# Verification of Infinite-State Systems Using Decidable Logic

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Supervised Verification of Infinite-State Systems

#### Model Checking [EC81,QS82]



\*Clarke, Emerson, and Sifakis won the 2007 Turing award for their contribution to Model Checking

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# **Technion Israel Institute of** Technology



2001-2009







# Infinite-State Systems

#### Programs:

- Dimensions of infinity:
  - unbounded number of dynamically allocated objects
  - unbounded number of threads
  - unbounded domain of variables (naturals, reals...)

#### Distributed systems:

- Dimensions of infinity:
  - unbounded number of hosts/switches
  - unbounded number of pending messages



```
pstmt.close()
comp_close();
```



#### Set of states is infinite



#### **Safety Verification**



System S is safe if all the reachable states satisfy the property  $P = \neg Bad$ 

#### **Inductive Invariants**



System S is safe if all the reachable states satisfy the property  $P = \neg Bad$ System S is safe iff there exists an **inductive invariant** *Inv*:

*Init*  $\subseteq$  *Inv* (Initiation) if  $\sigma \in$  *Inv* and  $\sigma \rightarrow \sigma'$  then  $\sigma' \in$  *Inv* (Consecution) *Inv*  $\cap$  *Bad* =  $\emptyset$  (Safety)

#### **Inductive Invariants**



System S is **safe** if all the **reachable** states satisfy the property  $P = \neg Bad$ System S is safe iff there exists an **inductive invariant** Inv:

*Init*  $\subseteq$  *Inv* (Initiation) if  $\sigma \in Inv$  and  $\sigma \rightarrow \sigma'$  then  $\sigma' \in Inv$  (Consecution)  $Inv \cap Bad = \emptyset$  (Safety)

- N pairs of players pass a ball:
  - 11 will pass to 1  $\downarrow$
  - − 1↓ will pass to 1 $\uparrow$
  - 21 will pass to 21
  - 2↓ will pass to  $2\uparrow$  ...



- N pairs of players pass a ball:
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. . .

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  - 11 will pass to 1  $\downarrow$
  - − 1↓ will pass to 1 $\uparrow$
  - 21 will pass to 21
  - 2↓ will pass to  $2\uparrow$  ...
- The ball starts at player 11
- Can the ball get to  $2\downarrow$ ?

11	21
1↓	2↓

- N pairs of players pass a ball:
  - 11 will pass to 1  $\downarrow$
  - − 1 $\downarrow$  will pass to 1 $\uparrow$
  - 21 will pass to 21
  - 2↓ will pass to  $2\uparrow$  ...
- The ball starts at player 1↑
- Can the ball get to  $2\downarrow$ ?
- Is "the ball is not at  $2\downarrow$ " an inductive invariant?



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  - − 11 will pass to 1↓
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  - 21 will pass to 21
  - 2↓ will pass to  $2\uparrow$  ...
- The ball starts at player 11
- Can the ball get to  $2\downarrow$ ?
- Is "the ball is not at  $2\downarrow$ " an inductive invariant? No!
  - Counterexample to induction



- N pairs of players pass a ball:
  - − 11 will pass to 1↓
  - − 1 $\downarrow$  will pass to 1 $\uparrow$
  - 21 will pass to 21
  - 2↓ will pass to  $2\uparrow$  ...
- The ball starts at player 1↑
- Can the ball get to  $2\downarrow$ ?
- Is "the ball is not at 2↓" an inductive invariant? No!
  - Counterexample to induction
- Inductive invariant: "the ball is not at 2↑ or 2↓"



#### Logic-based verification

- Represent *Init*, *Tr*, *Bad*, *Inv* by logical formulas
  - Formula ⇔ Set of states
- Automated solvers for logical satisfiability made huge progress
  - Propositional logic (SAT) industrial impact for hardware verification
  - First-order theorem provers
  - Satisfiability modulo theories (SMT) major trend in software verification
  - Z3, CVC4, iProver, Vampire ....

#### How can we **check** an inductive invariant?

Inv(V) is an inductive invariant if the following verification conditions are valid:

**Initiation**  $Init(V) \Rightarrow Inv(V)$  **unsat**( $Init(V) \land \neg Inv(V)$ )

**Cons**.  $Inv(V) \land TR(V,V') \Longrightarrow Inv(V')$  unsat( $Inv(V) \land TR(V,V') \land \neg Inv(V')$ )

**Safety**  $Inv(V) \Rightarrow \neg Bad(V)$ 

unsat( Inv(V) ABad(V) )



### What can we do about it?

#### Interactive theorem provers (Coq, Isabelle/HOL, LEAN)

- Programmer gives inductive invariant and proves it
- Huge programmer effort (~10-50 lines of proof per line of code)



#### SMT-based deductive verification (e.g. Dafny)

- Programmer provides ind. invariant
- VC's generated and discharged automatically
- SMT solver may diverge (matching loops, arithmetic)



# Alternative: Restrict VC's to decidable logic

Inv(V) is an inductive invariant if the following verification conditions are valid:

 $Init(V) \Longrightarrow Inv(V)$ unsat( $Init(V) \land \neg Inv(V)$ ) Initiation  $Inv(V) \land TR(V,V') \Longrightarrow Inv(V')$ unsat(  $Inv(V) \land TR(V,V') \land \neg Inv(V')$  ) Cons. Safety  $Inv(V) \implies \neg Bad(V)$ unsat( $Inv(V) \land Bad(V)$ ) Are the logical VC's valid ? E Decidable logic I can decide! Proof Counterexample

Challenges for verification with decidable logic

Formal specification

Modeling in a decidable logic

Deduction Checking validity of the VC's



Invariant inference

Finding an inductive invariant

# This talk

#### **Formal specification**

- Use EPR decidable fragment of first order logic
- Surprisingly expressive

#### Invariant inference

- Automatic (based on PDR)
  - Semi-algorithm: may diverge
- Interactive
  - Based on graphically displayed counterexamples to induction

## Effectively Propositional Logic – EPR

Decidable fragment of first order logic

+ Quantification  $(\exists^*\forall^*)$  - Theories (e.g., arithmetic)

☺ Allows quantifiers to reason about unbounded sets

-  $\forall x, y$ . leader(x)  $\land$  leader(y)  $\rightarrow$  x = y

- ☺ Satisfiability is decidable => Deduction is decidable
- ③ Small model property => Finite cex to induction
- © Turing complete modeling language
- ☺ Limited language for safety and inductive invariants

Suffices for many infinite-state systems

## Successful verification with EPR

#### • Shape Analysis [Itzhaky et al. CAV'13, POPL'14, CAV'14, CAV'15]

- Software-Defined Networks [Ball et al. PLDI'14]
- Distributed protocols [Padon et al. PLDI'16, OOPSLA'17, POPL'18, PLDI'18]
- Concurrent Modification Errors in Java programs [Frumkin et al. VMCAl'17]

# Example: Leader Election in a Ring

- Nodes are organized in a unidirectional ring
- Each node has a unique numeric id
- Protocol:
  - Each node sends its id to the next



- A node that receives a message passes it to the next if the id in the message is higher than the node's own id
- A node that receives its own id becomes the leader
- Theorem:
  - The protocol selects at most one leader

[CACM'79] E. Chang and R. Roberts. *An improved algorithm for decentralized extrema-finding in circular configurations of processes* 

• State: finite first-order structure over vocabulary V

- $\leq$  (ID, ID) total order on node id's
- **btw** (Node, Node, Node) the ring topology
- id: Node  $\rightarrow$  ID relate a node to its id
- **pending**(ID, Node) pending messages
- leader(Node) leader(n) means n is the leader

Axiomatized in EPR



structure



 $\langle n_5, n_1, n_3 \rangle \in I(btw)$ 

- State: finite first-order structure over vocabulary V (+ axioms)
- Initial states and safety property: EPR formulas over V
  - Init(V) initial states, e.g.,  $\forall$  id, n.  $\neg$  pending(id, n)
  - Bad(V) bad states, e.g.,  $\exists n_1, n_2$ . leader $(n_1) \land leader(n_2) \land n_1 \neq n_2$

 Transition relation: expressed as EPR formula TR(V, V'), e.g.: ∃n,s. "s = next(n)" ∧ ∀x,y. pending'(x,y)↔ (pending(x,y) ∨ (x=id[n]∧y=s))
 ∨ ∃n. pending (id[n],n) ∧ ∀x. leader'(x) ↔ (leader(x) ∨ x=n)

• State: finite first-order structure over vocabulary V (+ axioms)



Recv(n,msg): if msg = id(n) then leader(n) := true

if msg > id(n) then send(msg,next(n))

Transition relation: expressed as EPR formula TR(V, V'), e.g.:

 $\exists n,s. "s = next(n)" \land \forall x,y. pending'(x,y) \leftrightarrow (pending(x,y) \lor (x=id[n] \land y=s))$ 

 $\lor \exists n. pending (id[n],n) \land \forall x. leader'(x) \leftrightarrow (leader(x) \lor x=n)$ 

- State: finite first-order structure over vocabulary V (+ axioms)
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Specify and verify the protocol for **any** number of nodes in the ring

## Using EPR for Verification

- System model Init(V), Bad(V), TR(V, V') ∈ EPR
- Inductive invariant  $Inv(V) \in \forall^*$
- Verification conditions Initiation Init(V) $\Rightarrow$ Inv(V) unsat(Init(V) $\neg$ Inv(V)) Cons. Inv(V) $\land$ TR(V,V')  $\Rightarrow$  Inv(V') unsat(Inv(V) $\land$ TR(V,V') $\land$  $\neg$ Inv(V')) Safety Inv(V)  $\Rightarrow \neg$ Bad(V) unsat(Inv(V) $\land$ Bad(V))

# Verification conditions ∈ EPR→ Decidable to check

Safety property:



- ≤ (ID, ID) total order on node id's
- btw (Node, Node, Node) the ring topology
- id: Node → ID relate a node to its id
- pending(ID, Node) pending messages
- leader(Node) leader(n) means n is the leader

#### Safety property:



#### Safety property:



## So far

#### Formal specification

- Use EPR decidable fragment of first order logic
- Surprisingly expressive
  - Integers: numeric id's expressed with  $\leq$
  - Transitive closure: ring topology expressed with btw
  - Network semantics: pending messages
  - Sets and cardinalities (for consensus protocols)

Not in this talk

#### Next

#### Invariant inference: finding inductive invariants

- Automatically
  - Adapt techniques from finite-state model checking (PDR)
- Interactively
  - Based on graphically displayed counterexamples to induction

# How can we find a universally quantified inductive invariant?

I <sub>0</sub> Pad	$\forall n_1, n_2 : Node. leader(n_1) \land leader(n_2) \rightarrow n_1 = n_2$	At most one
⊐Ddu	$\neg \exists n_1, n_2$ : Node. leader( $n_1$ ) $\land$ leader( $n_2$ ) $\land$ $n_1 \neq n_2$	leader elected
I <sub>1</sub>	$\forall n_1, n_2$ : Node. leader( $n_1$ ) $\rightarrow id[n_2] \leq id[n_1]$	The leader has
	$\neg \exists n_1, n_2$ : Node. leader( $n_1$ ) $\land id[n_2] > id[n_1]$	the highest id
I <sub>2</sub>	$\forall n_1, n_2$ : Node. pnd(id[n_1], n_1) $\rightarrow$ id[n_2] $\leq$ id[n_1]	Only highest id
-	$\neg \exists n_1, n_2$ : Node. pnd(id[n_1], n_1) $\land$ id[n_2] > id[n_1]	can be self-pnd
I <sub>3</sub>	$\forall n_1, n_2, n_3$ : Node. $btw(n_1, n_2, n_3) \land pnd(id[n_2], n_1)$	Cannot bypass
	$\rightarrow id[n_3] \leq id[n_2]$	higher nodes
	$\neg \exists n_1, n_2, n_3$ : Node. $btw(n_1, n_2, n_3) \land pnd(id[n_2], n_1)$	
	$\wedge id[n_3] > id[n_2]$	$\frown$





#### Construct Inv by excluding "bad" states



- 1. How to find these states?
- 2. How to generalize into conjectures?



Absence, A. Karbyshev, N. Bjorner, S. Itzhaky, N. Rinetzky and S. Shoham.



#### ∀\* Invariant - excluded substructures



#### Leader election example



# (1) UPDR: Automatic inference

• Based on Bradley's IC3/PDR [VMCAI11,FMCAD11]

SAT-based verification of finite-state systems

- Abstracts concrete states using their logical diagram
- Backward traversal performed over diagrams
- Blocking of CTI excludes a *generalization* of its diagram → generates universally quantified lemmas

- [CAV'15, JACM'17] Property-Directed Inference of Universal Invariants or Proving Their Absence, A. Karbyshev, N. Bjorner, S. Itzhaky, N. Rinetzky and S. Shoham.
- [VMCAI'17] Property Directed Reachability for Proving Absence of Concurrent Modification Errors, A. Frumkin, Y. Feldman, O. Lhoták, O. Padon, M. Sagiv and S. Shoham.

#### **UPDR:** Possible outcomes

- Universal inductive invariant found
  - System is safe

Used to infer inductive invariants / procedure summaries of:

- Heap-manipulating programs, e.g.
  - Singly/Doubly/Nested linked list
  - Iterators in Java Concurrent modification error (CME)
- Distributed protocols
  - Spanning tree
  - Learning switch

No need for user-defined predicates/ templates!

#### **UPDR:** Possible outcomes

- Universal inductive invariant found
  - System is safe
- Abstract counterexample:
  - Safety not determined\*
  - But no universal inductive invariant exists!



\* can use Bounded Model Checking to find real counterexamples

# Proving the absence of universal invariant

Suppose that a universally quantified inductive invariant I exists. Then:



 $\sigma_{i+1} \vDash \neg \mathsf{Bad}$ 

If there is a universal inductive invariant  $I \in \forall^*$ , then any **abstract trace** does not reach Bad

➔ An abstract trace to Bad implies no universal inductive invariant exists

#### Termination?

#### Termination?

#### Is it decidable to **infer universal** inductive invariants? [POPL'16]

- No, in the general case
  - if the vocabulary contains at least one binary relation which is unrestricted
- Yes, for linked lists
  - if the vocabulary contains only one "transitive closure" binary relation, but as many constants and unary predicates as desired
  - UPDR will also terminate
  - proof uses well-quasi-order and Kruskal's tree theorem
- More decidable classes
- [POPL'16] Decidability of Inferring Inductive Invariants, O. Padon, N. Immerman, S. Shoham, A. Karbyshev, and M. Sagiv.

# Automatic Verification (e.g., UPDR)

Ultimately limited by undecidability

#### **Interactive Verification**



- Divide the problem between the human and the machine
- Find a suitable way to conduct the interaction



Supervised Verification of Infinite-State Systems

#### **Interactive Verification**



Automation



Supervised Verification of Infinite-State Systems

Expressiveness

# (2) Ivy: Interactive inference



[PLDI'16] IVy: Safety Verification by Interactive Generalization. O. Padon, K. McMillan, A. Panda, M. Sagiv, S. Shoham <u>https://github.com/Microsoft/ivy</u>

#### Summary 1

#### Verification with decidable logic

- EPR decidable fragment of FOL
  - Deduction is decidable
  - Finite counterexamples
- Can be made surprisingly powerful
  - Transitive closure: linked lists, ring topology [PLDI'16]
  - Paxos, Multi-Paxos, [OOPSLA'17]
  - Liveness and Temporal Properties [POPL'18]
  - Developing verified implementations [PLDI'18]

- Domain knowledge and axioms
- Derived relations
- Modularity
- Prophecy

### Summary 2

Invariant Inference





Ultimately limited by undecidability

Model Checking Static Analysis

- Automatic inference: UPDR
- Interactive inference: lvy
- Use logical diagram to infer  $\mathsf{Inv} \in \forall^*$
- Can also prove absence of  $Inv \in \forall^*$

# Decidable logic is useful!

- Other logics
- Theoretical understanding of limitations and tradeoffs
- Interactive verification
  - Dividing the problem between human and machine
  - Inference schemes
  - Forms of interaction

Seeking postdocs and students





Supervised Verification of Infinite-State Systems

