Exercise 1

We will call graph $G = (V, E)$ *square if*

- V *can be partitioned into* q *cliques* $C_1, \ldots, C_q$
- V *can be partitioned into* q *independent sets* $I_1, \ldots, I_q$
- $|C_i \cap I_j| = 1$ *for any* $1 \leq i, j \leq q$.

Prove that it is $\mathbf{NP}$*-hard to decide whether* G *is square.*

Exercise 2

Assume that $\mathbf{P} \neq \mathbf{RP}$. *What conclusions can you make?*

Exercise 3

In homework 2 exercise 2 you constructed the following reductions: $3SAT \preceq 4NEQ \preceq 3NEQ$.

(1) *Prove that this reduction can be resticted to* $E3SAT \preceq E4NAE \preceq E3NAE$.
(2) *Track the gap under this restricted reduction.*
(3) *For what* $\alpha, \beta \in [0, 1]$ *the* $gap - E3NAE\,[\alpha, \beta]$ *can be immediately concluded to be* $\mathbf{NP}$*-hard?*

Exercise 4

We define the following problems over compressed graphs:

- $CCSG_V$ *is a version of* CSG_V. *Thus, a* $CCSG_V$ *instance is a tuple* $U = (1^n, \Sigma, M)$, *where* M *is a poly-time Turing Machine calculating the constraints. Thus, for nodes* $u, v \in \{0,1\}^n$ *and colors* $i, j \in \Sigma$ *we say that coloring* u *with* i *and* v *with* j *satisfies the constrain on* (u, v) *if and only if* $M(u, v, i, j) = 1$.
- $CCSG_\Delta$ *is a version of* CSG_Δ. *Its instance is a tuple* $U = (1^n, q, M)$, *where* M *is a poly-time Turing Machine calculating the constraints. Thus, for nodes* $u, v \in \{0,1\}^n$ *and* $i \in \{0, 1, \ldots, q-1\}$ *we say that difference* i *is allowed on* (u, v) *if and only if* $M(u, v, i) = 1$.
- $CCSG_E$ *is a version of* CSG_E. *Thus, a* $CCSG_E$ *instance is a tuple* $U = (1^n, M_E, \Sigma, M)$, *where* M *and* M_E *are poly-time Turing Machines. For any nodes* $u, v \in \{0,1\}^2$ *we say that* (u, v) *is an edge if and only if* $M_E(u, v) = 1$. *For any edge* (u, v) *we say that coloring* (i, j) *satisfies the constrain if and only if* $M(u, v, i, j) = 1$.
- $C\chi$ *is a version of chromatic number problem. Its instance is just compressed graph* $U = (1^n, M_E)$, *where* M_E *is a poly-time Turing Machine computing the edges. For any nodes* $u, v \in \{0,1\}^2$ *we say that* (u, v) *is an edge if and only if* $M_E(u, v) = 1$.

Prove or disprove the following statements:

(1) $CCSG_\Delta \preceq C\chi$.
(2) $CCSG_V \preceq CCSG_\Delta$.

GOOD LUCK