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Phonon-mediated dissipation in micro- and nano-mechanical systems

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Abstract

We study phonon-mediated dissipation mechanisms in small-scale mechanical systems in light of recent efforts to design high-Q micrometer- and nanometer-scale electro-mechanical systems (MEMS and NEMS). We consider dissipation mechanisms which redistribute the energy within a resonator, such as thermoelastic damping, the Akhiezer effect, and the Landau–Rumer effect, as well as the radiation of energy away from the resonator into its surroundings.

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1. Introduction

Micro-electro-mechanical systems (MEMS) and more recently nano-electro-mechanical systems (NEMS) are being developed aggressively for a variety of applications, as well as for the experimental investigation of the quantum behavior of mesoscopic mechanical systems [1]. For these pursuits, it is desired to construct systems with very little loss of energy or very high quality factors Q. Unfortunately, even though one would naively expect otherwise, it has been consistently observed that the quality factors of resonators decrease with size significantly, even when made from pure single-crystal materials. It is therefore of great importance to develop a theoretical understanding of energy dissipation in mechanical resonators when one approaches submicron scales.

A variety of different mechanisms, some of which have been observed experimentally in mesoscopic systems [2–10], may contribute to the dissipation of energy in mechanical resonators, and thus impose limits on their quality factors. The dissipated energy is transferred from a particular mode of the resonator, which is driven externally, to energy reservoirs formed by all the other degrees of freedom of the system. Here, we focus our attention on phonon-mediated dissipation mechanisms, arising from energy transfer between the driven mode of the resonator and all other modes of *vibration*, assumed to be thermally excited. In the next two sections, we shall review these dissipation mechanisms distinguishing between an "internal phonon bath", occupying the normal modes of vibration of the resonator itself, and an "external phonon bath", occupying the

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vibrational degrees of freedom of the surrounding to which the resonator is connected. These thermal energy reservoirs will be present even in the most ideal resonator and should therefore impose fundamental and unavoidable upper limits on the quality factors of any resonator.

2. Internal phonons—redistribution of energy

The basic length scales associated with a vibrating resonator are set by its dimensions, e.g. length L, width w, and thickness t. The wavelength of the driven mode of the resonator is on the order of the overall size L of the resonator because, typically, only the lowest frequency modes of the resonator are driven. These *geometric* length scales are to be compared with two *thermal* length scales associated with the internal phonon bath—the dominant wavelength λ_T of thermal phonons, and the thermal phonon mean-free path ℓ_T—both depending on the average temperature T of the resonator.

Two very distinct regimes of behavior with respect to dissipation can be identified by comparing the thermal phonon mean-free path to the size of the system: (1) In the *diffusive regime* ($\ell_T \ll L$) the thermal phonons scatter so often that they can be treated as a collective or a gas using a hydrodynamic picture; (2) In the *ballistic regime* ($\ell_T \gg L$) the thermal phonons very rarely scatter off each other and are treated as individual particles using a kinetic picture. When the thermal phonon wavelength λ_T also becomes comparable to, or exceeds, one of the dimensions of the resonator the resonator crosses over to the mesoscopic regime where quantum behavior is expected.

As a first step we [11] have considered the extreme diffusive regime where the phonons thermalize so rapidly that they may be viewed simply as setting up a temperature field. In this regime, the complicated interaction between the resonator mode and the internal thermal phonon bath is captured by a single parameter—the material's thermal expansion coefficient α—which couples changes of length to changes of temperature. This coupling leads to a dissipation mechanism known as "thermoelastic damping" whereby local volume variations in the solid induce temperature gradients, and the irreversible flow of heat to compensate for these gradients brings about energy dissipation. We have established that for thin beams of silicon and gallium–arsenide at temperatures above a few hundreds Kelvin, thermoelastic damping is a relevant dissipation mechanism even down to the 10–100 nm scale, as long as the system remains in the diffusive phonon regime. We have also shown that thermoelastic dissipation has a non-trivial dependence on the dimensions of the beam.

We [12] are now investigating dissipation as the system deviates from the extreme diffusive regime and approaches ballistic behavior. We consider the driven resonator mode as setting up a time dependent modulation of the local lattice constant. The varying lattice constant takes the thermally distributed phonons out of equilibrium. Dissipation of energy should occur through the relaxation of the thermal phonon reservoir back to equilibrium via two channels: (a) by local phonon–phonon scattering which leads to thermalization corresponding to the new lattice constant (the so-called Akhiezer effect); and (b) by ballistic transport of phonons between "hot" and "cold" regions (as opposed to the diffusive transport of heat in the thermoelastic picture above). We have calculated the dissipation due to the Akhiezer effect in a finite one-dimensional chain, vibrating in three dimensions, and have observed a monotonic dependence on the length of the chain. We intend to calculate the dissipation in a more realistic three-dimensional model in the near future.

We [12] are also examining the ballistic regime where the driven mode of the resonator itself is viewed as a collection of "acoustic" phonons. Individual scattering events between these "acoustic" phonons and all other "thermal" phonons impose a finite lifetime on the acoustic phonons and lead directly to the dissipation of energy (the so-called Landau–Rumer effect). We have calculated this effect in a finite one-dimensional chain and have observed a striking resonant-like dependence on the length of the chain which is due to the unique quantization of the thermal phonon density of states as the system enters the mesoscopic regime. We intend to pursue this calculation in more realistic three-dimensional models as well.

3. External phonons—radiation of energy

In the most naive picture, the external phonon bath serves as an unlimited sink for energy that leaves the resonator by radiation of elastic waves or acoustic phonons. The decrease in the quality factor Q caused by radiation from a resonator's nth mode depends on T_n, the energy transmission coefficient from that mode into the surrounding structure, supporting the resonator. We [13] have calculated the transmission coefficient for the fundamental modes T_0 of a simple beam, in the case of an abrupt connection between the beam and its surroundings. We have found that for three of the fundamental modes (compression, torsion, and out-of-plane bending) the quality factor Q is proportional to the aspect ratio (L/t) of the beam. For the fourth fundamental mode (in-plane bending) $Q \sim (L/t)^3$. These results are inconsistent with experiments which show higher Q. We are therefore led to reconsider the naive picture in which the external phonon bath simply acts as an infinite energy sink, and conclude that it might be necessary to consider in detail the dissipative properties of the surrounding structure, supporting the resonator.

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