Exercise 1

Given languages A and B over Σ_A and Σ_B respectively we say that $f : \Sigma_A^* \to \Sigma_B^*$ is poly-time Karp' reduction from A to B (denoted $A \leq_P^{'} B$) if f is poly-time computable and one of the following holds:

(1) $\forall x \ : \ x \in A \Leftrightarrow f(x) \in B$.

(2) $\forall x \ : \ x \in A \Leftrightarrow f(x) \notin B$.

Prove that $\mathbf{NP}$ is closed under Karp' reduction if and only if $\mathbf{NP} = \mathbf{coNP}$.

Exercise 2

$k - NAE$ is a language of formulas such that:

- *ϕ is in k-CNF form*
- *ϕ has a satisfying assignment such that in each clause at least one literal is assigned to False.*

Prove that $3 - NAE$ is $\mathbf{NPC}$ using the following steps:

(1) *Show that $3 - SAT \leq_P 4 - NAE$ using the following reduction. Given a set of variables $x_1, \ldots, x_n$ add a new variable z. Every clause of the form $x_{i_1} \vee x_{i_2} \vee x_{i_3}$ is transformed to $x_{i_1} \vee x_{i_2} \vee x_{i_3} \vee z$.*

(2) *Show that $4 - NAE \leq_P 3 - NAE$ using the following reduction. Let $i - th$ clause be $x_1 \vee x_2 \vee x_3 \vee x_4$. Construct the following two clauses $x_1 \vee x_2 \vee w_i$ and $x_3 \vee x_4 \vee \overline{w_i}$, where w_i is a new variable that appears in these clauses only.*

Exercise 3

Recall that an undirected graph $G = (V, E)$ is called biparty *if one can represent V as a disjoint union of non-empty sets A and B such that any edge of G connects node from A with node from B.*

Let

$$BIPARTY = \{G \mid G \text{ is an undirected biparty graph } \}$$

and $\overline{BIPARTY}$ be its complement language.

(1) *Prove that $G \in BIPARTY$ if and only if G does not have a cycle with odd number of nodes.*

(2) *Construct a verifier to prove that $\overline{BIPARTY} \in \mathbf{NL}$.*

Exercise 4

Assume that in the definition of $\mathbf{NL}$ we allow the verifier to move on the witness tape in both left and right directions. Denote the resulting complexity class by $\mathbf{C}$. Prove that $3 - SAT \in \mathbf{C}$.

Exercise 5

Let $IND_{1003} = \{G \mid G$ is undirected graph containing an independent set of size 1003$\}$.

Prove that $IND_{1003} \in \mathbf{L}$.

GOOD LUCK