Country Portfolio Dynamics

Michael B Devereux (UBC)
and
Alan Sutherland (St Andrews)

November 2007

1. Introduction

- Many interesting research questions relating to composition of country portfolios and capital flows

- There is a need to incorporate portfolio allocation into open economy DSGE models - particularly in incomplete markets settings

- We develop a simple method which yields solutions for 'steady state' and first-order dynamics of equilibrium portfolio holdings

- Also yields solutions for the 'steady-state' and dynamics of asset returns and asset prices

- Enables analysis of capital flows, portfolio rebalancing, expected and unexpected valuation effects - all in the context of standard DSGE models

- Related Literature

  Samuelson (1970)

  Judd (1998)

  Judd and Guu (2001)

  Evans and Hnatkovska (2005)

  Engel and Matsumoto (2005)

  Kollmann (2006)

  Tille and van Wincoop (2007)

  Pavlova and Rigobon (2007)

2. General Outline of the Solution Approach

- Portfolio holdings

$$\alpha(Z_t) \simeq \alpha(\bar{Z}) + \frac{\partial \alpha}{\partial Z}\left(Z_t - \bar{Z}\right)$$

$$\simeq \quad \bar{\alpha} \quad + \quad \hat{\alpha}_t$$

2. General Outline of the Solution Approach

- Portfolio holdings

$$\alpha(Z_t) \simeq \alpha(\bar{Z}) + \frac{\partial \alpha}{\partial Z}\left(Z_t - \bar{Z}\right)$$
$$\simeq \quad \bar{\alpha} \quad + \quad \hat{\alpha}_t$$

- Samuelson (1970): to solve for portfolio holdings up to order *N* approximate the portfolio problem up to order *N+2*

2. General Outline of the Solution Approach

- Portfolio holdings

$$\alpha(Z_t) \simeq \alpha(\bar{Z}) + \frac{\partial \alpha}{\partial Z}\left(Z_t - \bar{Z}\right)$$
$$\simeq \quad \bar{\alpha} \quad + \quad \underbrace{\hat{\alpha}_t}_{=\gamma' \hat{Z}_t}$$

- Samuelson (1970): to solve for portfolio holdings up to order *N* approximate the portfolio problem up to order *N+2*

2. General Outline of the Solution Approach

- Portfolio holdings

$$\alpha(Z_t) \simeq \alpha(\bar{Z}) + \frac{\partial \alpha}{\partial Z}\left(Z_t - \bar{Z}\right)$$
$$\simeq \quad \bar{\alpha} \quad + \quad \underbrace{\hat{\alpha}_t}_{=\gamma' \hat{Z}_t}$$

- Samuelson (1970): to solve for portfolio holdings up to order *N* approximate the portfolio problem up to order *N+2*

- Expected excess returns

$$E_t\left[r_{xt+1}\right] \;=\; E_t\left[r_{1t+1} - r_{2t+1}\right] \;\simeq\; \bar{r}_x^{e} \;+\; \hat{r}_x^{e(1)}$$

2. General Outline of the Solution Approach

- Portfolio holdings

$$\alpha(Z_t) \simeq \alpha(\bar{Z}) + \frac{\partial \alpha}{\partial Z}\left(Z_t - \bar{Z}\right)$$
$$\simeq \bar{\alpha} + \underbrace{\hat{\alpha}_t}_{=\gamma' \hat{Z}_t}$$

- Samuelson (1970): to solve for portfolio holdings up to order *N* approximate the portfolio problem up to order *N+2*

- Expected excess returns

$$E_t\left[r_{xt+1}\right] = E_t\left[r_{1t+1} - r_{2t+1}\right] \simeq \underbrace{\bar{r}_x^e}_{=0} + \underbrace{\hat{r}_x^{e(1)}}_{=0}$$

2. General Outline of the Solution Approach

- Portfolio holdings

$$\alpha(Z_t) \simeq \alpha(\bar{Z}) + \frac{\partial \alpha}{\partial Z}\left(Z_t - \bar{Z}\right)$$
$$\simeq \quad \bar{\alpha} \quad + \quad \underbrace{\hat{\alpha}_t}_{=\gamma'\hat{Z}_t}$$

- Samuelson (1970): to solve for portfolio holdings up to order *N* approximate the portfolio problem up to order *N+2*

- Expected excess returns

$$E_t\left[r_{xt+1}\right] \;=\; E_t\left[r_{1t+1} - r_{2t+1}\right] \;\simeq\; \underbrace{\bar{r}_x^e}_{=0} \;+\; \underbrace{\hat{r}_x^{e(1)}}_{=0} \;+\; \hat{r}_x^{e(2)} \;+\; \hat{r}_{xt}^{e(3)}$$

2. General Outline of the Solution Approach

- Portfolio holdings

$$\alpha(Z_t) \simeq \alpha(\bar{Z}) + \frac{\partial \alpha}{\partial Z}\left(Z_t - \bar{Z}\right)$$
$$\simeq \quad \bar{\alpha} \quad + \quad \underbrace{\hat{\alpha}_t}_{=\gamma' \hat{Z}_t}$$

- Samuelson (1970): to solve for portfolio holdings up to order *N* approximate the portfolio problem up to order *N+2*

- Expected excess returns

$$E_t\left[r_{xt+1}\right] = E_t\left[r_{1t+1} - r_{2t+1}\right] \simeq \underbrace{\bar{r}_x^e}_{=0} + \underbrace{\hat{r}_x^{e(1)}}_{=0} + \underbrace{\hat{r}_x^{e(2)}}_{\text{constant}} + \hat{r}_{xt}^{e(3)}$$

2. General Outline of the Solution Approach

- Portfolio holdings

$$\alpha(Z_t) \simeq \alpha(\bar{Z}) + \frac{\partial \alpha}{\partial Z}\left(Z_t - \bar{Z}\right)$$
$$\simeq \quad \bar{\alpha} \quad + \quad \underbrace{\hat{\alpha}_t}_{=\gamma'\hat{Z}_t}$$

- Samuelson (1970): to solve for portfolio holdings up to order *N* approximate the portfolio problem up to order *N+2*

- Expected excess returns

$$E_t\left[r_{xt+1}\right] = E_t\left[r_{1t+1} - r_{2t+1}\right] \simeq \underbrace{\bar{r}_x^e}_{=0} + \underbrace{\hat{r}_x^{e(1)}}_{=0} + \underbrace{\hat{r}_x^{e(2)}}_{\text{constant}} + \underbrace{\hat{r}_{xt}^{e(3)}}_{=\delta'\hat{Z}_t}$$

2. General Outline of the Solution Approach

- Portfolio holdings

$$\alpha(Z_t) \simeq \alpha(\bar{Z}) + \frac{\partial \alpha}{\partial Z}\left(Z_t - \bar{Z}\right)$$
$$\simeq \quad \bar{\alpha} \quad + \quad \underbrace{\hat{\alpha}_t}_{=\gamma'\hat{Z}_t}$$

- Samuelson (1970): to solve for portfolio holdings up to order *N* approximate the portfolio problem up to order *N+2*

- Expected excess returns

$$E_t\left[r_{xt+1}\right] = E_t\left[r_{1t+1} - r_{2t+1}\right] \simeq \underbrace{\bar{r}_x^{e}}_{=0} + \underbrace{\hat{r}_x^{e(1)}}_{=0} + \underbrace{\hat{r}_x^{e(2)}}_{\text{constant}} + \underbrace{\hat{r}_{xt}^{e(3)}}_{=\delta'\hat{Z}_t}$$

- Once portfolio problem is solved the first-order behaviour of all other variables can be solved in the usual way - including the first-order behaviour of realised asset prices and returns

3. Example Model

- Preferences $$U = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{C_\tau^{1-\rho}}{1-\rho}$$

- Preferences over home and foreign goods

$$C = \left[\left(\frac{\chi}{2}\right)^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + \left(\frac{2-\chi}{2}\right)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

- Demand shocks $$\log \chi_t = \zeta_\chi \log \chi_{t-1} + \varepsilon_{\chi t}$$

- Endowments $$Y_t = \phi Y_{Lt} + (1-\phi) Y_{Kt} \qquad Y_t^* = \phi Y_{Lt}^* + (1-\phi) Y_{Kt}^*$$

$$\log Y_{Kt} = \zeta_K \log Y_{Kt-1} + \varepsilon_{Kt} \qquad \log Y_{Kt}^* = \zeta_K \log Y_{Kt-1}^* + \varepsilon_{Kt}^*$$

$$\log Y_{Lt} = \zeta_L \log Y_{Lt-1} + \varepsilon_{Lt} \qquad \log Y_{Lt}^* = \zeta_L \log Y_{Lt-1}^* + \varepsilon_{Lt}^*$$

- Y_K = "profits", Y_L = "labour income"
- Two tradeable financial assets - equity shares in Y_K and Y_K^*
- Asset returns $$r_{1t} = (Y_{Kt} + Q_{1t}) / Q_{1t-1} \qquad r_{2t} = (Y_{Kt}^* + Q_{2t}) / Q_{2t-1}$$
- Budget Constraint $$\alpha_{1t} + \alpha_{2t} = \alpha_{1t-1} r_{1t} + \alpha_{2t-1} r_{2t} + Y_t - C_t$$

- Parameter values

$$\beta = 0.95,\ \rho = 1,\ \theta = 2,\ \phi = 0.65$$

$$\zeta_K = \zeta_L = \zeta_\chi = 0.9$$

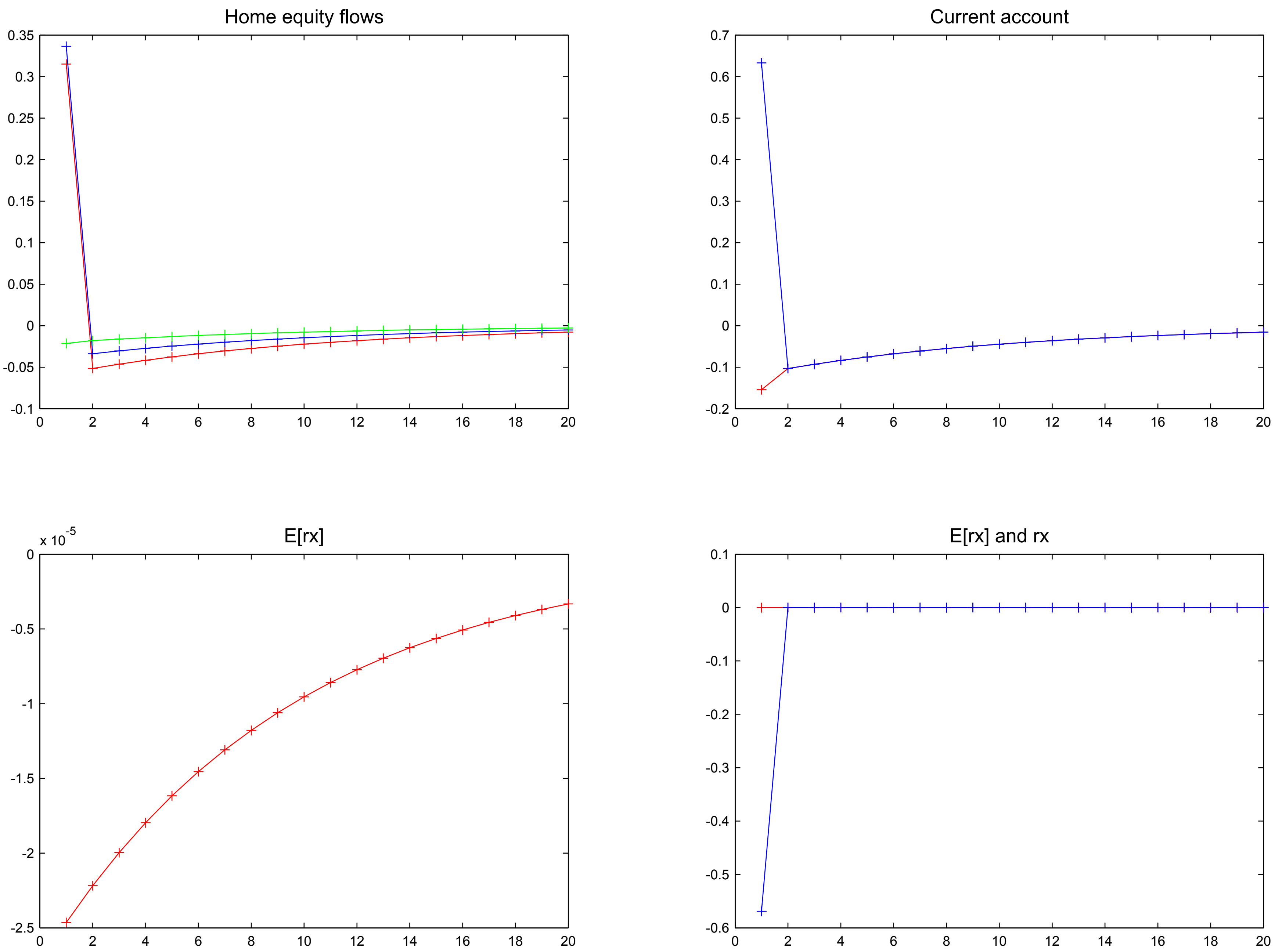
$$Var(\varepsilon_K) = Var(\varepsilon_\chi) = 0.01^2$$

$$Var(\varepsilon_L) = 4 \times 0.01^2$$

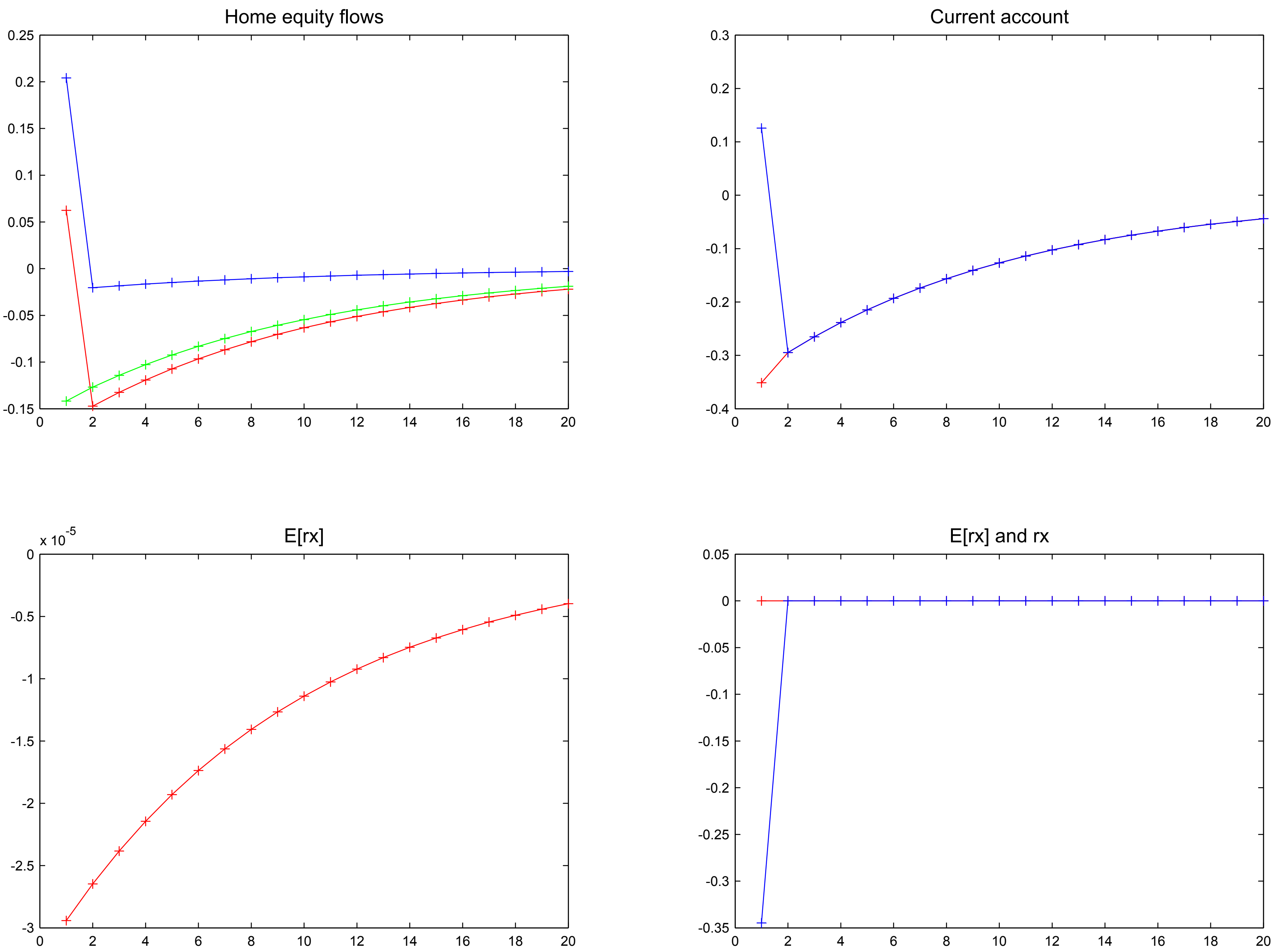
- Solution for zero-order portfolio - home households hold 80% of home equity and sell 20% to foreign households (and vice versa) - there is home equity bias

- The value of home capital income is negatively correlated with the value of foreign capital income - foreign equity is a good hedge for home capital income

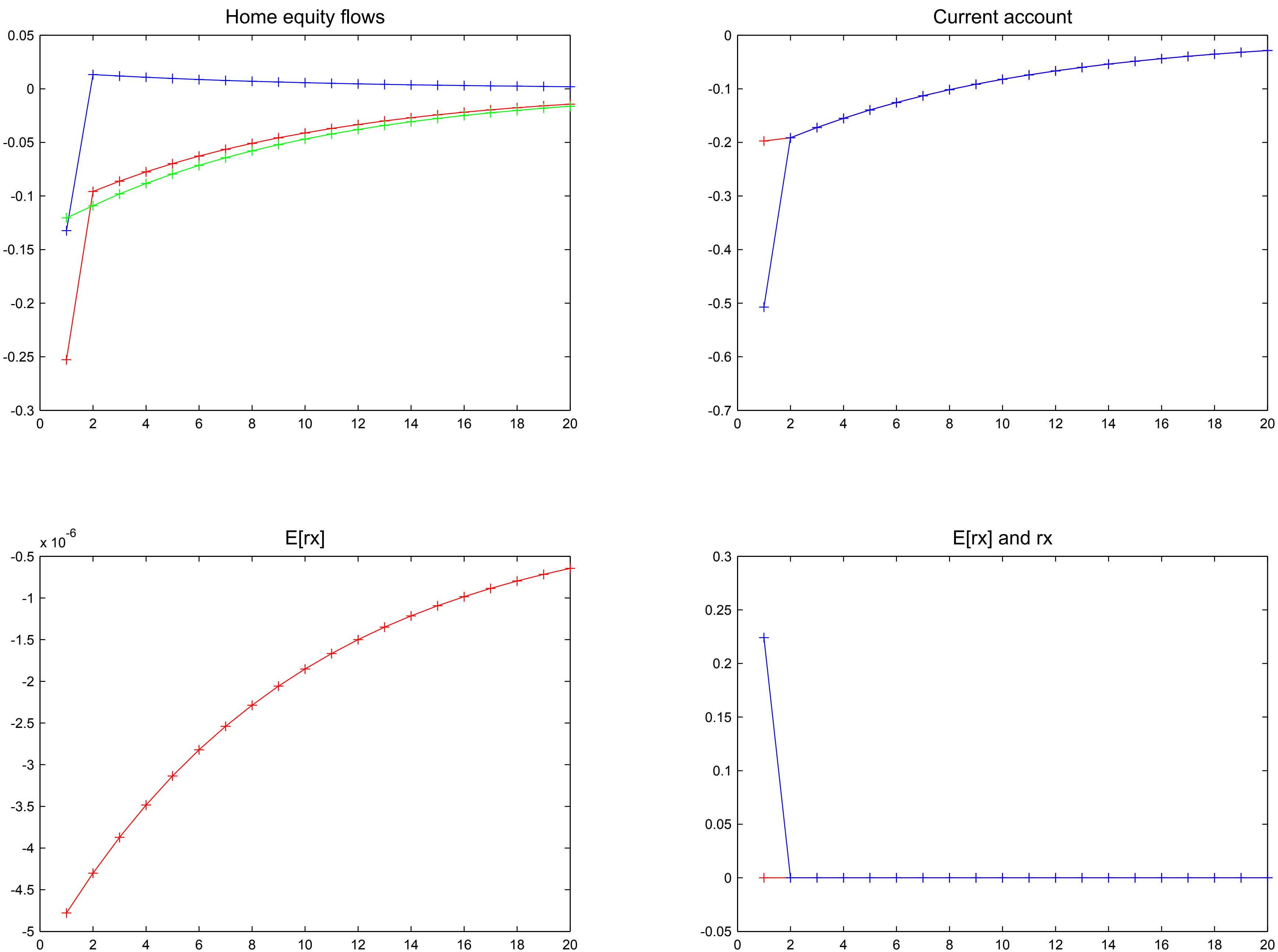
- The value of home capital income is also negatively correlated with the value of home labour income - home equity is a good hedge for home labour income



Home equity flows
Current account
E[rx]
x 10^-5
E[rx] and rx



Home equity flows
Current account
x 10-5
E[rx]
E[rx] and rx


Home equity flows
Current account
E[rx]
x 10^-6
E[rx] and rx

4. Summary

- Can solve for:
  - zero-order and first-order dynamics of portfolio holdings
  - second and third-order components of expected excess returns
  - first-order behaviour of realised asset prices and returns

- Makes is possible to analyse capital flows, unexpected and expected valuation effects and portfolio rebalancing - all within standard DSGE models

- Example model is illustrated in terms of numerical solutions - but analytical solutions can be obtained for many models