Verification of Distributed Protocols Using Decidable Logic

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Programming Languages Mentoring Workshop 2019



The research leading to these results has received funding from the European Research Council under the European Union's Horizon 2020 research and innovation programme (grant agreement No [759102-SVIS])

Why verify distributed protocols?

- Distributed systems are everywhere
 - Safety-critical systems
 - Cloud infrastructure
 - Blockchains



- Distributed protocols are notoriously hard to get right
 - Even small protocols can be tricky
 - Bugs occur on rare scenarios
 - Testing is costly and not sufficient

Verifying distributed protocols is hard

Verify distributed protocols for any number of nodes and resources



- Infinite state-space
 - unbounded #processes
 - unbounded #messages
 - unbounded #objects
- Asymptotic complexity of verification
 - Rice theorem



Safety of Infinite State Systems



System S is safe if all the reachable states satisfy the property $P = \neg Bad$

Inductive Invariants



System S is safe if all the reachable states satisfy the property $P = \neg Bad$ System S is safe iff there exists an **inductive invariant** *Inv*:

Init \subseteq *Inv* (Initiation) if $\sigma \in$ *Inv* and $\sigma \rightarrow \sigma'$ then $\sigma' \in$ *Inv* (Consecution) *Inv* \cap *Bad* = \emptyset (Safety)

Inductive Invariants



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- N pairs of players pass a ball:
 - 11 will pass to 1 \downarrow
 - − 1↓ will pass to 1 \uparrow
 - 21 will pass to 21
 - $-2\downarrow$ will pass to $2\uparrow$...



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- N pairs of players pass a ball:
 - 11 will pass to 1 \downarrow
 - − 1↓ will pass to 1 \uparrow
 - 21 will pass to 21
 - 2↓ will pass to $2\uparrow$...
- The ball starts at player 11
- Can the ball get to $2\downarrow$?

11	21
1↓	2↓

- N pairs of players pass a ball:
 - 11 will pass to 1 \downarrow
 - − 1 \downarrow will pass to 1 \uparrow
 - 21 will pass to $2\downarrow$
 - 2↓ will pass to $2\uparrow$...
- The ball starts at player 1↑
- Can the ball get to $2\downarrow$?
- Is "the ball is not at $2\downarrow$ " an inductive invariant?



- N pairs of players pass a ball:
 - − 11 will pass to 1↓
 - − 1 \downarrow will pass to 1 \uparrow
 - 21 will pass to 21
 - 2↓ will pass to $2\uparrow$...
- The ball starts at player 11
- Can the ball get to $2\downarrow$?
- Is "the ball is not at 2↓" an inductive invariant? No!
 - Counterexample to induction



- N pairs of players pass a ball:
 - − 11 will pass to 1↓
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 - 21 will pass to 21
 - 2↓ will pass to $2\uparrow$...
- The ball starts at player 1↑
- Can the ball get to $2\downarrow$?
- Is "the ball is not at 2↓" an inductive invariant? No!
 - Counterexample to induction
- Inductive invariant: "the ball is not at 2↑ nor 2↓"



Logic-based verification

Provers/solvers for different logics made huge progress

- Propositional logic (SAT) industrial impact for hardware verification
- Satisfiability modulo theories (SMT) major trend in software verification
- Automated first-order theorem provers
- Interactive theorem provers
- Z3, CVC4, iProver, Vampire, Coq, Isabelle/HOL

Logic-based verification



Represent *Init*, *Tr*, *Bad*, *Inv* by logical formulas: Formula \Leftrightarrow Set of states

Inv(V) is an inductive invariant if the verification conditions (VCs) are valid:InitiationInit(V) \Rightarrow Inv(V)unsat(Init(V) \neg Inv(V))Cons.Inv(V) \land TR(V,V') \Rightarrow Inv(V')unsat(Inv(V) \land TR(V,V') \land Inv(V'))SafetyInv(V) \Rightarrow \neg Bad(V)unsat(Inv(V) \land Bad(V))

Challenges for logic-based verification

Formal specification

Modeling the system and its invariants

Deduction Checking validity of the VCs

Inference Finding an inductive invariant

Deduction

Inv(V) is an inductive invariant if the following verification conditions are valid:

InitiationInit(V) \Rightarrow Inv(V)unsat(Init(V) $\land \neg$ Inv(V))Cons.Inv(V) \land TR(V,V') \Rightarrow Inv(V')unsat(Inv(V) \land TR(V,V') $\land \neg$ Inv(V'))

Safety $Inv(V) \Rightarrow \neg Bad(V)$

Church's Theorem

unsat($Inv(V) \land Bad(V)$)



Deduction

Interactive theorem provers (Coq, Isabelle/HOL, LEAN)

- Programmer proves the inductive invariant
- Huge programmer effort (~10-50 lines of proof per line of code)

e.g. Verdi



Automatic solvers/provers

(e.g. Z3, CVC4, Vampire)

- VCs discharged automatically
- Tools may diverge (for SMT: matching loops, arithmetic)
- Unpredictability (butterfly effect)

e.g. Ironfleet



Logic-based verification approaches



Logic-based verification approaches



This talk: Restrict VC's to decidable logic

Inv(V) is an inductive invariant if the following verification conditions are valid:

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Challenges for verification with decidable logic

Formal specification

Modeling in a decidable logic

Deduction Checking validity of the VC's



Invariant inference

Finding an inductive invariant

This talk

Logic: EPR – decidable fragment of first order logic

Formal specification

Surprisingly expressive

Invariant inference

- Automatic (based on PDR)
 - Semi-algorithm: may diverge
- Interactive
 - Based on graphically displayed counterexamples to induction

Effectively Propositional Logic – EPR

Decidable fragment of first order logic

+ Quantification $(\exists^*\forall^*)$ - Theories (e.g., arithmetic)

☺ Allows quantifiers to reason about unbounded sets

- $\forall x, y$. leader(x) \land leader(y) \rightarrow x = y

- ③ Satisfiability is decidable => Deduction is decidable
- Small model property => Finite cex to induction
- © Turing complete modeling language
- ☺ Limited language for safety and inductive invariants

Suffices for many infinite-state systems

Successful verification with EPR

Shape Analysis

[Itzhaky et al. CAV'13, POPL'14, CAV'14, CAV'15]

- Software-Defined Networks [Ball et al. PLDI'14]
- Distributed protocols [Padon et al. PLDI'16, OOPSLA'17, POPL'18, PLDI'18]
- Concurrent Modification Errors in Java programs [Frumkin et al. VMCAl'17]

Example: Leader Election in a Ring

- Nodes are organized in a unidirectional ring
- Each node has a unique numeric id
- Protocol:
 - Each node sends its id to the next



- A node that receives a message passes it to the next if the id in the message is higher than the node's own id
- A node that receives its own id becomes the leader
- Theorem:
 - The protocol selects at most one leader

[CACM'79] E. Chang and R. Roberts. *An improved algorithm for decentralized extrema-finding in circular configurations of processes*

State: finite first-order structure over vocabulary V

- \leq (ID, ID) total order on node id's
- id: Node \rightarrow ID relate a node to its id
- btw (Node, Node, Node) the ring topology
- **pending**(ID, Node) pending messages
- leader(Node) leader(n) means n is the leader

Axiomatized in EPR



structure



 $\langle n_5, n_1, n_3 \rangle \in I(btw)$

- State: finite first-order structure over vocabulary V (+ axioms)
- Initial states and safety property: EPR formulas over V
 - Init(V) initial states, e.g., \forall id, n. \neg pending(id, n)
 - Bad(V) bad states, e.g., $\exists n_1, n_2$. leader $(n_1) \land leader(n_2) \land n_1 \neq n_2$

 Transition relation: expressed as EPR formula TR(V, V'), e.g.: ∃n,s. "s = next(n)" ∧ ∀x,y. pending'(x,y)↔ (pending(x,y) ∨ (x=id[n]∧y=s))
∨ ∃n. pending (id[n],n) ∧ ∀x. leader'(x) ↔ (leader(x) ∨ x=n)

• State: finite first-order structure over vocabulary V (+ axioms)

Propose(n): send(id(n), next(n))

...

Recv(n,msg): if msg = id(n) then leader(n) := true

if msg > id(n) then send(msg,next(n))

Transition relation: expressed as EPR formula TR(V, V'), e.g.:

 $\exists n,s. "s = next(n)" \land \forall x,y. pending'(x,y) \leftrightarrow (pending(x,y) \lor (x=id[n] \land y=s))$

 $\lor \exists n. pending (id[n],n) \land \forall x. leader'(x) \leftrightarrow (leader(x) \lor x=n)$

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Specify and verify the protocol for any number of nodes in the ring

Using EPR for Verification

- System model Init(V), Bad(V), TR(V, V') ∈ EPR
- Inductive invariant $Inv(V) \in \forall^*$
- Verification conditions Initiation Init(V) \Rightarrow Inv(V) unsat(Init(V) \neg Inv(V)) Cons. Inv(V) \land TR(V,V') \Rightarrow Inv(V') unsat(Inv(V) \land TR(V,V') \land \neg Inv(V')) Safety Inv(V) $\Rightarrow \neg$ Bad(V) unsat(Inv(V) \land Bad(V))

Verification conditions ∈ EPR→ Decidable to check

Inductive Invariant for Leader Election

Safety property:



- (ID, ID) total order on node id's
- btw (Node, Node, Node) the ring topology
- id: Node → ID relate a node to its id
- pending(ID, Node) pending messages
- leader(Node) leader(n) means n is the leader

Inductive Invariant for Leader Election

Safety property:



Inductive Invariant for Leader Election

Safety property:



Axioms: Leader Election Protocol

- \leq (ID, ID) total order on node id's
- btw (a: Node, b: Node, c: Node) the ring topology
- id: Node \rightarrow ID relate a node to its unique id
- pending(ID, Node) pending messages
- leader(Node) leader(n) means n is the leader

	Intention	EPR Modeling
Node ID's	Integers	$ \begin{array}{l} \forall i: ID. \ i \leq i \ \text{Reflexive} \\ \forall i, j, k: ID. \ i \leq j \land j \leq k \rightarrow i \leq k \ \text{Transitive} \\ \forall i, j: ID. \ i \leq j \land j \leq I \rightarrow i = j \ \text{Anti-Symmetric} \\ \forall i, j: ID. \ i \leq j \lor j \leq i \ \text{Total} \\ \forall x, y: \text{Node.} \ id(x) = id(y) \rightarrow x = y \ \text{Injective} \end{array} $
Ring Topology	Next edges + Transitive closure	$\forall x, y, z: Node. btw(x, y, z) \rightarrow btw(y, z, x)$ Circular shifts $\forall x, y, z, w: Node. btw(w, x, y) \land btw(w, y, z) \rightarrow btw(w, x, z)$ Transitive $\forall x, y, w: Node. btw(w, x, y) \rightarrow \neg btw(w, y, x)$ Anti-Symmetric $\forall x, y, z, w: Node. distinct(x, y, z) \rightarrow btw(w, x, y) \lor btw(w, y, x)$
		"next(a)=b" = $\forall x: Node. x \neq a \land x \neq b \rightarrow btw(a,b,x)$

So far

Formal specification with EPR

- Surprisingly expressive
 - Integers: numeric id's expressed with \leq
 - Transitive closure: ring topology expressed with btw
 - Network semantics: pending messages
 - Sets and cardinalities (for consensus protocols) [OOPSLA'17]
 - Liveness properties [POPL'18, FMCAD'18]
 - Implementations [PLDI'18]

Not in this talk
Next

Invariant inference: finding inductive invariants

- (1) Automatically
 - Adapt techniques from finite-state model checking (PDR)
- (2) Interactively
 - Based on graphically displayed counterexamples to induction

How can we find a universally quantified inductive invariant?

Inductive Invariant for Leader Election

I₀ ⊣Bad	$\forall n_1, n_2 : Node. leader(n_1) \land leader(n_2) \rightarrow n_1 = n_2$ $\neg \exists n_1, n_2 : Node. leader(n_1) \land leader(n_2) \land n_1 \neq n_2$	At most one leader elected
I	$\forall n_1, n_2: \text{ Node. } \textbf{leader}(n_1) \rightarrow \textbf{id}[n_2] \leq \textbf{id}[n_1] \\ \neg \exists n_1, n_2: \text{ Node. } \textbf{leader}(n_1) \land \textbf{id}[n_2] > \textbf{id}[n_1] \end{cases}$	The leader has the highest id
I ₂	$\forall n_1, n_2: \text{ Node. } pnd(id[n_1], n_1) \rightarrow id[n_2] \leq id[n_1] \\ \neg \exists n_1, n_2: \text{ Node. } pnd(id[n_1], n_1) \land id[n_2] > id[n_1] \end{cases}$	
I ₃		higher nodes
	$\neg \exists n_1, n_2, n_3$: Node. $btw(n_1, n_2, n_3) \land pnd(id[n_2], n_1) \land id[n_3] > id[n_2]$	





Construct Inv by excluding "bad" states

- 1. How to find these states?
- 2. How to generalize into conjectures?



Absence, A. Karbyshev, N. Bjorner, S. Itzhaky, N. Rinetzky and S. Shoham.



∀* Invariant - excluded substructures



Leader election example



(1) Automatic inference: UPDR

• Based on Bradley's IC3/PDR [VMCAI11,FMCAD11]

SAT-based verification of finite-state systems

- Abstracts concrete states using their logical diagram
- Backward traversal performed over diagrams
- Blocking of CTI excludes a *generalization* of its diagram → generates universally quantified lemmas

- [CAV'15, JACM'17] Property-Directed Inference of Universal Invariants or Proving Their Absence, A. Karbyshev, N. Bjorner, S. Itzhaky, N. Rinetzky and S. Shoham.
- [VMCAI'17] Property Directed Reachability for Proving Absence of Concurrent Modification Errors, A. Frumkin, Y. Feldman, O. Lhoták, O. Padon, M. Sagiv and S. Shoham.

UPDR: Possible outcomes

- Universal inductive invariant found
 - System is safe

Used to infer inductive invariants / procedure summaries of:

- Heap-manipulating programs, e.g.
 - Singly/Doubly/Nested linked list
 - Iterators in Java Concurrent modification error (CME)
- Distributed protocols
 - Spanning tree
 - Learning switch

No need for user-defined predicates/ templates!

UPDR: Possible outcomes

- Universal inductive invariant found
 - System is safe
- Proof that no universal inductive invariant exists

Safety not determined*



* can use Bounded Model Checking to find real counterexamples

UPDR: Possible outcomes

- Universal inductive invariant found
 - System is safe
- Proof that no universal inductive invariant exists
 - Safety not determined*
- Divergence
 - In general, inferring universal ind. inv. is undecidable
 - For linked lists it is decidable, UPDR will also terminate
 - Proof uses well-quasi-order and Kruskal's tree theorem
- [POPL'16] Decidability of Inferring Inductive Invariants, O. Padon, N. Immerman, S. Shoham, A. Karbyshev, and M. Sagiv.

Automatic Inference (e.g., UPDR)

Ultimately limited by undecidability

(2) Interactive Inference



- Let the user guide the tool
 - User has intuition about the essence of the proof
 - Computer is good at handling corner cases



Supervised Verification of Infinite-State Systems



Automation



Supervised Verification of Infinite-State Systems

Ivy: Interactive Generalization

$$In\nu = I_0 \wedge \dots \wedge I_k$$



Displays "minimal" CTI to exclude

Generalizes to a partial state



• removes "irrelevant" facts (graphical interface - checkboxes)



Translates to universally quantified conjecture (via diagram) Provides auxiliary automated checks:

1. BMC(K): uses SAT solver to check if conjecture is true up to K

• User determines the right K to use

2. ITP(K): uses SAT solver to discover more facts to remove



Examines the proposed conjecture – it could be wrong Adds I_{k+1}

[PLDI'16] IVy: Safety Verification by Interactive Generalization. O. Padon,K. McMillan, A. Panda, M. Sagiv, S. Shoham https://github.com/Microsoft/ivy

Interactive Verification in IVy



Decidable Problems Predictable Automation

Proof intuition and creativity Graphical interaction

Summary 1

Verification with decidable logic

- EPR decidable fragment of FOL
 - Deduction is decidable
 - Finite counterexamples

- Domain knowledge and axioms
- Derived relations
- Modularity
- Prophecy

- Can be made surprisingly powerful
 - Transitive closure: linked lists, ring topology [PLDI'16]
 - Paxos, Multi-Paxos, [OOPSLA'17]
 - Liveness and Temporal Properties [POPL'18]
 - Developing verified implementations [PLDI'18]

Summary 2

Invariant Inference

- Automatic inference: UPDR [CAV'15, JACM]
- Interactive inference: lvy [PLDI'16]
- Use logical diagram to infer $\mathsf{Inv} \in \forall^*$
- Can also prove absence of $Inv \in \forall^*$

Take away

Decidable logic is useful
facilitates automation

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Proof Assistants	Supervised	
Ultimately limited by human proof/code: Verdi: ~10 IronFleet: ~4	Verification proof/code: IVy ~1/10 Ultimately limited by undecidab	
	Model Checking Static Analysis	

litv

- We need ways to guide verification tools
- How to divide the problem between human and machine?
- Different inference schemes
- Different Forms of interaction
- Other logics
- Theoretical understanding of limitations and tradeoffs



Supervised Verification of Infinite-State Systems