

Exercise No. 10: Second Quantization II

1. Construct explicit 4×4 matrices to represent the fermion creation and annihilation operators a_0 , a_0^\dagger , a_1 and a_1^\dagger for two one-particle states. Check the anti-commutation relations.
2. Suppose that the wave-function of an N -fermion system is a Slater determinant of orthonormal functions ϕ_i . Using the second quantization formalism show that the pair correlation function of the state factors as for plane waves.

Hint: consider the operators $a_i = \int d^3r \phi_i^*(\mathbf{r}) \psi(\mathbf{r})$.

3. (a) Compute the pair correlation function for a system of free non-interacting spinless bosons in a state $|\Phi\rangle$.
(b) Find the pair correlation function if the boson occupation numbers are given by

$$n_{\mathbf{p}} = ce^{-\alpha(\mathbf{p}-\mathbf{p}_0)^2/2},$$

where \mathbf{p} is the momentum of the bosons.