Exercise 1

Let L_ϵ to be the language of all 2CNF formulas φ, such that at least $\left(\frac{1}{2}+\epsilon\right)$ of φ's clauses can be satisfied. Prove that L_ϵ is $\mathbf{NP}$-hard for any $\epsilon > 0$.

Exercise 2

You proved in class that each undirected graph $G = (V, E)$ possess a cut with at least $\frac{|E|}{2}$ edges. Give a deterministic poly-time algorithm, which constructs such cut for a given G. Prove that your algorithm satisfies all the conditions. (hint: one of the possible approaches is to use algorithm, greedy on conditional expectations).

Exercise 3

Assume that there exists a universal TM $\mathcal{U}$, such that $\mathcal{U}(\langle M\rangle, x) = M(x)$ for any TM M and any $x \in \{0,1\}^*$. Moreover, if computation M on x requires $f(n)$ space then computation $\mathcal{U}(\langle M\rangle, x)$ requires $O(f(n))$ space.

(1) *Prove that $Space(logn) \subsetneq Space\left(log^2 n\right)$. (hint: use diagonalization method).*
(2) *Conclude that $\mathbf{L} \subsetneq \mathbf{PSPACE}$.*

GOOD LUCK