Exercise No. 4: Path Integrals

1. Find the propagator $U(x_b, T; x_a, 0)$ for a particle of mass m in one dimension under a constant external force f.
2. (a) Complete the computation of the propagator of a one-dimensional harmonic oscillator

   $$U(x_b, T; x_a, 0) = A(T) \exp\left\{ \frac{im\omega}{2\hbar \sin \omega T} \left[(x_a^2 + x_b^2) \cos \omega T - 2x_a x_b \right] \right\} \ ,$$

   where $A(T) = \left(\frac{m\omega}{2\pi i\hbar \sin \omega t} \right)^{1/2}$.

   (b) At $t = 0$ the system is in the coherent state $|\alpha\rangle$ with α being real. Find $|\psi(t)\rangle$ using the propagator found in 2a.
3. Compute the propagator $U(\mathbf{x}_b, t_b; \mathbf{x}_a, t_a)$ for a particle of charge e and mass m in a constant external magnetic field B in the z direction whose Lagrangian is

   $$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{eB}{2c}(x\dot{y} - y\dot{x}) \ .$$

   Hint: transform to a rotating coordinate system.
4. (a) Use the path integral approach in order to compute the partition function of a harmonic oscillator in equilibrium at a temperature T.

   (b) Extract the energy levels of the harmonic oscillator using 4a.