# Quasicrystals

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## I. Reading

- M. Senechal, *Quasicrystals and Geometry,* Chapter 4: "Order on the line."

## II. Problems

1. We showed in class that in the (perfect) Fibonacci crystal the ratio of the number $\#(L)$ of intervals of length $L$ and the number $\#(S)$ of intervals of length $S$ is

$$\#(L) : \#(S) = \tau : 1 = \frac{1+\sqrt{5}}{2} : 1. \tag{1}$$

Find, by direct use of (1), the relative numbers of the allowed two interval sequences $\#(LS)$ : $\#(SL)$ : $\#(LL)$. Compare with Senechal's calculation (pages 118-119) which is based on a subdivision of the projected strip, or "window," into three subintervals, corresponding to the three allowed sequences.

2. We showed in class that the points of $\mathbb{Z}^2$ which are projected onto the line $\ell$ of slope $1/\tau$ to form the Fibonacci crystal are given by

$$(||\frac{m}{\tau}||, m - ||\frac{m}{\tau}||), \quad m \in \mathbb{Z}, \tag{2}$$

where $||x||$ denotes the nearest integer to $x$.

(a) Using the fact that the projection of the unit vectors of $\mathbb{Z}^2$ on the line $\ell$ are given by $\tau/\nu$ and $1/\nu$, respectively, where $\nu^2 = \tau^2 + 1$, show that the coordinate of the $m^{th}$ point of the Fibonacci crystal is given by (Senechal Eq. 4.27)

$$x_m = \frac{m}{\nu} + \frac{1}{\tau\nu}||\frac{m}{\tau}||. \tag{3}$$

(b) Show that the Fibonacci crystal is closed under a rescaling by a factor of $\tau$, *i.e.* that if $x_m$ is in the crystal then so is $x_n = \tau x_m$. Determine the value of $n$.

(c) Show that the points $x_m$ can be expressed as a modulated periodic crystal

$$x_m = ma + f(m), \tag{4}$$

where the lattice spacing $a = \nu/\tau^2$, and $f(x)$ is a periodic function with period $\tau$.

(d) Using the Fourier series expansion of $f(x)$, calculate the Fourier transform of the Fibonacci crystal, as we did in class for the case of a simple modulation.