Exercise No. 7: Electromagnetic Interactions

1. (a) Prove that momentum is conserved in processes of photon emission and absorption in perturbation theory.

   (b) Show that a free electron cannot emit a single photon.

2. A hydrogen atom in its $2p$ state is at rest, i.e., has zero momentum, in a large box. Calculate the details of the spontaneous emission from the atom. In your calculation, devise the appropriate form of the dipole approximation for this situation. Compute the life-time of the $2p$ state.

3. A particle of mass m and charge e, whose unperturbed Hamiltonian is

$$H_0 = \frac{p_z^2}{2m} + \frac{1}{2}m\omega^2 z^2 \ ,$$

   is coupled the electromagnetic field.

   (a) Derive the selection rules for electric dipole transitions and state the possible energies and polarizations of the emitted or absorbed photon.

   (b) What is the cross-section for such a particle, initially in its ground state, to absorb a single photon of energy $n\hbar\omega$ and any polarization at an angle θ?

4. The quantized electromagnetic field is coupled to a classical current $\mathbf{j}(\mathbf{r}, t)$ with no net charge. At $t = 0$ the field is in the vacuum state. What is the state of the field at a later time t? Compute the probability of finding n photons with a wave-vector $\mathbf{k}$ and polarization λ.