Chapter 3

Elements of the Theory of Economic Decision Making under Uncertainty

In a world of certainty, actions imply in many instances unique consequences. Therefore a choice among consequences determines a choice among actions. However, under uncertainty, an action taken before the resolution of uncertainty does not uniquely determine the outcome. The outcome will also depend on the state of nature that realizes. The meaning of uncertainty is that the individual does not know the state of nature, although he may have a subjective probability belief over states of nature. In this chapter we describe some of the results from the theory of decision making under uncertainty. For the present purpose there can be continuum, countable, or finite states of nature.

3.1 EXPECTED UTILITY, RISK AVERSION, AND PORTFOLIO CHOICE

Assume that it is possible to attach numbers called utilities to consequences of actions in such a way that the *expected utility* measures the desirability of an action (von Neumann and Morgenstern, 1944).

A *risk averter* is defined as one who finds it unprofitable to participate in a *fair gamble*. A gamble is said to be fair if its expected value to the individual is zero. Let I and $u(I)$ be, respectively, income and the utility of income. Confronted with a choice among actions, an individual is supposed to choose that action which maximizes the expected utility of income, $Eu(I)$, where E is the expectation operator. No saturation of individual desires implies $u'(I) > 0$, where $u'(I)$ is the marginal utility of income.

Now consider a risk-averse individual who is offered a choice between a certain income I_0 and a chance gamble in which he would gain h_1 with probability π and lose h_2 with probability $1 - \pi$, where h_1 and h_2 are positive numbers. Being a risk averter, if $\pi h_1 - (1 - \pi)h_2 = 0$, he will choose the certain income. If this holds for all h_1, $h_2 > 0$, it implies that $u(\cdot)$ is a concave function of income; that is, $u''(I) < 0$ (see Figure 3.1).

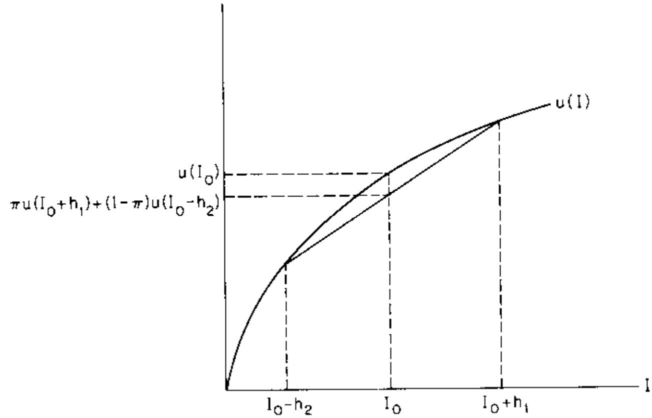

FIGURE 3.1

Arrow (1964) and Pratt (1964) suggested the following risk-aversion measures:

$$A(I) = -\frac{u''(I)}{u'(I)} = \text{absolute risk aversion}$$

$$P(I) = -\frac{Iu''(I)}{u'(I)} = \text{relative risk aversion}$$

where $u''(\cdot)$ is the second derivative of $u(\cdot)$. The measure of absolute (relative) risk aversion is said to be increasing if $A'(I) > 0$ $[P'(I) > 0]$ and it is said to be decreasing if $A'(I) < 0$ $[P'(I) < 0]$.

It can be shown (Arrow, 1964) that if a risk-averse individual is offered a certain income I_0 or a two-state gamble in which he gains h $(h > 0)$ in state 1 and loses h in state 2, then for small h he will not be indifferent between the two offers, unless the probability of state 1 exceeds $\frac{1}{2}$ (the probability associated with a fair gamble) by a number which is proportional to $A(I)$. Similarly, if a risk-averse individual is offered a certain income I_0 or a two-state gamble in which he gains hI_0 in state 1 and loses hI_0 in state 2, $h > 0$, then for small h he will not be indifferent between the two offers, unless the probability of the favorable state exceeds $\frac{1}{2}$ by a number which is proportional to $P(I)$.

The following are examples of utility functions with corresponding measures of risk aversion:

(a) $u = 1 - e^{-aI}, \qquad a > 0$

$$\Rightarrow \quad A(I) = a \quad \text{and} \quad P(I) = aI$$

(b) $u = aI - bI^2, \qquad a, b > 0, \qquad \text{for} \quad 0 \le I \le \dfrac{a}{2b},$

$$\Rightarrow \quad A(I) = \frac{2b}{a - 2bI} \quad \text{and} \quad P(I) = \frac{2bI}{a - 2bI}$$

(c) $u = \dfrac{1 - I^{1-a}}{a - 1}, \qquad a \ge 0,$

$$\Rightarrow \quad A(I) = \frac{a}{I} \quad \text{and} \quad P(I) = a$$

The first function exhibits constant absolute and increasing relative risk aversion; the second function exhibits increasing absolute and relative risk aversion; and the third function exhibits decreasing absolute and constant relative risk aversion.

The usefulness of the measures of risk aversion can be seen by considering changes in the optimal portfolio selected by an expected utility maximizer as his initial wealth changes. Consider a risk-averse individual who chooses his portfolio so as to maximize the expected utility of the return on the portfolio. The individual has an initial wealth W_0, which can be allocated between two assets—one safe and one risky. The risky asset yields a return of $R(\alpha)$ in state α, per dollar invested in it. The safe asset yields the same return in every state of nature, which for simplicity will be assumed to be unity. If the individual allocates a fraction s of his initial wealth to the risky asset, his return (income) in state α will be

$$I(\alpha) = W_0 + s[R(\alpha) - 1]W_0$$

Therefore, the investor's problem is

$$\text{(3.1)} \qquad \begin{array}{l} \text{choose} \quad s \\ \text{to maximize} \\ Eu\{W_0 + s[R(\alpha) - 1]W_0\} \end{array}$$

If no sign restrictions are imposed on s, the first-order condition for a maximum is

$$\text{(3.2)} \qquad Eu'[I(\alpha)][R(\alpha) - 1] = 0$$

We can now show that purchases of the risky asset increase, remain unchanged, or decrease with initial wealth, as there is decreasing, constant, or increasing absolute risk aversion.

To see this define $B = sW_0$ as the total investment in the risky asset, and differentiate (3.2) to obtain

$$\text{(3.3)} \qquad \frac{dB}{dW_0} = -\frac{Eu''[I(\alpha)][R(\alpha) - 1]}{Eu''[I(\alpha)][R(\alpha) - 1]^2}$$

By the assumption of risk aversion, the sign of the denominator of (3.3) is negative.

Let $A^* = A(W_0)$ be the value of the absolute risk-aversion measure when the portfolio consists of only the safe asset. The numerator of (3.3) can then be rewritten as

$$\begin{aligned} Eu''[I(\alpha)][R(\alpha) - 1] &= E\,\frac{u''[I(\alpha)]}{u'[I(\alpha)]}\,u'[I(\alpha)][R(\alpha) - 1] \\ &= -EA[I(\alpha)]u'[I(\alpha)][R(\alpha) - 1] \\ &= E\{A^* - A[I(\alpha)]\}u'[I(\alpha)][R(\alpha) - 1] \\ &\quad - A^*Eu'[I(\alpha)][R(\alpha) - 1] \\ &= E\{A(W_0) - A[I(\alpha)]\}u'[I(\alpha)][R(\alpha) - 1] \end{aligned}$$

where, in the last step use has been made of (3.2).

If $A'(I) > 0$, then when $R(\alpha) - 1 > 0$, $A(W_0) < A[I(\alpha)]$ and $[R(\alpha) - 1]\{A(W_0) - A[I(\alpha)]\} < 0$; and when $R(\alpha) - 1 < 0$, $A(W_0) > A[I(\alpha)]$, and $[R(\alpha) - 1]\{A(W_0) - A[I(\alpha)]\} < 0$. Therefore, the numerator in (3.3) is negative. Conversely, if $A'(I) < 0$, it can be shown to be positive. If $A'(I) = 0$, the numerator in (3.3) is equal to zero. This proves the assertion.

Analogously, wealth elasticities of the demand for assets are determined by the properties of the measure of relative risk aversion. It can be shown that the wealth elasticity of the demand for the safe asset is greater than, equal to, or less than unity as relative risk aversion is an increasing, constant, or decreasing function of wealth.[1]

Finally, it was shown by Pratt (1964) that a utility function $u^*(\cdot)$ is everywhere more risk averse than a utility function $u(\cdot)$ if and only if $u^*(\cdot)$ is a concave increasing transformation of $u(\cdot)$. This is equivalent to the statement that the absolute (relative) measure of risk aversion is everywhere larger for $u^*(\cdot)$ than for $u(\cdot)$.

3.2 INCREASING RISK

When is an investment venture said to be more risky than another investment venture? Rothschild and Stiglitz (1970) suggest three answers to this question. Let $R(\alpha)$ and $R'(\alpha)$ be random returns on two different investment projects. $R(\alpha)$ is said to be more risky than $R'(\alpha)$ if

(1) $R(\alpha)$ is equal to $R'(\alpha)$ plus some uncorrelated noise, that is, $R(\alpha) = R'(\alpha) + Z(\alpha)$, where $E[Z(\alpha)|R'(\alpha')] = 0$ for all α'.
(2) Given $ER'(\alpha) \geq ER(\alpha)$, every risk averter prefers $R'(\alpha)$ to $R(\alpha)$ that is, $Eu[R'(\alpha)] \geq Eu[R(\alpha)]$ for all concave $u(\cdot)$.
(3) $R(\alpha)$ and $R'(\alpha)$ have the same mean and $R(\alpha)$ has more weight in the tails than $R'(\alpha)$.

They proved that conditions (1)–(3) lead to a single definition of greater riskiness; that is, conditions (1)–(3) are equivalent. When we deal in Chapter 4 with increasing riskiness, we shall mean mean-preserving transformations of the original distribution, as in (3), which is also equivalent to the other two definitions of increasing riskiness just mentioned.

[1] See Arrow (1964). For an analysis of wealth effects on portfolios with more than two assets, see Cass and Stiglitz (1972).

3.3 MARKETS FOR RISK SHARING

A. *Contingent Commodity Markets*

Imagine an economy which consists of H individuals, and in which there are N commodities and S states of nature. In this economy, trading takes place before the resolution of uncertainty, and individuals can contract on the delivery of every good contingent upon the realization of a state of nature. Thus, if individual h buys 10 units of good 2 contingent on state 7, then he will get the 10 units of good 2 if state 7 realizes, and he will get nothing if another state of nature realizes.

Let $e_i^h(\alpha)$ be h's endowment of good i in state α, $c_i^h(\alpha)$ be his consumption of good i in state α, and $c^h(\alpha)$ his consumption vector. Then, given contingent commodity prices $g_i(\alpha)$, where $g_i(\alpha)$ is the unit price of good i to be delivered in state α, his budget constraint is

$$\sum_{\alpha=1}^{S} \sum_{i=1}^{N} g_i(\alpha) c_i^h(\alpha) \leq \sum_{\alpha=1}^{S} \sum_{i=1}^{N} g_i(\alpha) e_i^h(\alpha), \qquad h = 1, 2, \ldots, H \tag{3.4}$$

Individual h's tastes are represented by a von Neumann–Morgenstern utility function $u^h(\cdot)$, defined on the consumption vector at that state $c^h(\alpha)$ and the individual's probability beliefs, represented by a vector

$$[\pi^h(1), \pi^h(2), \ldots, \pi^h(S)], \pi^h(\alpha) \geq 0, \qquad \sum_{\alpha=1}^{S} \pi^h(\alpha) = 1$$

where $\pi^h(\alpha)$ is individual h's subjective probability assessment of state α. Let $u^h[c^h(\alpha)]$ be a concave function; that is, $u^h(\cdot)$ exhibits risk aversion. Expected utility of individual h, W^h, is a function of the contingent consumption vector $[c^h(1), c^h(2), \ldots, c^h(S)]$:

$$W^h[c^h(1), c^h(2), \ldots, c^h(S)] = \sum_{\alpha=1}^{S} \pi^h(\alpha) u^h[c^h(\alpha)] \tag{3.5}$$

Individual h's decision-making problem is to choose the vector of contingent commodity claims $[c^h(1), c^h(2), \ldots, c^h(S)]$ to maximize (3.5) subject to the budget constraint (3.4).

In equilibrium, aggregate demand for every contingent commodity claim has to equal its supply. Namely,

$$\sum_{h=1}^{H} c_i^h(\alpha) = \sum_{h=1}^{H} e_i^h(\alpha), \tag{3.6}$$

$$i = 1, 2, \ldots, N, \quad \alpha = 1, 2, \ldots, S$$

This is exactly analogous to the certainty case. Note, however, that the number of goods in the uncertainty model is SN instead of N in the certainty model, since here a good is distinguished by the state in which it is consumed in addition to its physical characteristics.

B. *Arrow Securities*

Instead of markets for commodity claims, assume now that there are securities which are payable in money. The amount of money paid by a security depends on the state of nature that realizes. Security α pays 1 dollar if state α occurs or zero if a different state occurs. There are precisely S such securities. Trade in securities takes place at the beginning of the period. Then, when a state α occurs, trade in commodities takes place.

Let $q(\alpha)$ be the price of security α, and $p_i(\alpha)$ the price of commodity i in state α. Consumer h solves a two-stage decision-making problem. In the first stage, before the resolution of uncertainty, he determines his portfolio; in the second stage, after the resolution of uncertainty, he uses portfolio returns to purchase commodities.

Suppose, for the moment, that the portfolio allocation $[A^h(1), A^h(2), \ldots, A^h(S)]$ has been chosen by individual h, where $A^h(\alpha)$ is his amount of security α holdings, $\alpha = 1, 2, \ldots, S$. Then, when the state of nature α realizes, commodity prices $[p_1(\alpha), \ldots, p_N(\alpha)]$ become known. In state α individual h solves the ordinary consumption problem:

(3.7) choose $c_1^h(\alpha), c_2^h(\alpha), \ldots, c_N^h(\alpha) \geq 0$

to maximize

$u^h[c^h(\alpha)]$

subject to

$$\sum_{i=1}^{N} p_i(\alpha) c_i^h(\alpha) \leq A^h(\alpha)$$

Note that only holdings of security α provide income in state α. The solution to this problem yields the indirect utility function $v^h[p_1(\alpha), \ldots, p_N(\alpha); A^h(\alpha)]$.

Turning to portfolio decisions, individual h chooses $[A^h(1), \ldots, A^h(S)]$ so as to maximize the expected value of his indirect utility function:

(3.8) choose $A^h(1), A^h(2), \ldots, A^h(S)$

to maximize

$$\sum_{\alpha=1}^{S} \pi^h(\alpha) v^h[p_1(\alpha), \ldots, p_N(\alpha); A^h(\alpha)]$$

subject to

$$\sum_{\alpha=1}^{S} q(\alpha)A^h(\alpha) \leq \sum_{\alpha=1}^{S} q(\alpha)\left[\sum_{i=1}^{N} p_i^h(\alpha)e_i^h(\alpha)\right]$$

where $\sum_{i=1}^{N} p_i(\alpha)e_i^h(\alpha)$ is h's endowment of security α, and it equals the value of his commodity endowment in state α.

In equilibrium the demand for good i in state α equals its supply. Hence,

$$\sum_{h=1}^{H} c_i^h(\alpha) = \sum_{h=1}^{H} e_i^h(\alpha), \tag{3.9}$$

$$i = 1, 2, \ldots, N, \quad \alpha = 1, 2, \ldots, S$$

In addition, the demand for every security equals its supply. Hence,

$$\sum_{h=1}^{H} A^h(\alpha) = \sum_{h=1}^{H} \sum_{i=1}^{N} p_i(\alpha)e_i^h(\alpha), \qquad \alpha = 1, 2, \ldots, S \tag{3.10}$$

To see the relationship between the contingent commodity claims and the Arrow securities models, suppose that

$$q(\alpha)p_i(\alpha) = g_i(\alpha) \tag{3.11}$$

In words, the price of security α times the spot price of commodity i in state α is equal to the price of a claim on one unit of commodity i in state α.

An individual facing those prices has the same opportunities under the two systems. In the securities framework he can effectively acquire a claim to a unit of commodity i in state α by paying $p_i(\alpha)q(\alpha)$. In the contingent commodity claims framework he can effectively acquire a unit of commodity i in state α by paying $q_i(\alpha)$. Hence, the effective price of a unit of a good in a given state is the same under both systems.

Arrow (1963 1964) showed that any Pareto optimal allocation can be realized by either a system of perfectly competitive markets in contingent claims on commodities or by a system of perfectly competitive markets in Arrow securities, provided there are self-fulfilling price expectations. In the former case there are NS markets, while in the latter case there are only $N + S$ markets. The two systems will be referred to as *complete market systems.*

In a complete market system the existing markets reveal an objective price for every good in every state of nature. This price is used by all market participants to evaluate goods in states of nature. If there is production, firms can use these prices to evaluate inputs and outputs so that they can maximize profits as in the deterministic environment. In such cases, producers do not bear risks; risks are borne only by consumers.

Now, one may have a complete market system even if there are no Arrow-type securities. What is important is to have sufficiently diverse securities in adequate numbers so that by an appropriate combination of these securities an investor will be able to assure himself of a dollar return in a particular state of nature and zero return in all other states, for all states of nature. Put differently, if the existing securities are capable of replicating the return patterns of Arrow-type securities, then we have a complete market system. This occurs if there exist S securities with independent patterns of return (in the algebraic sense).

In the real world, however, there are not enough securities to generate complete markets. We have stock markets, bond markets, etc., but the total number of traded securities falls short of the number of states of nature and the economy operates with less than complete markets.

3.4 INDIVIDUAL DECISION MAKING UNDER UNCERTAINTY: AN APPLICATION

In this section we elaborate on some elements of decision making under uncertainty in order to clarify some issues that were discussed in previous sections in general terms. Consider a simplified economy, with a single good, two firms, and two states of nature. A firm produces a state-dependent output with no input, so that the firm faces no decision problem. The firms are owned by individuals, and individuals trade in ownership shares before the resolution of uncertainty. After the resolution of uncertainty there is no incentive to trade in goods, because there is only one good.

We concentrate on a single individual and shall omit, therefore, the superscript. Since there is only one commodity, we also omit the subscript i.

The individual's preferences over consumption in different states of nature are represented by

$$W[c(1), c(2)] = \pi(1)u[c(1)] + \pi(2)u[c(2)] \tag{3.12}$$

The construction of indifference curves between $c(1)$ and $c(2)$ is shown in Figure 3.2.

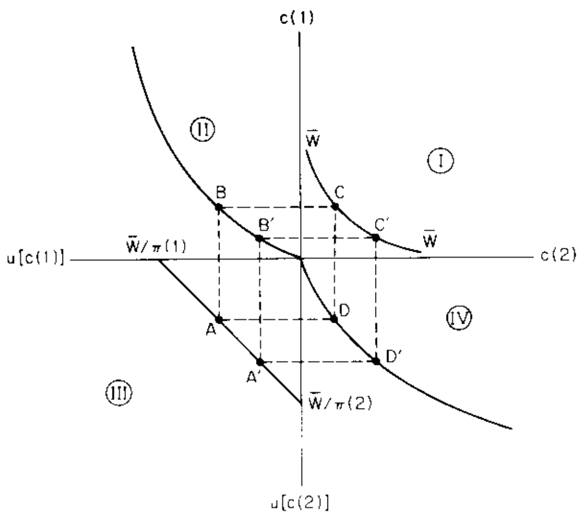

FIGURE 3.2

Quadrants II and IV in Figure 3.2 depict utility as a function of state-1 and state-2 consumptions, respectively. The line ranging from $\bar{W}/\pi(1)$ to $\bar{W}/\pi(2)$ in quadrant III describes all combinations of state-1 and state-2 utility levels for which the level of expected utility is fixed and equal to $\bar{W}$. Point A, in quadrant III, represents one such combination. Point C in quadrant I represents the combination of $c(1)$ and $c(2)$ which corresponds to point A. The same expected utility level is also achieved from the combination which is represented by points A' and C'. Concavity of the utility function implies that the indifference curve $\bar{W}\bar{W}$ in quadrant I, which connects points C and C', is convex to the origin. This shows that the preference function $W(\cdot)$ has convex to the origin indifference curves.

We turn now to the consumer's opportunity set. Let V_f be the market value of firm f, and let $R_f(\alpha)$ be its return (output) in state α, $f = 1, 2$. At the beginning of the period, V_f is known by every trader in the stock market but $R_f(\alpha)$ is unknown. $R_f(\alpha)$ is known only after the resolution of uncertainty at the end of the period.

At the beginning of the period the consumer buys or sells proportionate shareholdings in firm f at its going market value V_f. We denote by $\bar{s}_f$ his initial share ownership and by s_f his final share ownership in firm f. His portfolio investment is subject to the constraint

$$\sum_{f=1}^{2} s_f V_f = \sum_{f=1}^{2} \bar{s}_f V_f \equiv \bar{V} \tag{3.13}$$

The individual's consumption in state α equals the return on his portfolio; that is,

$$c(\alpha) = \sum_{f=1}^{2} s_f R_f(\alpha), \qquad \alpha = 1, 2 \tag{3.14}$$

Suppose that it is possible to sell short the ownership shares in firms. For our purposes a short sale is defined as an exchange in which an individual borrows units of a financial asset at the beginning of the period, agreeing to repay the lender the market value of these units at the end of the period. Thus, short sales of firm f shares mean $s_f < 0$.

In order to describe the consumer's two-dimensional opportunity set in consumption space, eliminate the s_f's from (3.14) and substitute them into (3.13) to obtain

$$c(1)\left[\frac{R_2(2)}{V_2} - \frac{R_1(2)}{V_1}\right] + c(2)\left[\frac{R_1(1)}{V_1} - \frac{R_2(1)}{V_2}\right] = \frac{[R_1(1)R_2(2) - R_1(2)R_2(1)]\bar{V}}{V_1 V_2} \tag{3.15}$$

Observe that in an equilibrium all terms in brackets have to be of the same sign. For suppose $R_2(2)/V_2 > R_1(2)/V_1$ and $R_2(1)/V_2 > R_1(1)/V_1$. Then shares of the second firm dominate the shares of the first firm as portfolio assets and there will be an excess demand for type-2 securities. Similarly, if the opposite inequalities hold, there will be an excess demand for type-1 securities. Hence, the terms in brackets on the left-hand side of (3.15) have the same sign. Now,

$$\frac{R_2(2)}{V_2} \geq \frac{R_1(2)}{V_1}$$

$$\frac{R_1(1)}{V_1} \geq \frac{R_2(1)}{V_2}$$

imply

$$R_1(1)R_2(2) \geq R_1(2)R_2(1)$$

and

$$\frac{R_2(2)}{V_2} \leq \frac{R_1(2)}{V_1}$$

$$\frac{R_1(1)}{V_1} \leq \frac{R_2(1)}{V_2}$$

imply

$$R_1(1)R_2(2) \leq R_1(2)R_2(1)$$

Hence, the terms in the brackets are all of the same sign. In the limiting case in which the vectors of returns are linearly dependent, all the terms in brackets are zero.

Assuming linear independence of the vectors of return, the consumption opportunity line described by (3.15) can be represented in Figure 3.3 by line A_1A_2.

Point a represents the bundle which obtains from a portfolio with zero holdings of firm-2 shares; that is, $s_2 = 0$ and $s_1 = \bar{V}/V_1$, while point b represents the bundle which obtains from a portfolio with zero holdings of firm-1 shares; that is, $s_1 = 0$ and $s_2 = \bar{V}/V_2$. The line segment which lies strictly between a and b represents portfolios with positive holdings of firm-1 and firm-2 shares; that is, $s_1 > 0$ and $s_2 > 0$, while points on the line segments aA_1 and bA_2 (excluding a and b) represent portfolios which include short sales of shares in firm-2 and firm-1, respectively.

It is seen from Figure 3.3 that with linearly independent patterns of returns across states and short selling, the consumer's opportunity set

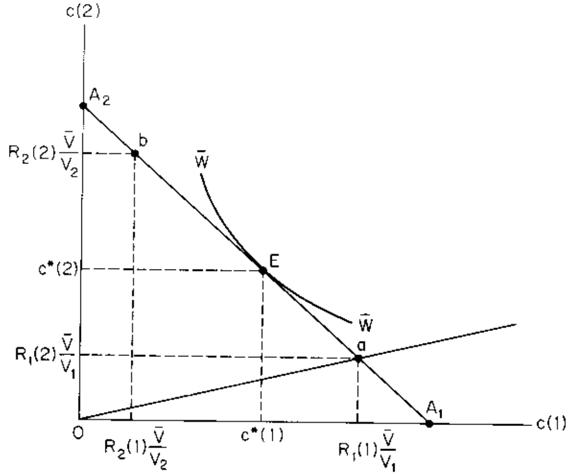

FIGURE 3.3

is $0A_1A_2$ for a given initial wealth $\bar{V}$. By increasing $\bar{V}$ to infinity, the entire nonnegative quadrant becomes the consumption opportunity set. This is equivalent to a situation of complete markets.

Now, if the pattern of returns is linearly dependent, that is, $R_1(1)R_2(2) - R_1(2)R_2(1) = 0$, then line A_1A_2 shrinks to a point, such as point a, and the investor's opportunity set with wealth $\bar{V}$ becomes the line segment $0a$. By increasing his wealth to infinity, his opportunity set becomes the ray from the origin passing through point a. This is a case of incomplete markets. Even by abandoning the assets–budget constraint, an investor is not able to obtain every combination of consumption; there are not sufficient market instruments to achieve it. In this case we have, in fact, only one type of security and two states of nature. Hence, there are too few types of securities compared with the number of states of nature in order to enable equivalence with contingent commodity markets.

Returning to the assumption of linear independence, the consumer's solution is represented by point E in Figure 3.3. The highest achievable expected utility level is shown by the indifference curve $\bar{W}\bar{W}$ and the maximizing expected utility bundle of state-1 and state-2 consumption levels is given by $[c^*(1), c^*(2)]$. The corresponding values of shares in firms 1 and 2, $s_1{}^*$ and $s_2{}^*$, can be determined from (3.14) by substituting $c^*(1)$ and $c^*(2)$ for $c(1)$ and $c(2)$, and solving for s_1 and s_2.

In order to gain more insight, an alternative way of representing the investor's decision-making problem is now discussed. By substituting (3.14) into (3.12), the level of expected utility can be expressed as a function of proportionate shareholdings:

$$U(s_1, s_2) \equiv W\left[\sum_{f=1}^{2} s_f R_f(1), \sum_{f=1}^{2} s_f R_f(2)\right] \tag{3.16}$$

The consumer's preferences over ownership shares in firms can be represented by assets–indifference curves. Since $W(\cdot)$ is concave, $U(\cdot)$ is concave in (s_1, s_2), and we can draw a set of indifference curves between the s_f's, which are convex to the origin.

A typical assets–indifference curve $\bar{U}\bar{U}$ is depicted in Figure 3.4. A similar indifference curve can be drawn even when the number of states of nature is larger than 2, while the number of firms is just 2. Thus, when two firms exist, Figure 3.4 accommodates also various situations of *incomplete markets.*

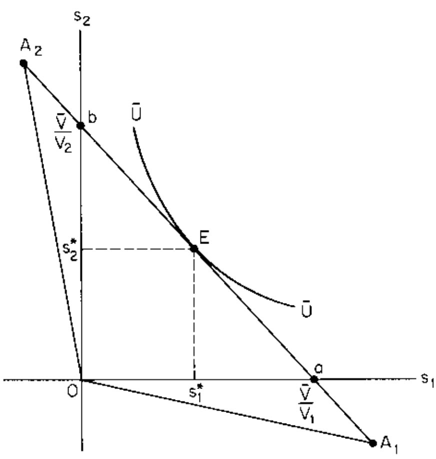

FIGURE 3.4

The opportunity set is described by $0A_2A_1$. The line segment *ab* describes all affordable combinations of s_1 and s_2 with $s_1 \geq 0$ and $s_2 \geq 0$ on the boundary of this set. It corresponds to the line segment *ab* in Figure 3.3. If the consumer is short in type-2 assets, that is, $s_2 < 0$, he must use his return on type-1 assets at the end of the period in order to make good his obligation to repay owners of firm 2. Line $0A_1$

describes all combinations of s_1 and s_2 at which $s_2 < 0$, and the consumer is just solvent at the end of the period in an adverse state, say state 2, where the slope of $0A_1$ is $R_1(2)/R_2(2) = \min_i[R_1(i)/R_2(i)]$. Beyond point A_1, along the extension of the line A_2A_1, the consumer cannot fulfill his contract in an adverse state. Similarly, the maximum nonbankrupt amount of short sales of firm-1 assets is indicated by point A_2. The line segments aA_1 and bA_2 in Figure 3.4 correspond to the line segments aA_1 and bA_2 in Figure 3.3.

A typical solution to the consumer decision-making problem is represented by point E in Figure 3.4. At this point the assets–indifference curve $\bar{U}\bar{U}$ is tangent to the budget line A_1A_2. The expected utility maximizing values of shareholdings in firm 1 and firm 2 are given by $s_1{}^*$ and $s_2{}^*$.

REFERENCES

Arrow, K. J. (1963–1964). The role of securities in the optimal allocation of risk bearing, *Review of Economic Studies* **31**, 91–96.

Arrow, K. J. (1964). "Aspects of the Theory of Risk-Bearing." Yrgö Jahnssonin säätio, Helsinki.

Cass, D., and Stiglitz, J. E. (1972). Risk aversion and wealth effects on portfolios with many assets, *Review of Economic Studies* **39**, 331–354.

Pratt, J. W. (1964). Risk aversion in the small and in the large, *Econometrica* **32**, 122–136.

Rothschild, M., and Stiglitz, J. E. (1970). Increasing risk: I. A definition, *Journal of Economic Theory* **2**, 225–243.

von Neumann, J., and Morgenstern, O. (1944). "Theory of Games and Economic Behavior." Princeton Univ. Press, Princeton, New Jersey.