Exercise 1

Prove or disprove:

(1) *There exists a function* $f:\mathbb{N}\to\mathbb{N}$ *such that* $f(n)=o(f(n))$.
(2) *There exists a function* $f:\mathbb{N}\to\mathbb{N}$ *such that* $f(n)=\omega(f(n))$.

Exercise 2

For any two languages L_1, L_2 *over alphabet* Σ *define their union by*

$$L_1 \cup L_2 = \{x\in\Sigma^* \mid x\in L_1 \text{ or } x\in L_2\}.$$

Complexity class $\mathbf{C}$ *is said to be closed under the union if* $L_1\cup L_2\in\mathbf{C}$ *whenever* $L_1, L_2\in\mathbf{C}$.

Prove or disprove the following statements:

(1) $\mathbf{P}$ *is closed under union.*
(2) $\mathbf{NP}$ *is closed under union.*

Exercise 3

For a language L *over alphabet* Σ *define*

$$L^* = \{x\in\Sigma^* \mid \exists k\in\mathbb{N} \text{ and } \exists x_1,\dots,x_k\in L \text{ such that } x=x_1\dots x_k\},$$
$$\overline{L} = \{x\in\Sigma^* \mid x\notin L\}.$$

Prove or disprove: $\overline{L^*}=\overline{L}^*$ *for any language* L.

Exercise 4

For any complexity class $\mathbf{C}$ *define its complement to be* $\mathbf{coC}=\{\overline{L}\mid L\in\mathbf{C}\}$.

Prove or disprove: $\mathbf{coNP}$ *is closed under* * *action.*

Exercise 5

For any complexity classes $\mathbf{C}_1,\mathbf{C}_2$ *prove that:* $\mathbf{C}_1\subseteq\mathbf{C}_2\Rightarrow\mathbf{coC}_1\subseteq\mathbf{coC}_2$.

Exercise 6

We say that graphs G *and* H *are isomorphic if nodes of* G *can be reordered to make it identical to* H. *Define a language*

$$ISO=\{\langle G,H\rangle \mid G \text{ and } H \text{ are isomorphic}\}.$$

Prove that $ISO\in\mathbf{NP}$.

Exercise 7

A triangle in undirected graph is a 3-clique. Define a language

$$TRIANGLE=\{G\mid G \text{ has a triangle}\}.$$

Prove that $TRIANGLE\in\mathbf{P}$.

GOOD LUCK