**Exercise 1**

*Let $A$ be a minimazation problem on graphs.*

*Let $kA$ be the language: $kA = \{G \mid A(G) \leq k\}$*

*(1) Prove that if there exists a poly-time (7)-approximation algorithm to $A$, then gap-$A[4, 28]$ is poly-time solvable.*

*(2) Prove that if $kA \in P$ for all $k$ then gap-$A[4, 28]$ is poly-time solvable.*

**Exercise 2**

*IS - the optimization problem of finding $\frac{|maximal\ independent\ set|}{|V|}$*

*VC - the optimization problem of finding $\frac{|minimal\ vertex\ cover|}{|V|}$*

*Prove:*

*gap-IS$[\alpha, \beta]$ is poly-time solvable iff gap-VC$[1-\beta, 1-\alpha]$ is poly-time solvable.*

**Exercise 3**

*Define the complexity class $C$:*

*$C$ is the class of all languages that can be verfied by a TM that has:*

*(a) Polynomial Work Tape*

*(b) Exponential Witness Tape that must be read bit by bit without going back*

*Prove: $C = PSPACE$*

**Exercise 4**

*Given the string: $1^n < M >$ (where $< M >$ is the encoding of a poly-time TM) define the following graph:*

$$\begin{aligned} G_M^n &= (V_M^n, E_M^n) \\ V_M^n &= \{0,1\}^n \\ E_M^n &= \{(u,v) \mid (u,v) \in L_M\} \end{aligned}$$

*(i.e. the edges are all the binary strings of length $2n$ accepted by $M$)*

*Prove: $CLIQUE - G_M^n \in NEXPTIME$*

GOOD LUCK