It is explained in the survey of the literature (see Chapter 4) that the introduction of uncertainty elements into the economic system may have devastating effects on many of the basic theorems of standard international trade theory. However, that literature has not considered markets for risk sharing. It is therefore important to reassess the fate of these theorems in a world of uncertainty when risk-sharing arrangements are allowed to take place.

When stock markets exist in situations of uncertainty, factor allocations are determined by real equity prices and factor endowments—not directly by commodity prices. The links between the competitive allocation of factors of production and commodity prices are real equity prices; these depend not only on commodity prices but also on risk attitudes and probability assessments. International risk sharing, such as international trade in equities, will equalize real equity prices between countries; in the absence of international trade in equities, real equity prices may not be equalized, even though trade in commodities will equalize commodity prices. Therefore, basic trade theorems such
as factor-price equalization are expected to hold under uncertainty if international trade in equities is allowed to take place, but not to hold if international trade in equities does not take place.

Using the model of international trade in goods and securities and the diagrammatic apparatus developed in Chapters 5 and 6, we shall reformulate the central theorems of the pure theory of international trade to accommodate elements of uncertainty. We explain why the basic trade theorems—specialization according to comparative advantage, factor-price equalization, Stolper–Samuelson and Rybczynski—do not carry over to uncertain environments in the absence of international trade in equities and why the Heckscher–Ohlin theorem is upset by uncertainty regardless of whether international trade in equities takes place. We also show that the theorems of specialization according to comparative advantage and factor-price equalization are restored in the presence of international trade in equities, and that there exist valid versions of the Stolper–Samuelson and Rybczynski theorems.

For expositional simplicity we analyze a model with two factors, two equities, and two commodities. However, recent extensions of the standard theorems of international trade to a world of many commodities and many factors of production (Etheh, 1974; Jones and Scheinkman, 1977) carry over to our model if international trade in equities is allowed to take place. The results of a many-commodity Ricardian world apply here as well.

7.1 COMPARATIVE COSTS THEORY

The comparative costs theory (which suggests that each country will specialize in the production and export of the commodity with the lowest relative labor costs) fails to hold when there is no international trade in equities. The reason is that the equilibrium production vector will then depend on preferences and subjective probability beliefs, in addition to relative commodity prices and output–labor ratios; the technological and price risks of the country will be fully borne by local residents, and production decisions could not be separated from preferences, including risk attitudes.

Figure 7.1 shows a typical single-country production equilibrium when international trade in equities does not take place. TT is the Ricardian production possibilities curve of the home country, and UU its highest affordable assets–indifference curve. Es represents a stock market equilibrium in which the country is incompletely specialized.
7. THE BASIC PROPOSITIONS REVISED

assets—indifference map depends on the distribution of commodity prices.

The slope of the assets—budget line $QQ$ is $q$—the relative world price of real equity $2$. $E_{sp}$ is the producers’ equilibrium point, which indicates a complete specialization in the production of commodity 1 and type-1 real equities. $E_{se}$ is the domestic investors’ equilibrium point. The economy imports $z_2$ units of type-2 real equities, and exports $L/a_{l,1} - z_1$ units of type-1 real equities. Therefore, if international trade in equities takes place, each country specializes according to its comparative advantage—which is well defined by output—input ratios, given that the countries have the same distribution of $\theta(x)$.

We now give a two-country example in which the absence of international trade in equities each country specializes in the commodity in which it has a comparative disadvantage. It is clear from our previous discussion that specialization according to comparative advantage will be restored with the introduction of trade in equities.

**EXAMPLE 7.1** Let the utility function of the home country be

(7.1) \[ u(c_1, c_2) = \log(c_1 + \log c_2) \]

and that of the foreign country be (asterisks denote variables of the foreign country)

(7.2) \[ u^*(c_1^*, c_2^*) = \gamma\left[(c_1^*)^{1/2}(c_2^*)^{1/2}\right]^\mu \]

\[ \mu = \frac{1}{2} \log 1.1, \quad \gamma > 0 \]

Hence, both countries are risk averse.

These utility functions yield the demand and indirect utility functions (with an appropriate choice of $\gamma$)

(7.3a) \[ c_1 = I - 1 \]

(7.3b) \[ c_2 = 1/p \]

(7.3c) \[ v = \log(I - 1 - \log p) \]

(7.4a) \[ c_1^* = I^*/2 \]

(7.4b) \[ c_2^* = I^*/2p \]

(7.4c) \[ v^* = p^{-\mu/2}(I^*)^\mu \]

7.1 COMPARATIVE COSTS THEORY

where $p$ is the relative price of the second commodity and $I$ stands for income, that is, $I = \theta z_1 + p\theta z_2$.

Let $a_{l,1}$ and $a_{l,1}^*$, $i = 1, 2$, be the labor–output ratio of the $i$th industry.

We assume that the home country has a comparative advantage in commodity 2, that is,

(7.5) \[ \frac{a_{l,2}}{a_{l,1}} < \frac{a_{l,2}^*}{a_{l,1}^*} \]

We choose

(7.6) \[ a_{l,2} = 19, \quad a_{l,1} = 72, \quad L = 72 \]

(7.7) \[ a_{l,2}^* = 20, \quad a_{l,1}^* = 72, \quad L^* = 40 \]

Assume that there are two states of the world with equal probabilities

(7.8) \[ \pi(1) = \pi(2) = \frac{1}{2} \]

These are both objective and subjective probabilities for each country.

We assume also that each country has the same distributions of the technological parameters, which are

(7.9a) \[ \theta_1(1) = 2, \quad \theta_1(2) = 5 \]

(7.9b) \[ \theta_2(1) = \theta_2^*, \quad \theta_2(2) = \theta_2^* \]

We show now that there is an equilibrium of the world economy in which there is no trade in equities and in which the home country specializes in the production of the first commodity while the foreign country specializes in the production of the second commodity, contrary to comparative advantage.

To see this, consider the allocation of resources

(7.10a) \[ z_1 = Z_1 = \frac{L}{a_{l,1}} = 1, \quad z_2 = Z_2 = 0 \]

(7.10b) \[ z_1^* = Z_1^* = 0, \quad z_2^* = Z_2^* = \frac{L^*}{a_{l,3}^*} = 2 \]

This implies, using (7.3) and (7.4), the following equilibrium condition in commodity markets in state $z$ (using only the market for good 2):

\[ c_2(z) + c_2^*(z) = \frac{1}{p(z)} + \theta_2(z) \]

\[ = \theta_2(z)(Z_2 + Z_2^*) \]

\[ = 2\theta_2(z) \]
which yields

\[(7.11) \quad p(\alpha) = \frac{1}{\theta_2(\alpha)}, \quad \alpha = 1, 2\]

The marginal rates of substitution between assets, evaluated at the allocation \((7.10)\) and equilibrium prices \((7.11)\), are given by

\[(7.12) \quad MRS = \frac{E[\theta_1(\alpha)z_1 + p(\alpha)\theta_2(\alpha)z_2 - 1 - \log p(\alpha)]^{-1}\theta_1(\alpha)p(\alpha)}{E[\theta_1(\alpha)z_1 + p(\alpha)\theta_2(\alpha)z_2 - 1 - \log p(\alpha)]^{-1}\theta_1(\alpha)}\]
\[= \frac{18}{72}\]

\[(7.13) \quad MRS^* = \frac{E[p(\alpha)^{-\frac{n}{2}}[\theta_1(\alpha)z_1^* + p(\alpha)\theta_2(\alpha)z_2^*]^{-1}p(\alpha)\theta_2(\alpha)}{E[p(\alpha)^{-\frac{n}{2}}[\theta_1(\alpha)z_1^* + p(\alpha)\theta_2(\alpha)z_2^*]^{-1}\theta_1(\alpha)}\]
\[= \frac{21}{72}\]

Hence, for

\[(7.14) \quad q = \frac{18}{72} < \frac{a_{l2}}{a_{l1}} < \frac{a_{l2}^*}{a_{l1}^*} < q^* = \frac{21}{72}\]

producers and investors choose the equilibrium values given in \((7.10)\), as shown in Figure 7.3.

The equilibrium consumption levels are

\[(7.15a) \quad c_1(\alpha) = \theta_1(\alpha) - 1 \Rightarrow c_1(1) = 1, \quad c_1(2) = 4\]
\[(7.15b) \quad c_2(\alpha) = \theta_2(\alpha) \Rightarrow c_2(1) = e^9, \quad c_2(2) = e\]
\[(7.16a) \quad c_1^*(\alpha) = p(\alpha)\theta_2(\alpha) \Rightarrow c_1^*(1) = 1, \quad c_1^*(2) = 1\]
\[(7.16b) \quad c_2^*(\alpha) = \theta_2(\alpha) \Rightarrow c_2^*(1) = e^9, \quad c_2^*(2) = e\]

These values are, of course, consistent with the commodity market clearing conditions. This completes the example.

So far we have discussed identical distributions of the technological parameters, which means that technological uncertainty is industry specific but not country specific. It may, of course, happen that two countries have different distributions of \(\theta_j(\alpha)\). In this case they produce different real equities while producing the same commodities. International trade in goods and securities will not lead to specialization in production if there exists a positive world demand for all securities. Furthermore, a country need not export that commodity in which its input–output ratio is lowest. However, we argue that in this case comparative advantage should be defined with respect to real equities—and, thus, a country exports that real equity in which comparative costs are lowest.

### 7.2 FACTOR-PRICE EQUALIZATION

The factor-price equalization theorem is fundamental to the Heckscher–Ohlin theory of international trade. It suggests that (under certain conditions) free trade in commodities is sufficient to cause factor prices to be equalized between countries even if there are no world markets for factors of production. This theorem fails to hold under uncertainty in the absence of international trade in securities.
In order to ensure factor-price equalization in our model, one has to ensure that the relative price of type-2 real equities, $q$, is equalized among the trading countries. But trade in commodities does not bring about equalization of $q$ even in the case in which the traditional conditions for factor-price equalization, such as the absence of specialization and factor intensity reversals, hold. Consider, for example, two trading countries which are identical in all respects, except that the second country has more physical capital than the first. Let the stock market equilibrium of the first country be described by $E_1$ in Figure 7.4.

Clearly, if the Rybczynski line for capital changes, RR, does not coincide with the wealth–portfolio line EE of the second country which passes through $E_1$, where the wealth–portfolio line is the collection of optimal portfolio points for given relative real equity prices and different levels of wealth (i.e., the asset space counterpart of the income-consumption line)—the stock market equilibrium of the second country would consist of a different relative price of type-2 real equities—and, thus, factor prices will not be equalized. Differences in tastes, attitudes toward risk, and subjective probability beliefs may also prevent factor-price equalization.

This does not mean that factor-price equalization is never expected to hold. For if the assets–indifference curves of both countries are linear with the same slope, then factor-price equalization will obtain. Neutrality toward risk by both countries makes their assets–indifference curves linear but not necessarily with the same slope, unless their ordinal preferences are the same. But even if countries exhibit risk aversion, the equilibrium distribution of prices may be such as to make the assets–indifference curves linear with the same slope.

Consider, for example, the case in which two countries have Cobb–Douglas-type utility functions which exhibit different degrees of risk aversion. The ordinal preferences may also differ. Given identical technologies, the equilibrium price distribution is in this case such that the vector $[p(1)\theta_1(1), p(2)\theta_1(2), \ldots, p(S)\theta_1(S)]$ is proportional to the vector $[\theta_1(1), \theta_1(2), \ldots, \theta_1(S)]$. This means that in the equilibrium the two types of equities—real equity 1 and real equity 2—are perfect substitutes in the investor’s portfolio, implying straight-line assets–indifference curves with a slope equal to the factor of proportionality between the two vectors. Hence, both countries have the same straight-line assets–indifference field, implying factor-price equalization under the standard conditions.

First, let us show that proportionality of the two vectors implies straight-line assets–indifference curves, with a slope equal to the factor of proportionality, for every utility function. Suppose that

$$\theta_1(\alpha)p(\alpha) = \lambda \theta_1(\alpha), \quad \alpha = 1, 2, \ldots, S, \quad \lambda > 0$$

Then

$$MRS = \frac{E_{\theta_1}(\alpha)\theta_1(\alpha)p(\alpha)}{E_{\theta_1}(\alpha)\theta_1(\alpha)} = \lambda$$

Second, let us show that (7.17) is implied by Cobb–Douglas utility functions. Let the utility functions be

$$u = (c_1^{1-\beta}c_2^\beta)^{\mu}, \quad 0 < \beta, \quad \mu < 1$$

$$u^* = [(c_1^{*1-\beta}c_2^{*\beta})^{\mu}], \quad 0 < \beta^*, \quad \mu^* < 1$$

Then, the market clearing condition for good 2 in state $\alpha$ is

$$\beta[\theta_1(\alpha)z_1 + p(\alpha)\theta_2(\alpha)z_2] + \beta^*[\theta_1(\alpha)z_1^* + p(\alpha)\theta_2(\alpha)z_2^*] = p(\alpha)\theta_2(\alpha)(Z_2 + Z_2^*)$$
It is clear from (7.20) that (7.17) holds with

\[ \lambda = \frac{\beta z_1 + \beta^* z_1^*}{z_2 + z_2^* - \beta z_2 - \beta^* z_2^*}, \quad \lambda > 0 \]

since \( z_2 + z_2^* = z_2 + z_2^* \) and \( 0 < \beta, \beta^* < 1 \). The Cobb-Douglas result holds also in a many-commodity world, as one can easily verify.

We have thus shown that in the case of Cobb-Douglas-type ordinal preferences, factor-price equalization holds under the standard conditions if there is no international trade in securities. Now, suppose that we open the economies to trade in real equities. Then, for general utility functions, if countries have the same distribution of \( \theta_j(x) \), \( j = 1, 2 \), that is, technological uncertainty is industry specific and not country specific (this is the generalization under uncertainty of the assumption of identical production technologies between countries), then independent of preferences, factor-price equalization will hold. International trade in equities will equalize relative real-equity prices across countries, and thus is sufficient to cause factor prices to be equalized under the standard conditions.

In the Heckscher-Ohlin model, commodity trade serves as a substitute for factor movements if countries do not specialize; this is due to the factor-price equalization theorem. However, it is clear from our discussion that in the presence of uncertainty, commodity trade does not substitute for factor movements—but trade in goods and securities does substitute for factor movements, provided the distribution of \( \theta_j(x) \), \( j = 1, 2 \), is the same in the trading countries in addition to the other assumptions. That is, if the conditions of the theorem are satisfied, the opening of international markets for factors of production will not induce international factor flows (Mundell, 1957).

It is important at this point to recognize the strength of the assumptions which assure factor-price equalization in the presence of international trade in equities. Apart from the standard Heckscher-Ohlin-type assumptions, it is required that countries have identical sectoral-specific perfectly correlated technological uncertainty. Thus, if India and the United States both produce the same crop, then not only do they have to have the same density function of rainfall, but also that when in fact 66.9 inches of rain falls in the United States, then exactly 66.9 inches of rain falls in India. This means that we can hardly expect factor-price equalization even in the presence of trade in equities. Clearly, if there is no trade in equities (or for this matter even some equities), then even under very strong assumptions factor-price equalization cannot be expected to take place.

7.3 THE STOLPER–SAMUELSON THEOREM

The Stolper–Samuelson theorem asserts that in the Heckscher-Ohlin model an increase in the price of a commodity induces an increase in the real reward of the factor in which this industry is relatively intensive, and a reduction in the reward of the other factor of production. Let us consider this theorem in the present framework.

Assume, first, that international trade in equities does not take place. For simplicity, consider a small country that faces state-independent commodity prices (that is, which are not subject to uncertainty). The production technology, however, is state dependent. Let \( E_1 \) in Figure 7.5 describe the stock market equilibrium of the country, at which the highest affordable assets-indifference curve \( UU \) is reached.

![Figure 7.5](image)

Suppose that the relative price of the second commodity rises. This shifts the entire assets-indifference map. In general, the marginal rates of substitution between type-1 and type-2 real equities and the assets-indifference curves pivot as a result of the increase in the relative price of good 2.

If the new assets-indifference curve which passes through \( E_1 \) is steeper than \( UU \) at \( E_1 \), as described by \( U'U' \), the relative price of type-2 real equities will rise—thereby inducing an increase in the real reward (in terms of real equities) of the factor that is used more intensively in the second industry. This is the Stolper–Samuelson theorem.
Hence, for sufficiently large Var[δ/ξ], we get

\[ \frac{\partial \text{MRS}}{\partial \theta} < 0 \]

This choice of real-equity holdings can be assessed by an appropriate choice of production technologies and factor endowments. Then the derivative of MRS with respect to \( \theta \) evaluated at the initial equilibrium is

\[ \text{MRS} = \frac{E[U\theta]}{E[U\theta]} \]

Assume now that \( \theta(x) = p \) for all \( x \), \( x = x_0 \), and that at the initial equilibrium

\[ \text{MRS} = \frac{E[U\theta(x)]}{E[U\theta(x)]} \]

which implies

\[ \frac{E[U\theta(x)]}{E[U\theta(x)]} = \frac{E[U\theta(x)]}{E[U\theta(x)]} = \frac{E[U\theta(x)]}{E[U\theta(x)]} \]

The yields the indirect utility function

\[ v = \log \left( \frac{\gamma}{1 + \log \gamma} \right) \]

EXAMPLE 7.2. Let the utility function be

Now, suppose that we open the economy to trade in real equities. Then, for our example, the relative price of a real equity is given. Figure 7.6 describes the new stock market equilibrium. The production point is \( E^* \), and the portfolio point is \( E^\pi \). The slope of the real money demand function is \( \gamma \) and the relative world price of real equity is \( p^* \). The slope of the real money demand function is \( \gamma \) and the relative world price of real equity is \( p^* \). The slope of the real money demand function is \( \gamma \) and the relative world price of real equity is \( p^* \).
using the other factor relatively intensively contracts. This theorem also fails to hold under conditions of uncertainty if there is no international trade in securities.

Consider Figure 7.4. Suppose $EE$ stands for the wealth–portfolio line of the home country, which is derived from the asset–indifference curves. It is clear that the Rybczynski line need not coincide with the wealth–portfolio line. The increase in the endowment of the factor used more intensively by the second industry at unchanged real-equity prices would cause output changes indicated by points on the Rybczynski line. However, if real-equity prices are unchanged, demand for real equities would be indicated by points on the wealth–portfolio line, which is not an equilibrium situation. It is seen from Figure 7.4 that real-equity prices have to change, although the distribution of commodity prices is kept constant, in order for the local demand for real equities to match local supply. The diagram indicates that the relative price of type-2 real equities will decrease; in the new equilibrium, expected output of the second industry will be larger—but the expected output of the first industry need not decline.

Furthermore, from portfolio theory we know that the wealth effect on securities is not unambiguously positive, so the wealth–portfolio line need not be positively sloped. If demand for type-2 real equities responds negatively to an increase in wealth, the expected output of the second industry could decline.

When international trade in equities is allowed, a small open economy faces given real-equity prices. In this case an increase in the endowment of a factor of production will generate the Rybczynski effect because equity prices do not change. Hence, in an uncertain world the Rybczynski theorem is saved if properly reformulated; namely, if what is kept constant is equity prices and not necessarily commodity prices. This is very clear if we remember that in an economy with stock markets, production decisions depend on equity prices and not on commodity prices.

7.5 THE HECKSCHER–OHLIN THEOREM

The Heckscher–Ohlin theorem links the pattern of trade to factor intensities and factor endowments. There are two definitions of relative factor abundance—the quantity and the value definitions. The quantity definition is based on relative factor endowments, while the value definition is based on relative factor prices. The Heckscher–Ohlin theorem under the quantity definition of relative factor abundance requires more stringent conditions than under the value definition of relative factor abundance. We concentrate, therefore, on the value version which states that a country exports the commodity which is relatively intensive in the factor whose relative reward prior to trade is lower than abroad.

That this theorem fails to hold under uncertainty, with or without international trade in equities, is clear from the fact that the pattern of trade may be state dependent (see Chapter 6); that is, a country may export one commodity in some states of nature, yet import this commodity in other states of nature.

Can the pattern of equity trade be linked to factor intensities and factor endowments? We were assured that the posttrade relative equity prices lie between the pretrade relative equity prices of the country, then we could have provided an affirmative answer using the standard argument of the value version of the Heckscher–Ohlin theorem. The preconditions on equity prices is satisfied if the pretrade preferences over securities prevail also in the posttrade situation. However, no such assurances exist. The opening of trade changes the distribution of commodity prices, which, in turn, changes the preferences over securities. The change in preferences may induce posttrade real-equity prices to lie outside the pretrade bounds.

REFERENCES

