Bootstrap and Resampling Methods, Spring 2023

Homework exercise 2

Due date: 30 April 2023 before class

Submission format: Please include your code in your submission as an appendix.

1. **Problem 11.13 from the book: Comparing bootstrap and jackknife empirically.** Generate 100 random samples of size 20 from a normal populations $N(\psi, 1)$, with parameter $\psi = 1$.

   (a) For each sample compute the bootstrap and jackknife estimate of $\mathrm{Var}(\hat{\psi})$ for $\hat{\psi} = \bar{X}$. Compute the mean and the standard deviation of these variance estimates over the 100 repetitions.

   (b) Repeat (a) for the (bad) estimate $\hat{\psi} = \bar{X}^2$ and compare the results. Give an explanation for your findings.

2. **Comparing confidence interval methods.** Generate 100 random samples of size 20 from an exponential distribution $\exp(1)$. The true mean is $\psi = 1$. Compute 100 standard, bootstrap-t and percentile intervals, and describe their coverage behavior: how often does 1 fall below the lower limit, how often above the upper? Explain your results.

3. **Problem 14.13 from the book: Behavior of BC$_a$ acceleration values.** Using the formulas $z[\alpha]$ we derived in class, and assuming $\hat{z}_0 = 0$ and $\hat{\psi} = 0$, do the following:

   (a) Set $\hat{a} = 0$ and plot $z[\alpha]$ against α for 100 equally spaced values of α (between $\alpha = 0.005$ and $\alpha = 0.995$). Verify that $z[\alpha]$ is monotone in α, so the CI size increases as the confidence level increases (as expected).

   (b) Now repeat (a) for $\hat{a} = \pm 0.1, \pm 0.2, ..., \pm 0.5$. For what values of $\hat{a}$ and α does $z[\alpha]$ fail to be monotone? Interpret this result.

   (c) To get some idea how large a value of $\hat{a}$ one might expect in practice, generate a standard normal sample $x_1, ..., x_{20}$. Compute the acceleration $\hat{a}$ for $\hat{\psi} = \bar{x}$. Create a more skewed sample by defining $y_i = \exp(x_i)$ and compute the acceleration $\hat{a}$ for $\hat{\psi} = \bar{y}$. Repeat this for $z_i = \exp(y_i)$. Repeat the exercise 10 times and summarize the results. How large a value of $\hat{a}$ seems likely to occur in practice?

4. **Problem 15.6 from the book: Uniformity of permutation p values under null.** The p values from a permutation test cannot have exactly a uniform distribution, because

there is a finite number of permutations, which we can denote by $M = \binom{n+m}{n}$. Show that:

$$\text{Prob}_{H_0}\left\{\text{ASL}\text{perm} = \frac{k}{M}\right\} = \frac{1}{M}\ , \quad \text{for } k = 1, 2, ..., M.$$