What if Achilles and the tortoise were to bargain? An argument against interim agreements

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Abstract

Zeno's paradoxes of motion, which claim that moving from one point to another cannot be accomplished in finite time, seem to be of serious concern when moving towards an agreement is concerned. Parkinson's Law of Triviality implies that such an agreement cannot be reached in finite time. By explicitly modeling dynamic processes of reaching interim agreements and using arguments similar to Zeno's, we show that if utilities are von Neumann-Morgenstern, then no such process can bring about an agreement in finite time in linear bargaining problems. To extend this result for all bargaining problems, we characterize a particular path illustrated by Raiffa (1953), and show that no agreement is reached along this path in finite time.

1 Introduction

1.1 Paradoxes of motion

Zeno of Elea, a philosopher of the 5th century BC, is known for four arguments showing that motion is illusory, or at least demonstrating that motion cannot explain the displacement of bodies in space. The second, and the most famous one is the paradox of Achilles and the tortoise, which claims that after giving a head start to the tortoise, Achilles can never reach the tortoise. Summarized in Aristotle's Physics, VI, 9, the argument amounts to saying:

> In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead.

The first argument, or paradox, (Aristotle, Physics, VI, 9), is even simpler, it

> ... asserts the non-existence of motion on the ground that that which is in locomotion must arrive at the half-way stage before it arrives at the goal.

Had Zeno lived in our happier times, or had he been interested in social interactions rather than physics, he would have concluded, in the same spirit, that moving towards an agreement is impossible, or at least that such motion can never terminate, as "that which is in locomotion towards an agreement must arrive at the half-way stage before it arrives at the goal", paraphrasing Aristotle's words. Thus, if Achilles and the tortoise are moving towards an agreement on how to split a drachma, they need to agree first on how to divide half of it but then they face a similar division problem of the remaining half. Thus, Zeno could conclude that moving towards an agreement can never end.

1.2 Zeno meets Parkinson

Trying to refute Zeno's arguments, Aristotle noted that after arriving at half the way, the problem of reaching the goal is similar to the original one, but not the same. The distances in Zeno's paradoxes become ever shorter, and likewise the time intervals required to pass them. With modern understanding of infinite sums we can complete Aristotle's argument by showing that the sum of the said time intervals converges. But can we make the same argument in the case of moving towards an agreement? Do the time intervals required to agree on smaller amounts also diminish, not to speak their sum converging? Here experience seems to indicate just the opposite. The celebrated Parkinson's Law of Triviality, which refers to decisions made by committees, states

> The time spent on any item of the agenda will be in inverse proportion to the sum involved. (Parkinson (1957), Chapter 3.)

In the example that illustrates this undisputed law, Parkinson describes a committee that spends two and a half minutes on a decision to spend $10,000,000 on the construction of an Atomic Reactor, forty five minutes on the decision on the material to be used for a bike shed costing $2350, and finally, an hour and a quarter on refreshment supplies worth $57 annually.

We can conclude from this law that a committee that allocates a certain amount of money in a gradual process will never completely achieve its goal, as described above in the bargaining between Achilles and the tortoise. Parkinson, being aware of the possibility of never ending deliberations, concludes that there is a small amount of money over which people lose interest, and conjectures that the point of vanishing interest is the amount people subscribe to charity. Here we analyze a stylized model where there is no point of losing interest, or, adopting Parkinson's conjecture, we study homo economicus, the self-interest motivated agent who never donates money to charity, by definition. We claim that for such agents, there is good theoretical support for the application of Zeno's argument to motion towards an agreement.

1.3 Moving in utility space

Parkinson explains his Law of Triviality by deep epistemological insights. A more direct explanation, which we try to advance here, would be to deny the distinction between trivial and big issues. Our starting point for this argument is the von Neumann-Morgenstern (vNM) theory of utility and the bargaining theory developed by Nash (1950), based on this type of utility. Moving towards an agreement means, in such a theory, reaching some interim agreements on utility vectors u_{t_1}, u_{t_2} and u_{t_3} at times t_1, t_2 and t_3, where $t_3 > t_2 > t_1$, and $u_{t_1} > u_{t_2} > u_{t_3}$. Assuming that the the interim agreement reached in the first time interval is more important than the one reached in the second means that $u_{t_2} - u_{t_1} > u_{t_3} - u_{t_2}$. Should we expect more time to be spent on the "bigger" issue, that is, should $t_2 - t_1 > t_3 - t_2$ hold? We keep in mind the pitfall against which Luce and Raiffa (1957) cautioned us in what they call Fallacy 3: It is meaningless to say that since $u_{t_2} - u_{t_1} > u_{t_3} - u_{t_2}$, moving from u_{t_1} to u_{t_2} is *preferred* to moving from u_{t_2} to u_{t_3}. But if such a preference is meaningless why should times spent deliberating these movements be different?

1.4 Bargaining forever

We claim that while Zeno's paradox of motion in physical space does not represent a real pehnomenon, it is alive and kicking when bargainers move in the space of vNM utility vectors. To show this we use Nash's model of bargaining, but our solution to Nash's bargaining problem is a *dynamic process* rather than a specification of a Pareto agreement. More specifically, we introduce path-solutions that assign to each problem a time-parameterized path of interim agreements.

We start our study of path-solutions by assuming two simple axioms. First, we require that the path reflects restarting of bargaining at each moment of time. Second, since we assume vNM utility functions, which are defined up to positive affine transformations of individuals' utility function, we want the path-solution to be covariant under such transformations. These two requirements are enough to guarantee that such a path-solution—achieving continuously new interim agreements—can never reach in finite time a full-blown agreement on the Pareto frontier for bargaining problems with linear Pareto frontier.

The argument is simple. By the continuity assumption, there exists some time at which the bargainers reach a new interim agreement which is *not* on the Pareto frontier. By the first assumption on a path-solution, this interim agreement serves as a new status quo point for further bargaining. But due to the linearity, the new problem looks exactly as the original problem save re-scaling which by the second property of a path-solution is immaterial. Thus, it should take the bargainers the same time to reach the frontier from the new status quo point as it took them when they started. This is impossible, unless the frontier is never reached.

This reflects the same intuition as in Zeno's first argument. After arriving half the way, one faces the *same* problem, namely, to pass the *remaining* way. For physical distance the argument is flawed, because the remaining problem is *not* the same as the first one, as the remaining distance is shorter than the original one. But for moving in vNM utility space, the remaining problems is indeed the same as the original one, because there is are no shorter distances, as distances are defined up to multiplication by positive constant.

To reach the conclusion that the Pareto frontier is not reached for general problems, we assume more axioms on the path-solution. These axioms imply that the path-solution is the one described by Raiffa (1953). We show that this path solution fails to reach an agreement in finite time for any bargaining problem.

1.5 And in reality?

We do not study here empirical evidence on the theoretical claims we make. We note however, that international relationships provide a plethora of examples of bargaining, which may serve as test cases. In many conflicts an agreement is achieved directly, without going through a sequence of interim agreements. Yet some international conflicts are marked by a bargaining dynamics in which interim agreements are reached sequentially.

One example is the dozen of treaties reached in the last fifty years, mostly between the US and the USSR, over control and reduction of nuclear armaments. The most famous of these are the SALT I and II treaties and more recently the START treaty.

In this example one can argue that the Pareto frontier is not well defined. That is, it is not clear what the possible final agreements are. But in the Israeli-Palestinian conflict it is quite obvious what constitutes a final agreement. This conflict is managed through what is almost officially called the “peace process” (Quandt (2005)). Since 1993, several interim agreements have been reached, yet an agreement, namely a peace treaty between the parties, seems to be as far in the future as it was more than twenty five years ago when the process started. Of course, the political situation is complicated, but one wonders what is the causal relationship; does the complexity of the conflict require a gradual process of interim agreements, or does this type of process prevent the reaching of a solution of the conflict, as indicated by the theoretical results here.

1.6 Related work

The dynamic aspects of bargaining has been dealt with in several works, starting with an axiom of step-by-step negotiation in Kalai (1977). Later work emphasized axioms that involve the change of disagreement point while keeping the bargaining set fixed (Thomson (1987), Peters and van Damme (1993), Livne (1989), Anbarci and Sun (2009)). But none of these works introduced time explicitly into the theory. The image of the Raiffa path for two players was axiomatize by Livne (1989) and Peters and van Damme (1993). This image is described by a differential equation that relates the change of utility of one player in terms of the utility of the other player. Thus, the dynamic, temporal aspect of the path is not expressed is these works. A time parameterized path of interim agreements is described in O’Neill *et al.* (2004), but bargaining is described there by a continuum of Pareto frontiers rather then one bargaining problem.

The discrete Raiffa solution is characterized axiomatically in Anbarci and Sun (2009). In Diskin *et al.* (2010), a family of discrete generalized Raiffa solutions is axiomatized. Moreover, in this work the Raiffa time parameterized solution was introduced and has been shown to be the limit of the discrete solutions in this family.

2 Bargaining dynamics

Bargaining theory suggests various types of agreements for bargaining problems of n players. More specifically, the theory considers a family $\mathcal{B}$ of bargaining problems (S, d), where $S \subseteq R^n$ is a set of utility vectors and d is a status quo, or a disagreement point in S. It then looks for a solution function σ, which assigns to each problem (S, d) an agreement— a Pareto point, $\sigma(S, d)$ in S.

Here we are interested in dynamic processes of bargaining. Thus, we are looking for a path-solution function Π which assigns to each problem (S, d) a time parameterized path, $\pi = \Pi(S, d)$, in S. More specifically, $\pi(t)$ is a function defined for $t \geq 0$, the values of which are in S, and $\pi(0) = d$. We think of $\pi(t)$ as an interim agreement achieved at time t, which serves as the status quo point for further bargaining.

The paths π and $\hat{\pi}$ are *similar* if each is obtained from the other by linearly speeding up or slowing down. That is, if there exists $c > 0$ such that $\hat{\pi}(t) = \pi(ct)$. Two solutions Π and $\hat{\Pi}$ are *similar*, if there exists $c > 0$ such that for each (S, d), $\hat{\Pi}(S, d)(t) = \Pi(S, d)(ct)$ for all t.

We say that the path π *does not reach an agreement* if for all t, $\pi(t)$ is not Pareto. Obviously, any path which is similar to such a path π has the same property.

We say that a solution Π is continuous (differentiable) if for each problem (S, d) in $\mathcal{B}$ the path $\Pi(S, d)$ is continuous (differentiable).[1]

2.1 Bargaining forever: the linear case

A bargaining problem (S, d) is linear if there is $a > 0$ in R^n, such that $S = \{x \in R^n \mid ax \leq 1\}$ and $ad < 1$. The linear bargaining problem $(S_0, 0)$, where S_0 is defined by $a = (1, \ldots, 1)$ and $d = 0$ is the *division of a drachma* problem.

To show that an agreement is never reached in linear problems we need only two axioms. The first states that the path $\Pi(S, d)$ means that bargaining restarts every moment. That is, the interim agreement reached at time t, $\Pi(S, d)(t)$, is the status quo point from which the bargaining process restarts.

Axiom 1 (Restarting)
Let $d' = \Pi(S, d)(t)$. *Then* $\Pi(S, d')(t') = \Pi(S, d)(t + t')$.

The second axiom reflects the assumption that the utility of each player is given by a vNM utility function, and therefore it is determined up to a positive

[1] Only derivative from the right is required at $t = 0$.

affine transformation. Thus, the path should be covariant with respect to such transformations to which we refer, in the sequel, as utility re-scaling.

Axiom 2 (Scale covariance)
If $a, b \in R^n$, $a > 0$ and $\sigma(x) = (a_i x_i + b_i)_{i \in N}$, then $\Pi(\sigma(S), \sigma(d)) = \sigma(\Pi(S, d))$.

Theorem 1 *Suppose that $\mathcal{B}$ contains all linear problems. If Π is a continuous solution that satisfies axioms 1 and 2, then for each linear problem (S, d), $\Pi(S, d)$ does not reach an agreement.*

2.2 How slowly is an agreement approached?

We now give an explicit expression of path solutions for the division of the drachma problem when axioms 1 and 2 are satisfied. For simplicity we assume symmetry. A point, set, or path in R^n are *symmetric* if they are invariant under permutations of coordinates.

Axiom 3 (Symmetry)
If S and d are symmetric, then $\Pi(S, d)$ is also symmetric.

We also assume that the players improve upon their initial status quo point. This is expressed in the next axiom.

Axiom 4 (Individual rationality)
For $t > 0$, $\Pi(S, d)(t) \geq d$, and if d is not Pareto, then $\Pi(S, d)(t) \neq d$.

Observe, that if Π satisfies the axioms of individual rationality and restarting, then for any $t' > t$, $\Pi(S, d)(t') \geq \Pi(S, d)(t)$, and if $\Pi(S, d)(t)$ is not Pareto, then $\Pi(S, d)(t') \neq \Pi(S, d)(t)$.

Theorem 2 *Suppose that $\mathcal{B}$ contains all linear problems, and let $(S_0, 0)$ be the division of the drachma problem. If Π is a continuous solution that satisfies axioms 1-4, then $\Pi(S_0, 0)$ is similar to the path π which satisfies for each i,*

$$\pi_i(t) = (1/n)[1 - e^{-t}].$$

The explicit formula in Theorem 2 demonstrates the claim of Theorem 1: since $1 - e^{-ct} < 1$ for each t, the path does not have any Pareto point on it. Yet, the path converges at infinity to a Pareto agreement.[2]

Corollary 1 *Under the conditions of Theorem 2, for the division of a drachma problem $(S_0, 0)$, $\lim_{t \to \infty} \Pi(S_0, 0)(t) = (1/n, \ldots, 1/n)$.*

[2] Axiom 4 is not essential for Theorem 2. The theorem holds without this axiom if we allow the constant c to be negativ. But for Corollary 1, axiom 4 is required. Without it, the path can get further away from the Pareto frontier as $t \to \infty$.

The proof of Theorem 1 hinges on the fact that changing the status quo point in the interior of a linear problem results in a linear problem and that all linear problems are the "same" in the sense that they can be transformed into each other by re-scaling of utility. This echoes Zeno's first argument: After making a certain way towards an agreement we face exactly the same problem and therefore it is impossible to reach an agreement in finite time.

Such an argument cannot be used for non-linear problems, and indeed the theorem does not hold in general unless we assume some restrictions on the path, which we do next.

2.3 Bargaining forever on the Raiffa path-solution

We define the Raiffa path for general bargaining problems and show that agreement is never reached on this path. The path computed in Theorem 2 for the division of the drachma problem is a special case of the Raiffa path. In order to define the Raiffa path, we first describe the set $\mathcal{B}$ in detail.

A pair (S, d) is in $\mathcal{B}$, if S is closed, convex, comprehensive[3], and positively bounded.[4] In addition we require that all the boundary points of S are Pareto.

For each problem (S, d) the function $m_i(S, d) = \max\{x_i \mid (x_i, d_{-i}) \in S\}$ is well defined, and obviously, $m(S, d) \geq d$. The *Utopia point* for a bargaining problem (S, d) is $m(S, d) = (m_i(S, d))_{i \in N}$.

Definition 1 *The* Raiffa path-solution, Π^R, *assigns to each problem* (S, d) *the path* $\pi = \Pi^R(S, d)$ *defined by the differential equation*

$$\pi'(t) = m(S, \pi(t)) - \pi(t), \tag{1}$$

with the initial condition $\pi(0) = d$. *Thus, starting in d at time 0, the path moves at each point of time towards the Utopia point.*

It is shown in Diskin *et al.* (2010) that for each problem $(S, d) \in \mathcal{B}$ this differential equation has a unique solution, and it converges to a Pareto point of S.

Let $\hat{\pi}(t) = (1/n)[1 - e^{-nt}]$ be a path similar to the solution for the division of the drachma problem, $(S_0, 0)$, in Theorem 2. Then for each i, $\hat{\pi}_i'(t) = 1 - n\hat{\pi}_i(t) = 1 - (n-1)\hat{\pi}_i(t) - \hat{\pi}_i(t)$. Since $1 - (n-1)\hat{\pi}_i(t) = m_i(S_0, \hat{\pi}(t))$, $\hat{\pi}$ is the Raiffa path solution of the division of the drachma problem. As the right hand side of the differential equation (1) is covariant with utility re-scaling, it follows that if Π is a differentiable path solution that satisfies axioms 1-4, then there exists $c > 0$ such that for each linear problem (S, d), $\Pi(S, d)(t) = \Pi^R(S, d)(ct)$.

The Raiffa path-solution which extends the solution of linear problems to all problems, also suffers from the deficiency of not being able to bring the bargainers to an agreement.

Theorem 3 *The Raiffa path-solution of any problem does not reach an agreement.*

[3]That is, for each $x \in S$, $\{y \mid y \leq x\} \subseteq S$.

[4]That is, there exists $a > 0$ in R^n and a constant α, such that $bx \leq \alpha$ for each $x \in S$.

2.4 Axiomatizing the Raiffa path-solution

By adding two axioms to the previous ones, we can characterize the family of solutions that are similar to the Raiffa path-solution. The first requires that the larger the bargain set is, the higher that bargainers aspire to.

Axiom 5 (Monontonicity)
If (S, d) and (T, d) are two problems in $\mathcal{B}$ such that $S \subseteq T$, then $\Pi'(T, d)(0) \geq \Pi'(S, d)(0)$.

The next axiom says that the only part of the bargaining problem which is relevant to the determination of the path is the set of individually rational outcomes.

Axiom 6 (Relevance)
If (S, d) and (T, d) are two problems in $\mathcal{B}$ such that $\{x \mid x \in S, x \geq d\} = \{x \mid x \in T, x \geq d\}$, then $\Pi(S, d) = \Pi(T, d)$.

Theorem 4 *If Π is a differentiable path-solution that satisfies axioms 1-6, then it is similar to the Raiffa path-solution Π^R.*

3 Proofs

Proof of Theorem 1. By the scale covariance axiom it is enough to prove the claim for the division of drachma problem $(S, 0)$. Let $\pi = \Pi(S, 0)$. Suppose that the set $\{t \mid \pi(t) \text{ is Pareto}\}$ is not empty. By the continuity of π it has a minimal point T which is the first time the path reaches the Pareto frontier of S. Since $\pi(0) = 0$, $T > 0$. Choose t, $0 < t < T$, and let $d' = \pi(t)$. The problem $(S, 0)$ can be transformed by re-scaling into the problem (S, d'). Therefore, by the scale covariance axiom the path π is transformed by the same function into $\Pi(S, d')$. In particular, T is also the first time the path $\Pi(S, d')$ reaches the Pareto frontier of S. But this is contradicted by the restarting axiom, since $\Pi(S, d')(T - t) = \pi(t + (T - t)) = \pi(T)$ which is Pareto, and $T - t < T$. ∎

Proof of Theorem 2. Let $\pi = \Pi(S_0, 0)$ be a path that satisfies the conditions in the theorem. By the symmetry there exists a real valued function $f(t)$, such that $\pi_i(t) = f(t)$ for each i. Fix t and let $d' = \pi(t)$. Then, $d'_i = f(t)$. The transformations $(1 - nf(t))x_i + f(t)$ transform the problem $(S_0, 0)$ into the problem (S_0, d'). Hence, by axiom 2, $\pi = \Pi(S_0, 0)$ is transformed by these transformations into $\Pi(S_0, d')$. Thus, for any t', $\Pi_i(S_0, d')(t') = [1 - nf(t)]f(t') + f(t) = f(t) + f(t') - nf(t)f(t')$. By axiom 1, $\Pi_i(S_0, d')(t') = f(t + t')$. We conclude that for each non-negative t and t',

$$f(t + t') = f(t) + f(t') - nf(t)f(t') \tag{2}$$

By Theorem 1, for each $t \geq 0$, $f(t) < 1/n$. By the axiom of individual rationality for each $t > 0$, $f(t) > 0$. Thus, for each $t > 0$, $0 < 1 - nf(t) < 1$.

Hence, the function $g(t) = \ln[1 - nf(t)]$ is well defined, continuous, and for each $t > 0$, $g(t) < 0$. It is easy to check that by (2), for each $t > 0$ and $t' > 0$, $g(t+t') = g(t) + g(t')$. The continuity of g implies that there is $c > 0$ such that $g(t) = -ct$. Thus, $f(t) = (1/n)[1 - e^{-ct}]$, and π is similar to the path described in the theorem. ∎

Proof of Theorem 3. Let $(S, 0)$ be a problem for which $m_i(S, 0) = 1$ for each i, and let $\pi = \Pi^R(S, 0)$. Then, for each t such that $\pi(t) \in S$, $\pi(t) = \int_0^t m(S, \pi(\tau)) - \pi(\tau) d\tau$. Then, for $||\cdot||$, the L_1 norm in R^n,

$$||\pi(t)|| = ||\int_0^t m(S, \pi(\tau)) - \pi(\tau) d\tau|| \leq \int_0^t ||m(S, \pi(\tau)) - \pi(\tau)|| d\tau.$$

By the axioms of individual rationality and restarting, for each t, $\pi(t) \geq 0$. By the comprehensiveness of S, $m(S, \pi(t)) \leq m(S, 0)$. Thus, $m(S, \pi(\tau)) - \pi(\tau) \leq m(S, 0) - \pi(0)$. We conclude that the integrand in the above integral satisfies $||m(S, \pi(\tau)) - \pi(\tau)|| \leq ||m(S, 0) - \pi(0)|| = ||(1, \ldots, 1)|| = n$. Hence, $t \geq ||\pi(t)||/n$.

Suppose that for some t, $\pi(t)$ is Pareto, and T is the first time this happens. Note, that $m(S, \pi(T)) = \pi(T)$. Else, for some i, $m_i(S, \pi(T)) > \pi_i(T)$, contrary to $\pi(T)$ being Pareto. Observe also that the simplex—the convex hull of the unit vectors in R^n—is contained in S. This shows that $||\pi(T)|| \geq 1$, because otherwise, $m(S, \pi(T)) \neq \pi(T)$. Thus $T \geq 1/n$. Since every problem (S, d) can be transformed by utility re-scaling into a problem of the type now discussed, it follows that for any problem (S, d), the first time T that $\Pi^R(S, d)$ is Pareto must be at least $1/n$.

Suppose now that T is the first time that $\Pi^R(S, d)$ is Pareto. Let $t = T - 1/(2n)$ and $d' = \Pi^R(S, d)(t)$. By the axiom of restarting, $\Pi^R(S, d')(1/(2n)) = \Pi^R(S, d)(t + 1/(2n)) = \Pi^R(S, d)(T)$. Thus, $\Pi^R(S, d')$ reaches a Pareto point in a time which is less then $1/n$, contrary to what we have proved. This shows that a Raiffa path can never reach a Pareto point. ∎

Proof of Theorem 4. Suppose Π is a differentiable path-solution that satisfies axioms 1-6. We have shown that for the split the drachma problem, $(S_0, 0)$, there exists $c > 0$ such that $\Pi(S_0, 0)(t) = \Pi^R(S_0, 0)(ct)$. Since, $m(S, x) - x$ is covariant under utility re-scaling, it follows that for each linear problem (S, d), $\Pi(S, d)(t) = \Pi^R(S, d)(ct)$. For a general problems (S, d) we fix i and define two problems, (S^-, d) and (S^+, d) such that $S^- \subseteq S \subseteq S^+$.

The problem (S^+, d) is a linear problem and therefore, as we have shown, $\Pi'(S^+, d)(0) = c(\Pi^R)'(S^+, d)(0)$. By the monotonicity axiom $\Pi'(S, d)(0) \leq c(\Pi^R)'(S^+, d)(0)$.

The problem (S^-, d) agrees with a linear problem on the set of individually rational points, and thus $\Pi'(S^-, d)(0) = c(\Pi^R)'(S^-, d)(0)$ by the relevance axiom. By monotonicity, $\Pi'(S, d)(0) \geq c(\Pi^R)'(S^-, d)(0)$.

Finally, the problems are constructed such that

$$(\Pi_i^R)'(S^-, d)(0) = (\Pi_i^R)'(S^+, d)(0) = c(\Pi_i^R)'(S, d)(0).$$

Thus we conclude that $\Pi'_i(S,d)(0) = c(\Pi_i^R)'(S,d)(0)$. Since this is true for each i, it follows that $\Pi'(S,d)(0) = c(\Pi^R)'(S,d)(0)$.

The details of the construction of S^+ and S^- are in the proof of Theorem 1 in Diskin *et al.* (2010). For $t > 0$, let $d' = \Pi(S,d)(t)$. Then, by the restarting axiom, $\Pi'(S,d)(t) = \Pi'(S,d')(0) = c(\Pi^R)'(S,d')(0) = c(\Pi^R)'(S,d)(t)$. ∎

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