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CHAPTER 1: CAPITAL MOBILITY AND PRINCIPLES OF INTERNATIONAL TAXATION

1 Introduction

To be completed.

2 A Stylized Model of International Capital Mobility

We present a stripped-down model of international capital mobility which enables us to explore key issues of international taxation without being sidetracked by irrelevant complications. Consider an economy that lives for two periods, indexed by $t = 1, 2$. There is one aggregate, all-purpose good in each period, serving for both consumption and investment.

Following Saint-Paul (1994) and Razin and Sadka (1995), we assume a stylized economy in which there are two types of workers: Skilled workers who have high productivity and provide one efficiency unit of labor per unit of labor time and unskilled workers who provide $\beta < 1$ efficiency units of labor per unit of time.. Workers have one unit of labor time during each one of their two periods of life. They are born without skills and thus with low productivity. In the first period each worker chooses whether to acquire an education and become a skilled worker, or instead remain unskilled.

There is a continuum of individuals, characterized by an innate ability parameter, e, which is the time needed to acquire a skill. By investing e units of labor time in education, in the first period, a worker becomes skilled, after which the remaining $(1-e)$ units of labor time in the first period provide an equal amount of efficiency units of labor in the balance of the first period. She also provides one efficiency unit of labor in the second period. Less capable individuals require more time to become skilled and thus find education more costly in terms of lost income. We assume a positive pecuniary cost of acquiring skills, γ, which is not tax deductible. The cumulative distribution function of innate ability is denoted by $G(\cdot)$ with the support being the interval $[0,1]$. The density function is denoted by $g = G'$. If one individual chooses not to acquire skill in the first period she provides β efficiency units of labor in each one of these periods. For the sake of notational simplicity, we normalize the size of the population to one.

Given these assumptions, there exists a cutoff level, e^*, such that those with education cost parameters below e^* will invest in education and become skilled, while everyone else remains unskilled. The cutoff level is determined by the equality between the present value of the payoff to education and the cost of education (including lost income):

$$(1-\tau_L)(1-\beta)\left[w_1 + \frac{w_2}{1+(1-\tau_K)r_D}\right] = (1-\tau_2)w_1e^* + \gamma \tag{2.1}$$

where w_t is the wage rate per efficiency unit of labor in period $t = 1, 2$, r_D is the domestic rate of interest, τ_L is the tax rate on labor income and τ_K is the tax rate on capital income. Note that $1-\beta$ is the gain in efficiency units of labor from acquiring skill. Therefore, the expression on the right-hand-side of equation (1) is the present value of the payoff to education in the two periods of life. The left-hand-side of equation (1) is the cost of education, consisting of lost income and pecuniary costs. Rearranging (2.1) yields:

$$e^{*} = (1-\beta)\left[w_{1} + \frac{w_{2}/w_{1}}{(1-\tau_{K})r_{D}}\right] - \frac{\gamma}{(1-\tau_{L})w_{1}}. \tag{2.2}$$

Note that the two taxes, the tax on labor income and the tax on capital income, have opposite effects on the decision to acquire skill. The tax on labor income reduces the foregone (net) of tax income component of the cost of education. It also reduces the payoff to education by the same proportion. Were the pecuniary cost γ equal to zero (or else tax-deductable), the labor income tax would have no effect on the decision to acquire skill. However, with a positive pecuniary cost of education, the labor income tax has a negative effect on acquiring skills: It reduces e^{*} and, consequently, the proportion of the population who becomes skilled [namely, $G(e^{*})$]. On the other hand, the tax on capital income has a positive effect on education, because it reduces the (net-of-tax) discount rate and thereby raises the present of the future payoff to education.

In our model the leisure that an individual consumer is taken to be exogenously given for the sake of simplicity. Total labor supply is distorted by the taxes as can be seen from (2.2). Note that there are $G(e^{*})$ skilled individuals and $1-G(e^{*})$ unskilled in each period. The labor supply of each one of the unskilled individuals in efficiency units is β in each period. Therefore, total labor supply in efficiency units of the unskilled individuals is $\beta\left[1-G(e^{*})\right] in$ each period. However, a skilled individual devotes e units of her time in the first period to acquire education and hence works only $1-e$ units of time in the first period. Thus, $e-$individual labor supply in the first period varies over ε and the labor supply or skilled individuals is equal to $\int_0^{e^*}(1-e)dG$. Any skilled individual supplies as labor all of his unit time in the second period. Thus, total labor supply in efficiency units in period $t=1,2$, is defined as follows:

$$L_1 = \int_0^{e^*}(1-e)dG + \beta[1-G(e^*)] \tag{2.3}$$

and

$$L_2 = G(e^*) + \beta[1-G(e^*)]. \tag{2.4}$$

For the sake of simplicity, assume that all individuals have the identical preferences over first and second-period consumption [$c_1(e)$ and $c_2(e)$ respectively], represented by a common, concave utility function $u[c_1(e), c_2(e)]$.[1] Each individual has initial income (endowment) in the first period of I_1 units of the consumption capital good. The total amount of the initial endowments (which is I_1 because the size of the population was normalized to one), serves as the capital shock employed in the first period. (This initial endowment was generated by past savings or is inherited). Because taxation of the fixed initial endowment is not distortionary, we may assume that the government taxes away the entire value of the

[1] Because leisure is exogenously given, it is dropped out from the utility function.

initial endowments. Thus, an individual of type e faces the following budget constraints in periods one and two, respectively:

$$c_1(e) + s_D(e) + s_F(e) = F_1(e) + T_1 \tag{2.5}$$

and

$$\begin{aligned} c_2(e) \quad = \quad & T_2 + E_2(e) + s_D(e)[1 + (1 - \tau_K)r_D] \\ & + s_F(e)[1 + (1 - \tau_{FD} - \tau_N^*)r_F] \end{aligned} \tag{2.6}$$

where $E_t(e)$ is after-tax labor income, net of the cost of education, $t = 1, 2$, and where T_t is a uniform lump-sum transfer (demogrant) in period $t = 1, 2$. That is:

$$E_1(e) = \begin{cases} (1 - \tau_L)(1 - e)w_1 - \gamma & \text{for } \; e \; 5 \; e^* \\ (1 - \tau_L)\beta w_1 & \text{for } \; e = e^* \end{cases} \tag{2.7}$$

and

$$E_2(e) = \begin{cases} (1 - \tau_L)w_2 & \text{for } \; e \; 5 \; e^* \\ (1 - \tau_L)\beta w_2 & \text{for } \; e = e^* \end{cases} \tag{2.8}$$

The economy is open to international capital flows, so that an individual can channel savings to either the domestic or foreign capital market. We denote by $s_D(e)$ and $s_F(e)$ the savings channelled by an $e-$individual to the domestic and foreign capital market, respectively. We denote by r_D and r_F the real rate of return in these markets respectively.[2] The government levies a tax at the rate τ_K on capital (interest) income from domestic sources. Capital (interest) income from foreign sources is subject to a non-resident tax at the rate of τ_N^* levied by the foreign government. The domestic government may levy an additional tax on its residents on their foreign-source income at the rate of τ_{FD}. Note that $\tau_{FD} + \tau_N^*$ is the effective tax rate on foreign-source income of residents.

For the sake of brevity, we consider in the text the case of a capital-exporting country that is, its national savings exceed domestic investment, with the difference (defined as the current account surplus) invested abroad. (The analogous case of a capital-importing country can be worked out similarly). By arbitrage possibilities, the net-of-tax rate of interest earned at home and abroad are equalized, that is:

$$(1 - \tau_K) r_D = (1 - \tau_{FD} - \tau_N^*) r_F . \tag{2.9}$$

Employing (2.9), one can consolidate the two one-period budget constraints (2.5) and (2.6) into one lifetime budget constraint:

[2] These rates (r_D and r_F) hold in essence between periods one and two and we therefore assign no time subscript (one or two) indices to these rates.

$$c_1(e) + qc_2(e) = E_1(e) + qE_2(e) + T, \tag{2.10}$$

where

$$q = [1 + (1 - \tau_K)r_D]^{-1}, \tag{2.11}$$

is the relative after-tax price of second-price consumption and

$$T \equiv T_1 + qT_2 \tag{2.12}$$

is the discounted sum of the two transfers (T_1 and T_2).[3]

As usual, the consumer maximizes her utility function, subject to her lifetime budget constraint. A first-order condition for this optimization is that the intertemporal marginal rate of substitution is equated to the tax-adjusted interest factor:

$$MRS(e) \equiv u_1 [c_1(e), c_2(e)] / u_2[c_1(e), c_2(e)] = 1 + (1 - \tau_K)r_D = q^{-1}, \tag{2.13}$$

[3]Note that even though T may seem at first glance to be dependent on τ_K (through the discount factor q), we may nevertheless assume that these are two independent policy tools because the government can always change either T_1 and T_2 in order to keep T constant when it changes τ_K.

where u_i denotes the partial-derivative of u with respect to its ith argument, $i = 1, 2$. Equations (2.13) and (2.10) yield of course the consumption demand functions $\overline{c}[q, E_1(e) + qE_2(e) + T]$ and $\overline{c}_2[q, E_1(c) + qE_2(e) + T]$ of an $e-$individual. The maximized value of the utility function of an $e-$individual MV[q,E$_1(e) + qE_2(e) + T]$, denotes the indirect utility function.

Denote the aggregate consumption demand, in period $t = 1, 2$, by:

$$C_t(q, (1 - \tau_L)w_1, (1 - \tau_L)w_2T) \equiv \int_0^1 \overline{c}_t[q, E_1(e) + qE_2(e) + T]dG + \tag{2.14}$$

$$\int_0^{e^*} \overline{c}_t\{q(1 - e)(1 - \tau_L)w_1 + q(1 - \tau_L)w_2 + T - \gamma dG +$$

$$[1 - G(e^*)]\overline{c}_t\{q, \beta(1 - \tau_L)w_1 + q1 - \tau_L)w_2] + T\}$$

where use is made of (2.7) and (2.8). Note that e^* is also a function τ_L, $q(1 - \tau_L)w_1$ and $(1 - \tau_L)w_2$ [see equation (2.2)].

All firms are identical so that with no further loss of generality we assume that there is only one firm. Its objective, dictated by the firm's shareholders, is to maximize the discounted sum of cash flows according to the firm. We assume that the firm finances its investment by issuing debt. In the first period it has a cash flow of $(1 - \tau_K)F(K_1, L_1) - [K_2 - (1 - \delta)K_1] + \tau_K \delta K$, where $F(\cdot)$ is a neo-classical, constant-returns-to-scale, production function. In the second period the firm has a cash flow of $(1 - \tau_K)F(K_2, L_2) + L)1 - \delta)K_2 + \tau_K \delta K_2$. We denote by δ both the physical and the economic rate of depreciation (assumed for the sake of simplicity to be equal one to the other). The depreciation rate is also assumed to apply for tax purposes. We further assume that the corporate income tax is fully integrated into the individual income tax. Specifically, we assume that the firm pays a corporate tax at the rate τ_K, which applies also to individuals, and that no additional taxes are levied on the firm's distributions (dividends) to its shareholders.[4] With such integration of the individual income tax and the corporate tax, there is no difference between debt and equity finance. The firm's discounted sum of its after-tax cash-flow is therefore:

$$\begin{aligned}\pi &= (1-\tau_K)[F(K_1,L_1)-w_1L_1]-[K_2-(1-\delta)K_1]+\{\tau_K\delta K_1 \\ &\quad +(1-\tau_K)[F(K_2,L_2)-w_2L_2]+\tau_K\delta K_2+(1-\delta)K_2\}/(1+(1-\tau_K)r_D).\end{aligned} \tag{2.15}$$

Note that K_1 is exogenous as it was set already in period zero. Maximizing (2.15) with respect to K_2, L_1 and L_2 yields the familiar marginal productivity conditions:

$$F_L(K_1,L_1)=w_1 \tag{2.16}$$

$$F_L(K_2,L_2)=w_2 \tag{2.17}$$

[4] Note nevertheless that all such dividends are fully taxed away by the government as no distortion is entailed by taxing away poor profits.

and

$$F_K(K_2, L_2) - \delta = r_D. \tag{2.18}$$

Note that although taxes do not affect the investment rule of the firm, nevertheless, the taxes are distortionary. To see this distortion consider the intertemporal marginal rate of transformation of (MRT) second-period consumption (namely, c_2) for first-period consumption (namely, c_1) which is equal to $(1 - \delta) + F_K(K_2, L_2)$. The latter expression is the opportunity cost of c_1 in terms of c_2. From equation (18), we can see that:

$$MRT = 1 + r_D.$$

However, from equation (2.13) we can see that the common intertemporal marginal rate of substitution to all individuals is equal to:

$$MRS = 1 + (1 - \tau_K)r_D.$$

Hence, the MRT is not equal (in fact, larger than) the MRS.

The government has also a consumption demand of C_t^G in period $t = 1, 2$.and the government can lend or borrow at market rates. With no loss of generality we assume that the government operates only in the domestic capital market.

Therefore, the government does not have to balance its budget period by period but only over the two-period horizon:

$$\begin{aligned} & C_1^{\mathrm{G}} + \frac{C_2^{\mathrm{G}}}{1 + (1 - \tau_{\mathrm{N}}^*) r_{\mathrm{F}}} + T_1 + \frac{T_2}{1 + (1 - \tau_{\mathrm{N}}^*) r_{\mathrm{F}}} \\ = \; & \tau_{\mathrm{L}} w_1 L_1 + \frac{\tau_{\mathrm{L}} w_2 L_2}{1 + (1 - \tau_{\mathrm{N}}^*) r_{\mathrm{F}}} + \frac{\tau_{\mathrm{K}} r_{\mathrm{D}} S_{\mathrm{D}}}{1 + (1 - \tau_{\mathrm{N}}^*) r_{\mathrm{F}}} \\ & + \frac{\tau_{\mathrm{F\,D}} r_{\mathrm{F}} S_{\mathrm{F}}}{1 + (1 - \gamma_{\mathrm{N}}^*) r_{\mathrm{F}}} + \pi_1, \end{aligned} \tag{2.19}$$

where:

$$S_{\mathrm{D}} = \int_0^1 s_{\mathrm{D}}(e) dG$$

is the aggregate private savings chanelled into the domestic capital market (and is equal, at equilibrium, to the domestic capital stock in the second period, namely, K_2);

$$S_{\mathrm{F}} = \int_0^1 s_{\mathrm{F}}(e) dG$$

is the aggregate savings chanelled into the foreign capital market (and is equal, at equilibrium to aggregate private savings (namely, $T_1 + (1 - \tau_{\mathrm{L}}) w_1 L_1 - C_1 - G(e^*)\gamma$ minus the aggregate private savings chanelled into the domestic capital market, namely S_{D}); and

$$\pi_1 = F(K_1, L_1+) + (1-\delta)K_1 - w_1 L_1$$

is the firm's surplus in the first period (which is also equal to the aggregate private initial endowments. Note that we assumed that this surplus is fully taxed away by the government.

The left-hand-side of equation (2.19) represents the discounted sum of the government expenditures on public consumption and transfers. Note that the appropriate discount rate for the government is $(1-\tau_N^*)r_F$ which is the rate at which the domestic economy can lend abroad. The right-hand-side of (2.19) represents the revenues from the labor income taxes, the interest income taxes, and the pure surplus of the firm.

Market clearance in the first period requires that:

$$CA + C_1 + C_1^G + K_2 - (1-\delta)K_1 + G(e^*)\gamma = F(K_1, L_1) \tag{2.20}$$

where CA is the current account surplus.[5] This surplus is equal also to the aggregate private saving chanelled abroad, that is $\int_0^A s_F(e)dG$. Market clearance in the second period requires that:

$$C_2 + C_2^G = F(K_2, L_2) + (1-\delta)K_2 + CA[1 + (1-\tau_N^*)r_F]. \tag{2.21}$$

[5] For notational simplicity we assume that the net external assets are initially equal to zero so that there is no external debt payment term in CA.

Note that the tax at the rate τ_N^* is levied by the foreign country on the interest income of the residents of the home country, and must therefore be subtracted from the resources available to the home country.

We can substitute the current account surplus (CA) from (2.20) into (2.19) in order to get one condition:

$$C_1 + \frac{C_2}{1+(1-\tau_N^*)r_F} + C_1^G + \frac{C_2^G}{1+(1-\tau_N^*)r_F} + K_2 - (1-\delta)K_1 + G(e^*)\gamma = F(K_1, L_1) + \frac{F(K_2, L_2)}{1+(1-\tau_N^*)r_F} + \frac{(1-\delta)K_2}{1+(1-\tau_N^*)r_F}. \tag{2.22}$$

Note that by Walras Law, we may ignore the government budget constraint (2.19), because it will be satisfied when equation (2.22) (the economy-wide "budget" constraint) and equation (2.10) (the individual budget constraints) hold. This is shown in the Appendix.

The government (social planner) preferences are represented by a social welfare function (V). With no loss of generality, this function is assumed to be a weighted average of the individual utility functions. That is:

$$\begin{aligned} V(q, w_1^N, w_2^N, T) &= \int_0^1 \theta(e) v[q, E_1(e) + qE_2(e) + T] dG \\ &= \int_0^{e^*} \theta(e) v[q(1-e)w_1^N + qw_2^N + T - \gamma] dG \\ &\quad + [1 - G(e^*)]\, v\{q, \beta(w_1^N + qw_2^N) + T\} \int_{e^*}^1 \theta(e) dG \end{aligned} \tag{2.23}$$

where $\theta(e) = 0$ is the weight assigned to the $e-$type individual, and $w_t^N = (1-\tau_L)w_t$ is the after-tax wage per efficiency unit of labor in period $t = 1, 2$.

Note that this social welfare function is individualistic: It depends only on individual utilities. It thus respects individual preferences. Individuals differ in their innate abilities and, consequently, in their incomes and wealth levels. Our specification of V implies that the government is concerned about both equity and efficiency. For instance, when $\theta(e) \equiv 1$, our social welfare function is a Benthamite utilitarian function, exhibiting a preference for equality. Such a preference is even stronger when θ rises in e, reaches to the extreme Rawlsian max-min criterion when $\theta(e) = 0$ for all $e < 1$ and $\theta(1) = 1$. Note that the lump-sum transfer T, being uniform across individuals, has a limited redistribution potential. Another important example is when $\theta(e) = 0$ for all e, except for the median e, e_{M}, for which $\theta(e_{M)}) = 1$. In this case the social welfare function reflects the median voter, and our model describes a majority voting system.

The government has to employ also taxes on labor income and capital income in order to redistribute income (and raise revenues). However, these taxes are distortionary: A labor income tax distorts human capital accumulation, and the tax on capital income creates as usual a saving-investment distortion. The optimal or the median voter tax policy is in essence an optimal tradeoff between equity and efficiency.

The policy tools at the government's disposal are inter alia labor income taxes and capital income taxes. We therefore assume that the government can choose the after-tax wage rates (w_1^N and w_2^N) and the after-tax discount factor (q). The government can choose also T, the discounted sum of the lump-sum transfers (T_1 and T_2). Once w_1^N, w_2^N, q and T are chosen, then private consumption demands $[C_1\ (q, w_1^N, w_2^N, T)]$ and $C_2(q, w_1^N, w_2^N, T)$ are determined. The cutoff level e^* and the labor supplies L_1 and L_2 are also determined:

$$e^*(w_1^N.w_2^N,q) = (1-\beta)[1+qw_2^N/w_1^N] - \gamma/w_1^N \tag{2.2'}$$

$$L_1(w_1^N,w_2^N,q) = \int_0^{e^*(w_1^N,w_2^N,q)} (1-e)dG + \tag{2.3'}$$
$$\beta\{1-G\left[e^*(w_1^N,w_2^N,q)\right]\}$$

and

$$L_2(w_1^N,w_2^N,q) = G\left[e^*(w_1^N,w_2^N,q)\right] + \tag{2.4'}$$
$$\beta\{1-G[e^*(w_1^N,w_2^N,q)]\}.$$

In choosing its policy tools $(q, w_1^N, w_2^N$, and $T)$, and its public consumption demand $(C_1^G$ and $C_2^G)$, the government is constrained by the economy-wide "budget" constraint (2.22), where C_1, C_2, L_1, L_2 and e^* are replaced by the functions $C_1(\cdot)$, $C_2(\cdot)$, $L_1(\cdot)$, $L_2(\cdot)$ and $e^*(\cdot)$, given by equations (2.14) and (2.2′) – (2.4′), respectively. Note that the capital stock in the first period (K_1) is exogenously given. The capital stock in the second period (K_2) must satisfy the investment rule of the firm [equation (2.18)]. Note that since the economy is fiinancially open, the individuals, by the arbitrage condition (2.9), are indifferent between chanelling their savings domestically or abroad. This means that the government can choose K_2 and then r_D and the pre-tax wages (w_1 and w_2) are determined so as to clear the capital market and labor market in each period from equations (2.18), (2.16) and (2.17), respectively.

To sum up, the government chooses C_1^G, C_2^G, q, w_1^N, w_2^N, T and K_2, so as to maximize the social welfare function (2.23), subject to the economy-wide "budget" constraint (2.22). Note that C_1, C_2, L_1, L_2 and e^* (in the latter constraints are replaced by the functions $C_1(\cdot)$, $C_2(\cdot)$, $L_1(\cdot)$, $L_2(\cdot)$ and $e^*(\cdot)$ respectively.

In this optimization, K_2 appears only in the economy-wide "budget" constraint (2.22). Thus, the first-order condition for the optimal level of K_2 does not depend on the specification of individual preferences and the weight function, [namely $\theta(e)$:

$$1 - q^* F_K(K_2, L_2) - q^*(1 - \delta) = 0. \tag{2.24}$$

Substituting the firm's investment rule (2.18) and rearranging terms yield:

$$1 - \delta + F_K(K_2, L_2) = 1 + (1 - \tau_N^*) r_F . \tag{2.25}$$

The optimal stock of capital (implicitly determined from equation (2.25))ascertains the Diamond-Mirrlees (1971) aggregate production efficiency theorem: The intertemporal marginal rate of transformation [which is $1 - \delta + F_K(K_2, L_2)$] must be equated to the world intertemporal mrginal rate of transformation [which is equal to $1 + (1 - \tau_N^*) r_F$]. This rule can be seen in Figure 1, where first-period consumption $(C_1 + C_1^G)$ is plotted on the horizontal axis and second-period consumption $(C_2 + C_2^G)$ on the vertical axis. Suppose that L_1, L_2 and e^* are given. The production possibility frontier is described by the curve ABD whoe slope is equal (in absolute value) to $(1-\delta) + F_F(K_2, L_2)$. The optimal-tax level of K_2 is HD; which gives rise to the consumption possibility frontier given by MBN. Any other level of $K_{2,}$ say D', must generate a lower consumption possibility frontier - the curve $M'N'$.

Employing the firm's investment rule (2.18) and the arbitrage condition (2.19), we can conclude from (2.25) that:

$$r_D = (1 - \tau_N^*) r_F, \tag{2.26}$$

or that:

$$\tau_{FD} = \tau_K (1 - \tau_N^*). \tag{2.27}$$

3 Appendix

In this appendix we demonstrate that the government budget constraint [equation (2.19)] is redundant, as it is satisfied when (2.22) and (2.10) both hold (Walras's Law).

Substituting the definitions of $E_1(e)$ and $E_2(e)$ in (2.7) and (2.8) rspectively into the individual budget constraint [equation (2.10)], aggregating over all individuals and dividing by q yields:

$$C_1/q + C_2 = T_1/q + T_2 + (1 - \tau_L)w_1 L_1/q + (1 - \tau_2)w_2 L_2 - Ge^*)G(e^*)\gamma/q. \tag{A1}$$

(Recall the definitions of L_1 and L_2 in equations (2.3) and (2.4), respectively.) Now divide the economy-wide "budget constraint", equation (2.22), by $q^* = [1 + (1 - \tau_N^*)r_F]^{-1}$, the discount rate faced by the domestic economy, and subtract it from (A1) to get:

$$\begin{aligned} &C_1\left(\frac{1}{q} - \frac{1}{q^*}\right) - \frac{T_1}{q} - T_L - \frac{(1-\tau_2)w_1 L_1}{q} \\ &-(1-\tau_L)w_2 L_2 + G(e^*)\gamma\left(\frac{1}{q} - \frac{1}{q^*}\right) \\ &\frac{C_1^G}{q^*} - C_2^G - \frac{K_2 - (1-\delta)K_1}{q^*} + \frac{F(K_1, L_1)}{q^*} + \\ &F(K_2, L_2) + (1-\delta)K_2 = 0. \end{aligned} \tag{A2}$$

Note that:

$$\frac{1}{q} - \frac{1}{q^*} = 1 + (1 - \tau_K)r_D - [1 + (1 - \tau_N^*)r_F] = -\tau_{FD} r_F \tag{A3}$$

by equation (2.9). Substituting (A3) into (A2) yields:

$$-\tau_{FD} r_F C_1 - \frac{T_1}{q^*} + \tau_{FD} r_F T_1 - T_2 \tag{A4}$$
$$-\frac{(1-\tau_L) w_1 L_1}{q^*} + \tau_{FD} r_D (1-\tau_L) W_1 L_1$$
$$-(1-\tau_L) w_2 L_2 - \tau_{FD} r_F G(e^*)\gamma$$
$$-\frac{C_1^G}{q^*} - C_2^G - \frac{K_2 - (1-\delta) K_1}{q^*} +$$
$$\frac{F(K_1, L_1}{q^*} + F(K_2, L_2) + (1-\delta) K_2 = 0.$$

Substituting the definition of π_1 [namely, $F(K_1, L_1) + (1-\delta)K_1 - w_1 L_1$], and Euler's equation [namely, $F(K_2, L_2) = F_K(K_2, L_2)K_2 + F_L(K_2, L_2)L_2 = (r_D + \delta)K_2 + w_2 L_2$, by the marginal productivity conditions (2.17) and (2.18)] into (A4) yield:

$$\tau_{FD} r_F [T_1 + (1-\tau_L) w_1 L_1 - C_1 - G(e^*)\gamma] - \frac{T_1}{q^*} \tag{A5}$$
$$-T_2 + \frac{\pi_1}{q^*} + \frac{\tau_L w_1 L_1}{q^*} + \tau_L w_2 L_2 + r_D K_2 - \frac{K_2}{q^*} +$$
$$K_2 - \frac{C_1^G}{q^*} - C_2^G = 0.$$

Substituting:

$$r_D K_2 - \frac{K_2}{q^*} + K_2 = -\tau_{FD} r_F K_2 + \tau_K r_D K_2$$

into (A5), then multiplying it by q^* and rearranging terms, yields:

$$q^*\tau_{FD}r_F\left[T_1 + (1-\tau_L)w_1L_1 - e_1 - G(e^*)\gamma - K_2\right] \tag{A6}$$

$$+q^*\tau_K r_D K_2 + \gamma_L w_1 L_1 + q^*\tau_L w_2 L_2 +$$

$$\pi_1 = C_1^G + q^* C_2^G + T_1 + q^* T_2.$$

Recalling that $S_D = K_2$ and

$$S_F = T_1 + (1-\tau_L)w_1L_1 - C_1 - G(e^*)\gamma - K_2$$

we can see that the government budget constraint, equation (2.19), follows from equation (A6).

References

[1] Saint-Paul, (1994)

[2] Razin and Sadka, AER, 1995

[3] Diamond and Mirrlees (1971)