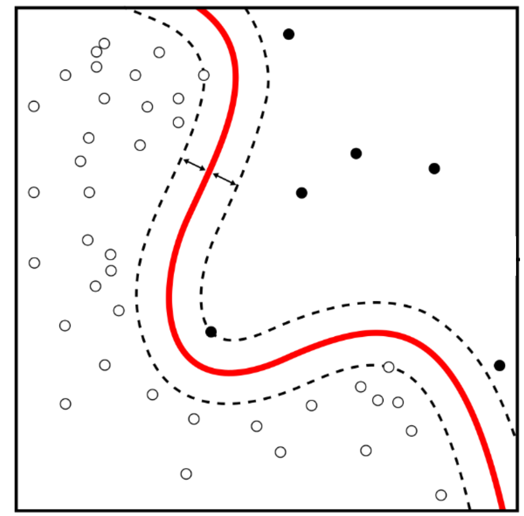
This Talk

Invariant Inference

Exact Concept Learning



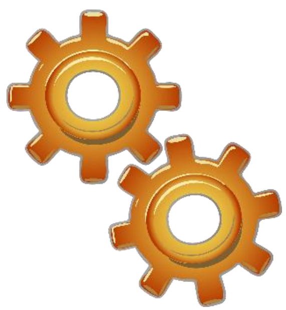
vs.



- Query-based learning models for invariant inference
- Complexity lower and upper bounds for each model
- Invariant inference is harder than concept learning
- Complexity results for invariant inference algorithms from concept learning algorithms

Inductiveness-Query Model

inference algorithm



α_1 inductive?

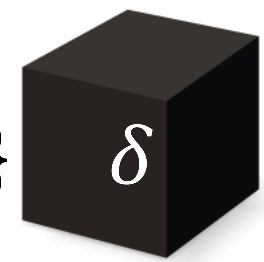
✓ / ✗+counterexample

…

α_m inductive?

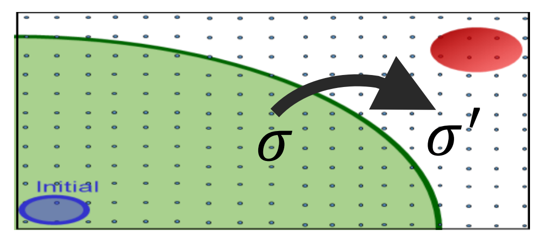
✓ / ✗+counterexample

inductiveness-query oracle

$\{\alpha_i\}$  $\{\alpha_i\}$

$\boldsymbol{\alpha_i \wedge \delta \wedge \neg\alpha_i'}$ unsat?

Cex to Induction (CTI):
Transition (σ, σ') of δ s.t.
$\sigma \vDash \alpha_i, \quad \sigma' \vDash \neg\alpha_i$



ICE framework - Learn from examples:

Positive : $\sigma \vDash I$ (e.g., initial)

Negative: $\sigma \nvDash I$ (e.g., bad)

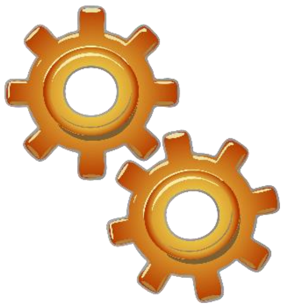
Implication: $\sigma \vDash I$ implies $\sigma' \vDash I$ (CTI)

* $\alpha_i' = \alpha_i[V \mapsto V']$

[CAV'14] ICE: A Robust Framework for Learning Invariants. Garg, Löding, Madhusudan, Neider

Inductiveness-Query Model

inference algorithm



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…

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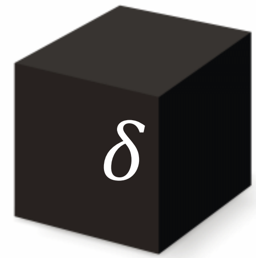
inductiveness-query oracle

$\{\alpha_i\}$  $\{\alpha_i\}$

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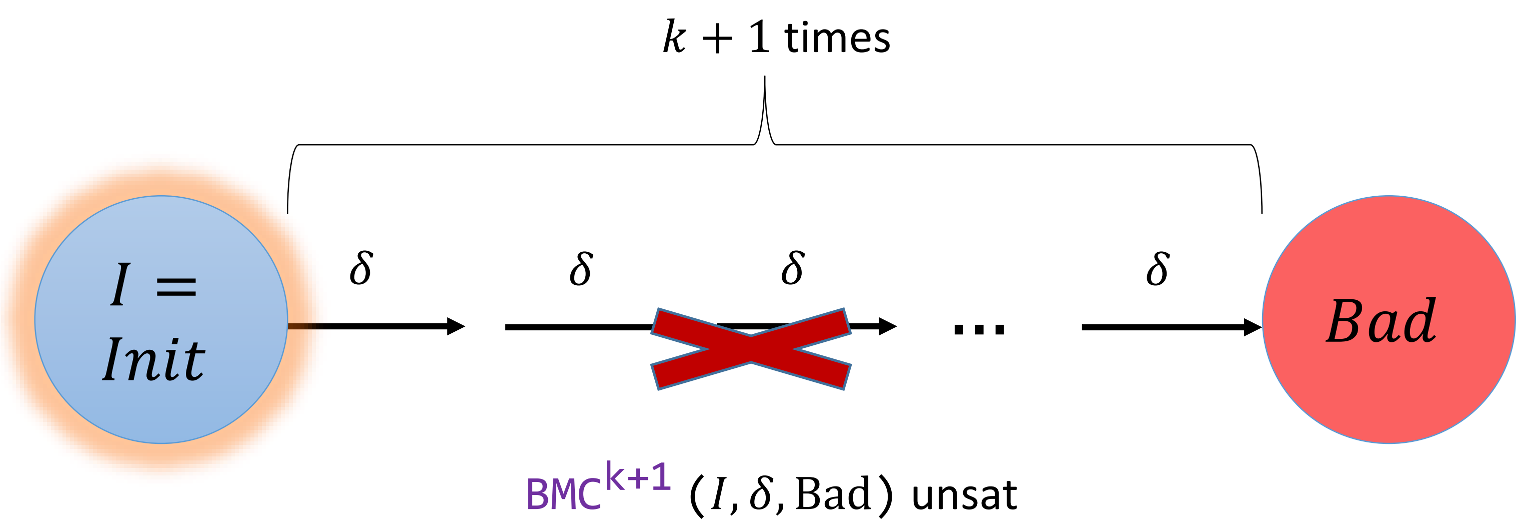
Is it sufficient to capture existing SAT-based algorithms?

* α_i'=$\alpha_i[V \mapsto V']$

[CAV'14] ICE: A Robust Framework for Learning Invariants. Garg, Löding, Madhusudan, Neider

Interpolation-Based Inference

$I = Init$



[CAV'03] Interpolation and SAT-Based Model Checking, McMillan

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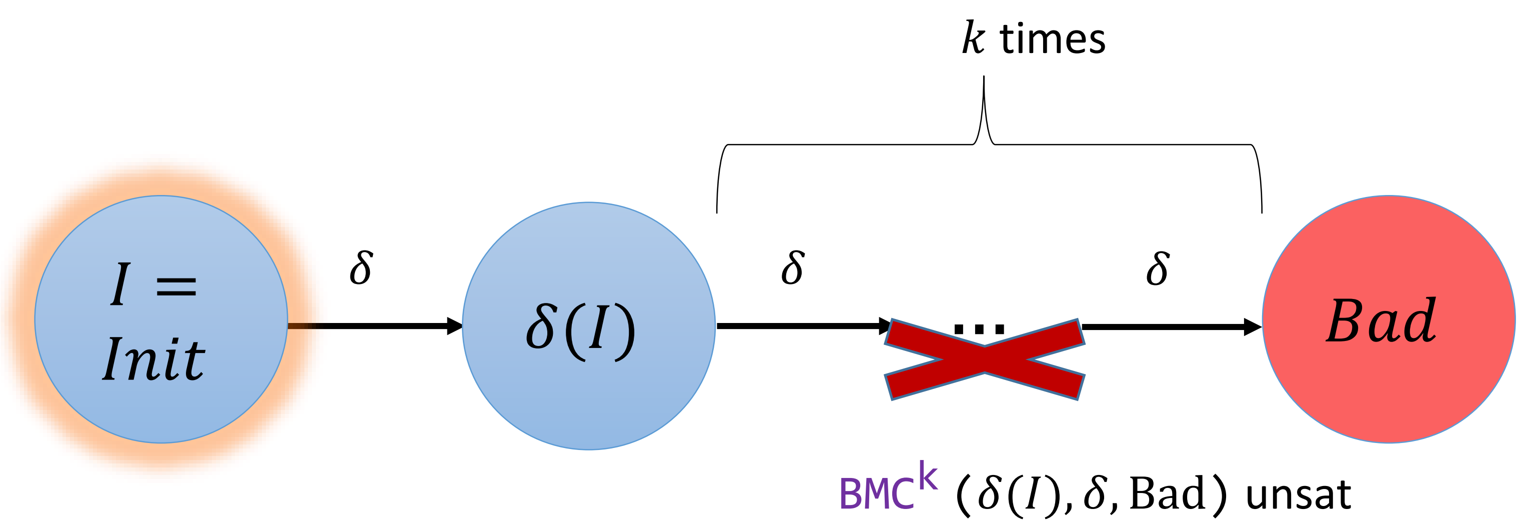

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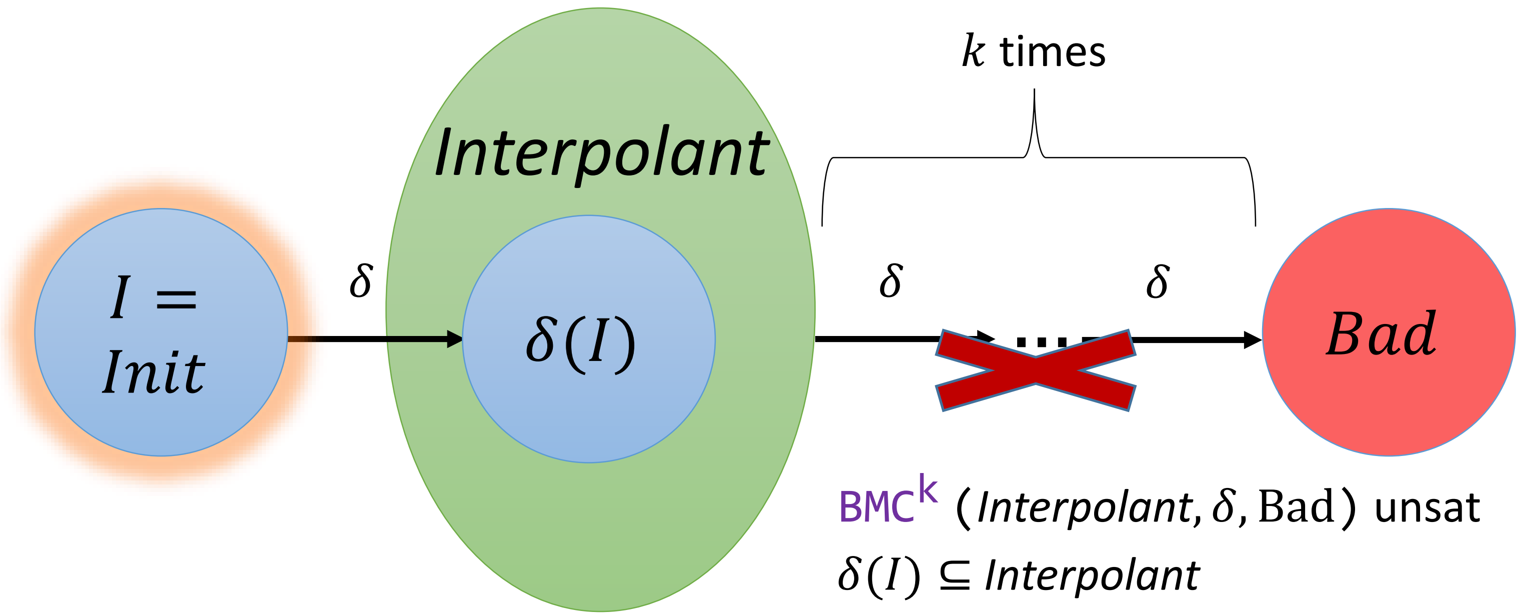


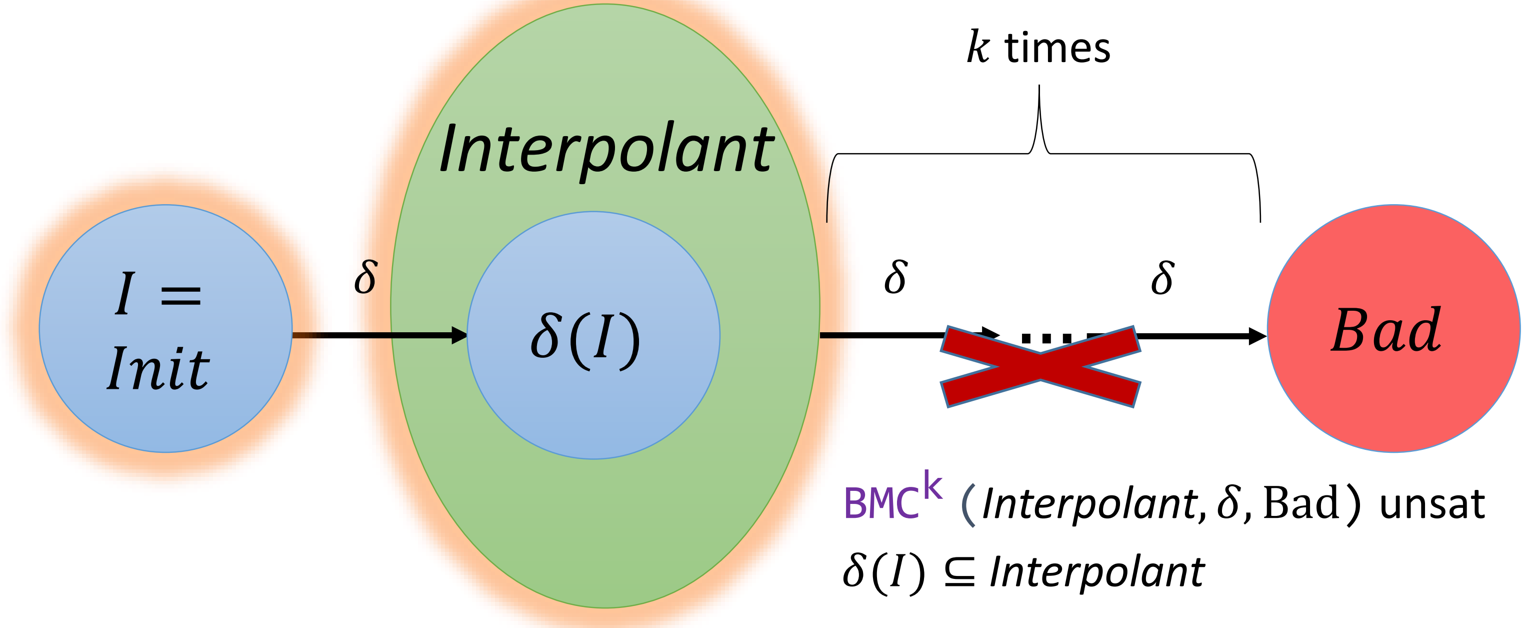
BMC^k $(Interpolant, \delta, \mathrm{Bad})$ unsat

$\delta(I) \subseteq Interpolant$

[CAV'03] Interpolation and SAT-Based Model Checking, McMillan

Interpolation-Based Inference

$$I = Init \lor Interpolant$$



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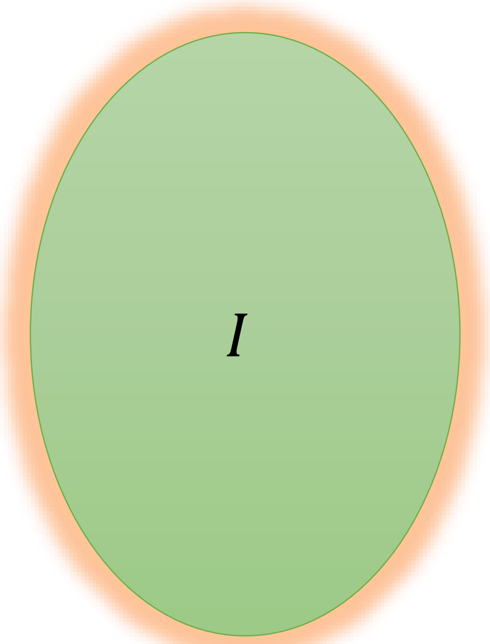
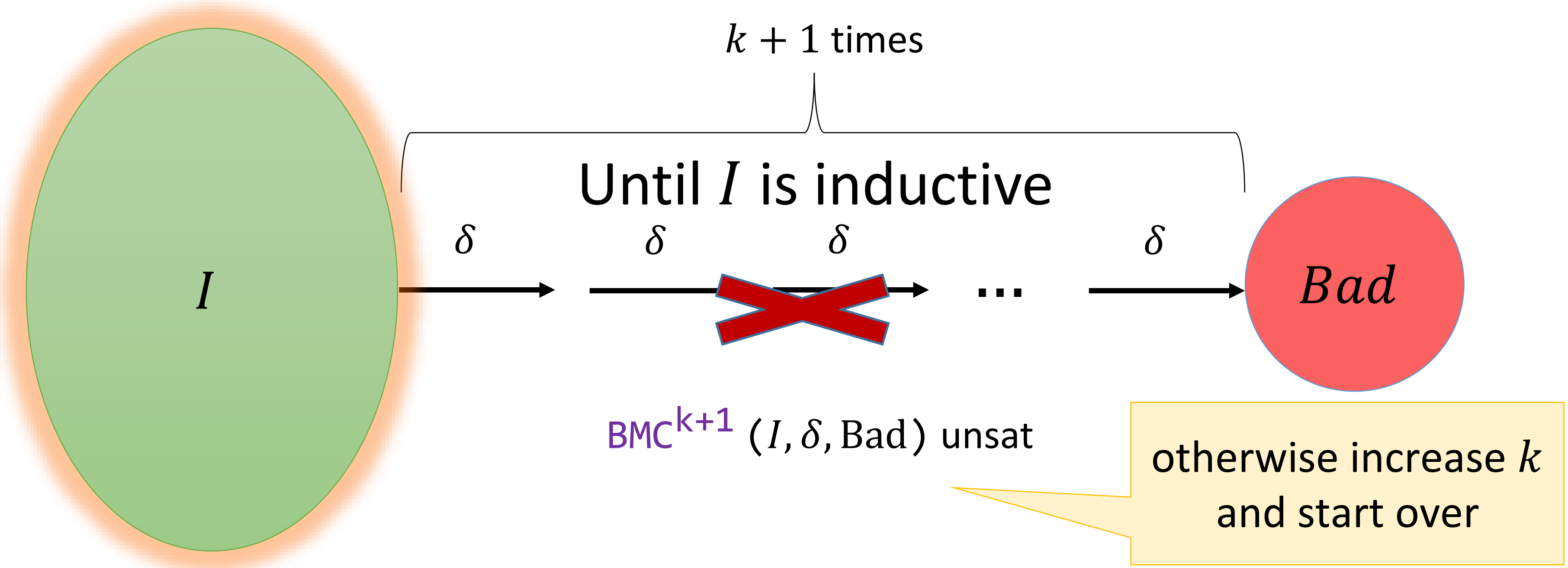


Inductive ?

[CAV'03] Interpolation and SAT-Based Model Checking, McMillan

Interpolation-Based Inference

$$I = Init \lor Interpolant \lor Interpolant_2 \lor \ \dots$$



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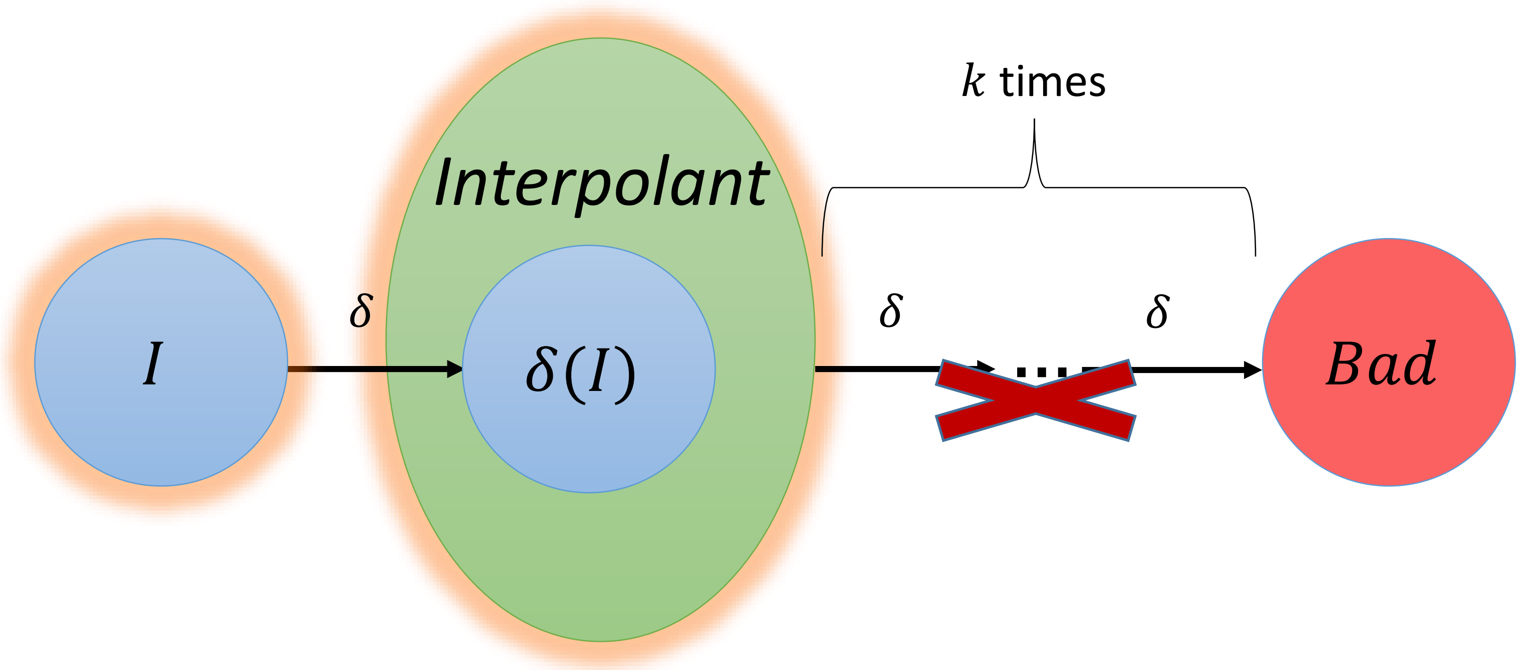
Computing an Interpolant



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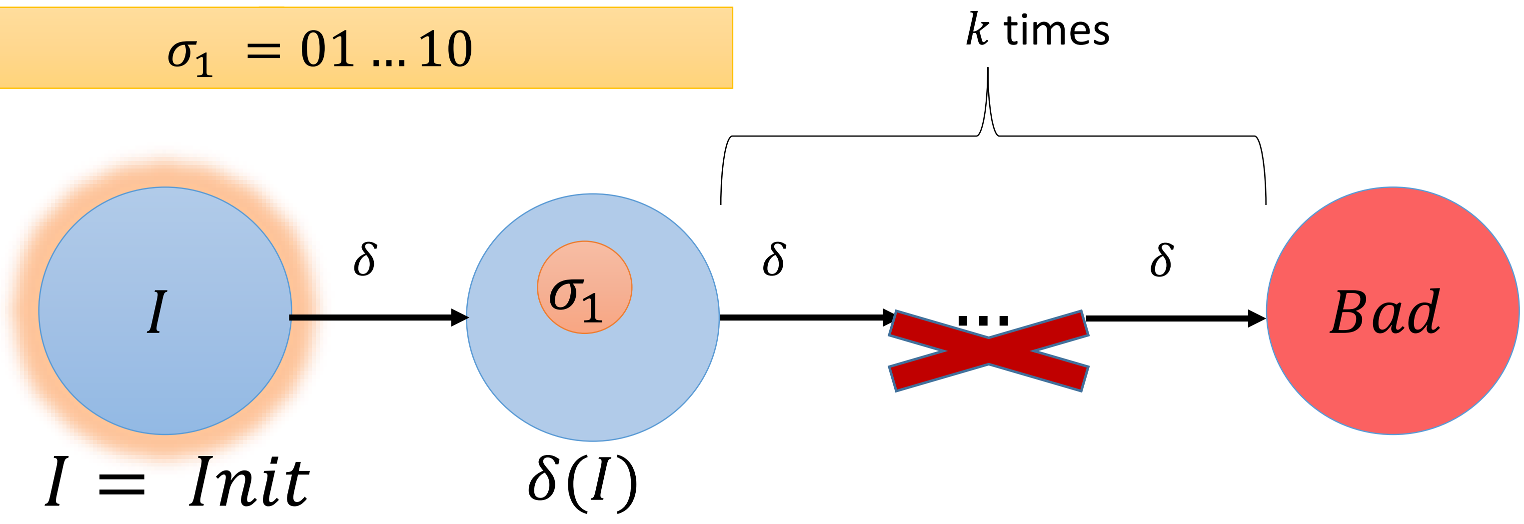
$Interpolant_1 = (x_1 = 0 \land x_2 = 1 \land \cdots \land x_{n-1} = 1 \land x_n = 0)$

$\sigma_1 = 01 \dots 10$



[HVC'12] Computing Interpolants without Proofs. Chockler, Ivrii, Matsliah
[LPAR'13] Instantiations, Zippers and EPR Interpolation. Bjørner, Gurfinkel, Korovin, Lahav

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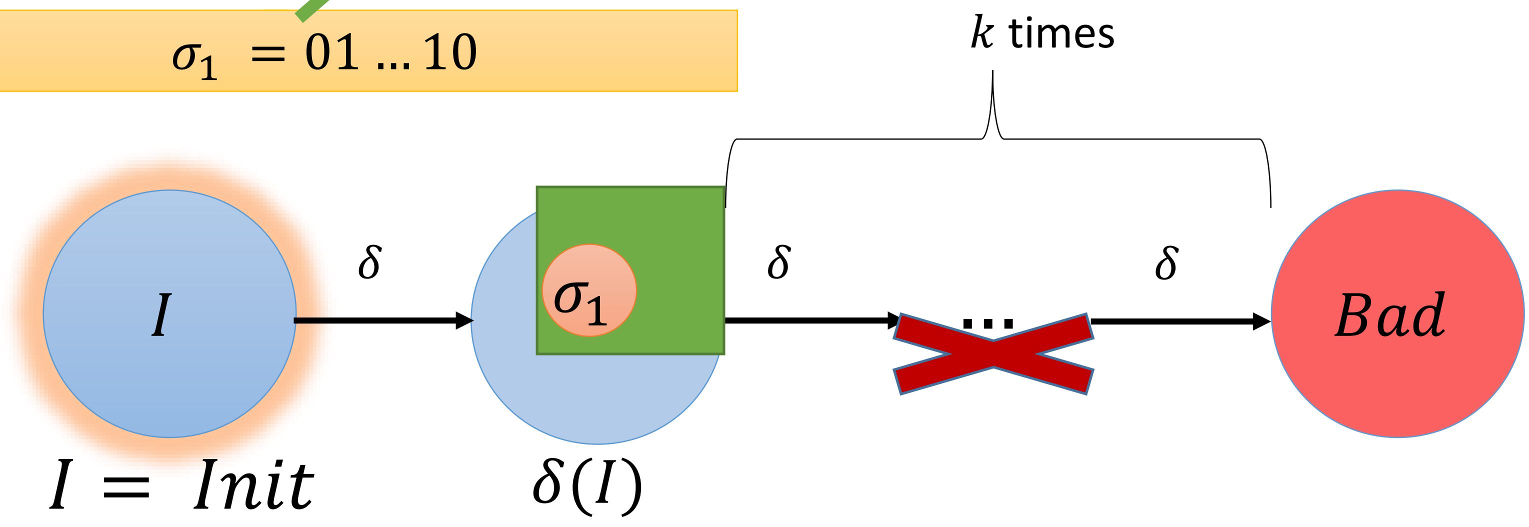

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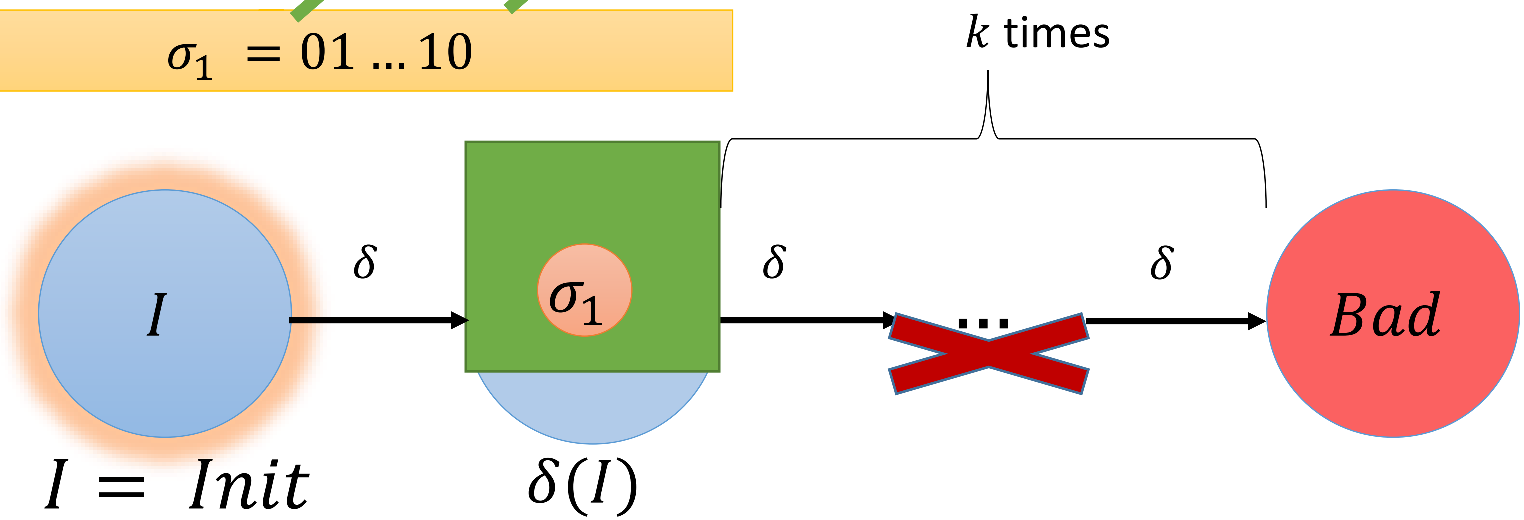

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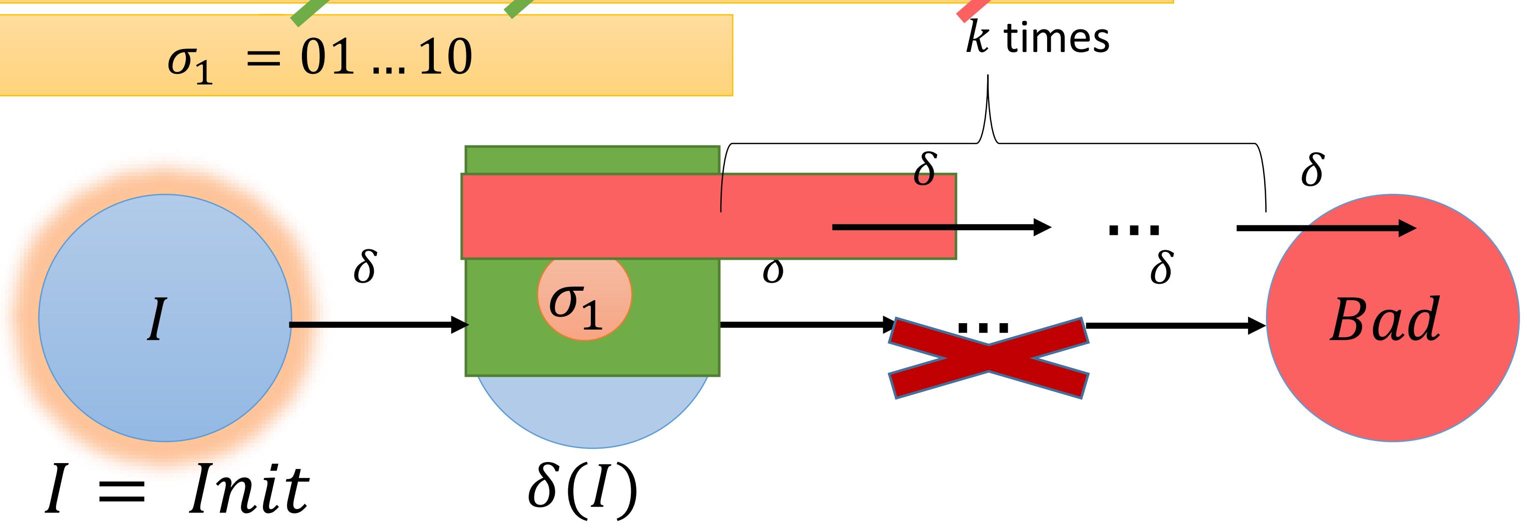

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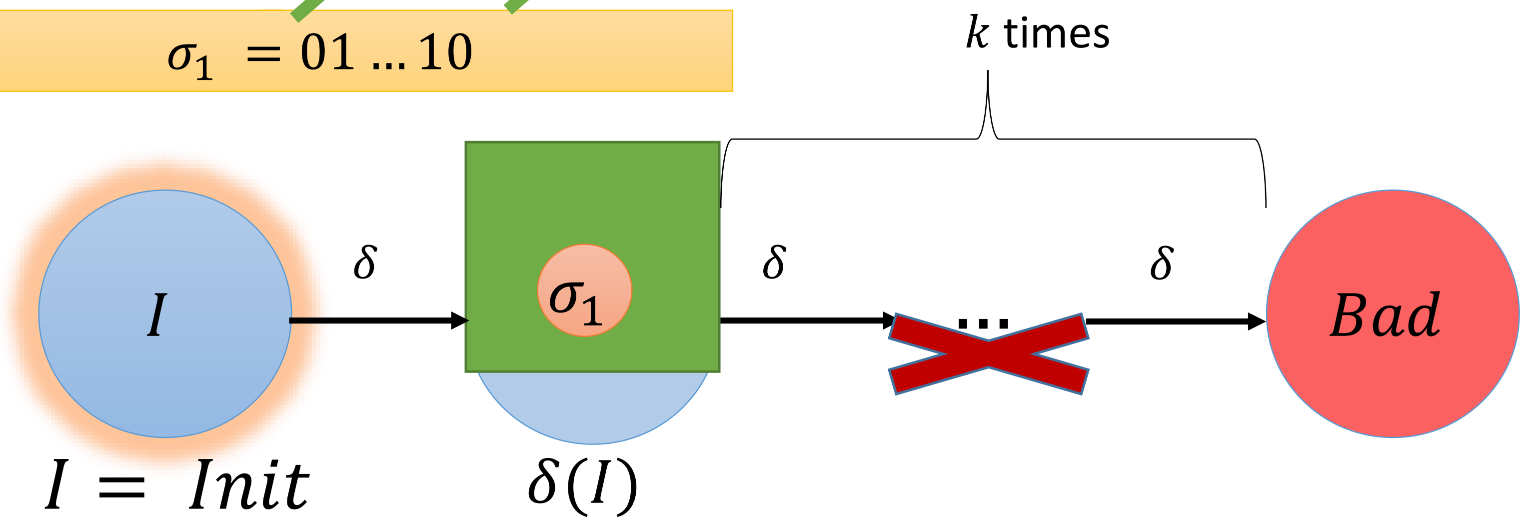

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Init:
$(x_1, \ldots, x_n) := 0 \ldots 0$

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δ:
$y_1, \ldots, y_n := *$
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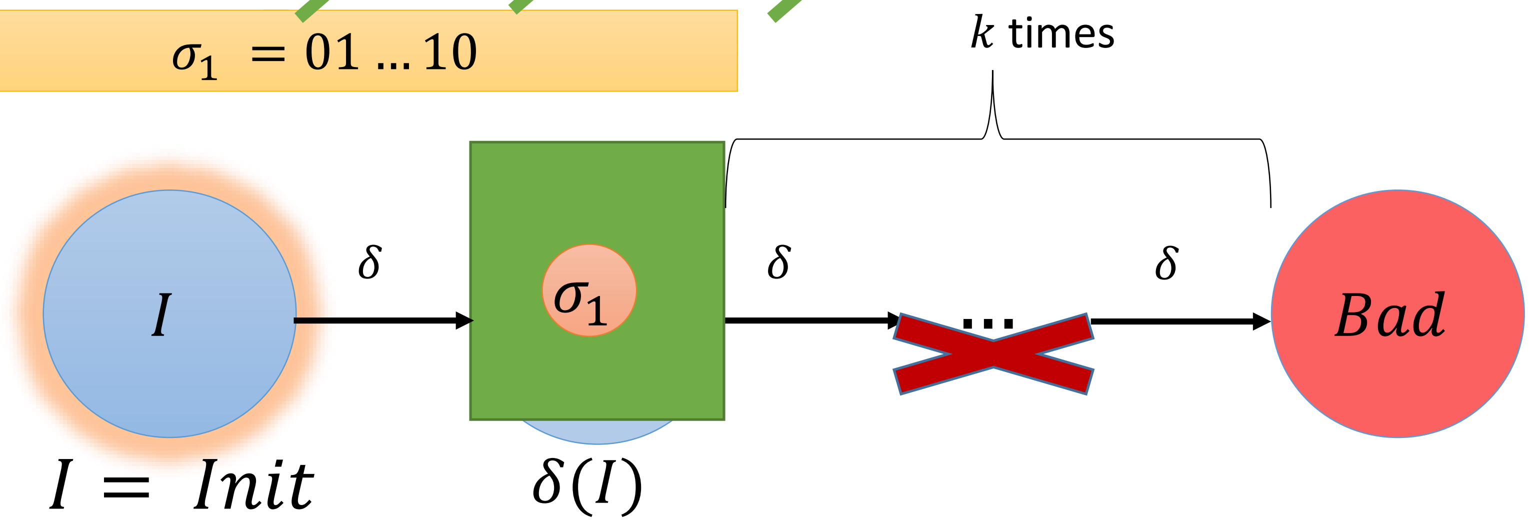

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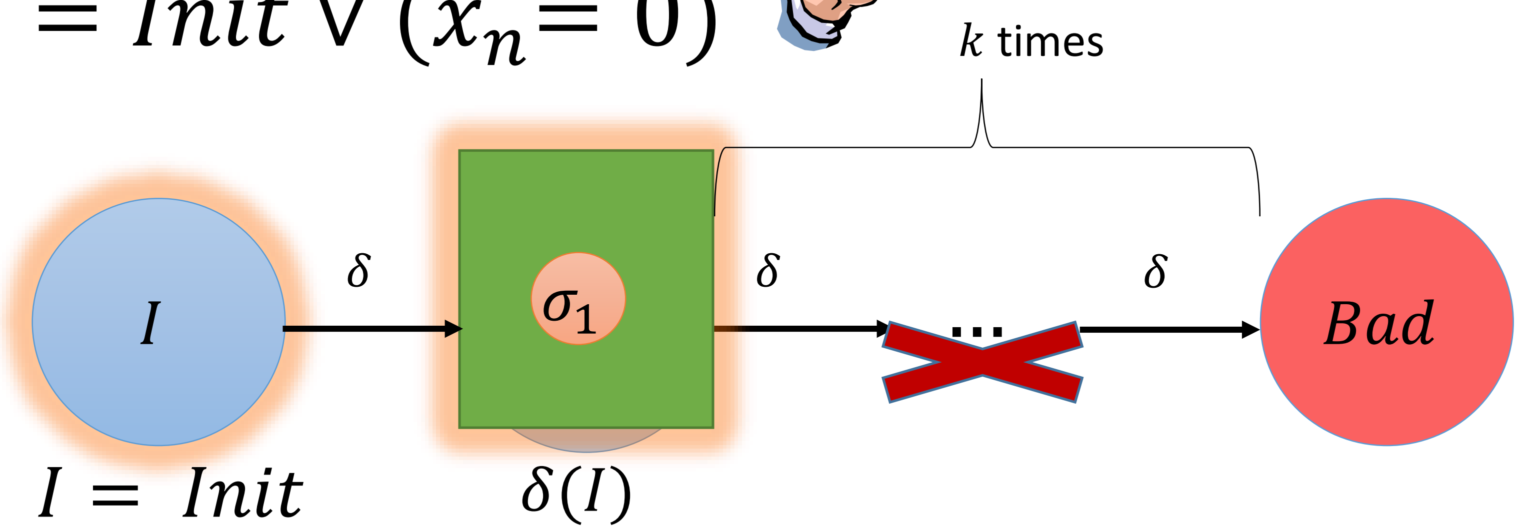
$$I = Init \vee (x_n = 0)$$



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Model-Based Interpolation

Inferring invariant in DNF:

$$\underbrace{\left(\ell_1^1 \wedge \cdots \wedge \ell_{k_1}^1\right)}_{\text{gen}(\sigma_1)} \vee \;\ldots\; \vee \underbrace{\left(\ell_1^m \wedge \cdots \wedge \ell_{k_m}^m\right)}_{\text{gen}(\sigma_m)}$$

ITP-k:

$I \ := \ \text{false}$

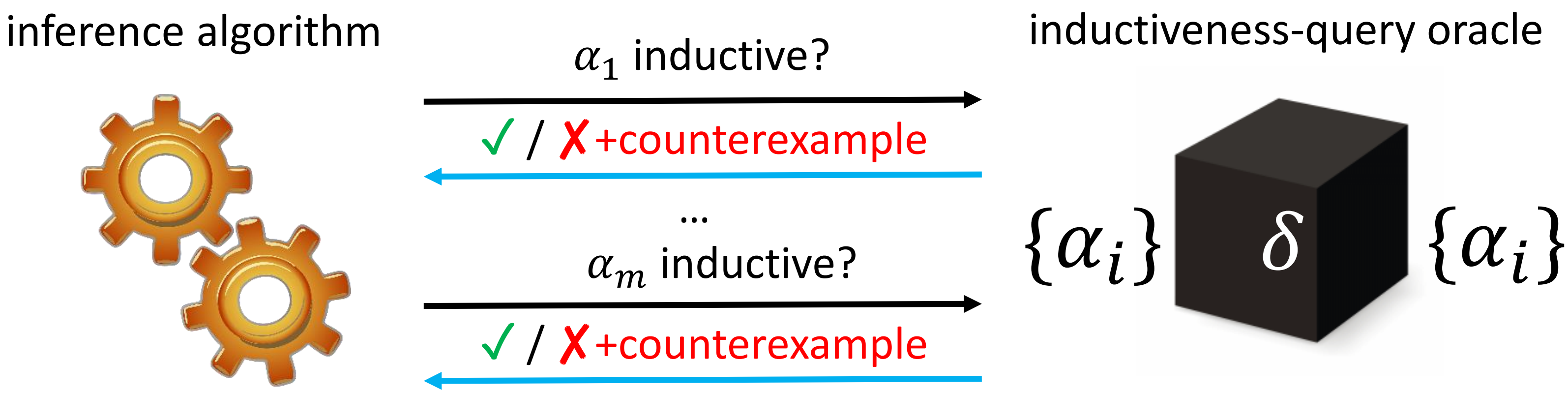
`while` $(\_, \sigma')$ `counterexample`
`to Inductive`(δ, I)`:`

$I \ := \ I \vee \textbf{generalize}(\sigma')$

$\textbf{generalize}(\sigma')$:

drop literals from σ'
while BMC$^k(\sigma', \delta, \text{Bad})$ unsat

Inductiveness-Query Model



I := false

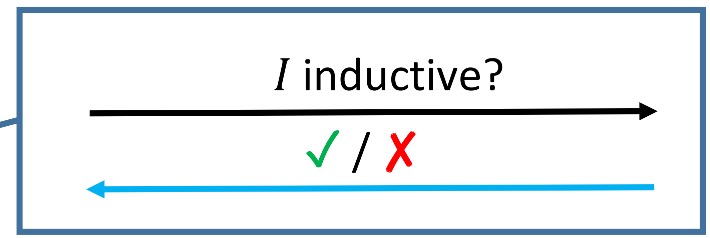
while $(\_, \sigma')$ counterexample to Inductive(δ, I):

  I := $I \lor$ **generalize**(σ')

generalize(σ'):

  drop literals from σ'

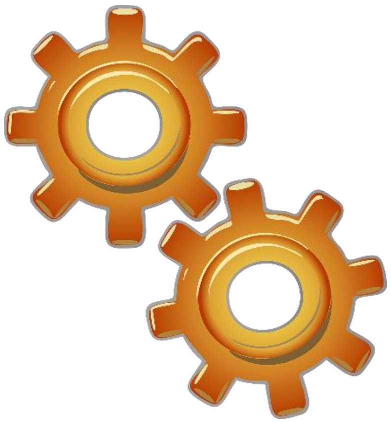
  while BMC$^k(\sigma', \delta, \text{Bad})$ unsat



?

Hoare-Query Model

inference algorithm



Hoare-query oracle

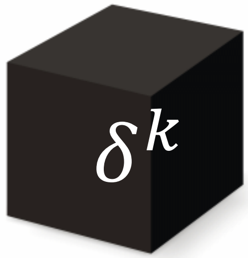
$\{\alpha_1\}\delta^k\{\beta_1\}?$

✓ / ✗ +counterexample

…

$\{\alpha_m\}\delta^k\{\beta_m\}?$

✓ / ✗ +counterexample

$\{\alpha_i\}$ 

$\{\beta_i\}$

$\text{BMC}^k(\alpha_i, \delta, \neg\beta_i)$ unsat?

Trace $(\sigma_0, \dots, \sigma_k)$ of δ s.t.
$\sigma_0 \vDash \alpha_i, \quad \sigma_k \vDash \neg\beta_i$

Capable of modeling several interesting algorithms

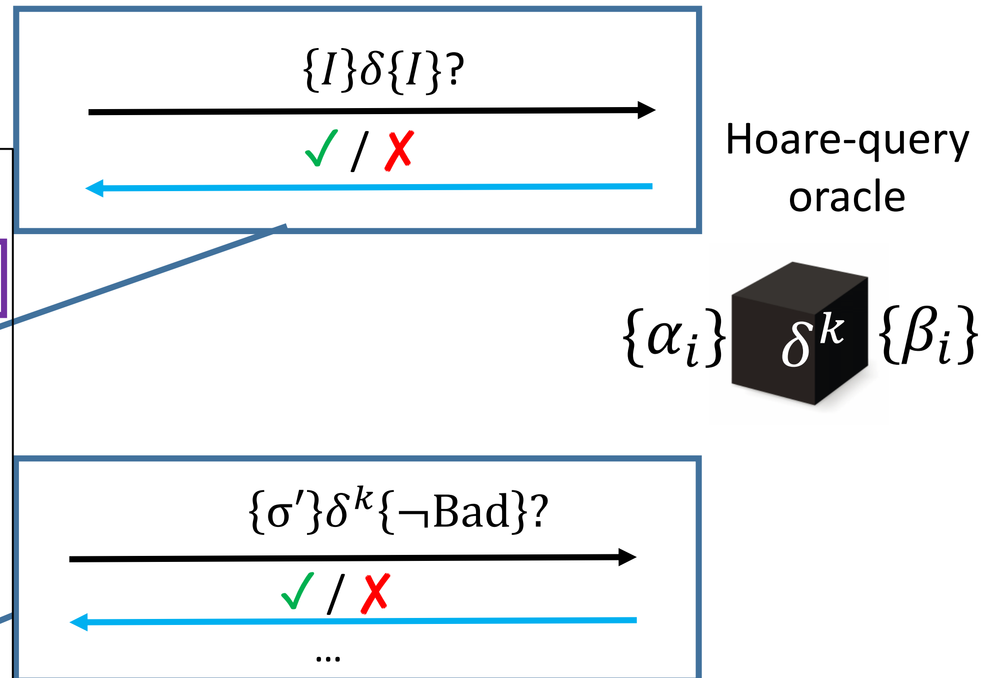
Hoare-Query Model

ITP-k:

I := false

while $(\_, \sigma')$ counterexample
    to Inductive(δ, I):
    I := $I \lor$ **generalize**(σ')

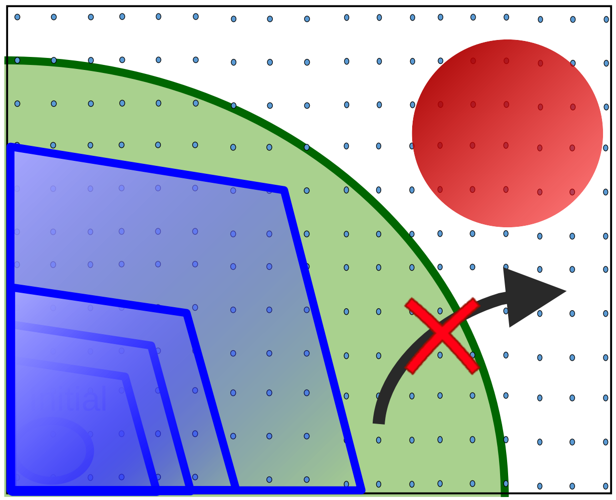
generalize(σ'):
    drop literals from σ'
    while BMC$^k(\sigma', \delta, \text{Bad})$ unsat



Also captures IC3/PDR

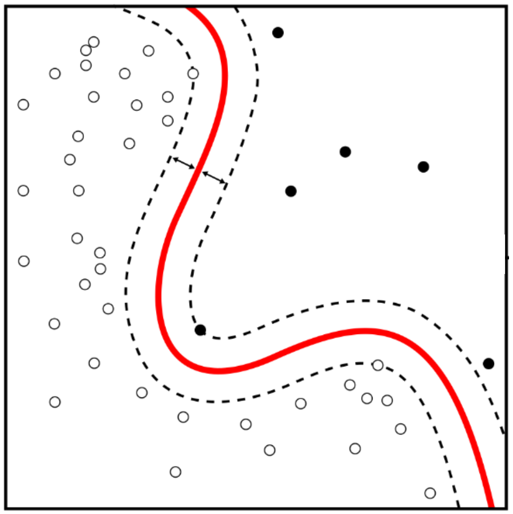
Outline

Invariant Inference



vs.

Exact Concept Learning



- Query-based learning models for invariant inference
- Complexity lower and upper bounds for each model
- Invariant inference is harder than concept learning
- Complexity results for invariant inference algorithms from concept learning algorithms

Hoare-Query Complexity

Thm: Every Hoare-query algorithm requires $\mathbf{2^{\Omega(n)}}$ queries in the worst case for inferring $I \in$ DNF s.t. $|I| \leq \mathrm{poly}(n)$

n is the vocabulary size, $k = \mathrm{poly}(n)$

Throughout the talk

- even with unlimited computational power
- unconditional lower bound

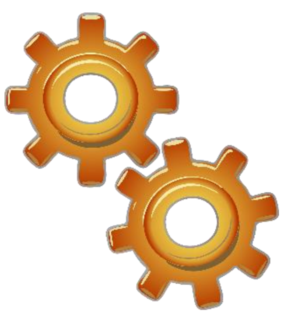
[POPL'20] Complexity and Information in Invariant Inference. Feldman, Immerman, Shoham, Sagiv

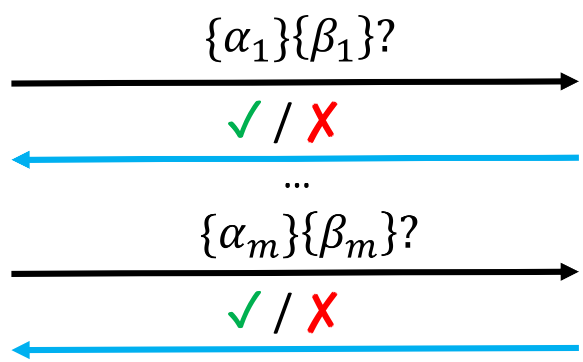
Hoare-Query Complexity

<u>Thm</u>: Every Hoare-query algorithm requires $\mathbf{2^{\Omega(n)}}$ queries in the worst case for inferring $I \in$ DNF s.t. $|I| \leq \mathrm{poly}(n)$

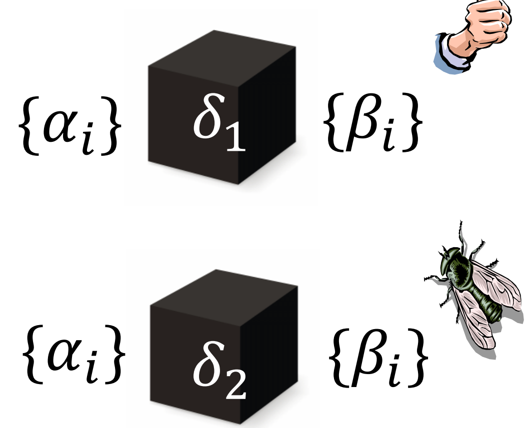
<u>Proof sketch:</u>

given
inference algorithm
with <$\mathbf{2^{poly(n)}}$queries





construct δ_1, δ_2 for
Hoare-query oracle



1. δ_1 has an inductive invariants with at most n cubes
2. δ_2 does not (in fact, unsafe)
3. all queries return the same answer for δ_1, δ_2

Hoare-Query Complexity

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Proof sketch:

given **construct δ_1, δ_2 for**

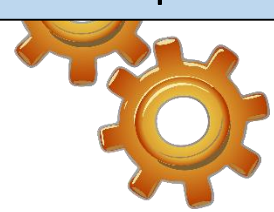
infe

wit

take δs checking validity of Boolean quantified formulas

$\exists x_1, \dots, x_n.\ \forall y_1, \dots, y_n.\ \phi(x_1, \dots, x_n, y_1, \dots, y_n)$

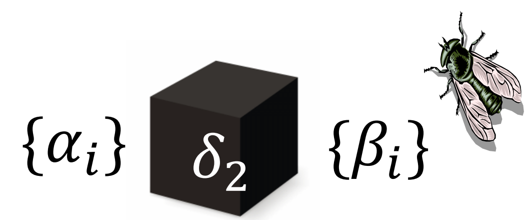
a sub-exponential number of valuations do not determine validity!



…

$\{\alpha_m\}\{\beta_m\}?$

✓ / ✗



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Hoare-Query Complexity

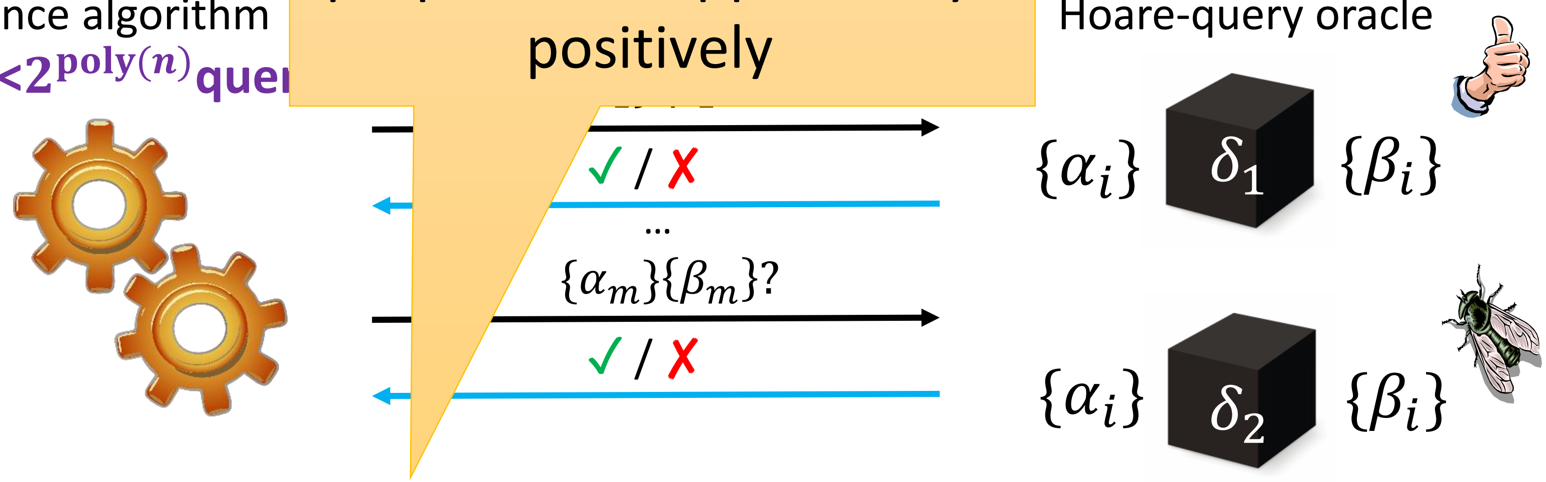
Thm: Every Hoare-query algorithm requires $2^{\Omega(n)}$ queries in the worst case for inferring $I \in$ DNF s.t. $|I| \leq \mathrm{poly}(n)$

Proof sketch:

given inference algorithm **with $<2^{\mathrm{poly}(n)}$ quer**

I is monotone: propositions appear only positively

construct δ_1, δ_2 for Hoare-query oracle



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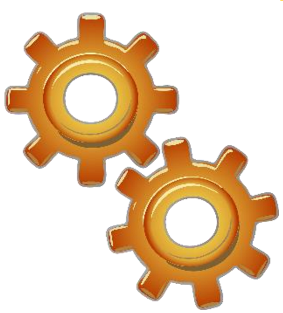

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✓ / ✗

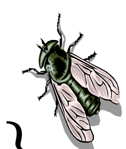
…

$\{\alpha_m\}\{\beta_m\}$?

✓ / ✗

construct δ_1, δ_2 for Hoare-query oracle

$\{\alpha_i\}$ δ_1 $\{\beta_i\}$



$\{\alpha_i\}$ δ_2 $\{\beta_i\}$

Cor: Every Hoare-query algorithm requires $\mathbf{2^{\Omega(n)}}$ queries in the worst case for inferring short monotone DNF invariants

Hoare > Inductiveness

Thm: There exists a class of transition systems $\mathcal{P}$, so that for solving inference:

1. ∃ Hoare-query algorithm (with k=1) with $\mathrm{poly}(n)$ queries
2. ∀ inductiveness-query algorithm requires $2^{\Omega(n)}$ queries

[POPL'20] Complexity and Information in Invariant Inference. Feldman, Immerman, Shoham, Sagiv

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1. ∃ Hoare-query algorithm (with k=1) with $\mathrm{poly}(n)$ queries
2. ∀ inductiveness-query algorithm requires $2^{\Omega(n)}$ queries

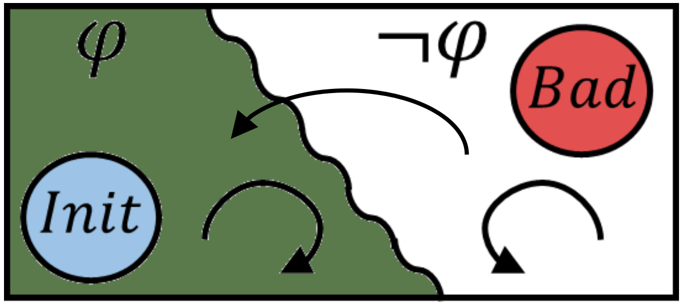
Proof:

$\mathcal{P}$ = maximal transition systems for monotone DNF with n cubes

propositions appear only positively

$$\varphi = x_1 \lor (x_2 \land x_3)$$

Maximal system for φ:



Hoare > Inductiveness

Upper bound:

∃ Hoare-query algorithm (with k=1) with $\mathrm{poly}(n)$ queries

Proof: **ITP-1** takes $\boldsymbol{O(n^2)}$ queries

```
I := false
while (_,σ') counterexample
        to Inductive(δ,I):
    I := I ∨ generalize(σ')

generalize(σ'):
    drop literals from σ'
    while BMC¹(σ',δ,Bad) unsat
```

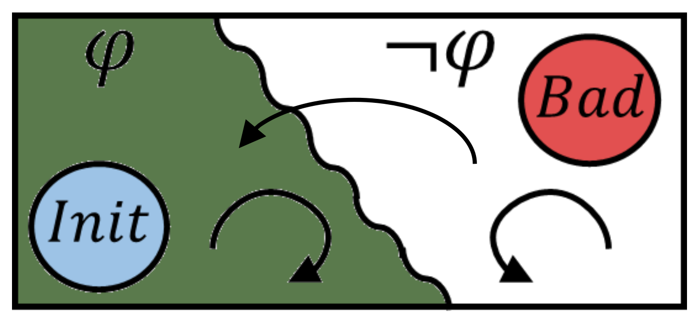
minimal

$\sigma' \Rightarrow \varphi$

φ is monotone

1 iteration 1 iteration

$$\varphi = x_1 \lor (x_2 \land x_3)$$

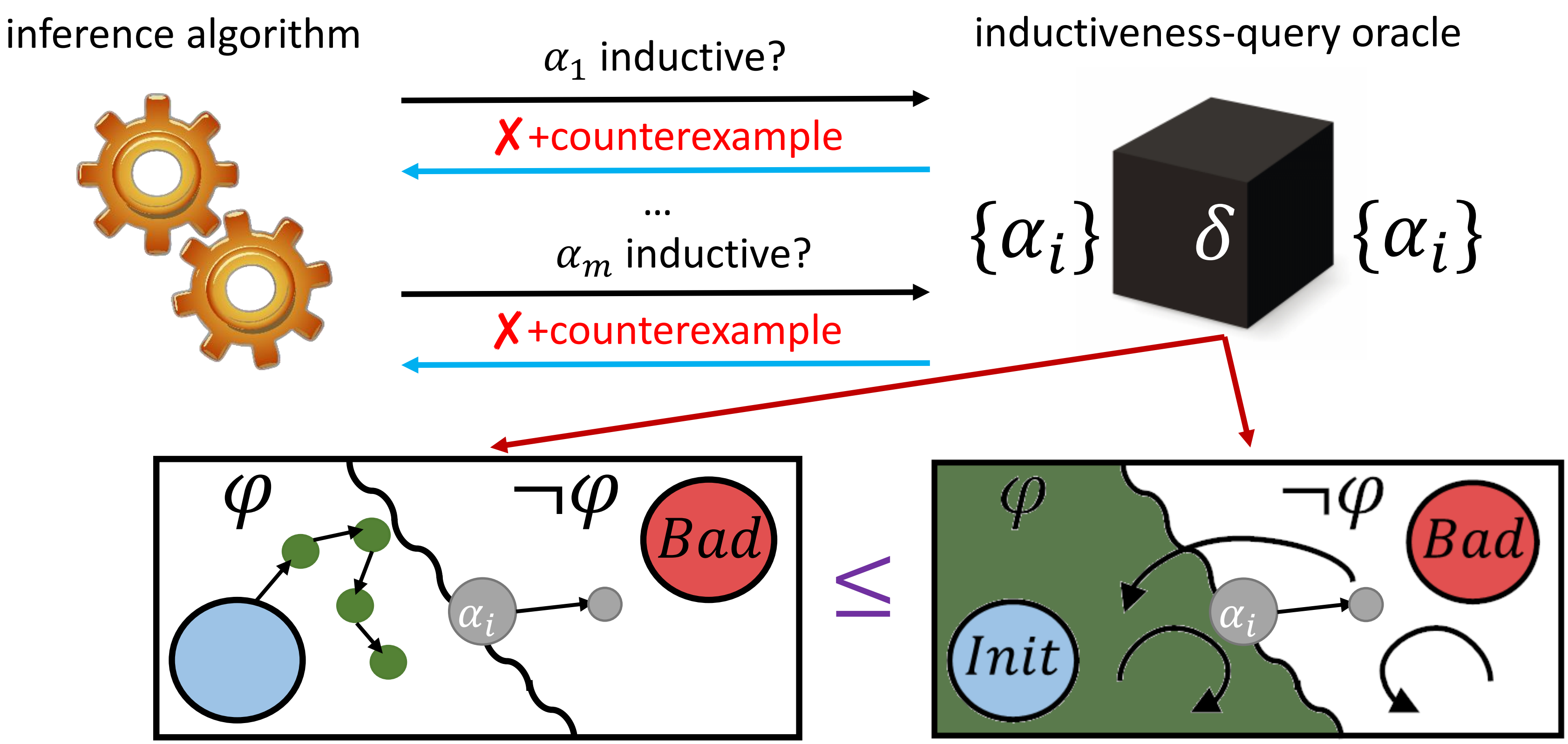


Hoare > Inductiveness

Lower bound:

∀ inductiveness-query algorithm requires $2^{\Omega(n)}$ queries

Proof:



Hoare > Inductiveness

Lower bound:

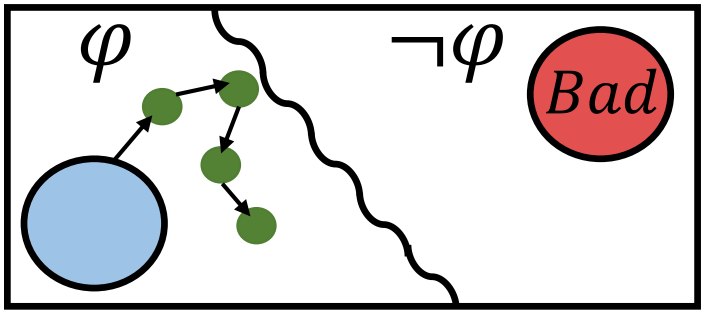
∀ inductiveness-query algorithm requires $2^{\Omega(n)}$ queries

Proof:

previous Corollary

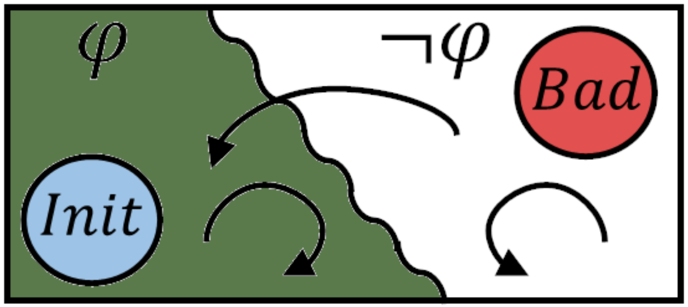
$2^{\Omega(n)} \leq$

general systems
monotone DNF invariants



$\leq$

maximal systems
monotone DNF invariants



Hoare > Inductiveness

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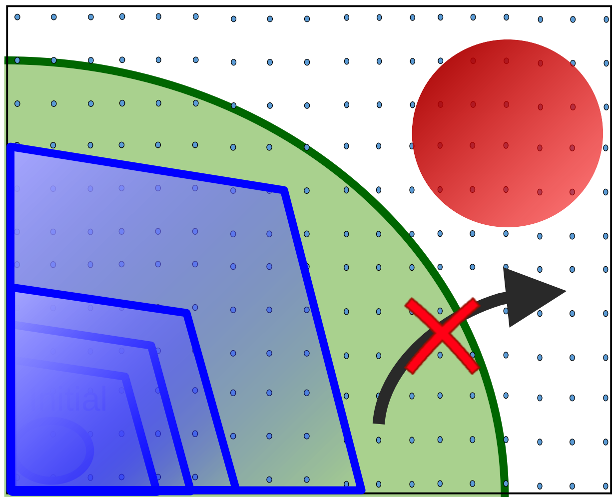
1. ∃ Hoare-query algorithm (with k=1) with $\mathrm{poly}(n)$ queries
2. ∀ inductiveness-query algorithm requires $2^{\Omega(n)}$ queries

Similar proof works with a simple case of IC3/PDR
⇒ ICE cannot model PDR,
and the extension of [VMCAI'17] is necessary

[POPL'20] Complexity and Information in Invariant Inference. Feldman, Immerman, Shoham, Sagiv
[VMCAI'17] IC3 - Flipping the E in ICE. Vizel, Gurfinkel, Shoham, Malik.

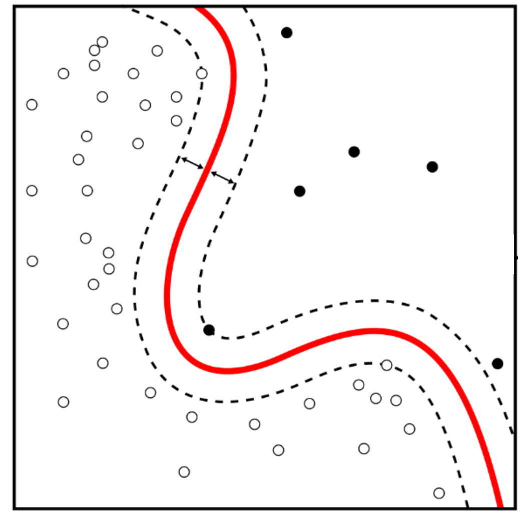
Outline

Invariant Inference



vs.

Exact Concept Learning



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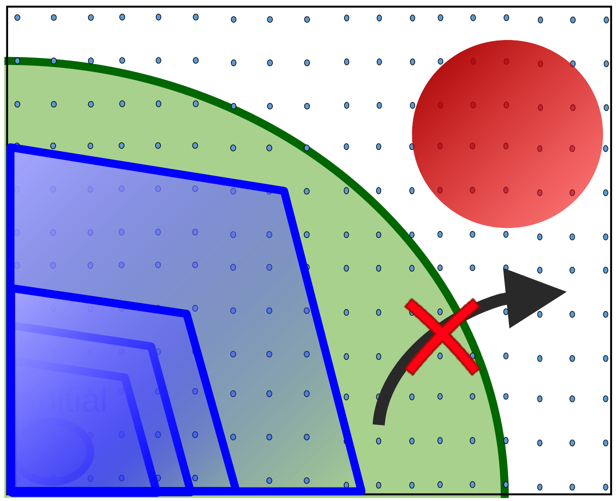
Inferring Monotone DNF

Invariant Inference



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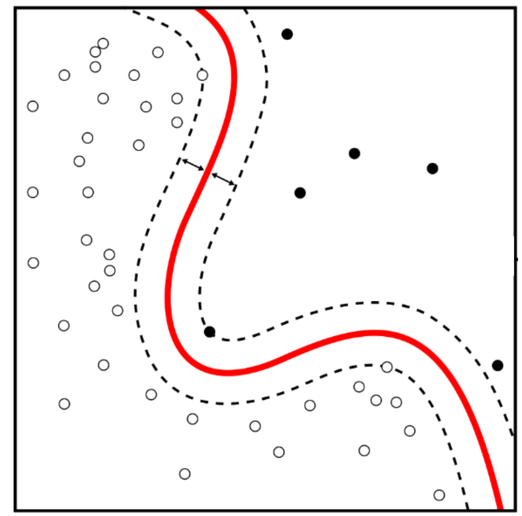


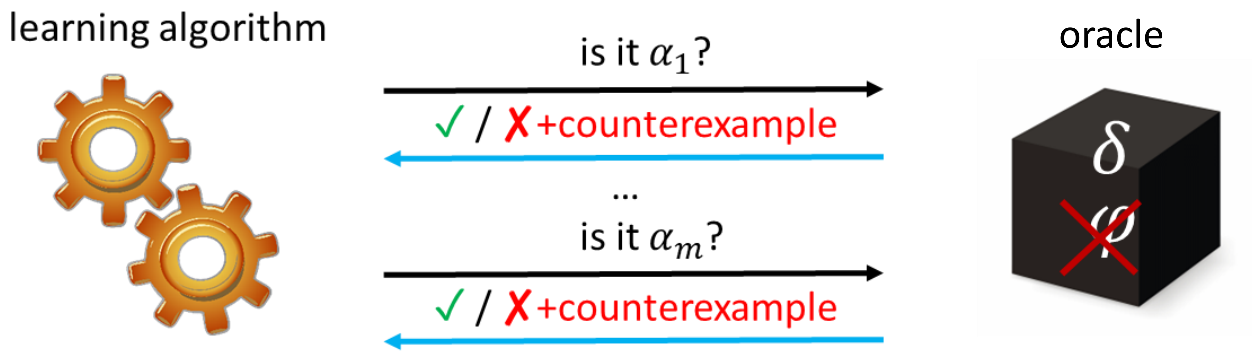
	Maximal	General
Inductive	$2^{\Omega(n)}$	$2^{\Omega(n)}$
Hoare	poly	$2^{\Omega(n)}$

Equiv	sub-exponential
Equiv + mem	poly

[ML’87] Queries and Concept Learning, Angluin
[COLT’12] Tight Bounds on Proper Equivalence Query Learning of DNF, Hellerstein et al.

Inductiveness vs. Equivalence Queries

Invariant Inference — Exact Concept Learning



Counterexamples to induction:

$\sigma \vDash \neg\varphi$ **or** $\sigma' \vDash \varphi$

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Inductive	$2^{\Omega(n)}$	$2^{\Omega(n)}$
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Positive/negative examples:

$\sigma^+ \vDash \varphi$, $\sigma^- \vDash \neg\varphi$

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Equiv + mem	poly

[ML'87] Queries and Concept Learning, Angluin

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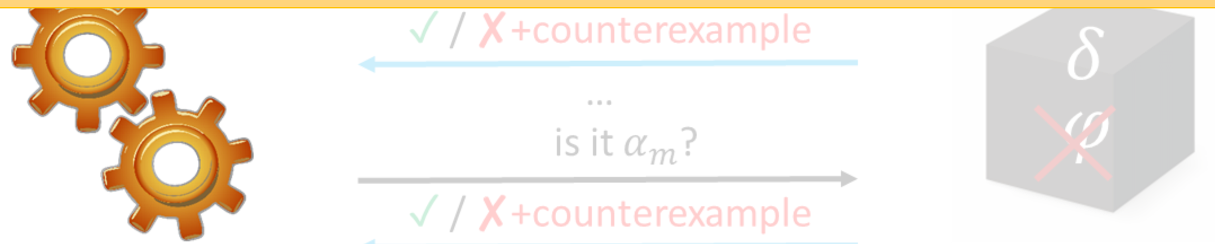
Inductiveness vs. Equivalence Queries

<u>Thm</u>: Learning from counterexamples to induction is **harder** than learning from positive/negative examples.



<u>Counterexamples to induction:</u>

$\sigma \models \neg\varphi$ **or** $\sigma' \models \varphi$

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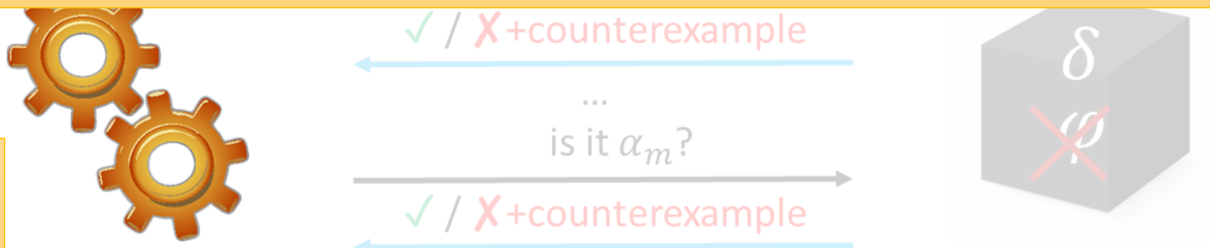
ICE



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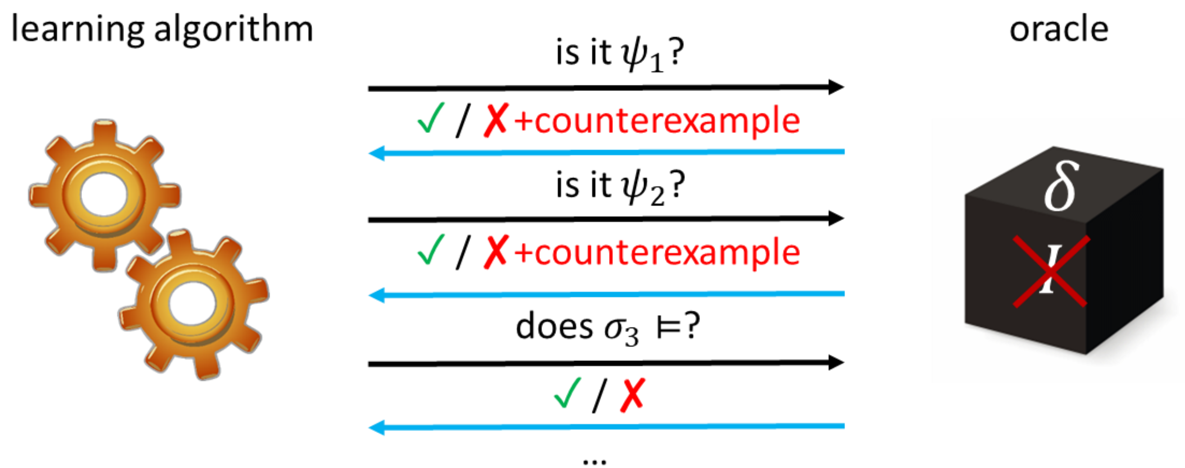
[ML'87] Queries and Concept Learning, Angluin
[COLT'12] Tight Bounds on Proper Equivalence Query Learning of DNF, Hellerstein et al.
[CAV'14] ICE: A Robust Framework for Learning Invariants. Garg, Löding, Madhusudan, Neider

Invariant Inference with Equivalence & Membership Queries



Invariant Inference

	Maximal	General
Inductive	$2^{\Omega(n)}$	$2^{\Omega(n)}$
Hoare	poly	$2^{\Omega(n)}$

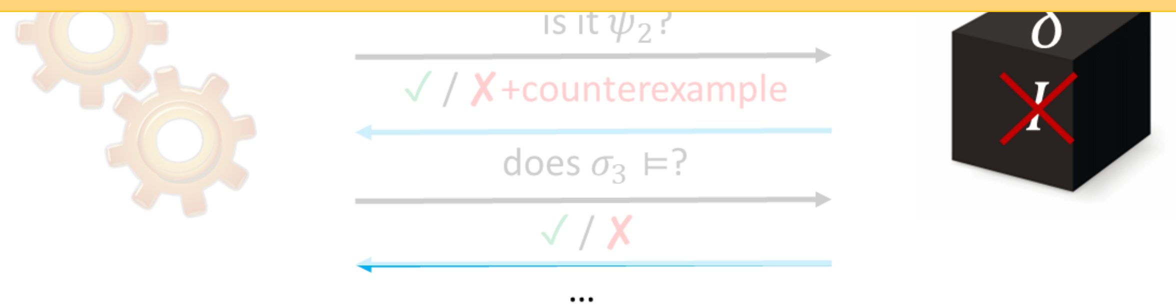
Exact Concept Learning

Equiv	sub-exponential
Equiv + mem	poly

[ML'87] Queries and Concept Learning, Angluin
[COLT'12] Tight Bounds on Proper Equivalence Query Learning of DNF, Hellerstein et al.

Invariant Inference with Equivalence & Membership Queries

Thm. In general, in the Hoare-query model, **no efficient way** to implement a teacher for equivalence and membership queries



Invariant Inference

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[POPL’20] Complexity and Information in Invariant Inference. Feldman, Immerman, Shoham, Sagiv

Invariant Inference with Equivalence & Membership Queries

Thm. In general, in the Hoare-query model, **no efficient way** to implement a teacher for equivalence and membership queries

Sufficient conditions for

exact learning algorithms $\Longrightarrow$ invariant inference algorithms

	Maximal	General
Inductive	$2^{\Omega(n)}$	$2^{\Omega(n)}$
Hoare	poly	$2^{\Omega(n)}$

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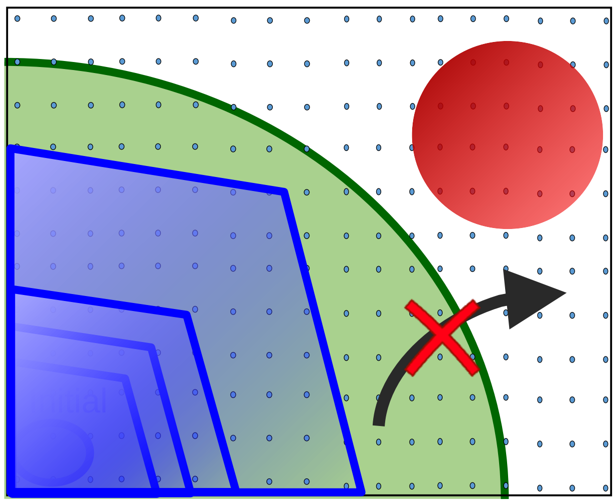
Outline

Invariant Inference



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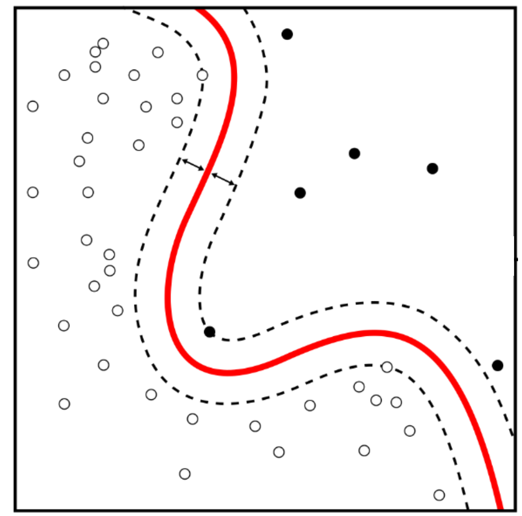

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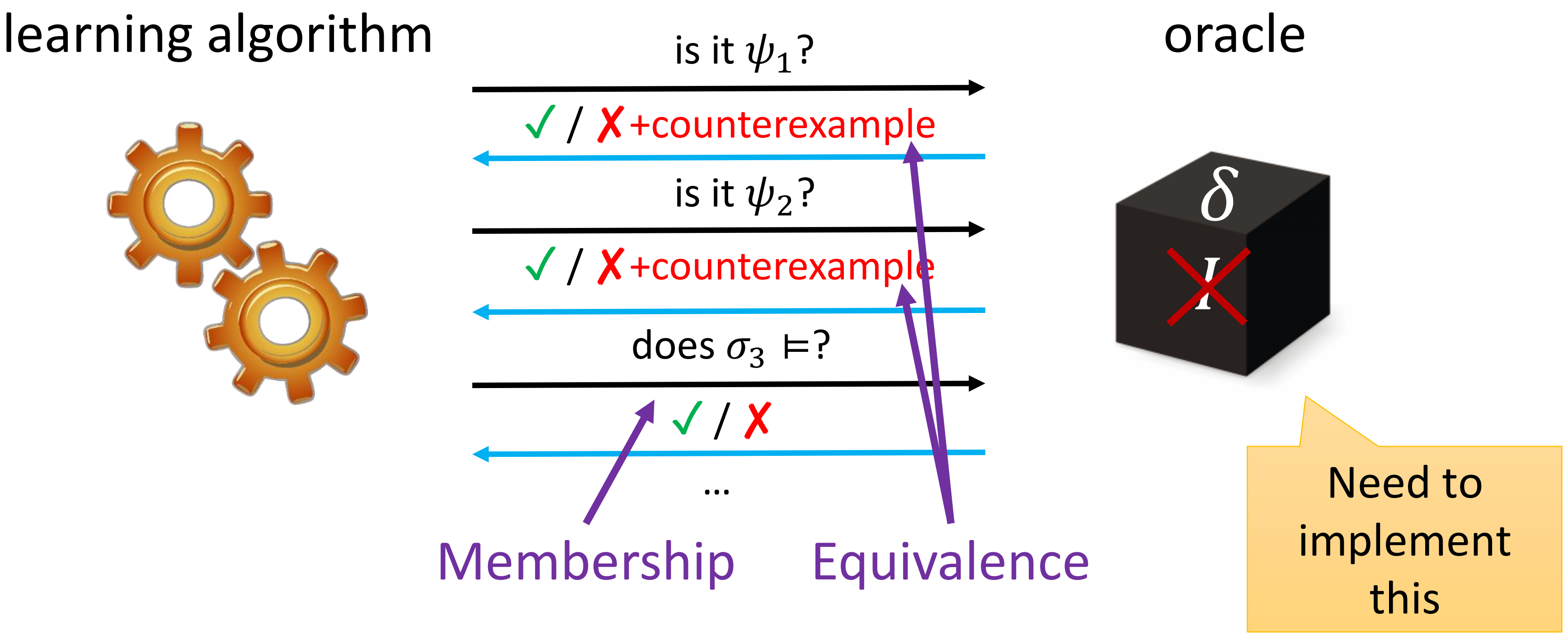
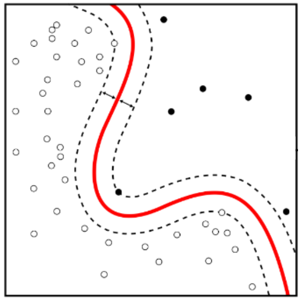
From Learning to Inference



From Learning to Inference



Exact **learning**
DNF formulas

ψ := false

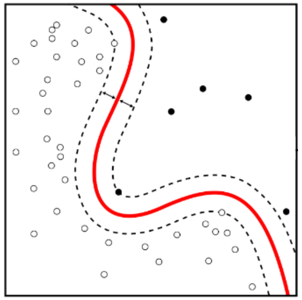
while σ' counterexample
to Equivalence(ψ):

ψ := $\psi \vee$ **generalize**(σ')

generalize(σ'):
drop literals from σ'
while Membership(σ')= ✓

[CACM'84] A Theory of the Learnable. Valiant
[ML'87] Queries and Concept Learning. Angluin
[ML'95] On the Learnability of Disjunctive Normal Form Formulas. Aizenstein and Pitt

From Learning to Inference



Exact **learning** DNF formulas $\Longrightarrow$

ψ := false

while σ' counterexample to **Equivalence**(ψ): → **Inductive**(I)

$\psi := \psi \vee$ **generalize**(σ')

generalize(σ'):

drop literals from σ'

while **Membership**(σ')= ✓ → $\text{BMC}^k(\sigma', \delta, \text{Bad})$ **unsat**

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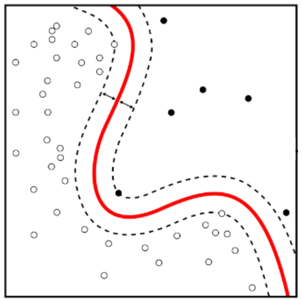
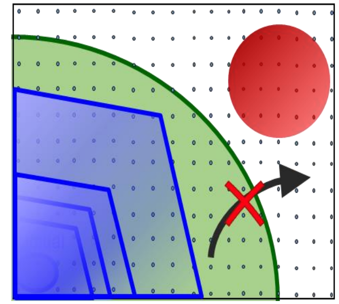
From Learning to Inference



Exact **learning**
DNF formulas

$\Longrightarrow$



Inferring
DNF invariants

ψ := false	I := false
while σ' counterexample to **Equivalence**(ψ):	while $(\_, \sigma')$ counterexample to **Inductive**(I):
ψ := $\psi \vee$ **generalize**(σ')	I := $I \vee$ **generalize**(σ')
generalize(σ'): drop literals from σ' while **Membership**(σ')= ✓	**generalize**(σ'): drop literals from σ' while $\mathrm{BMC}^k(\sigma', \delta, \mathrm{Bad})$ **unsat**

[CACM'84] A Theory of the Learnable. Valiant
[ML'87] Queries and Concept Learning. Angluin
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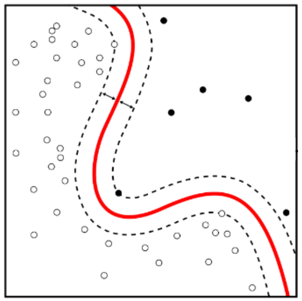
[CAV'03] Interpolation and SAT-Based Model Checking, McMillan
[HVC'12] Computing Interpolants without Proofs. Chockler, Ivrii, Matsliah

From Learning to Inference

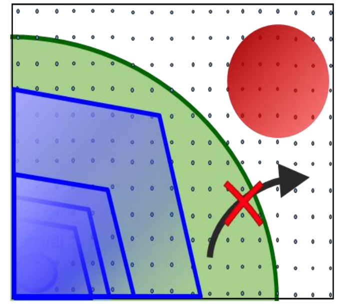


Efficiently

Exact **learning**

DNF formulas

$\Longrightarrow$



Efficiently

Inferring

DNF invariants

When is the transformation correct?

ψ := false

while σ' counte…

to **Equiva…**

ψ := ψ ∨ **ge…**

generalize(σ'):

drop literals from σ'

while **Membership**(σ')= ✓

→

I := false

…′) counterexample

…**Inductive**(I):

… ∨ **generalize**(σ')

generalize(σ'):

drop literals from σ'

while **BMC**^{k}(σ', δ, **Bad**) **unsat**

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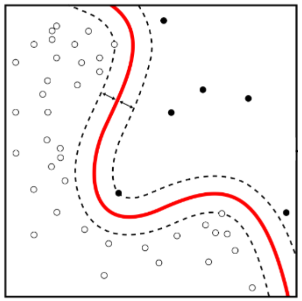
[HVC'12] Computing Interpolants without Proofs. Chockler, Ivrii, Matsliah

From Learning to Inference

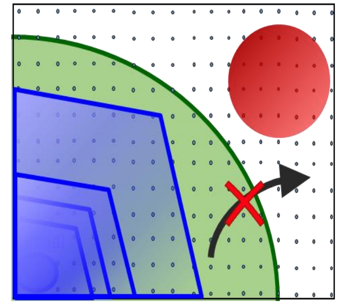


Efficiently

Exact **learning**

DNF formulas

$\Longrightarrow$



Efficiently

Inferring

DNF invariants

Thm: can implement queries when
the invariant is k-fenced
and the algorithm's queries are one-sided

generalize(σ'):
drop literals from σ'
while **Membership**$(\sigma')= ✓$

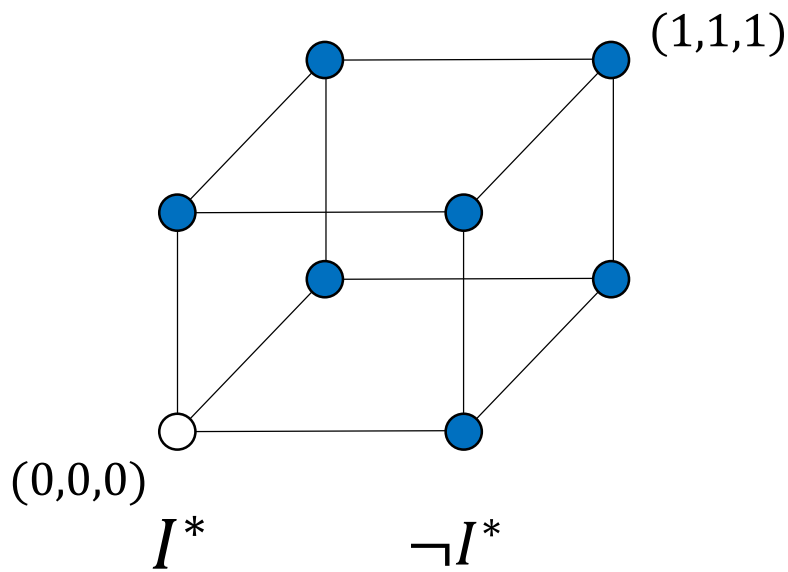
→

generalize(σ'):
drop literals from σ'
while **BMC**$^k(\sigma', \delta, \mathbf{Bad})$ **unsat**

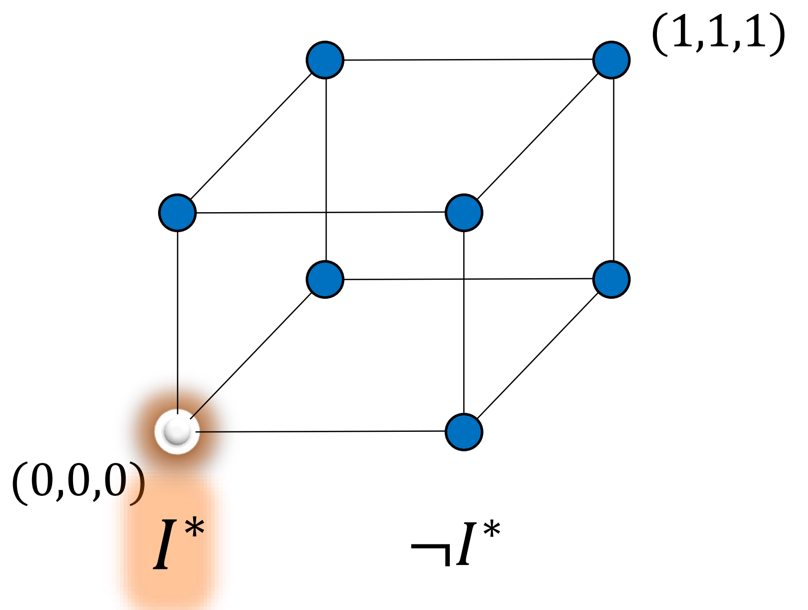
[CACM'84] A Theory of the Learnable. Valiant
[ML'87] Queries and Concept Learning. Angluin
[ML'95] On the Learnability of Disjunctive Normal Form Formulas. Aizenstein and Pitt

[CAV'03] Interpolation and SAT-Based Model Checking, McMillan
[HVC'12] Computing Interpolants without Proofs. Chockler, Ivrii, Matsliah

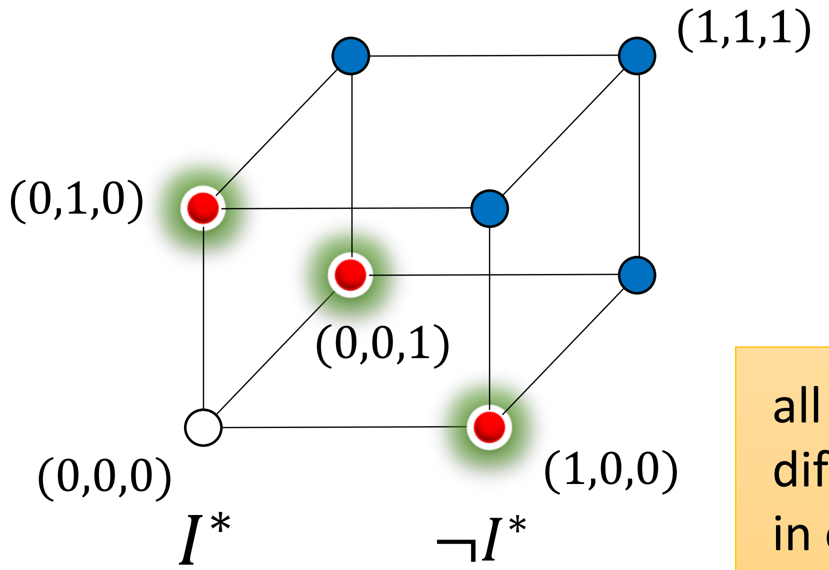
k-Fenced Invariants



k-Fenced Invariants



k-Fenced Invariants

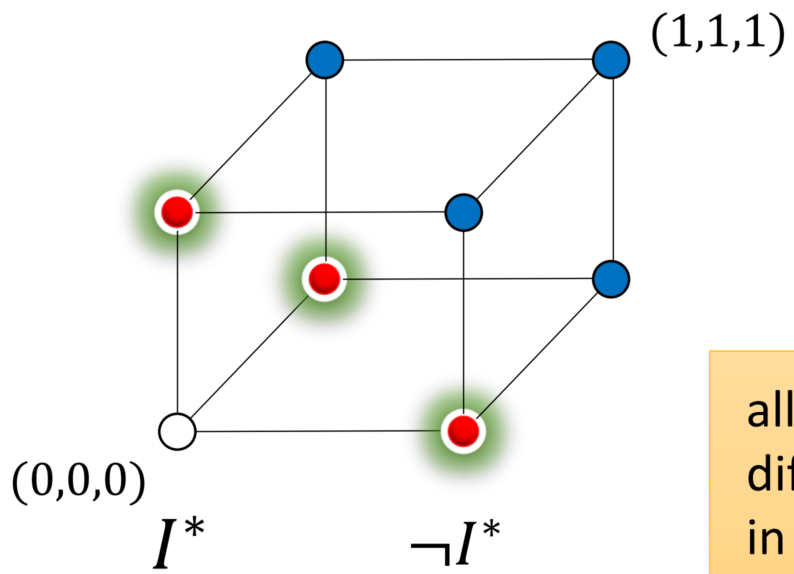


all states $\sigma \notin I^*$ that differ from some $\sigma' \in I^*$ in one bit

$$\partial^-(I^*)$$

Outer boundary

k-Fenced Invariants



all states $\sigma \notin I^*$ that differ from some $\sigma' \in I^*$ in one bit

I^* is k-fenced if

all the states in $\partial^-(I^*)$

can reach a bad state in at most k steps

Example: k-Fenced Invariant

Init:
$(x_1, \dots, x_n) \coloneqq 0 \dots 0$

Bad:
$(x_1, \dots, x_n) = 1 \dots 1$

δ:
$y_1, \dots, y_n \coloneqq *$
$x_1, \dots, x_n \coloneqq (x_1, \dots, x_n) +$
$2 \cdot (y_1, \dots, y_n) \pmod{2^n}$

$$I^*: \; x_n \neq 1$$

all the states in $\partial^-(I^*) = \{x_n = 1\}$
can reach a bad state in at most k steps $= 1$

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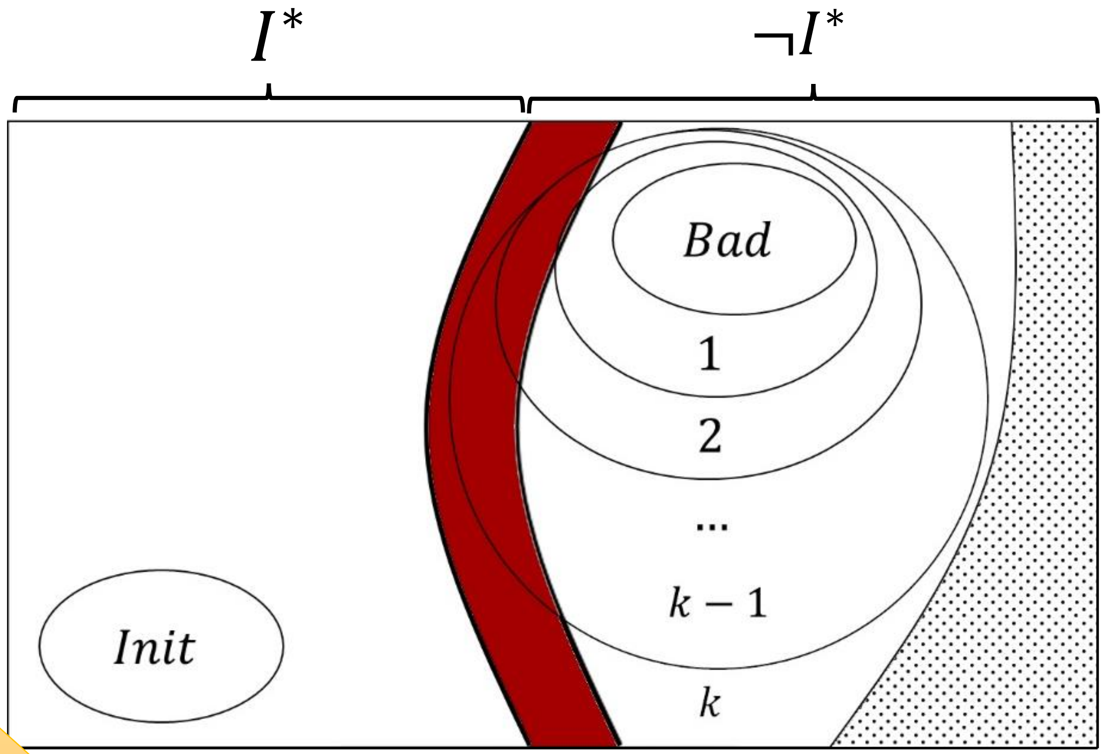
$I^*: \; x_n \neq 1$

In general **not** all states in $\neg I^*$ need to reach bad

In this example $\neg I^*$

all the states in $\partial^-(I^*) = \{x_n = 1\}$
can reach a bad state in at most k steps $= 1$

k-Fenced Invariants



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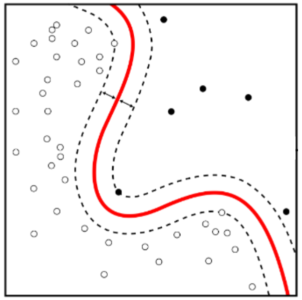
From Learning to Inference



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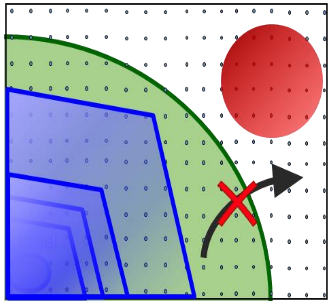
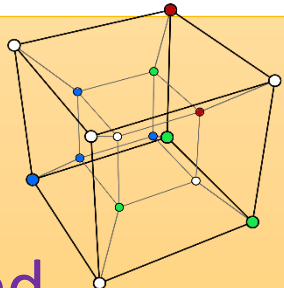


Efficiently

Inferring

DNF invariants

Thm: can implement queries when
the invariant is k-fenced ✓
and the algorithm's queries are one-sided



`One-Sided Equivalence`(ψ): $\psi \Longrightarrow \varphi$

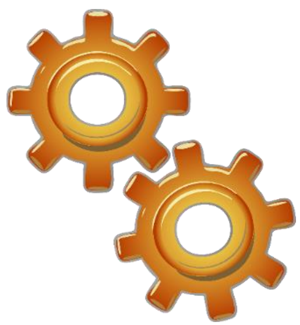
`One-Sided Membership`(σ): $\sigma \in \varphi \cup \partial^{-}(\varphi)$

One-Sided Equivalence Queries to Invariants

inference algorithm

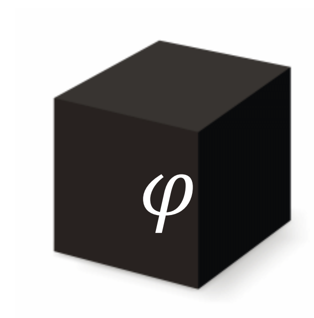
$\psi \Longrightarrow \varphi$

teacher



is it ψ?

✓ / ✗ +counterexample



is ψ an inductive invariant?

✓ yes hooray!

✗ +counterexample transition:

(σ, σ') s.t. $\sigma \vDash \psi, \sigma' \vDash \neg\psi$

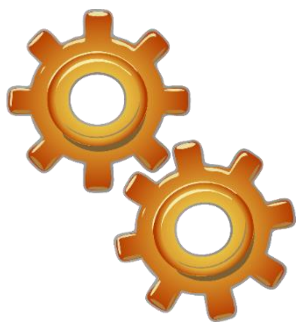
Always return σ' as positive example

One-Sided Membership Queries to k-Fenced Invariants

inference algorithm

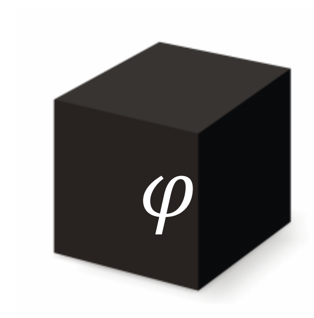
$\boldsymbol{\sigma \in \varphi \cup \partial^{-}(\varphi)}$

teacher



is $\sigma_3 \vDash$?

✓ / ✗



can't σ_3 reach bad states **in k steps?**

BMC^{k}$(\sigma_3, \delta, \mathrm{Bad})$ **unsat?**

✓ then yes

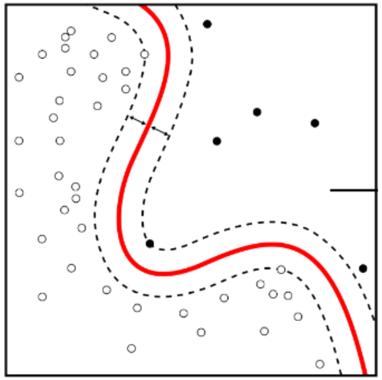
✗ then no

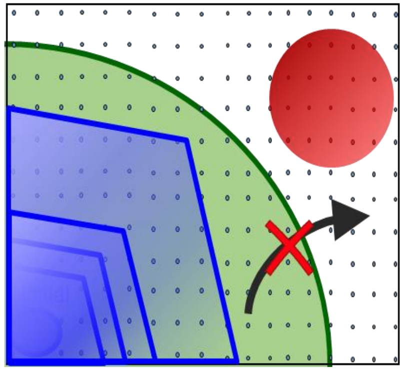
Doesn't always imply that $\sigma_3 \vDash I^*$

From Learning to Inference

<u>Thm</u>: Let $\mathcal{C}$ be a class of formulas.

$\exists\mathcal{A}$ **identifying** $\varphi \in \mathcal{C}$ with polynomially-many one-sided queries $\Longrightarrow$ $\exists\mathcal{A}$ **inferring** $I^* \in \mathcal{C}$ with polynomially-many SAT queries whenever I^* is k-fenced





[POPL'21] Learning the Boundary of Inductive Invariants. Feldman, Sagiv, Shoham, Wilcox

Efficient Inference

Thm 1: $\mathcal{C}$ = monotone DNF

$\exists\mathcal{A}$ **identifying** $\varphi \in \mathcal{C}$ with polynomially-many one-sided queries

$\Longrightarrow$

$\exists\mathcal{A}$ **inferring** $I^* \in \mathcal{C}$ with polynomially-many SAT queries whenever I^* is k-fenced

[CACM'84] A Theory of the Learnable. Valiant
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Efficient Inference

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$\psi \Longrightarrow \varphi$

```
ψ  :=  false
while σ′ counterexample
      to Equivalence(ψ):
   ψ  :=  ψ ∨ generalize(σ′)

generalize(σ′):
   drop literals from σ′
   while Membership(σ′)= ✓
```

$\sigma' \in \varphi \cup \partial^-(\varphi)$

```
I  :=  false
while (_,σ′) counterexample
      to Inductive(I):
   I  :=  I ∨ generalize(σ′)

generalize(σ′):
   drop literals from σ′
   while BMC^k(σ′, δ, Bad) unsat
```

Efficient Inference

Thm 1: $\mathcal{C}$ = monotone DNF

$\exists\mathcal{A}$ **identifying** $\varphi \in \mathcal{C}$ with polynomially-many one-sided queries $\Longrightarrow$ $\exists\mathcal{A}$ **inferring** $I^* \in \mathcal{C}$ with polynomially-many SAT queries whenever I^* is k-fenced

Thm 1: The **interpolation-based algorithm** converges in a polynomial number of SAT queries if I^* is

- k-fenced, and
- has a short monotone DNF representation

[CACM'84] A Theory of the Learnable. Valiant
[ML'87] Queries and Concept Learning. Angluin
[ML'95] On the Learnability of Disjunctive Normal Form Formulas. Aizenstein and Pitt

Efficient Inference

Thm 2: $\mathcal{C}$ = almost-monotone DNF

$\exists\mathcal{A}$ **identifying** $\varphi \in \mathcal{C}$ with polynomially-many one-sided queries $\implies$ $\exists\mathcal{A}$ **inferring** $I^* \in \mathcal{C}$ with polynomially-many SAT queries whenever I^* is k-fenced

Thm 2: A **different algorithm** converges in a polynomial number of SAT queries if If I^* is

- k-fenced, and
- has a short almost-monotone DNF representation

at most $O(1)$ terms include negated variables

[Inf. Comput. '95] Exact Learning Boolean Function via the Monotone Theory. Bshouty

Inference from Unrestricted Queries

Thm′: Let $\mathcal{C}$ be a class of formulas.

two-sided

$\exists\mathcal{A}$ **identifying** $\varphi \in \mathcal{C}$ with polynomially-many ~~one-sided~~ queries $\Longrightarrow$ $\exists\mathcal{A}$ **inferring** $I^* \in \mathcal{C}$ with polynomially-many SAT queries whenever I^* is k-fenced

Thm 3: A **different algorithm** converges in a polynomial number of SAT queries if I^* is

- two-sided k-fenced, and
- has a short DNF *and* a short CNF representation

  e.g., I^* is expressible as a short decision tree

[Inf. Comput. '95] Exact Learning Boolean Function via the Monotone Theory. Bshouty

Inference from Unrestricted Queries

Thm′: Let $\mathcal{C}$ be a class of formulas.

two-sided

$\exists\mathcal{A}$ **inferring** $^*\in\mathcal{C}$ with

ia y-many

que ries

* is k-fenced

Thm: also when $I^*$is one-sided k-fenced
but **not by transformation from learning**

Thm 3: A **different algorithm** converges in a polynomial number of SAT queries if I^* is

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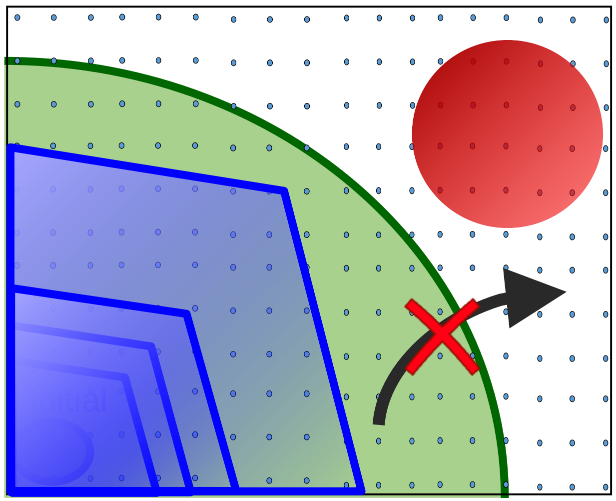
e.g., I^* is expressible as a short decision tree

[Inf. Comput. '95] Exact Learning Boolean Function via the Monotone Theory. Bshouty
[SAS '22] Invariant Inference With Provable Complexity From the Monotone Theory. Feldman, Shoham

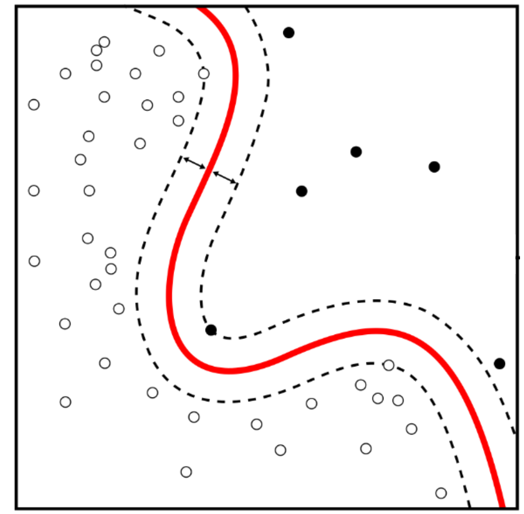
Conclusion (1)

Invariant Inference



vs.

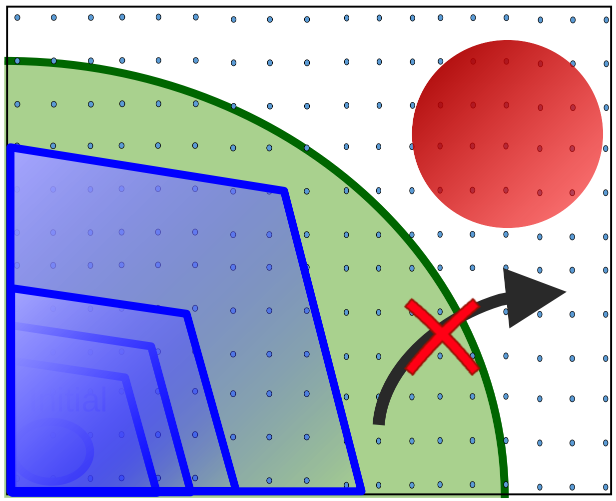
Exact Concept Learning



- Query-based learning models for invariant inference
- Complexity lower and upper bounds for each model
- Invariant inference is harder than concept learning
- Complexity results for invariant inference algorithms from concept learning algorithms

Conclusion (2)

Invariant Inference



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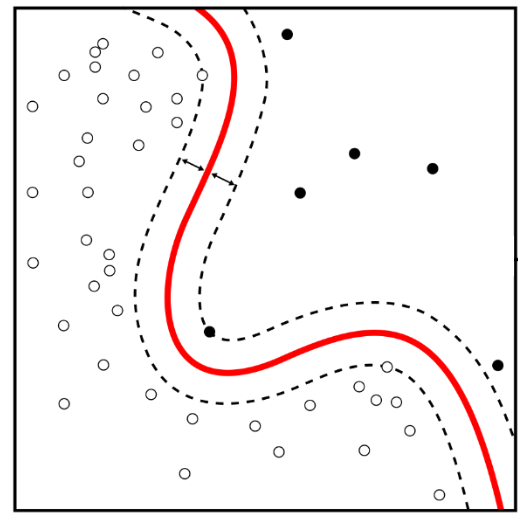

- What about IC3/PDR?
- Impact of k in the Hoare query model?
- Is the fence condition necessary?
- Other conditions?
- Beyond Boolean programs