The “New Keynesian” Phillips Curve in the Open Economy

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Abstract

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1 Introduction

In this paper, we examine how open capital market policies would interact with the degree of price rigidity in the domestic economy to affect the output-inflation tradeoffs and the related volatilities of output and inflation to nominal shocks. The analysis will be conducted in an optimization-based “New Keynesian” framework a la Blanchard and Kiyotaki (1987). In the discussion, we extend to the open economy the succinct exposition of Woodford (2000) in the context of a closed economy.

Why is such extension potentially useful? Evidently, the degree of price stickiness is related to the organization of markets—for instance, whether they are common or segmented. Similarly, it can be affected by the openness of the economy both in commodity trade and capital mobility.

As an illustration, consider the evidence in Figure 1 below. The left panel measures the extent to which 4 groups of countries restrict capital movements based on the IMF’s Annual Report on Exchange Rate Arrangements and Exchange Restrictions. The right panel provides their corresponding average output-inflation tradeoff parameters as estimated by Ball, Mankiw, and Romer (1988). The figure shows clearly that countries with greater restrictions on capital mobility tend to have steeper Phillips curves. This finding has been substantiated by more serious econometric estimations by Loungani, Razin, and Yuen (2001).

2 The analytical framework

Consider a small open economy with a representative household that is endowed with a continuum of goods-specific skills—uniformly distributed on the unit interval [0; n]—to be supplied to a differentiated product industry. The household seeks to maximize a discounted sum of expected utilities:

$$ E_0 \sum_{t=0}^{\infty} \left[ u(C_t; M_t = P_t; \pi_t) \right] \delta^t v(h_t(j); \pi_t) dj $$
where \( \gamma \) is the subjective discount factor, \( C \) is the Dixit-Stiglitz index of household consumption, \( P \) the Dixit-Stiglitz price index, \( M = P \) the demand for real balances, \( \gamma \) a preference shock, and \( h(j) \) the supply of type-\( j \) labor to the production of good of variety \( j \). Like Obstfeld and Rogoñ (1996), we define the consumption index and its corresponding price index respectively as

\[
C_t = \frac{1}{n} \sum_{j=0}^{n-1} c_t(j) \mu_j^{1/\mu} dj + \frac{1}{n} \sum_{j=0}^{n-1} c^F_t(j) \mu_j^{1/\mu} dj \;
\]

and

\[
P_t = \frac{1}{n} \sum_{j=0}^{n-1} p_t(j) \mu_j^{1/\mu} dj + \frac{1}{n} \sum_{j=0}^{n-1} [p^F_t(j)] \mu_j^{1/\mu} dj \;
\]

where \( c(j) \) represents domestic consumption of the \( j \)th domestically produced good, \( c^F(j) \) domestic consumption of the \( j \)th foreign-produced good, \( p(j) \) the domestic-currency price of \( c(j) \), \( p^F(j) \) the foreign-currency price of \( c^F(j) \), \( \gamma \) the nominal exchange rate (domestic-currency price of foreign currency), and \( n \) the fraction of goods that are produced domestically.

The budget constraint facing the household is given by:

\[
Z_n \sum_{j=0}^{n-1} p_t(j) c_t(j) dj + \sum_{j=0}^{n-1} [p^F_t(j)] c^F_t(j) dj + \frac{\mu}{1 + i_t} M_t + B_t + \gamma B^F_t \;
\]

\[
= M_{t-1} + (1 + i_t) B_{t-1} + f_{t-1} (1 + i^F_t) B^F_{t-1} + \sum_{j=0}^{n-1} w_t(j) h_t(j) dj + \sum_{j=0}^{n-1} \frac{1}{F(j)} dj \;
\]

where \( B \) is the domestic-currency value of domestic borrowing, \( B^F \) the foreign-currency value of foreign borrowing, \( f_{t-1} \) the forward exchange rate for foreign currencies purchased/sold at time \( t-1 \) for delivery at time \( t \), \( i \) and \( i^F \) the domestic and foreign interest rates, \( w(j) \) the wage rate per unit labor of type \( j \), and \( \frac{1}{F(j)} \) profit income from firms of type \( j \). With perfect capital mobility, covered interest parity prevails:

\[
1 + i_t = (1 + i^F_t) \frac{\mu_{f_{t+1}}} {n_t} \;
\]
From now on, we shall focus on the relation between aggregate supply of goods and consumption smoothing made possible by international capital mobility. For this purpose, we would not be concerned about the details of aggregate demand (including the demand for money), international commodity trade, and the determination of the exchange rate. For simplicity, separability between consumption and real money balances is assumed for the utility function.

The relevant utility-maximizing conditions for our purpose include an intratemporal condition for the choice of labor supply of type \( j \):

\[
\frac{v_h(h_t; \pi_t)}{u_c(C_t; \pi_t)} = \frac{w_t(j)}{p_t}
\]

and an intertemporal condition for the consumption-saving choice:

\[
\frac{u_c(C_t; \pi_t)}{u_c(C_{t+1}; \pi_{t+1})} = (1 + r^w);
\]

where \( r^w \) is the world real rate of interest, assumed for simplicity to be time-invariant. This latter equality is a consequence of both the purchasing power parity and the covered interest parity.

As in the Dixit-Stiglitz model, demand for good \( j \) satisfies

\[
c_t(j) = C_t \frac{\mu}{p_t(j)} \frac{p_t}{p_t} \theta_t \mu : 
\]

The production function assumes the form

\[
y_t(j) = A_t f(\theta_t(j));
\]

where \( A \) is a random productivity shock. The variable cost of supplying \( y_t(j) \) is \( w_t(j)f^{(1)}(y_t(j)) = A_t \), which implies a (real) marginal cost of

\[
s_t(j) = \frac{w_t(j)}{p_t A_t f^{(1)}(f^{(1)}(y_t(j)) = A_t)}:
\]
Using (2), we can replace the real wage above by the marginal rate of substitution. Defining production indices for home goods and foreign goods as $Y^H_t = R_n \sum_0 (p_t(y)) \frac{p_t}{p_t} (j)$ and $Y^F_t = R_n \sum_0 (p_t(y)) \frac{p_t}{p_t} (j)$ so that the index for all goods produced around the world is $Y^W_t = Y^H_t + Y^F_t$ and imposing symmetry across firms (so that we can drop the index $j$), the above equation can be rewritten as

$$s(y; C; \sigma; A) = \frac{v_h(f; A)}{u_c(C; \sigma) A f (f - 1)}$$

(5)

Trade-wise, price-making firms face world demand for its products so that equation (4) implies

$$y_t(j) = Y^W_t \mu p_t(j) \frac{\mu_i}{\mu}$$

(4)

where $y_t(j)$ is the quantity of good $j$ supplied by the firm to meet the world demand.

The goods markets are monopolistically competitive. A fraction $\delta$ of the firms sets their prices flexibly at $p_1 t$, supplying $y_1 t$ whereas the remaining $1 - \delta$ of firms sets their prices one period in advance (in period $t - 1$) at $p_2 t$; supplying $y_2 t$. In the former case, the price is marked up above the marginal cost by a factor of $1(= \frac{\mu}{\mu - 1} > 1)$ so that

$$p_1 t \delta \frac{1}{1 + \delta} s(y_1 t; C_t; \sigma_t; A_t) = 0$$

(6a)

In the latter case, $p_2 t$ will be chosen to maximize expected discounted profits $E_t^{1+\delta} (p_2 t, y_2 t, h_t)$

$$E_t^{1+\delta} \frac{1}{1+\delta} Y^W_t p_t^{\mu_i} \frac{1}{1+\mu_i} (Y^W_t p_t^{\mu_i} p_2 t^{\mu_i} - A_t)$$

where we have used the inverse demand function from (4) for $y_2 t$ and the inverse production function for $h_t$: One can show that $p_2 t$ satisfies

$$E_t^{1+\delta} \frac{1}{1+\delta} Y^W_t p_t^{\mu_i} \frac{1}{1+\mu_i} p_2 t^{\mu_i} \frac{1}{1+\mu_i} s(y_2 t; C_t; \sigma_t; A_t) = 0$$

(6b)

Given $p_1 t$ and $p_2 t$, the aggregate price index (1) can be rewritten as:
\[ P_t = \frac{n[p_t^{1,1} \mu + (1 + \delta) p_t^{1,1} \mu] + (1 + \mu) n'' t^{-1,1} \mu_1^{1,1} \mu^1_{1,1}}{(1 + \mu) n} = 1 s(Y_t^n; C_t^n; \pi_t; A_t); \quad (6a) \]

In the extreme case where all prices are fully flexible (i.e., \( \delta = 1 \)), output will attain its natural level \( Y_t^n \) implicitly defined by

\[ \frac{p_t}{n p_t^{1,1} + (1 + \mu)n'' t^{-1,1} \mu_1^{1,1} \mu^1_{1,1}} = 1 s(Y_t^n; C_t^n; \pi_t; A_t); \quad (6a) \]

Among other things, \( Y_t^n \) depends on the level of home consumption under flexible prices (\( C_t^n \)), domestic and foreign prices (\( p_t \) and \( p_t^n \)), as well as the exchange rate (\( \pi_t \)). For later purpose, we can denote \( s(Y_t^n; C_t^n; \pi_t; A_t) \) as \( s_t^n \).

In the absence of capital flows, \( C_t^n = Y_t^n \) so that the natural output level is defined by

\[ \frac{p_t}{n p_t^{1,1} + (1 + \mu)n'' t^{-1,1} \mu_1^{1,1} \mu^1_{1,1}} = 1 s(Y_t^n; Y_t^n; \pi_t; A_t); \quad (7) \]

When the economy is completely closed in terms of both commodity trade and capital flows (\( n = 1 \) and \( C_t^n = Y_t^n \)), \((6a)\) further simplifies to

\[ 1 = 1 s(Y_t^n; Y_t^n; \pi_t; A_t); \quad (8) \]

In this last case, equilibrium output is completely independent of monetary policy.

### 2.1 The Phillips curve

This section derives the expectations-augmented Phillips curve of the kind hypothesized by Phelps (1967) and Friedman (1968) for both open and closed economies.

In order to obtain a tractable solution, we log-linearize the equilibrium conditions around the steady state. We assume that \( (1 + r^n) = 1 \), which is necessary for the existence of the steady states. In particular, we consider a deterministic steady state where \( \pi_t = 0 \) and
When capital is perfectly mobile, consumption smoothing can be achieved and it will be trendless given the assumption that $(1 + r^n) = 1$. As a result, $\theta_t = 0 = \theta_t^n$: The Phillips
Curve therefore simplifies to
\[
\frac{1}{4} i E_{t_i} 1(\frac{1}{4}) = \mu \cdot \frac{1}{1 + \frac{1}{i} \cdot \frac{1}{n} \log(e_i) i E_{t_i} [\log(e_i)]}
\]

\[\text{(9)}\]

2.3 Closing the capital account

In the absence of capital flows, consumption smoothing can no longer be achieved and consumption will fluctuate with domestic output (i.e., \(C_t = \psi_t^H\) and \(C_t^n = \psi_t^n\)). As a result, the Phillips curve assumes the form
\[
\frac{1}{4} i E_{t_i} 1(\frac{1}{4}) = \mu \cdot \frac{1}{1 + \frac{1}{i} \cdot \frac{1}{n} \log(e_i) i E_{t_i} [\log(e_i)]}
\]
\[\text{(9\text{a})}\]

2.4 Closed economy

If we further close the trade account, the economy will be self-sufficient and \(n = 1\). In this case, the Phillips curve will take an even simpler form
\[
\frac{1}{4} i E_{t_i} 1(\frac{1}{4}) = \mu \cdot \frac{1}{1 + \frac{1}{i} \cdot \frac{1}{n} \log(e_i) i E_{t_i} [\log(e_i)]}
\]
\[\text{(9\text{b})}\]

which is exactly identical to equation (1.23) in Woodford (2000).

3 Short-run aggregate supply

This section examines how exogenous shocks to nominal GDP defined as
\[n = \rho_1 y_{1t} + (1 - \rho_2) y_{2t} = P_t^H Y_t^H \quad Y_t, \quad Q_t\] would affect the relative responses of domestic output and producer prices. From the Phillips curve equation (9), we can show that the sensitivity of \(\log(Y_t^H)\) and \(\log(Y_t^n)\) with respect to innovations in the exogenous process, viz., \(\log(Q_t) i E_{t_i} [\log(Q_t)]\),
in the case of perfect capital mobility is

\[
\text{output-elasticity}^{\text{open}} = \frac{3^\circ}{1 + n \frac{\circ}{\ddagger}} \frac{1^\circ}{1 + \frac{\circ}{\ddagger} + \frac{\circ}{\ddagger} + \frac{\circ}{\ddagger}}
\]

while the sensitivity of \( \log(P_t^H) + E_{t'} \log(P_{t'}^H) \) is

\[
\text{price-elasticity}^{\text{open}} = \frac{n^\circ}{1 + n \frac{\circ}{\ddagger} + \frac{\circ}{\ddagger} + \frac{\circ}{\ddagger}} \frac{\circ}{1 + \frac{\circ}{\ddagger}}
\]

Similarly, the sensitivity parameters in the case of a closed economy are given by

\[
\text{output-elasticity}^{\text{closed}} = \frac{3^\circ}{1 + n \frac{\circ}{\ddagger} + \frac{\circ}{\ddagger} + \frac{\circ}{\ddagger}}
\]

and

\[
\text{price-elasticity}^{\text{closed}} = \frac{3^\circ}{1 + n \frac{\circ}{\ddagger} + \frac{\circ}{\ddagger} + \frac{\circ}{\ddagger}} \frac{\circ}{1 + \frac{\circ}{\ddagger} + \frac{\circ}{\ddagger} + \frac{\circ}{\ddagger}}
\]

As discussed in Woodford (2000), these sensitivity parameters are related to the degree of strategic complementarity among price setters. In turn, the latter depends on the organization of markets. For instance, strategic substitutability (complementarity) will prevail if all factor prices are (cannot be) instantaneously equalized across the suppliers of different goods, the case of common (segmented) factor markets. In our case, we show another example where the organization of—in particular, the integration or not of the domestic capital market into—the world capital markets matters. Consumption smoothing, which comes with the opening of the capital market, will increase the degree of strategic complementarity, thus rendering prices more sticky and magnifying output responses.
4 Appendix

Let us start with the two price-setting equations:

\[ \log(p_{1t}) = \log(P_t) + \frac{1}{1 + \mu!} \left( \psi_{1t}^{l} \psi_{t}^{n} \right) + \frac{\mu}{1 + \mu!} \left( \phi_{1t}^{l} \phi_{t}^{n} \right); \]  \hspace{1cm} (A.1a)

and

\[ \log(p_{2t}) = E_{t_1} \log(P_t) + \frac{1}{1 + \mu!} \left( \psi_{2t}^{l} \psi_{t}^{n} \right) + \frac{\mu}{1 + \mu!} \left( \phi_{2t}^{l} \phi_{t}^{n} \right); \]  \hspace{1cm} (A.1b)

Log-linearizing the demand functions facing the firm (4) (where we can replace \( c_t \) and \( C_t^{W} \) by \( y_t \) and \( Y_t^{W} \) respectively), we get

\[ \psi_{jt} = \psi_{jt}^{W} \mu \left[ \log(p_{jt}) - \log(P_t) \right]; \] \hspace{1cm} (A.2)

where \( \psi_{jt}^{W} = n \psi_{jt}^{H} + (1 - n) \psi_{jt}^{F} \). Substituting (A.2) into (A.1a) and rearranging terms, we have

\[ \log(p_{1t}) = \log(P_t) + \frac{\mu}{1 + \mu!} \left( \psi_{1t}^{W} \psi_{t}^{n} \right) + \frac{\mu}{1 + \mu!} \left( \phi_{1t}^{W} \phi_{t}^{n} \right); \]  \hspace{1cm} (A.1a_1)

and

\[ \log(p_{2t}) = E_{t_1} \log(P_t) + \frac{\mu}{1 + \mu!} \left( \psi_{2t}^{W} \psi_{t}^{n} \right) + \frac{\mu}{1 + \mu!} \left( \phi_{2t}^{W} \phi_{t}^{n} \right); \]  \hspace{1cm} (A.1b_1)

Together, (A.1a_1) and (A.1b_1) imply that

\[ \log(p_{2t}) = E_{t_1} \log(p_{1t}); \] \hspace{1cm} (A.3)

From the aggregate price index equation (19), we have an approximate relation of the
following kind

\[
\log(P_t) = n [\log(p_{1t}) + (1_i - n) \log(p_{2t})] + (1_i - n) \log(p_{t}^{p}) ;
\]  

(A.4)

From this, the unanticipated rate of inflation is given by

\[
\log(P_t) - E_t \log(P_t) = n \cdot f \log(p_{1t}) + (1_i - n) \cdot f \log(p_{2t}) + (1_i - n) \cdot \log(p_{t}^{p}) ;
\]

where \( E_t \) is the real exchange rate. Substituting (A.1a) into the above expression yields an open-economy Phillips curve of the form

\[
\log(P_t) - E_t \log(P_t) = \phi_t^{W} + (1_i - n) \phi_t^{F} ;
\]

Equation (9) in the text can be obtained by noting that \( \phi_t^{W} = n \phi_t^{H} + (1_i - n) \phi_t^{F} ; \)

5 References

References

1-65.


