# FIXED EXCHANGE RATES and Foreign Exchange Intervention

## Central Bank Balance Sheet

### Assets

1. Foreign Assets
2. Domestic Assets

### Liabilities

1. Deposits held by Private Banks
2. Currency in circulation

\[ H = \text{Base Money} \]

- Foreign Assets Sale $\rightarrow$ Base Money contracts
- Foreign Assets Purchase $\rightarrow$ Base Money Expands
Fixed Exchange Rate

\[ \bar{S} = \text{fixed exchange rate} \]

\[ i^* + \frac{\bar{S} - S}{S} \]

return

Automatic increase in M following CB intervention in the foreign exchange market.

Result: Base Money is endogenous
The Sustainability of Fixed Exchange Rate Regime

\[ \frac{M}{P} = L(i) = e^{-\eta i} \]  

Demand for money

\[ \log \left( \frac{M}{P} \right) = -\eta i \]

(1) \[ m_t - p_t = -\eta i_t \]

\[ i_t = i^* + \frac{dS_t}{S_t dt} = i^* + \frac{d \log S_t}{dt} \approx i^* + \Delta s_t \]  

Interest parity

\[ \Delta s_t = s_{t+1} - s_t \]

(2) \[ i_t = i_t^* + (s_{t+1} - s_t) \]
Purchasing Power Parity

\[ P_t = S_t P_t^* \]

\[ \log P_t = \log S_t + \log P_t^* \]

(3) \[ p_t = s_t + p_t^* \]

Substitute (2) & (3) into (1):

(4) \[ m_t = s_t + p_t^* - \eta (s_{t+1} - s_t) - \eta i_t^* \]

\[ P_t^* = P^* , \quad i_t^* = i^* \]

\[ m_t = \text{constant } + s_t - \eta (s_{t+1} - s_t) \]
Fixed Exchange Rate

\[ m_t = -s + p^*_t \]
money supply is totally endogenous

\[ \bar{m} = \bar{s} + p^* \]
if \( P^*_t = P^* \)

A Simple Model (Krugman 1979) \( P^* = i^* = 0 \)

\[ m_t = m = s \]
fixed exchange rate

\[ m_t - s_t = -\eta (s_{t+1} - s_t) = -\eta \mu \]
if \( s_{t+1} - s_t = \mu \)

flexible exchange rate
International Reserves

\[ B_{H,t} + \bar{S} B_{F,t} = \bar{M} \]

\[ B_{H,t+1} + \bar{S} B_{F,t+1} = \bar{M} \]

\[ B_{H,t+1} - B_{H,t} = -\bar{S} \left[ B_{F,t+1} - B_{F,t} \right] \]

\[ B_{F,t+1} - B_{F,t} = -\frac{1}{\bar{S}} \left[ B_{H,t+1} - B_{H,t} \right] \]

\[ B_{F,t+1} - B_{F,t} = -\frac{1}{\bar{S}} \mu B_{H,t} \]
Central Bank Balance Sheet

\[ M_t = B_{H,t} + \bar{S}B_{F,t} \]

Domestic Credit Expands Indefinitely

\[ \frac{B_{H,t+1} - B_{H,t}}{B_{H,t}} = \mu \]

rate of expansion

\[ b_{H,t+1} - b_{H,t} \cong \mu \]

“Shadow” Exchange Rate

\[ \tilde{s}_t = b_{H,t} + \eta \mu \]
Logarithmic Approximation

\[ B_{H,t+1} = (1 + \mu)B_{H,t} \]

\[ b_{H,t+1} = \log (1 + \mu) + b_{H,t} \]

\[ f(x) = f'(x^0)(x - x^0) \]

\[ \log(1 + \mu) = 1(1 + \mu - 1) = \mu \]

\[ x = 1 + \mu, \quad x^0 = 1 \]

\[ b_{H,t+1} = b_{H,t} + \mu \]
The “Shadow” exchange rate is:

a market-based exchange rate when the central bank has no international reserves:

\[ \tilde{s}_t = b_{H,t} + \eta \mu \]
Implications:

(1) Instantaneous Collapse

\[ \tau \approx 0 \text{ s} \]

(2) Calculations:

\[ \tilde{s}_T = \bar{s} \Rightarrow b_{H,T} + \eta\mu = \bar{s} \Rightarrow b_{H,T} \]

\[ b_{H,t+1} = b_{H,t} + \mu \Rightarrow \]

\[ b_{H,T} = b_{H,T-1} + \mu = b_{H,T-2} + 2\mu = T\mu b_{H,0} \Rightarrow T \]
\[ b_{H,t+1} = b_{H,t} + \mu \]

\[ B_{F,t+1} - B_{F,t} = -\frac{\mu}{S} B_{H,t} \]
Sustainability of Fixed Exchange Rate

(1) \( \tilde{S} \)

\( S \)

\( \sim \)

\( S \)

no budget deficit (\( \mu = 0 \))

(2) imperfect asset substitutability
   (a) regulating capital inflows
   (b) risk premium

\[ i_t = i_t^* + \frac{ES_{t+1} - S_t}{S_t} + \rho \]

\( \rho \) is a function of external debt
if $\rho$ is a function of external debt (B) minus domestic assets (A) a sterilized intervention which keeps $M$ constant switches reserves (negative external debt) for domestic assets would change the risk premium, and change domestic interest rate. Sales of reserves accompanied by purchase of domestic bonds will raise $\rho$ and $i$. 