Exercise No. 1: Classical Field Theory and QM

Classical Field Theory

1. Using the Lagrangian of the electromagnetic field

$$\mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} \ ,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and taking $c = 1$,

   (a) Compute the energy density in the absence of any charges or currents in terms of **E** and **B**. Bring the expression to the known form.
   Hint: remember that the Hamiltonian density can be shifted by a total divergence.

   (b) The Lagrangian in the presence of charges and currents is

$$\mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + j_\mu A^\mu \ .$$

   Find the equations of motion in the relativistic notation and then write them in terms of **E** and **B**.

Basic Quantum Mechanics

2. Find the energy spectrum and the eigenstates of a non-relativistic spinless electron in a constant magnetic field.

3. A spin-$\frac{1}{2}$ particle is placed in a rotating magnetic field

$$\mathbf{B} = B_1 \left(\hat{\mathbf{x}} \cos \omega t + \hat{\mathbf{y}} \sin \omega t \right) + B_0 \hat{\mathbf{z}} \ .$$

   (a) What are the energy levels and the eigenstates of the system at $t = 0$?

   (b) Calculate the exact time evolution of a spin pointing initially up in the z-direction.