Optimum Investment in Human Capital [1, 2]

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I. INTRODUCTION

Recent empirical studies (Schultz [8] and [9]) show that an accumulation of knowledge contributes much more to the growth of *per capita* income than does an increase in the capital-labour ratio. Major parts of the increase in productivity come from investment in human capital and learning by doing. While the learning-by-doing theory has been well formulated (Arrow [1]), there is no theoretical formulation of the link between productivity change and human investment. The purpose of this paper is to provide such a discussion by incorporating the theory of investment in human capital (Becker [2], Ben-Porath [3]) into a model of economic growth.

It is shown that for a simple form of a return-to-labour function the existence of non-Harrod-neutral technical change, when it is the result of investment in human capital, is a source of externality. A general result which relates the form of the return-to-labour function to the form that technical progress must take if externalities are to be avoided is then established.

II. CONSUMPTION AND INVESTMENT IN HUMAN CAPITAL BY AN INDIVIDUAL HOUSEHOLD

In this section we analyze consumption and investment in human capital by an individual household, i, in competitive markets. Competitive markets imply that unlimited borrowing and lending by the individual household can take place at a constant rate of interest ρ_t; the individual household possessing an amount of human capital A_t^i, which is a small fraction of the aggregate stock of human capital, has no influence on the rental rate w_t. We assume throughout the paper that a fixed fraction of time is assigned to leisure; thus, only the allocation of the remainder between work and accumulation of human capital is analyzed.

The technology of producing human capital is described by a function which relates the rate of increase in the amount of human capital $\alpha_t^i = \dot{A}_t^i/A_t^i$ to the fraction of non-leisure time assigned to education g_t^i. This increasing function $g_t^i = g^i(\alpha_t^i)$ is strictly convex (exhibiting diminishing returns) and invariant with respect to time. We assume that time is the single input in the process of education, and that the horizon of the individual household is infinite.[3] Depreciation of human capital, which under the latter assumption is mainly

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[3] The assumption that the horizon of the decision-making unit is infinite simplifies the analysis of the aggregate amounts of consumption and investment, which otherwise depends upon the age distribution of the population. Another simplification that the return-to-labour function has a multiplicative form, will be related later.

because members of the household have finite life spans, is denoted $\underline{\alpha}^i$ (i.e., $g^i(\underline{\alpha}^i) = 0$). Maximum rate of increase in human capital is $\bar{\alpha}^i$ (i.e., $g^i(\bar{\alpha}^i) = 1$). The size of the household N_t^i is expected to increase at a constant relative rate $n^i = \dot{N}_t^i/N_t^i$. Current income consists of labour income $w_t A_t^i N_t^i(1-g_t^i)$, and non-labour income $\rho_t B_t^i$ where the latter is derived from possession of B_t^i units of financial assets. The individual household seeks to maximize intertemporal utility derived from *per capita* consumption

$$\int_0^\infty e^{-\delta^i t} u^i(C_t^i/N_t^i)dt \qquad \ldots(1)$$

subject to

$$\dot{B}_t^i = w_t N_t^i A_t^i (1-g_t^i) + \rho_t B_t^i - C_t^i$$

and initial amounts

$$A_0^i > 0,\ N_0^i > 0,\ B_0^i,$$

where $u^i(\ )$ is instantaneous utility which is an increasing and concave function.

In what follows, we avoid corner solutions for the consumer problem by assuming that marginal cost is zero when no time is devoted to investment[1] $[(g^i(\underline{\alpha}^i))' = 0]$ and is infinite when all available time is devoted to investment $[(g^i(\bar{\alpha}^i))' = \infty]$. Positive consumption is assured by assuming $[(u^i(0))' = \infty]$.

To solve the individual household's problem, we apply the Pontryagin Maximum Principle to the Hamiltonian[2]

$$\mathscr{H}(A^i, \lambda^i, \alpha^i, B^i, \theta^i, C^i)e^{\delta t} = u^i(C^i/N^i) + \theta^i[wN^iA^i(1-g^i(\alpha^i)) + \rho B^i - C^i] + \lambda^i\alpha^i A^i$$

to get the following necessary conditions:

$$\theta^i N^i = [u^i(C^i/N^i)]' \qquad \ldots(2)$$

$$\lambda^i = wN^i\theta^i[g^i(\alpha^i)]' \qquad \ldots(3)$$

$$\dot{\theta}^i = (\delta^i - \rho)\theta^i \qquad \ldots(4)$$

$$\dot{\lambda}^i = \lambda^i(\delta^i - \alpha^i) - wN^i\theta^i[1-g^i(\alpha^i)]. \qquad \ldots(5)$$

We assume that the transversality conditions are fulfilled:

$$\lim_{t\to\infty} \lambda_t^i A_t^i e^{-\delta^i t} = \lim_{t\to\infty} \theta_t^i B_t^i e^{-\delta^i t} = 0. \qquad \ldots(6)$$

Equations (2)-(6) together with the feasibility conditions constitute a set of sufficient conditions. The differential equations for *per capita* consumption and investment in human capital can be solved from (2)-(6) to get

$$\frac{(u^i(C^i/N^i))''}{(u^i(C^i/N^i))'}\,(C^i/N^i)^{\cdot} = \rho - \delta^i - n^i \qquad \ldots(7)$$

$$\frac{(g^i(\alpha^i))''}{(g^i(\alpha^i))'}\,\dot{\alpha}^i + \frac{\dot{w}}{w} = \rho - n^i - \alpha^i - \frac{1-g^i(\alpha^i)}{(g^i(\alpha^i))'} \qquad \ldots(8)$$

$$\lim_{t\to\infty} w_t A_t^i (u^i(C_t^i/N_t^i))'(g^i(\alpha_t))' e^{-\delta^i t} = 0 \qquad \ldots(9)$$

$$\lim_{t\to\infty} B_t^i (u^i(C_t^i/N_t^i))' e^{-(\delta^i + n^i)t} = 0$$

where $\dot{w}/w$ is the *expected* rate of increase in the rental on human capital by the individual.

We remark that (8) is a necessary condition for maximizing the present value of labour income where the rate of interest is used to discount. This is anticipated because human

[1] Prime superscripts denote derivates.

[2] Hereafter we shall omit time subscripts except when a model is introduced or when ambiguity might arise.

capital in this model does not enter the utility function, and serves only as a source of income. Some further implications[1] of this model with respect to the theory of life cycle earnings when the horizon is finite are analyzed in Razin [5].

III. ECONOMIC GROWTH AND THE NATURE OF TECHNICAL PROGRESS

We now aggregate the individual magnitudes described in the previous section in order to analyze the general equilibrium of the economy.

Let a simple model for the economy with m identical individuals[2] be

$$Y_t = G(K_t, A_t, L_t) \quad \text{(Production Function)} \qquad \ldots(10)$$

$$L_t = (1-g_t)N_t \quad \text{(Labour Force)} \qquad \ldots(11)$$

$$\alpha_t = \dot{A}_t/A_t,\ z_t = \dot{K}_t/K_t,\ n = \dot{N}_t/N_t \quad \text{(Rates of change in stocks)} \qquad \ldots(12)$$

$$I_t = \phi(z_t)K_t \quad \text{(Firms' Investment)} \qquad \ldots(13)$$

$$C_t + I_t = Y_t \quad \text{(Equality of Aggregate Demand and Supply)} \qquad \ldots(14)$$

where K_t, A_t and N_t are aggregate stocks of physical capital, human capital and population respectively; C_t is total consumption; g_t is the fraction of population engaged in investment in human capital; I_t is total investment by firms; and α_t is the rate of technical progress. The production function is assumed to have constant returns to scale and positive and diminishing marginal productivities. The investment function $\phi(\ )$ in (13) is assumed to be strictly convex.[3]

An equilibrium of the market economy may be described by (10)-(14) together with the behavioural relations (7)-(9).[4]

Let the objective function for the socially optimum planning problem of the economy be

$$\int_0^\infty u(C_t/N_t)e^{-\delta t}dt \qquad \ldots(15)$$

where the utility function $u(\ )$ and discount rate δ are identical to those of the representative consumer. The optimum problem is to maximize (15), subject to (10)-(14) and initial values of the stocks, where the control variables are z_t and α_t (or, alternatively, C_t and α_t). We assume throughout that there exists a unique optimum path which is the interior solution of the above problem, and that the necessary conditions for the optimum are also sufficient. For a market economy with perfect foresight[5] the following proposition can now be stated:

Proposition I. *If the household's return-to-labour function is* $wA^iN^i(1-g^i)$, *then individual optimizing decisions lead to a social optimum if and only if the accumulation of human capital leads to Harrod-neutral technical progress.*

[1] The results are more general than (although similar to) those obtained by Ben-Porath [3] and Oniki [4].

[2] The assumption of identical individuals means $g^i(\ ) = g(\ )$, $u^i(\ ) = u(\ )$, $\delta^i = \delta$, $n^i = n$, and also that individuals have the same initial endowments and expectations. In a market economy, therefore, we have $C_t = mC_t^i$, $\alpha_t^i = \alpha_t$, $g_t^i = g_t$, $A_t = mA_t^i$ and $N_t = mN_t^i$.

[3] This investment relation which was introduced and analyzed by Uzawa in [10] and [11], reflects adjustment costs that are associated with installing and operating new capital. Upon maximizing the present value of the firms' net cash flow, the rate z can be shown to be dependent on the rate of interest and the rental on physical capital when firms behave competitively. This problem is described briefly in the Appendix. All results of the paper hold for an investment function of the form $\phi = \phi(z, k)$. Observe that K_t may include human capital which is specific to the firm (see Becker [2]).

[4] Equations (7)-(9), where superscripts i are omitted, describe rates of change of aggregate magnitudes in the market economy. See footnote 2, above.

[5] A market economy is said to have perfect foresight if expectations on the part of individual agents are always realized.

Proof. Since individuals are identical, the function $g(\ )$ is strictly convex, and the marginal productivity of labour is positive—we must have for the optimum solution[1] $\alpha^i = \alpha$ and therefore we can write in (11)

$$g = g(\alpha). \qquad \text{...(16)}$$

Let the Hamiltonian of the optimum problem be

$$\mathscr{H}(K, z, \lambda_K, A, \alpha, \lambda_A)e^{\delta t} = u\left(\frac{G(K, A, L) - K\phi(z)}{N}\right) + \lambda_A \alpha A + \lambda_K zK.$$

The optimum conditions are[2]

$$A\lambda_A = G_L(K, A, L)U'(C/N)g'(\alpha) \qquad \text{...(17)}$$

$$(G_A(K, A, L)/N)u'(C/N) + \alpha\lambda_A = -\dot{\lambda}_A + \delta\lambda_A \qquad \text{...(18)}$$

$$N\lambda_K = u'(C/N)\phi'(z) \qquad \text{...(19)}$$

$$u'(C/N)[G_K(K, A, L) - \phi(z)]/N + \lambda_K z = -\dot{\lambda}_K + \delta\lambda_K \qquad \text{...(20)}$$

$$\lim_{t\to\infty} \lambda_A A e^{-\delta t} = \lim_{t\to\infty} \lambda_K K e^{-\delta t} = 0 \qquad \text{...(21)}$$

together with (10)-(14) and initial values $A_0 > 0$, $N_0 > 0$, K_0.

Let the imputed rental on human capital be $w^0 = G_L(K, A, L)/A$, the imputed rental on physical capital be $r^0 = G_K(K, A, L)$ and the imputed rate of interest ρ^0 be

$$\rho^0 = \delta + n - \frac{U''(C/N)}{U'(C/N)}(C/N). \qquad \text{...(22)}$$

From (11), (16), (17), (18) and (22) we get

$$\frac{\dot{w}^0}{w^0} + \frac{g''(\alpha)}{g'(\alpha)}\dot{\alpha} = \rho^0 - n - \alpha - \frac{1-g(\alpha)}{g'(\alpha)}\frac{AG_A(K, A, L)}{LG_L(K, A, L)}. \qquad \text{...(23)}$$

It is well known that if and only if technical progress is Harrod-neutral (i.e., $G(K, A, L) = F(K, AL)$), then $AG_A(K, A, L)/LG_L(K, A, L) = 1$, in which case (23) reduces to

$$\frac{\dot{w}^0}{w^0} + \frac{g''(\alpha)}{g'(\alpha)}\dot{\alpha} = \rho^0 - n - \alpha - \frac{1-g(\alpha)}{g'(\alpha)}. \qquad \text{...(24)}$$

Differentiating (19) with respect to time and substituting into (20), using (22) and (19), we get

$$\frac{\phi''(z)}{\phi'(z)}\dot{z} = \rho^0 - z - \frac{r^0 - \phi(z)}{\phi'(z)}. \qquad \text{...(25)}$$

Using (17) and (19), the terminal conditions (21) can be rewritten as follows:

$$\lim_{t\to\infty} w^0 A_t g'(\alpha_t)u'(C_t/N_t)e^{-\delta t} = \lim_{t\to\infty} K_t\phi'(z_t)U'(C_t/N_t)e^{-(\delta+n)t} = 0. \qquad \text{...(26)}$$

Comparing (22), (24)-(26) with (7)-(9) and expression (A6) in the Appendix the market economy with perfect foresight will satisfy the optimum conditions.[3]

Thus the proof is completed.[4]

[1] Otherwise, if $\alpha^i \neq \alpha^j$ and $A^i = A^j$ by a convex combination of α^i and α^j the planner will increase the labour force while maintaining the same rate of α.

[2] $G_i\ (K, A, L)$ denotes the partial derivative of $G\ (K, A, L)$ with respect to i, $i = K, A, L$.

[3] The second terminal condition (26) can be shown to combine two terminal conditions—one for consumers and another for firms in a market economy with perfect foresight (by using the identity $\rho_t B_t = r_t K_t$).

[4] Note that in our model, contrary to Samuelson [7], population growth, under the objective function (15), is not a source of externality.

Proposition I can now be given an intuitive explanation. The effect of the assumption that the return-to-labour function is multiplicative (i.e., represented by $wA^iN^i(1-g^i)$) is that the individual household is indifferent between all $(A^i, N^i(1-g^i))$ pairs with a constant product. If there are to be no externalities the social return to education must be given by the same functional form as the private return: otherwise the self-interested actions of individual households will not optimize social returns. But this equality of functional forms requires society to be indifferent between all (A, L) pairs with a constant product, hence the need for Harrod-neutrality.[1]

This discussion suggests an immediate extension of Proposition I for the general case where the return-to-labour function is given by $wH(A^i, N^i(1-g^i))$ with the function $H(\ )$ representing services of labour by the household.

Proposition II. *If the household's return-to-labour function is $wH(A^i, N^i(1-g^i))$ then individual optimizing decisions lead to a social optimum if and only if the accumulation of human capital leads to technical progress which is representable by a production function of the form $G(K, A, L) = F(K, H(A, L))$.*

The proof of this proposition follows step by step the proof of Proposition I where the imputed rental on human capital is given by $w^0 = G_L/H_L$.

Intuitively, the individual household is assumed to be indifferent between all $(A^i, N^i(1-g^i))$ pairs with a constant value for the function $H(\ )$. Therefore, if externalities are to be avoided, society should also be indifferent between all (A, L) pairs with a constant value for $H(\ )$. Hence the production function necessarily takes the form $F(K, H(A, L))$.

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[1] Detailed descriptions of the optimum path and the market path when technical progress is Harrod-neutral are given in Razin [5] and [6].

APPENDIX

The Firm's Investment Function

The net cash flow of the representative firm is denoted by $S = Y - I - WL$, where $Y = F(K, A, L)$ and $I = \phi(z)K$.

The firm will seek to maximize

$$\int_0^\infty S_t e^{-\int_0^t \rho_s ds} dt \qquad \ldots(A1)$$

subject to $K_0 > 0$.

The Hamiltonian of this problem is

$$\mathscr{H}(K, \lambda_K, z) = [F(K, A, L) - \phi(z)K - WL + \lambda_K zK]e^{-\int_0^t \rho_s ds}$$

where ρ and W are determined in the market.

Necessary conditions for maximization are given by

$$F_L = W \qquad \ldots(A2)$$

$$\lambda_K = \phi'(z) \qquad \ldots(A3)$$

$$F_k - \phi(z) + \lambda_K z = -\dot{\lambda}_K + \rho\lambda_K. \qquad \ldots(A4)$$

We assume that the transversality condition is fulfilled,

$$\lim_{t \to \infty} \lambda_K K e^{-\int_0^t \rho_s ds} = 0. \qquad \ldots(A5)$$

Combining (A3) and (A4) we get the investment function

$$\frac{\phi''(z)}{\phi'(z)} \dot{z} = \rho - z - \frac{r - \phi(z)}{\phi'(z)} \qquad \ldots(A6)$$

where $r = F_K$.