Volatility of FDI and Portfolio Investments: The Role of Information, Liquidation Shocks and Transparency

Itay Goldstein† and Assaf Razin‡

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Abstract

The paper develops a model of foreign direct investments (FDI) and foreign portfolio investments. FDI is characterized by hand-on management style which enables the owner to obtain relatively refined information about the productivity of the firm. This superiority, relative to portfolio investments, comes with a cost: A firm owned by the relatively well-informed FDI investor has a low resale price because of asymmetric information between the owner and potential buyers. Consequently, investors, who have a higher (lower) probability of getting a liquidity shock that forces them to sell early, will invest in portfolio (direct) investments. This result can explain the greater volatility of portfolio investments relative to direct investments. We show that this pattern may become weaker as the transparency in the capital market or the corporate governance in the host economy increase.

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†Fuqua School of Business, Duke University, Box 90120, Durham, NC 27708. Tel: 1-919-660-7858. E-mail: itayg@duke.edu
‡Eitan Berglas School of Economics, Tel Aviv University; Department of Economics, Cornell University; CPER and NBER. E-mail: razin@post.tau.ac.il.
1 Introduction

Foreign direct investment, FDI, has proven to be resilient during financial crises. For instance, FDI in East Asian countries was remarkably stable during the global financial crises of 1997-98. In sharp contrast, other forms of private capital flows – portfolio equity and debt flows – were subject to large reversals during the same period (as documented by Lipsey (2001)). The resilience of FDI during financial crises was also evident during the Mexican crisis of 1994-95 and the Latin American debt crisis of the 1980s. As a result, FDI inflows into developing countries are often viewed as stable "cold" money, which are generated by long term considerations. In contrast, foreign portfolio flows are often deemed as unstable "hot" money, which are triggered by short term considerations.

A similar conclusion follows the analysis of UNCTAD data on FDI net inflows. This data shows that total net inflows of FDI into developing countries were $187 billions in 1997, $188 billions in 1998, $222 billions in 1999, and $240 billions in 2000 (UNCTAD (2001)), while at the same time net inflows of portfolio investments were much more volatile (see, for example, World Bank (2002)). Using World Bank data on 111 countries, Albuquerque (2001) shows that 88% of the countries in his sample have lower coefficient of variation of net FDI inflows than that of other net inflows.1 Interestingly, Lipsey (1999) shows that the ratio of the volatility of net FDI inflows to the volatility of other net long term inflows is smaller in developing countries than in developed countries: The ratio of FDI’s volatility to other long-term flows’ volatility is 0.59 in Latin America, 0.74 in South East Asia, 0.86 in Europe, and 0.88 in the US.2

This paper develops a model which can explain the persistence of foreign direct investment compared to the volatility of international portfolio flows, characterized by large withdrawals of funds during liquidity crises. The model endogenize the choice of foreign investors between direct investment and portfolio investment in the host country . The model highlights a key difference between the two types of investment: Direct investment investors, who take both ownership and control positions in the domestic firms are in effect the managers of the firm, whereas portfolio

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1 See also Bachetta and van Wincop (2000).
2 The literature also has plenty of other sources of related evidence: Frankel and Rose (1996) show that the size of FDI flows reduces the probability of currency crises. Chuhan, Perez-Quiros and Popper (1996) show that FDI flows are less sensitive to shocks in other countries and to shocks in other types of investments. Sarno and Taylor (1999) show that FDI flows are more persistent than other flows.
investors, who gain ownership without control of domestic firms, must delegate decisions to managers, whose agenda may not be always consistent with that of the owners. Consequently, due to an agency problem between managers and owners, portfolio investment projects are managed less efficiently than direct investment projects. To be more specific, direct investors, who act effectively as managers of their own projects, are more informed than portfolio investors regarding the prospects of their projects. This information enables them to manage their projects more efficiently. This effect generates an advantage, with an added value in the capital markets, to direct investments relative to portfolio investments.

However, there is also a disadvantage to the information that is gained by investing directly. When investors are forced to sell their projects before maturity due to a liquidation shock, the price they can get will be lower if they have more information on the economic fundamentals of the investment project. Thus, if potential buyers know that the seller has more information, they will suspect that the sale results from bad information on the prospects of the investment, and will thus be willing to pay a lower price. In our model, investors sometimes need to sell their investments before maturity because of liquidation shocks. Thus, if they invest directly, the investors bear the cost of getting a lower price if and when they are forced to sell the project before maturity.

Thus, our model describes a key trade-off between management efficiency and liquidity. Both sides of this trade-off are driven by the effect of information. When they invest directly, investors get more information about the fundamentals of the investment, and can manage the project more efficiently, than their portfolio-investors counterparts. However, this also generates an asymmetric-information "lemons" type problem when they try to sell the investment before maturity. Therefore, this superior information effect reduces the price they can get when they are forced to sell the project prematurely. The trade-off generates differences between volatility of direct investment flows and volatility of portfolio investment flows: Investors with high expected liquidation needs are affected by the low price more than they are affected by the management efficiency, and thus, in equilibrium, will choose to become portfolio investors. Similarly, investors with low expected liquidation needs will choose to become direct investors. As a result, portfolio investments will be characterized by a higher probability of early liquidation, and greater volatility, compared to direct investments.

We start by analyzing a simple model with a continuum of identical investors. Each investor

\footnote{For a recent survey on agency problems and their effect on financial contracting, see: Hart (2000).}
has the same ex-ante probability of receiving a liquidation shock, and this probability is known in the market. This model describes an industry that consists of investors with identical expected liquidation needs. We use the model to analyze the differences in direct-portfolio investment patterns across different industries; each industry is characterized by industry-specific expected liquidation needs. We show that when there are some fixed set-up costs to investing directly (e.g., costs of acquiring information), industries in which investors are more vulnerable to liquidation risks, will be owned by portfolio investors. As a result, these industries will be characterized by higher probabilities of early liquidations and greater flow volatility.

Thus, this basic model demonstrates a key trade-off between direct investment and portfolio investment. However, it also has two limitations: First, it cannot explain differences in volatility between direct investments and portfolio investments when the two types coexist. Second, it understates the disadvantage of direct investments to investors with high expected liquidation needs. This is because the expected liquidation need of an individual investor is known in the market, and thus investors who have a high expected liquidation need do not get a very low price when they sell their direct investments prematurely. This is because, potential buyers know that the sale is likely to be triggered by a liquidation shock and not by inferior information on the part of the owner. (This is also the reason for the result that investors with high expected liquidation needs choose portfolio investments only when there are some fixed set-up costs to direct investments.)

To relax these two limitations, we extend the basic model to include two types of investors. One type has a higher probability of getting a liquidation shock, and the other type has a lower probability of getting a liquidation shock. In this framework, the type of an individual investor is not necessarily known in the market. As a result, investors with high expected liquidation needs might be perceived as having low expected liquidation needs, and suffer from a very low price when they want to sell prematurely. This generates an additional force that pushes investors with a high probability of getting liquidation shocks to invest in portfolio investments: They try to separate themselves from the other type of investors. They do so by investing in portfolio investments.

Thus, in the model with heterogeneous investors, there exists a separating equilibrium (for some parameter values), in which investors with high expected liquidation needs invest only through portfolio investments, and investors with low expected liquidation needs invest only through direct

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4Since investors in an industry are identical, they all choose the same type of investment in equilibrium.
investments. This may occur even with no fixed set-up costs associated with direct investments. This pattern can explain the differences between the proportion of reversals in the two types of investment. Interestingly, for some parameter values, the model generates multiple equilibria: Either both types of investors choose direct investments (a pooling equilibrium), or investors with high expected liquidation needs choose portfolio investments, and investors with low expected liquidation needs choose direct investments (a separating equilibrium). This multiplicity of equilibria results from asymmetric-information externalities between investors with high expected liquidation needs: When more investors of their type choose direct investments, the re-sale price of these investments will increase, and the incentive of each investor of this type to choose these investments will increase.\(^5\) This multiplicity can explain why some countries have more direct investments than others, and why some periods of time are characterized by more direct investments than others. Another interesting point is that in the case of multiple equilibria, the equilibrium with portfolio investments is Pareto dominated by the other equilibrium. This may generate interesting policy implications, which are discussed in the concluding section of the paper.

Finally, we analyze the effect of transparency on the pattern of investments observed in equilibrium. Our motivation is to explain why the differences in volatility between the two types of investment are smaller in developed economies than in developing economies. We introduce two measures of transparency: Transparency between sellers and buyers, that we call capital-market transparency; and transparency between managers and owners (when the manager and the owner are not the same person), that we call corporate-governance transparency. When the degree of capital-market transparency increases, buyers know more about the reason of a sale, when a sale takes place. This reduces the degree of asymmetric information, and thus reduces the disadvantage of direct investments. As a result, the likelihood of an equilibrium, in which investors with high expected liquidation needs choose portfolio investments, decreases. Thus, the model predicts that developed economies, in which capital-market transparency is expected to be higher, will have smaller differences between the volatility of portfolio investments, and the volatility of direct investments. When the degree of corporate-governance transparency increases, portfolio investors can sometime get information on the fundamentals of their projects, and manage them efficiently. They

\(^5\)The forces that lead to the existence of a separating equilibrium and a pooling equilibrium here are similar to those in Stiglitz (1975).
can also decide to sell them when they observe low realizations of the fundamentals. As a result, the differences between the two types of investment become smaller, and for some parameter values, the separating equilibrium is eliminated, generating a smaller difference between the volatilities of the two types of investment.

We now briefly indicate the relation between this paper and recent literature. Albuquerque (2001) develops a model aimed at explaining differences between the volatility of direct investments and the volatility of portfolio investments. His paper, relies on expropriation risks and the inalienability of direct investments, and thus is different from the information-based mechanism developed here. Another possible explanation that is often mentioned to the difference in volatility between direct investments and portfolio investments is that direct investments are irreversible for some exogenous reason. However, as Albuquerque (2001) suggests, this argument is problematic for two reasons: First, because firms have many various ways to withdraw funds that was invested as direct investment.6 Second, because in times of a crisis, not only FDI, but also other types of flows might dry up, and thus the relative stability of FDI remains a puzzle. Importantly, the liquidity of each type of investment is endogenous in our model. In addition, our model sheds light on the difference in the volatility ratio between developed economies and developing economies, and this cannot be explained when FDI is assumed to be irreversible.

Our paper is also related to the literature that use the asymmetric information hypothesis to address different issues related to FDI. Among them, Gordon and Bovenberg (1996) use asymmetric information between domestic investors and foreign investors to explain the home bias phenomenon. Razin, Sadka and Yuen (1998) explain the pecking order of international capital flows with a model of asymmetric information. Razin and Sadka (2002) analyze the gains from FDI when foreign investors have superior information on the fundamentals of their investments. None of these papers, however, analyzes the effects of asymmetric information on the volatility of FDI and portfolio investments.

Although we write this paper in the context of international capital flows, we believe the mechanism we suggest here is more general, and can serve to analyze the trade-off between direct investments and portfolio investments, or between management efficiency and liquidity, in other contexts.7 In a related paper, Bolton and von-Thadden (1998) analyze a trade-off between direct

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6For more details on this point, see Albuquerque (2001) and Hausman and Fernandez-Arias (2000).

7The reason we think the model fits best in the context of international flows is the strong empirical evidence on
investments and portfolio investments. Their model, however is not based on the differences in information that each one of these investments provides. They also do not analyze the volatility of different investments in equilibrium. To the best of our knowledge, our paper is the first paper that looks at an information-based trade-off between direct investments and portfolio investments, and analyzes the effect of this trade-off on the volatility of the different investments. Our paper also touches on other issues that have been discussed in the finance literature. Admati and Pfleiderer (1991) discuss the incentive of traders to reveal the fact that they are trading for liquidity reasons and not because of bad information. Admati and Pfleiderer (1988) and Foster and Viswanathan (1990) point to the existence of externalities between traders who trade for liquidity reasons.

The remainder of this paper is organized as follows: Section 2 presents the basic model with one type of investor. In Section 3, we study the basic trade-off between direct investments and portfolio investments, and determine the type of investment that is chosen in equilibrium in different industries. In Section 4, we study the implications of our model for the probabilities of early withdrawals of direct investments and portfolio investments. In Section 5, we extend the model, and analyze the pattern of investments and withdrawals when there are two types of investors. Section 6 studies the effect of transparency on investment patterns in equilibrium. Section 7 concludes, and highlights additional implications of our model. Proofs are relegated to the Appendix.

2 Analytical Framework

A small economy is faced by a continuum of foreign investors. Each investor has an opportunity to invest in one investment project. Investment can occur in two forms. The first form is a direct investment (FDI). The second form is a portfolio investment. The only difference between the two forms of investment is that a direct investor will effectively act like a manager, whereas in case of a portfolio investment, the investor will not be the manager, and the project will be managed by an "outsider". We assume that investors are risk neutral, and thus each investor chooses the form of investment that maximizes her ex-ante expected payoff.

There are three periods of time: 0, 1, and 2. In period 0, each investor decides whether to make a direct investment or a portfolio investment. In period 2, the project matures. The payoff from volatility of international flows, which can be explained by our trade-off.
the project is denoted as $R$, where $R$ is given by:

$$R = (1 + \varepsilon) k - \frac{1}{2} Ak^2.$$  \hfill (1)

Here, $\varepsilon$ denotes a random productivity factor (technology shock) that is realized in period 1; $k$ is the level of capital input invested in the project in period 1, after the realization of $\varepsilon$. We assume that $\varepsilon$ is distributed between $-1$ and $1$ according to a cumulative distribution function $G(\cdot)$ and a density function $g(\cdot) = G'(\cdot)$. We also assume that $E(\varepsilon) = 0$.

2.1 Management and Efficiency

In period 1, after the realization of the technology shock, the manager of the project observes $\varepsilon$. Thus, if the investor owns the project as a direct investment, she observes $\varepsilon$, and chooses $k$, so as to maximize the payoff. The chosen level of $k$ will then be equal to $k^*(\varepsilon)$, which is given by:

$$k^*(\varepsilon) = \frac{1 + \varepsilon}{A}.$$  \hfill (2)

Thus, the ex-ante (expected) payoff from the investment in case of a foreign direct investment is given by:

$$E\left[\frac{(1 + \varepsilon) \cdot (1 + \varepsilon)}{A} - \frac{1}{2} A \left(\frac{1 + \varepsilon}{A}\right)^2\right] = \frac{E\left((1 + \varepsilon)^2\right)}{2A}.$$  \hfill (3)

In case of a portfolio investment, the owner is not the manager, and thus she does not observe $\varepsilon$. In this case, the manager follows earlier instructions as for the level of $k$. A possible rationale behind this sequence of firm decisions, whereby the level of capital input $k$ is determined \textit{ex ante}, has to do with a potential agency problem between the owner and the manager (who is responsible for making these decisions). Loosely speaking, the latter is not exclusively interested in the net worth of the firm as is the former. For example, with no explicit instructions at hand, the manager may wish to set $k$ at the highest possible level in order to gain power.\footnote{We use this specification to simplify the exposition.} As a result, when the owner does not have information about the firm’s productivity, she will have to set investment guidelines.

\footnote{In this case, if the owner cannot verify the information that the manager had at the time of the decision, she will not be able to prove that the manager acted to maximize his own objective function.}
for the manager (who knows more about $\varepsilon$ than she does) so as to protect her own interests. This agency problem is not modelled explicitly here because we want to focus instead on the control and management characteristics of FDI (which we discuss below) and on the asymmetric information between the firm’s "insiders" and "outsiders". What we do, however, capture in our model is the spirit of the agency problem, and the inefficiency associated with the fact that the owner of the project is not the manager.

The earlier instruction is chosen by the owner to maximize the expected return absent any information on $\varepsilon$, and is thus given by: $\frac{1}{A}$. Then, the ex-ante (expected) payoff from a portfolio investment is:

$$E\left(\frac{(1 + \varepsilon)}{A} - \frac{1}{2A}\right) = E\left(\frac{1 + 2\varepsilon}{2A}\right) = \frac{1}{2A}.$$ (4)

Clearly, the expected payoff from the project is higher in case of a direct investment rather than in case of a portfolio investment. This result reflects the inefficiency in management in case of a portfolio investment. The disadvantage of direct investment will follow from the possibility of a liquidation shock in period 1.

### 2.2 Liquidation Shocks and Resales

In period 1, before the value of $\varepsilon$ is known to those who will be later informed about it, the owner of the project gets a liquidation shock with probability $\lambda$ ($0 < \lambda < 1$). An investor that got a liquidation shock needs to sell the project in period 1. The underlying assumption behind this sequence is similar to the assumption made by Diamond and Dybvig (1983). Thus, an investor that got a liquidation shock, derives utility only from period-1 consumption. If she does not get a liquidation shock, she derives utility from period-2 consumption. As a result, an investor that got a liquidation shock will sell the project in period 1, as she cannot wait to collect the payoff in period 2. The project can be sold to outside investors, who are not informed about $\varepsilon$, but are familiar with the other parameters of the problem.

An investor that did not get a liquidation shock will sell the project in period 1, only if she observed bad information about the level of productivity $\varepsilon$. This is because, in this case, she has

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10 Recall that in this section we analyze a model where all the investors have the same $\lambda$. This assumption is relaxed in Section 5.
superior information over the buyer, and can exploit it. Since portfolio investors do not observe \( \varepsilon \) in period 1, only direct investors will sell the project at that time absent a liquidation shock. This fact reduces the price that direct investors can get when they try to sell the project at period 1.

2.3 Period-1 Prices

We now derive the price that the direct investor gets if she sells the project in period 1. This price is equal to the expected value of the project from the perspective of the buyer, given that the buyer knows that the owner is trying to sell, and given that she does not know the reason for the sale. We denote the threshold level of \( \varepsilon \), under which the direct owner is selling the project without a liquidation shock as \( \varepsilon_D \). Then, the buyer knows that with probability \((1 - \lambda) G(\varepsilon_D)\), the owner is selling the project because of a low realization of \( \varepsilon \), whereas with probability \( \lambda \), she is selling it because of a liquidation shock. We assume that if the project is sold because of a liquidation shock (that is, before the realization of \( \varepsilon \) is revealed to the owner), the new owner will not observe the value of \( \varepsilon \) after the sale (thus she will own the project as a portfolio investment). However, if the project is sold because of a low realization of the technology parameter, the new owner will know the value of \( \varepsilon \) after the sale (thus she will own the project as a direct investment). Then, using Bayes’ rule, the price that the direct investor gets for the project in period 1 is given by:

\[
P_{1,D} = \frac{(1 - \lambda) \int_{-1}^{\varepsilon_D} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) d\varepsilon + \lambda \int_{-1}^{1} \frac{1 + 2\varepsilon}{2A} g(\varepsilon) d\varepsilon}{(1 - \lambda) G(\varepsilon_D) + \lambda}.
\]

(5)

The owner, in turn, will set \( \varepsilon_D \), such that given \( P_{1,D} \), when observing \( \varepsilon_D \), she will be indifferent between selling the project and not. This yields the following equation:

\[
P_{1,D} = \frac{(1 + \varepsilon_D)^2}{2A}.
\]

(6)

Thus, we have two equations that determine \( P_{1,D} \) and \( \varepsilon_D \). We show in the Appendix that for each \( \lambda \) between 0 and 1, there is a unique solution for \( \varepsilon_D \) (denoted as \( \varepsilon_D(\lambda) \)) between \(-1\) and \(0\). As a result there is also a a unique solution for \( P_{1,D} \) (denoted as \( P_{1,D}(\lambda) \)). Importantly, \( \varepsilon_D(\lambda) \) and \( P_{1,D}(\lambda) \) are increasing in \( \lambda \). Thus, when \( \lambda \) is higher, the probability that an early sale results from a liquidation shock (and not from a bad realization of the technology parameter) is also higher, and the price of the project in period 1 increases.
As for the portfolio investor, if she sells the project in period 1, everybody knows she does it because of a liquidation shock. Thus, the price she gets for the project is given by:

\[ P_{1,P} = \int_{-1}^{1} \frac{1 + 2\varepsilon}{2A} g(\varepsilon) d\varepsilon = \frac{1}{2A}. \] (7)

When we solve for \( P_{1,D} \) and \( \xi_D \), we can see that \( \xi_D < 0 \), and thus that \( P_{1,D} < \frac{1}{2A} \). Thus, the price that the direct investor can get on the project in period 1 is lower than the price that the portfolio investor can get on the project in this period. The reason is that when the direct investor tries to sell the project, the price will change, reflecting the possibility that the sale originates from bad information on the prospects of the investment project.

To sum up, there is a trade-off between holding the project as a direct investment and holding it as a portfolio investment. On one hand, a direct investment enables the investor to manage the project more efficiently. This increases the return that she gets in case she does not have to sell early. On the other hand, when she holds the project as a direct investment, the investor will get a lower price for the project if she sells it in the short term. This is because potential buyers know that with some probability the project is being sold because of bad information on the prospects of the investment. Thus, the information that is associated with a direct holding of the investment is not necessarily beneficial, as it harms the investor when she tries to sell the project early.

### 3 The Basic Trade-Off between Direct Investments and Portfolio Investments

We now study the basic trade-off between direct investments and portfolio investments in a model with homogeneous investors. We start by writing down the expected payoff in each case while taking into account the possibility of observing a liquidation shock.

When the investor holds the investment as a direct investment, with probability \( \lambda \), she receives a liquidation shock, and sells the project in period 1. Then, her payoff is:

\[ P_{1,D}(\lambda) = \frac{(1 + \xi_D(\lambda))^2}{2A}. \]

With probability \( 1 - \lambda \), the investor does not get a liquidation shock. Then, she will sell the project if the realization of \( \varepsilon \) is below \( \xi_D(\lambda) \), and she will not sell it if the realization of \( \varepsilon \) is above
\( \varepsilon_D(\lambda) \). \( \varepsilon_D(\lambda) \) is determined by equations (5) and (6)). The total expected payoff in this case can thus be written as:

\[
\int_{-1}^{\varepsilon_D(\lambda)} \frac{(1 + \varepsilon_D(\lambda))^2}{2A} g(\varepsilon) d\varepsilon + \int_{\varepsilon_D(\lambda)}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) d\varepsilon.
\]

Thus, the ex ante expected payoff from a direct investment is given by:

\[
EV_{\text{Direct}} (\lambda) = \lambda \frac{(1 + \varepsilon_D(\lambda))^2}{2A} + (1 - \lambda) \left( \int_{-1}^{\varepsilon_D(\lambda)} \frac{(1 + \varepsilon_D(\lambda))^2}{2A} g(\varepsilon) d\varepsilon + \int_{\varepsilon_D(\lambda)}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) d\varepsilon \right). \tag{8}
\]

We now derive the ex ante expected payoff from a portfolio investment. When the investor holds the investment as a portfolio investment, with probability \( \lambda \), she receives a liquidation shock, and sells the project in period 1. Then, her payoff is:

\[
P_{1,P} = \frac{1}{2A}.
\]

With probability \( 1 - \lambda \), the investor does not receive a liquidation shock. Then, her expected payoff is:

\[
E \frac{(1 + 2\varepsilon)}{2A} = \frac{1}{2A}.
\]

Thus, the ex ante expected payoff from a portfolio investment is given by:

\[
EV_{\text{Portfolio}} = \frac{1}{2A}. \tag{9}
\]

In order to determine whether, in period 0, investors choose a direct investment or a portfolio investment, we need to compare \( EV_{\text{Direct}} (\lambda) \) with \( EV_{\text{Portfolio}} \). At this point, we make an additional assumption: We assume that, in period 0, there is an additional cost to make a direct investment. This represents the initial cost of acquiring information via a direct investment. We denote this cost as \( c \). Then, it is clear that, in period 0, investors will choose a direct investment if:

\[
Dif(\lambda) \equiv EV_{\text{Direct}} (\lambda) - EV_{\text{Portfolio}} > c.
\]

Similarly, they will choose a portfolio investment if:
\[ \text{Dif}(\lambda) \equiv \text{EV}_{\text{Direct}}(\lambda) - \text{EV}_{\text{Portfolio}} < c. \]

Proposition 1 studies the properties of the function \( \text{Dif}(\lambda) \).\(^{11}\)

**Proposition 1** For any \( \lambda \) between 0 and 1, \( \text{Dif}'(\lambda) < 0 \). Moreover, \( \text{Dif}(1) = 0 \), and \( \text{Dif}(0) > 0 \).

We now explain the intuition behind the proposition. When \( \lambda = 1 \), the investors know they will have to sell the project in period 1. Thus, the (gross) return they get on the investment is the price they will get in period 1. Moreover, when they sell in period 1, the price is not adjusted to reflect any information on the prospects of the project. This is because potential buyers know that the sale is a result of a liquidation shock. As a result, in this case the expected return on a direct investment is equal to the return on a portfolio investment.

As \( \lambda \) decreases from 1 to 0, there are two opposite effects on the value of \( \text{Dif}(\lambda) \). First, agents know that with a higher probability, they will not observe a liquidation shock, and thus will continue to own the project until maturity. As a result, they value more the higher efficiency that results from more information, and care less about the lower period-1 price that results from this information. This effect increases the difference between \( \text{EV}(\text{Direct}) \) and \( \text{EV}(\text{Portfolio}) \). Second, when \( \lambda \) decreases from 1 to 0, the period-1 price will be lower if a direct investor tries to sell. This is because potential buyers know that the sale is more likely to reflect bad information about the prospects of the investment and less likely to reflect a liquidation shock. This effect reduces the difference between \( \text{EV}(\text{Direct}) \) and \( \text{EV}(\text{Portfolio}) \). According to Proposition 1 the first effect is stronger than the second effect. As a result, \( \text{Dif}'(\lambda) < 0 \). Finally, following the above analysis, we can tell that \( \text{Dif}(0) > 0 \).

In Proposition 2, we study the optimal investment vehicle that is chosen in period 0.

**Proposition 2** If \( c \geq \text{Dif}(0) \), investors will always choose a portfolio investment in period 0. If \( c = 0 \), investors will always choose a direct investment in period 0. If \( 0 < c < \text{Dif}(0) \), there is a threshold level of \( \lambda \): \( \lambda^*(c) \) (\( 0 < \lambda^*(c) < 1 \)), such that if \( \lambda < \lambda^*(c) \), investors will choose a direct investment in period 0, and if \( \lambda > \lambda^*(c) \), investors will choose a portfolio investment in period 0.

\(^{11}\)Clearly, if \( \text{Dif}(\lambda) \equiv \text{EV}_{\text{Direct}}(\lambda) - \text{EV}_{\text{Portfolio}} = c \), the investors will be indifferent between the two types of investment. We ignore this case here.
Figure 1: The Choice between Direct Investment and Portfolio Investment

The intuition behind Proposition 2 is straightforward, given Proposition 1. We thus get that when $c$ is in an intermediate range, a direct investment will occur if and only if the probability of observing a liquidation shock is below a certain threshold. Figure 1 demonstrates the choice between direct investments and portfolio investments.

Note that in the current model, investors choose portfolio investments only when $c > 0$. This is a result of the fact that the specification with homogeneous investors understates the disadvantage of direct investments to investors with high expected liquidation needs. We address this problem in Section 5, when we analyze a model with heterogeneous investors.

4 The Probability of Midstream Sales of the Investment Project

We now analyze the probability that foreign investors will sell their investment in period 1, and withdraw their money out of the economy before the maturity of the investment. In case of a direct investment, this probability is given by:

$$
\lambda + (1 - \lambda) G_{\bar{\varepsilon}}((\lambda)),
$$
where $\lambda$ is the probability of a sale that results from a liquidation shock, and $(1 - \lambda)G_{\xi_D}(\lambda)$ is the probability of a sale that results from a technology shock. In case of a portfolio investment, an early sale can result only from a liquidation shock, and thus the probability of an early sale is simply given by:

$$\lambda.$$}

We now consider two industries. One industry is characterized by a lower probability of liquidation shocks (a lower $\lambda$), and the other is characterized by a higher probability of liquidation shocks (a higher $\lambda$). Both industries are characterized by the same cost of acquiring information ($c$ is the same). Suppose that the differences between the two industries are such that in the first industry, investors invest via a direct investment, and in the second industry they invest via a portfolio investment (that is, in the first industry, $\lambda < \lambda^*(c)$, and in the second industry, $\lambda > \lambda^*(c)$). What will be the difference between the probabilities of early sales in the two industries? Here, there are two effects. First, the higher probability of a liquidation shock in the second industry increases the probability of an early sale in this industry (which is the industry with portfolio investments). Second, the possibility of a sale that is based on a technology shock exists only in the first industry (which is the industry with direct investments). This effect increases the probability of an early sale in the industry with a direct investment. When the differences between the $\lambda$'s in the two industries are large enough, the probability of an early sale in the industry with a portfolio investment will be higher.

To sum up, our model shows that industries that are characterized by a high probability of receiving a liquidation shock are more likely to have portfolio investments. As a result, we can explain the higher probability of an early withdrawal of portfolio investments: Since these investments are owned by investors that are more vulnerable to a liquidation risk, they will be liquidated more often.

We now present a numerical example that demonstrates our model. For the purpose of this example, we assume that $\varepsilon$ is uniformly distributed between $-1$ and $1$, and that $A = 0.5$. Table 1 shows the optimal type of investment and the probability of an early sale for different levels of $\lambda$, and for different levels of $c$, under the assumption of a uniform distribution. In the table, the optimal type of investment is determined according to the rule in Proposition 2. Then, the probability of
an early sale is equal to $\lambda$ in case of a portfolio investment, and is equal to $\lambda + (1 - \lambda) G_{\mathcal{D}}(\lambda)$ in case of a direct investment.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\varepsilon_{\mathcal{D}}(\lambda)$</th>
<th>$c = 0.1$</th>
<th>$c = 0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Investment</td>
<td>Probability of Early Sale</td>
<td>Investment</td>
</tr>
<tr>
<td>0.1</td>
<td>Direct</td>
<td>0.37</td>
<td>Direct</td>
</tr>
<tr>
<td>0.2</td>
<td>Direct</td>
<td>0.49</td>
<td>Direct</td>
</tr>
<tr>
<td>0.3</td>
<td>Direct</td>
<td>0.58</td>
<td>Direct</td>
</tr>
<tr>
<td>0.4</td>
<td>Direct</td>
<td>0.65</td>
<td>Direct</td>
</tr>
<tr>
<td>0.5</td>
<td>Direct</td>
<td>0.72</td>
<td>Portfolio</td>
</tr>
<tr>
<td>0.6</td>
<td>Direct</td>
<td>0.78</td>
<td>Portfolio</td>
</tr>
<tr>
<td>0.7</td>
<td>Direct</td>
<td>0.84</td>
<td>Portfolio</td>
</tr>
<tr>
<td>0.8</td>
<td>Portfolio</td>
<td>0.8</td>
<td>Portfolio</td>
</tr>
<tr>
<td>0.9</td>
<td>Portfolio</td>
<td>0.9</td>
<td>Portfolio</td>
</tr>
</tbody>
</table>

Table 1: Numerical Example - Probabilities of Early Sales

Using the table, we compare the probabilities of early sales between different industries that have different levels of $\lambda$, but the same level of $c$. We can see that in most cases, industries that have portfolio investments will have a higher probability of an early sale. The reason is that the industries with portfolio investments are characterized by higher $\lambda$’s, since higher $\lambda$’s make portfolio investments more attractive. Only in a few cases, the probability of an early sale in an industry with direct investments will be higher than the probability of an early sale in an industry with portfolio investments. In these cases, the levels of $\lambda$ in the two industries are close, and thus the dominant effect is the possibility of an information-based early sale in the industry with direct investments (a possibility that does not exist in industries with portfolio investments). However, in most of the cases the dominant effect is the difference in $\lambda$ between industries with portfolio investments and industries with direct investments, and thus in most cases, industries with portfolio investments will have higher probabilities of early sales.

In the remainder of the paper, when we analyze the volatility of the two types of investment, we consider only the volatility that results from liquidation shocks. This is because of two reasons. One, as we saw in this section, the liquidation shock is usually the dominant effect behind the
reversal of flows. Two, the volatility of international flows that many commentators are trying to explain is usually not associated with information on the fundamentals of the investments. Thus, in this paper, we are more interested in reversals that are driven by liquidation shocks.

5 Heterogeneous Investors

So far we have analyzed an economy (industry), where all the investors had the same probability of getting a liquidation shock. This framework was efficient in demonstrating the basic trade-off between direct investments and portfolio investments. However, it also had two main limitations: One, in equilibrium, all the investors in the economy (industry) followed the same investment strategy. Thus, we could not analyze the differences in volatility between direct investments and portfolio investments, when both of them coexist. Two, since all the investors were identical, the probability that an individual investor received a liquidation shock was known to potential buyers. This limited the disadvantage of direct investments: Investors with a very high $\lambda$ (who have the lowest benefit from direct investments) knew that if they sell a direct investment in period 1, the price will not be very low, as the market knows their $\lambda$, and thus assesses a high probability that the sale results from a liquidation shock. As a result, even for these investors, portfolio investments dominated direct investments only when we introduced a fixed positive cost $c$ to investing directly.

We now extend the model to allow for two types of investors in an economy (industry). We will analyze the pattern of investments and withdrawals when the two types of investors coexist. As we will see, this extension sheds light on other effects that determine the pattern of investments and withdrawals. The main difference in the analysis results from the fact that when different investors have different $\lambda$’s, it is not always known in the market what is the $\lambda$ of each individual investor. As a result, when they want to sell a project in period 1, investors will sometimes face a price that does not reflect their true $\lambda$. This may create an incentive to signal the true $\lambda$, by choosing an investment vehicle.

5.1 The New Framework

Suppose again that there is a continuum $[0, 1]$ of investors in the economy. Proportion $\frac{1}{2}$ of them have high expected liquidation needs, and proportion $\frac{1}{2}$ have low expected liquidation needs. For-
mally, assume that the first type of agents face a liquidation need with probability $\lambda_H$, whereas the second type of agents face a liquidation need with probability $\lambda_L$. For simplicity, we assume that $1 > \lambda_H > \frac{1}{2} > \lambda_L > 0$, and that $\frac{\lambda_H + \lambda_L}{2} = \frac{1}{2}$. We also assume that $c = 0$. Investors know their type ex ante, however this is their private information.

The existence of heterogeneous investors does not affect the payoffs from portfolio investments. Since owners of portfolio investments never observe $\varepsilon$, they sell the project in period 1 only when they receive a liquidation shock. Since this is known to potential buyers, the price they will pay for a portfolio investment in period 1 is $\frac{1}{2A}$, as we had in (7). Similarly, the ex-ante expected payoff from a portfolio investment is $\frac{1}{2A}$, as we had in (9).

However, the expected payoffs from direct investments change. When there are heterogeneous investors, potential buyers do not know the type of an individual investor. As a result, the ex ante probability that an individual investor receives a liquidation shock may be different from the probability that is perceived by the market. The price of direct investments in period 1, and the threshold level of $\varepsilon$, below which investments are sold, will depend on the probability that is perceived by the market. Denoting the probability of a liquidation shock for an individual investor as $\lambda_i$, and the probability that is perceived by the market as $\lambda_m$, we get that the expected payoff from a direct investment for this individual investor is:

$$
(1 - \lambda_i) \left[ \int_{-1}^{\varepsilon_D(\lambda_m)} \frac{(1 + \varepsilon_D(\lambda_m))^2}{2A} g(\varepsilon) d\varepsilon + \int_{\varepsilon_D(\lambda_m)}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) d\varepsilon \right] + \lambda_i \cdot \frac{(1 + \varepsilon_D(\lambda_m))^2}{2A}.
$$

In this expression, the value of $\varepsilon_D(\lambda_m)$ is determined according to: (5) and (6). The expected payoff from direct investments here is clearly different from (8). This difference will have an important effect on the type of investment that will be chosen in equilibrium.

### 5.2 Symmetric Equilibria

We analyze symmetric equilibria, i.e. equilibria in which agents of the same type will choose the same type of investment. Thus, there are four potential equilibria, to which we refer here as four different cases: 1. All investors invest in the direct investment. 2. All investors invest in the portfolio investment. 3. $\lambda_H$ investors invest in the portfolio investment, and $\lambda_L$ investors invest in the direct investment. 4. $\lambda_H$ investors invest in the direct investment, and $\lambda_L$ investors invest in

---

12 Note that our results hold in a more general setting, for any $\lambda_H > \lambda_L$, and for $c > 0$. 

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18
the portfolio investment. Towards the end of the section, we analyze the possibility of existence of other (non-symmetric) equilibria.

We start by analyzing the conditions that are required to establish each one of the four cases as an equilibrium. Then, we characterize the equilibrium outcomes under different parameter values.

**Case 1: Both $\lambda_H$ and $\lambda_L$ investors invest in direct investments**

In this case, in the proposed equilibrium, when an investor wants to sell her project in period 1, potential buyers assess a probability of $\frac{1}{2}$ that the investor is trying to sell because of a liquidation need. This is because all the investors use the same investment vehicle, and thus in case of an early sale, potential buyers do not know the type of the investor that is trying to sell (recall that $\frac{\lambda_H + \lambda_L}{2} = \frac{1}{2}$). Thus, both types of investors will sell because of bad information when they observe a signal telling them that $\epsilon < \xi_D(\frac{1}{2})$. The price they will get will be: $\frac{(1 + \xi_D(\frac{1}{2}))^2}{2A}$. If an investor diverged from this proposed equilibrium strategy and invested in a portfolio investment, potential buyers would know that if she sells the project it is because of a liquidation shock. Then, the price she will get will be: $\frac{1}{2A}$. In equilibrium, $\lambda_H$ investors will invest in a direct investment if the expected payoff from a direct investment is higher than the expected payoff from a portfolio investment. The condition is:

\[
(1 - \lambda_H) \left[ \int_{-1}^{\xi_D(\frac{1}{2})} \frac{(1 + \xi_D(\frac{1}{2}))^2}{2A} g(\epsilon) d\epsilon + \int_{\xi_D(\frac{1}{2})}^{1} \frac{(1 + \epsilon)^2}{2A} g(\epsilon) d\epsilon \right] + \lambda_H \frac{(1 + \xi_D(\frac{1}{2}))^2}{2A} \geq \frac{1}{2A}.
\]

A similar condition applies for $\lambda_L$ investors:

\[
(1 - \lambda_L) \left[ \int_{-1}^{\xi_D(\frac{1}{2})} \frac{(1 + \xi_D(\frac{1}{2}))^2}{2A} g(\epsilon) d\epsilon + \int_{\xi_D(\frac{1}{2})}^{1} \frac{(1 + \epsilon)^2}{2A} g(\epsilon) d\epsilon \right] + \lambda_L \frac{(1 + \xi_D(\frac{1}{2}))^2}{2A} \geq \frac{1}{2A}.
\]

**Case 2: Both $\lambda_H$ and $\lambda_L$ investors invest in portfolio investments**

In this case, in the proposed equilibrium, there are only portfolio investments, and thus investors sell their project in period 1 only because of a liquidation shock. If an investor diverges from this equilibrium and holds a direct investment, she may try to sell in period 1 following a low realization of $\epsilon$. In this case, given that the equilibrium behavior of all the investors is identical, potential buyers will not know her type, and the price she will get will be: $\frac{(1 + \xi_D(\frac{1}{2}))^2}{2A}$. As a result, the conditions that we need in order to establish this case as an equilibrium, are the opposite conditions than the ones we had in case 1:
\[(1 - \lambda_H) \left[ \int_{-1}^{\xi_D(\frac{1}{2})} \frac{(1 + \xi_D(\frac{1}{2}))}{2A} g(\varepsilon) d\varepsilon + \int_{\xi_D(\frac{1}{2})}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) d\varepsilon \right] + \lambda_H \frac{(1 + \xi_D(\frac{1}{2}))}{2A} \leq \frac{1}{2A} \]

\[(1 - \lambda_L) \left[ \int_{-1}^{\xi_D(\frac{1}{2})} \frac{(1 + \xi_D(\frac{1}{2}))}{2A} g(\varepsilon) d\varepsilon + \int_{\xi_D(\frac{1}{2})}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) d\varepsilon \right] + \lambda_L \frac{(1 + \xi_D(\frac{1}{2}))}{2A} \leq \frac{1}{2A} \]

**Case 3:** \( \lambda_H \) investors invest in portfolio investments, \( \lambda_L \) investors invest in direct investments

In this case, in the proposed equilibrium, there is separation between \( \lambda_H \) investors and \( \lambda_L \) investors. Thus, when a \( \lambda_L \) investor wants to sell her direct investment in period 1, potential buyers know her type, and assess a probability of \( \lambda_L \) that she is selling because of a liquidation need. \( \lambda_H \) investors, who follow the proposed-equilibrium strategy and invest in portfolio investments, will get a price of \( \frac{1}{2A} \) in case they sell in period 1. However, if a \( \lambda_H \) investor diverges from the equilibrium strategy and invests in a direct investment, potential buyers will think she is a \( \lambda_L \) investor, and then when she tries to sell, the price she will get will be \( \frac{(1 + \xi_D(\lambda_H))}{2A} \). Note that this price is lower than \( \frac{(1 + \xi_D(\lambda_L))}{2A} \) and lower than \( \frac{(1 + \xi_D(\frac{1}{2}))}{2A} \). Thus, a \( \lambda_H \) investor is punished when she diverges from the equilibrium strategy. The condition, under which \( \lambda_H \) investors will invest in a portfolio investment, is:

\[(1 - \lambda_H) \left[ \int_{-1}^{\xi_D(\lambda_L)} \frac{(1 + \xi_D(\lambda_L))}{2A} g(\varepsilon) d\varepsilon + \int_{\xi_D(\lambda_L)}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) d\varepsilon \right] + \lambda_H \frac{(1 + \xi_D(\lambda_L))}{2A} \leq \frac{1}{2A} \]

Similarly, the condition, under which \( \lambda_L \) investors will invest in a direct investment, is:

\[(1 - \lambda_L) \left[ \int_{-1}^{\xi_D(\lambda_L)} \frac{(1 + \xi_D(\lambda_L))}{2A} g(\varepsilon) d\varepsilon + \int_{\xi_D(\lambda_L)}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) d\varepsilon \right] + \lambda_L \frac{(1 + \xi_D(\lambda_L))}{2A} \geq \frac{1}{2A} \]

**Case 4:** \( \lambda_H \) investors invest in direct investments, \( \lambda_L \) investors invest in portfolio investments

Following the same line of argument that we used in Case 3, we derive the conditions, under which this equilibrium can hold. The condition for \( \lambda_H \) investors is:

\[(1 - \lambda_H) \left[ \frac{1 + \xi_D(\lambda_H)}{2} \cdot \frac{(1 + \xi_D(\lambda_H))}{2A} + \frac{1}{2} \int_{\xi_D(\lambda_H)}^{1} \frac{(1 + \varepsilon)^2}{2A} d\varepsilon \right] + \lambda_H \frac{(1 + \xi_D(\lambda_H))}{2A} \geq \frac{1}{2A} \]
The condition for $\lambda_L$ investors is:

$$(1 - \lambda_L) \left[ \frac{1 + \varepsilon_D(\lambda_H)}{2} \ast \frac{(1 + \varepsilon_D(\lambda_H))^2}{2A} + \frac{1}{2} \int_{\xi_D(\lambda_H)}^{1} \frac{(1 + \varepsilon)^2}{2A} \, d\varepsilon \right] + \lambda_L \frac{(1 + \varepsilon_D(\lambda_H))^2}{2A} \leq \frac{1}{2A}.$$

5.3 Equilibrium Outcomes

Proposition 3 characterizes the equilibrium outcomes under different parameter values. The characterization of equilibrium outcomes is based on two threshold values of $\lambda_H$, which are defined below:

$\lambda_H^*$ is given by: (Here, $\lambda_L^* \equiv (1 - \lambda_H^*)$)

$$(1 - \lambda_H^*) \left[ \int_{-1}^{\xi_D(\lambda_L^*)} \left( (1 + \varepsilon_D(\lambda_L^*))^2 \, g(\varepsilon) \, d\varepsilon + \int_{\xi_D(\lambda_L^*)}^{1} (1 + \varepsilon)^2 \, g(\varepsilon) \, d\varepsilon \right) + \lambda_H^* (1 + \varepsilon_D(\lambda_L^*)^2 = 1, \quad (11)$$

and $\lambda_H^{**}$ is given by:

$$(1 - \lambda_H^{**}) \left[ \int_{-1}^{\xi_D(\lambda_L)} \left( (1 + \varepsilon_D(\lambda_L))^2 \, g(\varepsilon) \, d\varepsilon + \int_{\xi_D(\lambda_L)}^{1} (1 + \varepsilon)^2 \, g(\varepsilon) \, d\varepsilon \right] + \lambda_H^{**} \left( (1 + \varepsilon_D(\lambda_L))^2 \right)^2 = 1. \quad (12)$$

As we show in the proof of Proposition 3 in the Appendix, $\frac{1}{2} < \lambda_H < \lambda_H^{**} < 1$.

**Proposition 3** When $\frac{1}{2} < \lambda_H < \lambda_H^*$, only Case 1 is an equilibrium. That is, both $\lambda_H$ and $\lambda_L$ investors invest in direct investments.

When $\lambda_H^* \leq \lambda_H \leq \lambda_H^{**}$, both Case 1 and Case 3 are equilibria. That is, either both $\lambda_H$ and $\lambda_L$ investors invest in direct investments, or $\lambda_H$ investors invest in portfolio investments and $\lambda_L$ investors invest in direct investments.

When $\lambda_H^{**} < \lambda_H < 1$, only Case 3 is an equilibrium. That is, $\lambda_H$ investors invest in portfolio investments and $\lambda_L$ investors invest in direct investments.

Proposition 3 shows that the only patterns of investment that can exist in equilibrium are represented by Case 1 - all investors invest in direct investments - and Case 3 - $\lambda_H$ investors invest in portfolio investments and $\lambda_L$ investors invest in direct investments. Case 3 is the case where portfolio investments are more volatile than direct investments. In this case, investors with high expected liquidation needs invest in portfolio investments, whereas those with low expected liquidation needs invest in direct investments. Interestingly, in contrast to the model with homogeneous investors, here we have an equilibrium, in which some investors choose direct investments, whereas
others choose portfolio investments. Moreover, here, investors choose portfolio investments even if there is no immediate cost associated with the direct investments ($c = 0$).

As we noted above, a main difference between the current model and the model presented in the previous sections, is that here potential buyers do not know the type of an individual investor. As a result, an investor with a very high expected liquidation need may have to sell the project at a very low price, because the market perceives the expected liquidation need to be low. Thus, in some cases, investors with high $\lambda$’s choose portfolio investments in order to distinguish themselves from investors with low $\lambda$’s and avoid the low period-1 prices.

As Proposition 3 shows, the level of $\lambda_H$ affects the set of possible equilibria. When $\lambda_H$ is higher than $\lambda_H^{**}$, the difference between $\lambda_L$ and $\lambda_H$ is so large that investors with high expected liquidation needs ($\lambda_H$ investors) never invest in direct investments. In this case, $\lambda_H$ investors prefer to invest in a less efficient investment in order not to be perceived as low-$\lambda$ investors and get a low price when they need to sell in period 1.

When $\lambda_H$ is between $\lambda_H^*$ and $\lambda_H^{**}$, the difference between $\lambda_L$ and $\lambda_H$ is smaller. As a result, we have an equilibrium, in which $\lambda_H$ investors invest in direct investments. However, there is also another equilibrium, in which they invest in portfolio investments. The reason for the multiplicity of equilibria is the existence of externalities among $\lambda_H$ investors. Thus, a $\lambda_H$ investor benefits from having other investors of her type investing in the same type of investment. This is because, then, when she tries to sell the project, the price does not decrease that much as the market knows that the sale is very likely to be driven by a liquidation shock. As a result, when all $\lambda_H$ investors invest in portfolio investments, an individual $\lambda_H$ investor would like to do the same thing in order to avoid the low price when she needs to sell (given that she needs to sell quite often). Similarly, when all $\lambda_H$ investors invest in direct investments, an individual $\lambda_H$ investor would like to invest in a portfolio investment as well.

The existence of multiple equilibria here implies that we may have jumps from an equilibrium with a lot of direct investments to an equilibrium with much less direct investments. This may explain why some countries have more direct investments than others, and why some periods of time are characterized by more direct investments than others. Interestingly, the equilibrium represented by Case 3 is Pareto-dominated by the one represented by Case 1. In Case 3, there is a loss of efficiency, caused by the fact that investors do not monitor their projects directly. Thus,
when $\lambda_H$ investors invest in portfolio investments, we have a coordination failure.

When $\lambda_H$ is lower than $\lambda_H^*$, $\lambda_H$ investors always invest in direct investments. Here, the difference between $\lambda_L$ and $\lambda_H$ is small, and the dominant factor that determines the behavior of $\lambda_H$ investors is the greater efficiency associated with direct investments.

Interestingly, for all values of $\lambda_H$, $\lambda_L$ investors invest in direct investments. This is because these investors care less about the price they will get in period 1, and as a result prefer to stick with the more efficient investment. Thus, there is no issue of coordination regarding these investors.

Table 2 presents a numerical example using the case of a uniform distribution. The table shows the possible equilibria for different values of $\lambda_H$, and demonstrates the results of Proposition 3.

<table>
<thead>
<tr>
<th>$\lambda_L$, $\lambda_H$</th>
<th>Possible Equilibria</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_L = 0$, $\lambda_H = 1$</td>
<td>A: Case 3</td>
</tr>
<tr>
<td>$\lambda_L = 0.1$, $\lambda_H = 0.9$</td>
<td>A: Case 3</td>
</tr>
<tr>
<td>$\lambda_L = 0.2$, $\lambda_H = 0.8$</td>
<td>A: Case 3</td>
</tr>
<tr>
<td>$\lambda_L = 0.3$, $\lambda_H = 0.7$</td>
<td>A: Case 3, B: Case 1</td>
</tr>
<tr>
<td>$\lambda_L = 0.4$, $\lambda_H = 0.6$</td>
<td>A: Case 1</td>
</tr>
<tr>
<td>$\lambda_L = 0.5$, $\lambda_H = 0.5$</td>
<td>A: Case 1</td>
</tr>
</tbody>
</table>

Table 2: Numerical Example - Equilibrium Outcomes for different values of $\lambda_H$

Before we close this section, we wish to discuss the possibility of existence of non-symmetric equilibria in the model, and show that our conclusions do not change when such equilibria are considered. The proof of Proposition 3 demonstrates that $\lambda_L$ investors will choose direct investments under all circumstances. Thus, a non-symmetric equilibrium can be only a case where some $\lambda_H$ investors choose direct investments, whereas others choose portfolio investments. When $\frac{1}{2} < \lambda_H < \lambda_H^*$, and Case 1 is the only symmetric equilibrium, we can show that the model does not have a non-symmetric equilibrium. This is because in this range of parameters, a $\lambda_H$ investor will choose a direct investment even when all other $\lambda_H$ investors choose portfolio investments, (this is the reason why Case 3 is not an equilibrium in this range). Thus, given that in this range, $\lambda_H$ investors choose direct investments even when portfolio investments are most beneficial, we can say that they will always choose to invest directly. Similarly, we can show that when $\lambda_H^{**} < \lambda_H < 1$, Case 3 is the
only possible equilibrium of the model. When $\lambda_H^* \leq \lambda_H \leq \lambda_{H}^{**}$, we have two symmetric equilibria - Case 1 and Case 3 - and one asymmetric equilibrium. In this asymmetric equilibrium, all $\lambda_L$ investors choose direct investments, whereas $\lambda_H$ investors split between direct investments and portfolio investments. This third equilibrium does not change our conclusions in any significant way, as it also features more reversals of portfolio investments than reversals of direct investments.

6 The Effect of Transparency

As we noted in Section 1, the ratio between the volatility of foreign portfolio investments and the volatility of foreign direct investments is higher in developing economies than in developed economies. In this section, we try to shed light on this phenomenon by analyzing the effect of transparency on the pattern of investments in our model. As we will show below, in some cases, more transparency will lead to less separation between investors with high $\lambda$ and investors with low $\lambda$, and thus to lower differences in volatility between the two types of investment.

In this section we use the framework with heterogeneous investors developed in the last section, and analyze two measures of transparency. The first one measures the degree of transparency between buyers and sellers; we denote it: Capital-market transparency. When this measure is higher, buyers are more informed about the reason, for which the investor is selling the project. The second one measures the degree of transparency between managers and owners; we denote it: Corporate-governance transparency. When this measure is higher, owners, who do not act as managers, are more informed about the fundamentals of their projects. Our hypothesis is that both measures of transparency are higher in developed economies than in developing economies.

6.1 Capital-Market Transparency

Assume that with probability $\beta$ ($0 < \beta < 1$), the reason for an early sale in period 1 is known to all potential buyers. If this event happens, potential buyers know what triggers the liquidation of a project (that was held either as a direct investment or as a portfolio investment): a liquidation shock or a productivity shock. Assume also that sellers know whether the reason for a sale is revealed to buyers or not. Clearly, when $\beta$ is higher, the ex-ante level of transparency in the economy is higher.
The addition of the parameter $\beta$ to the model does not change the payoffs from portfolio investments. Thus, for any value of $\beta$ between 0 and 1, buyers know that a portfolio investment is always sold because of a liquidation shock. This is because the owner of a portfolio investment does not observe information on the realization of the productivity level. As a result, the expected payoff from a portfolio investment is independent of $\beta$ and is equal to $\frac{1}{2A}$, as we showed before.

However, the analysis of direct investments is affected considerably by the introduction of $\beta$. Suppose that buyers know the reason for an early sale. If they know the reason is a liquidation shock, the price they will be willing to pay in period 1 will be $\frac{1}{2A}$, which is equal to the price they pay on a portfolio investment. If they know the reason is a productivity shock, no sale will take place. This is due to the classic 'lemons' problem: When a seller has superior information on the quality of the project, and the buyer knows that the sale is driven by this information, there will be no price that will satisfy both the buyer and the seller. More technically, it is easy to verify that if the reason for the sale is known to the buyer, $\varepsilon_D$ (which is determined by (5) and (6)) is equal to $-1$, meaning that there are no sales in equilibrium. Then, denoting the probability of a liquidation shock for an individual investor as $\lambda_i$, and the probability that is perceived by the market as $\lambda_m$, we get that the expected payoff from a direct investment for this individual investor is:

\[
(1 - \beta) \left[ (1 - \lambda_i) \left[ \int_{-1}^{1} \frac{(1 + \varepsilon_D(\lambda_m))^2}{2A} \, g(\varepsilon) \, d\varepsilon + \int_{-1}^{1} \frac{(1 + \varepsilon)^2}{2A} \, g(\varepsilon) \, d\varepsilon \right] + \lambda_i \cdot \frac{1}{2A} \right] + \beta \left[ (1 - \lambda_i) \cdot \int_{-1}^{1} \frac{(1 + \varepsilon)^2}{2A} \, g(\varepsilon) \, d\varepsilon + \lambda_i \cdot \frac{1}{2A} \right].
\]

As in the last section, there are four potential symmetric equilibria, denoted as cases 1-4. Proposition 4 characterizes the equilibrium outcomes that will hold under different parameter values. Again we make use of two threshold values of $\lambda_H$, which are defined below:

$\lambda_H^E(\beta)$ is given by:

\[
(1 - \beta) \left[ (1 - \lambda^*_H(\beta)) \left[ \int_{-1}^{1} \frac{\varepsilon_D(\lambda^*_L(\beta))^2}{2A} \, g(\varepsilon) \, d\varepsilon + \int_{-1}^{1} \frac{(1 + \varepsilon)^2}{2A} \, g(\varepsilon) \, d\varepsilon \right] \right] + \beta \left[ (1 - \lambda^*_H(\beta)) \cdot \int_{-1}^{1} \frac{(1 + \varepsilon)^2}{2A} \, g(\varepsilon) \, d\varepsilon + \lambda^*_H(\beta) \right] = 1,
\]
and \( \lambda_{**}^H (\beta) \) is given by:

\[
(1 - \beta) \left[ \left(1 - \lambda_{**}^H (\beta) \right) \left[ \int_{-1}^{1} \left(1 + \varepsilon D\left(\frac{\varepsilon}{2}\right)\right)^2 g(\varepsilon) \, d\varepsilon + \int_{1}^{1} \left(1 + \varepsilon\right)^2 g(\varepsilon) \, d\varepsilon + \right] + \lambda_{**}^H (\beta) \cdot (1 + \varepsilon D\left(\frac{\varepsilon}{2}\right))^2 \right]
+ \beta \left[ (1 - \lambda_{**}^H (\beta)) \cdot \int_{-1}^{1} (1 + \varepsilon)^2 g(\varepsilon) \, d\varepsilon + \lambda_{**}^H (\beta) \right] = 1.
\]

(15)

Again, \( \frac{1}{2} < \lambda_{H}^* (\beta) < \lambda_{**}^H (\beta) < 1 \). (Here, \( \lambda_{L}^* (\beta) \equiv (1 - \lambda_{H}^* (\beta)) \)).

**Proposition 4**  
For every \( 0 < \beta < 1 \):

- When \( \frac{1}{2} < \lambda_{H} < \lambda_{H}^* (\beta) \), only Case 1 is an equilibrium.
- When \( \lambda_{H}^* (\beta) \leq \lambda_{H} \leq \lambda_{**}^H (\beta) \), both Case 1 and Case 3 are equilibria.
- When \( \lambda_{**}^H (\beta) < \lambda_{H} < 1 \), only Case 3 is an equilibrium.

Both \( \lambda_{H}^* (\beta) \) and \( \lambda_{**}^H (\beta) \) are increasing in \( \beta \).

Proposition 4 shows that the characterization of equilibrium outcomes provided in Proposition 3 holds for every value of \( \beta \) between 0 and 1. Thus, at low levels of \( \lambda_{H} \), Case 1 is the only equilibrium, at high levels of \( \lambda_{H} \), Case 3 is the only equilibrium, and at intermediate levels, both Case 1 and Case 3 are possible equilibria. The main point of Proposition 4 is that as \( \beta \) increases, Case 3 becomes less likely to be an equilibrium and Case 1 becomes more likely to be an equilibrium. More technically: As \( \beta \) increases, the range of \( \lambda_{H} \), in which Case 3 is an equilibrium (between \( \lambda_{H}^* (\beta) \) and 1), becomes smaller, and the range, in which Case 1 is an equilibrium (between 0 and \( \lambda_{**}^H (\beta) \)), becomes larger. Figure 2 illustrates the possible equilibrium outcomes as a function of \( \lambda_{H} \) and \( \beta \).

The implication of Proposition 4 is that in economies with more transparency between sellers and buyers, \( \lambda_{H} \) investors and \( \lambda_{L} \) investors are more likely to make the same investments rather than separate and invest in different types of investments, as they do in Case 3. As a result, in these economies, the difference between the volatility of direct investments and the volatility of portfolio investments is expected to be, on average, smaller. This may explain why developed economies are characterized by a lower ratio between the volatility of portfolio investments and the volatility of direct investments.

The intuition behind the result is strongly related to the basic trade-off in our model. As we suggest in the paper, when investors make direct investments, they get more information on
Figure 2: Possible Equilibria for Different Values of $\lambda_H$ and $\beta$

the fundamentals of the project, and can manage it more efficiently. However, this also has a disadvantage, since if they need to sell the project, this additional information will create a problem of asymmetric information between them and potential buyers, and will reduce the price they can get. As a result, in equilibrium, investors that expect to sell more often, might make portfolio investments. When there is more transparency between buyers and sellers regarding the reason for an early sale, the problem of asymmetric information becomes smaller, and thus investors that expect to sell more often, have lower incentives to make portfolio investments.

6.2 Corporate-Governance Transparency

Assume that in case of a portfolio investment, the owner of the project observes $\varepsilon$ in period 1, with probability $\alpha$ ($0 < \alpha < 1$). If this happens, the owner of the project can act as a direct investor: She can instruct the manager to choose the optimal level of $k$, and she can decide to sell the project if the realization of $\varepsilon$ is below a certain threshold. Our interpretation is that a higher $\alpha$ represents a better flow of information between the manager and the owner when the two are not the same person. Thus, when $\alpha$ is higher, there is a higher probability that the information that is available to the manager will also be available to the owner. As a result, a higher $\alpha$ represents a higher level
of ex ante transparency. For simplicity, in this subsection, we assume that $\beta = 0$.

The introduction of the parameter $\alpha$ into the model does not change the analysis of direct investments. It only changes the analysis of portfolio investments. When owners of portfolio investments observe $\varepsilon$, they will be able to achieve the same management efficiency as direct investors. Thus, if they don’t sell the project in period 1, their investment will yield an expected payoff of: $\frac{E(1+\varepsilon)^2}{2A}$ (see (3)). Additionally, they also may decide to sell the project if the realization of $\varepsilon$ is below a certain level. We denote this threshold level of $\varepsilon$ as $\varepsilon_{P}$. Using the same principles as in (5) and (6), we derive the following equation that determines $\varepsilon_{P}$ as a function of $\alpha$ and $\lambda_{m}$ (the later is the probability of a liquidation shock that is perceived by the market):

$$
(1 - \lambda_{m}) \alpha \int_{-1}^{1} \varepsilon_{P}^{(\alpha, \lambda_{m})} (1 + \varepsilon)^2 g(\varepsilon) d\varepsilon + \lambda_{m} = (1 + \varepsilon_{P}^{(\alpha, \lambda_{m})})^2.
$$

(16)

This equation is slightly different from the one implied by (5) and (6), as it considers the fact that a portfolio investor observes $\varepsilon$ with probability $\alpha$. As we show in the Appendix, for every $\lambda_{m}$ and $\alpha$ between 0 and 1, there is a unique solution for $\varepsilon_{P}$ between $-1$ and 0. As we also show in the Appendix, the analysis of the equation reveals that $\varepsilon_{P}^{(\alpha, \lambda_{m})}$ is increasing in $\lambda_{m}$ and decreasing in $\alpha$. Thus, when the probability of a liquidation shock increases, there is, on average, a smaller problem of asymmetric information between sellers and buyers, and investors will sell their projects under a larger range of parameters. Similarly, when the probability of information being revealed to owners increases, there is, on average, a greater problem of asymmetric information, and investors will sell their projects under a smaller range of parameters. A direct result of the last property is that $\varepsilon_{P}^{(\alpha, \lambda_{m})} > \varepsilon_{D}^{(\lambda_{m})}$ for every $0 < \alpha < 1$.

Following (6), we know that the price of portfolio investments in period 1 will be: $\frac{(1+\varepsilon_{P}^{(\alpha, \lambda_{m})})^2}{2A}$. Then, the ex ante expected payoff from a portfolio investment for a $\lambda_{i}$ investor will be:

$$
(1 - \lambda_{i}) \left[ (1 - \alpha) \cdot \frac{1}{2A} + \alpha \left( \int_{-1}^{1} \frac{\varepsilon_{P}^{(\alpha, \lambda_{m})} (1+\varepsilon_{P}^{(\alpha, \lambda_{m})})^2}{2A} g(\varepsilon) d\varepsilon + \int_{\varepsilon_{P}^{(\alpha, \lambda_{m})}}^{1} \frac{(1+\varepsilon)^2}{2A} g(\varepsilon) d\varepsilon \right) \right] + \lambda_{i} \cdot \frac{(1 + \varepsilon_{P}^{(\alpha, \lambda_{m})})^2}{2A}.
$$

(17)

When $0 < \alpha < 1$, the basic trade-off between direct investments and portfolio investments is similar to the trade-off in Section 5. On average, direct investments still provide more information to owners. This enables owners to achieve more management efficiency, but also reduces the price they can get when they sell their investments in period 1. As $\alpha$ increases, the differences between the expected payoffs from the two types of investment become smaller.
We now provide a partial characterization of equilibrium outcomes as a function of $\alpha$ and $\lambda_H$. Proposition 5 characterizes the range of parameters, for which Case 3 is an equilibrium. This range is also illustrated in Figure 3.

**Proposition 5** For every $\lambda_H$, Case 3 is an equilibrium if and only if $\alpha \in [\alpha^*(\lambda_H), \alpha^{**}(\lambda_H)]$. Here $\forall \lambda_H \in [\lambda_H^*, 1)$, $\alpha^*(\lambda_H) = 0$; $\forall \lambda_H \in (\frac{1}{2}, \lambda_H^*)$, $\alpha^*(\lambda_H) \in (0, 1)$; $\forall \lambda_H \in (\frac{1}{2}, 1)$, $\alpha^{**}(\lambda_H) \in (0, 1)$ and $\alpha^{**}(\lambda_H) > \alpha^*(\lambda_H)$. $\lambda_H^*$ is defined by (11).

As the proposition shows, when $\alpha$ increases from an intermediate level to a high level, Case 3 ceases to be an equilibrium. The reason is that as $\alpha$ becomes high, the difference in efficiency between direct investments and portfolio investments becomes small. At the same time, if $\lambda_H$ investors invest in portfolio investments, period-1 prices of these investments will be high, and $\lambda_L$ investors will prefer to invest in portfolio investments themselves. As a result, when $\alpha$ is high, there cannot be an equilibrium, in which $\lambda_H$ investors make portfolio investments and $\lambda_L$ investors make direct investments. The implication of this result is that countries with high levels of $\alpha$ are less likely to be in an equilibrium, in which there is a full separation between $\lambda_H$ investors and $\lambda_L$ investors and the difference in volatility between the two types of investment is large.

The proposition also shows that for some range of $\lambda_H$ (between $\frac{1}{2}$ and $\lambda_H^*$), an increase in $\alpha$ from a low level to an intermediate level will generate the opposite result. In this range, as $\alpha$ increases, Case 3 becomes a possible equilibrium, since $\lambda_H$ investors have a higher incentive to make portfolio investments. Since in this range, $\alpha$ is still relatively small, $\lambda_L$ investors still prefer to make the direct investments.
To sum up, the proposition shows that for all values of $\lambda_H$, an increase in $\alpha$ from an intermediate level to a high level will eliminate Case 3 as a possible equilibrium. However, in a partial range of $\lambda_H$, an increase in $\alpha$ from a low level to an intermediate level will make Case 3 a possible equilibrium. Thus, the proposition provides partial support to the hypothesis that countries with higher levels of $\alpha$ are characterized by smaller differences in volatility between direct investments and portfolio investments. The proposition does suggest, however, that countries with intermediate levels of $\alpha$ are more likely to have Case 3 as an equilibrium than countries with high levels of $\alpha$.

As for the other potential symmetric equilibria: We can easily show that for every $\alpha$ between 0 and 1, Case 2 and Case 4 do not satisfy equilibrium conditions. In the general set-up, we could not find an analytical characterization of the values of $\alpha$, for which Case 1 is an equilibrium. In order to shed more light on this point, we ran a simulation for a uniform distribution of $\varepsilon$. Table 3 shows the possible symmetric equilibria for different values of $\alpha$ and $\lambda_H$, when $\varepsilon$ is uniformly distributed.

<table>
<thead>
<tr>
<th>$\alpha \setminus \lambda_H$</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Case 1</td>
<td>Case 1, Case 3</td>
<td>Case 3</td>
<td>Case 3</td>
<td>Case 3</td>
</tr>
<tr>
<td>0.1</td>
<td>Case 1</td>
<td>Case 1, Case 3</td>
<td>Case 3</td>
<td>Case 3</td>
<td>Case 3</td>
</tr>
<tr>
<td>0.2</td>
<td>Case 1</td>
<td>Case 1, Case 3</td>
<td>Case 3</td>
<td>Case 3</td>
<td>Case 3</td>
</tr>
<tr>
<td>0.3</td>
<td>Case 1, Case 3</td>
<td>Case 1, Case 3</td>
<td>Case 3</td>
<td>Case 3</td>
<td>Case 3</td>
</tr>
<tr>
<td>0.4</td>
<td>Case 1, Case 3</td>
<td>Case 1, Case 3</td>
<td>Case 3</td>
<td>Case 3</td>
<td>Case 3</td>
</tr>
<tr>
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<td>Case 1, Case 3</td>
<td>Case 3</td>
<td>Case 3</td>
<td>—</td>
</tr>
<tr>
<td>0.6</td>
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<td>Case 1</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
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<td>Case 1</td>
<td>Case 1</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>0.8</td>
<td>Case 1</td>
<td>Case 1</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
<td>0.9</td>
<td>Case 1</td>
<td>Case 1</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 3: Numerical Example - Symmetric Equilibria for Different Values of $\alpha$ and $\lambda_H$

As Table 3 shows, under a uniform distribution, $\alpha$ does not seem to have an effect on the possibility of Case 1 being an equilibrium. Only the level of $\lambda_H$ determines whether Case 1 will be an equilibrium or not. Thus, the only effect of $\alpha$ on investment patterns in equilibrium is achieved through its effect on the possibility of observing Case 3 as an equilibrium, which was stated in Proposition 5. Interestingly, the table shows that for some parameters, the model will not have any
symmetric equilibria. One can show, that when this happens, the model will have one asymmetric equilibrium, in which $\lambda_H$ investors will choose portfolio investments, whereas $\lambda_L$ investors will split between direct investments and portfolio investments. Clearly, this case exhibits smaller differences in volatility between direct investments and portfolio investments than Case 3.

7 Concluding Remarks

The model we developed in this paper describes an information-based trade-off between direct investments and portfolio investments. According to the model, direct investors will be more informed about the fundamentals of their projects. This information will enable them to manage their projects more efficiently. However, it will also create an asymmetric-information problem in case they need to sell their projects prematurely, and will reduce the price they can get in that case. As a result, for some parameter values, investors, who know they are more likely to get a liquidation shock that forces them to sell early, will choose to make portfolio investments, whereas investors, who know they are less likely to get a liquidation shock, will choose to make direct investments.

This result can explain the empirical finding, according to which foreign portfolio investments are more volatile and exhibit much more reversals than foreign direct investments. Moreover, the model shows that transparency - both capital-market transparency and corporate-governance transparency - sometimes reduces the difference between the volatility of direct investments and the volatility of portfolio investments. This result is in accord with the empirical finding that the ratio of FDI’s volatility to other long-term flows’ volatility is smaller in developing countries than in developed countries.

To conclude, we wish to highlight four additional implications of our model.

One, the information-based trade-off between direct investments and portfolio investments has implications for the expected yields on each type of investment. Thus, in case of a liquidation shock, direct investors get a very low return on their investment. Investors will be willing to bear that risk and make direct investments only if they are compensated in the form of a higher expected yield. In order to address this issue in an appropriate way, our model should be adjusted to include risk averse agents. As for empirical evidence, we are not aware of any empirical study that looked at the differences between the expected yield on direct investments and the expected yield on portfolio investments.
investments. We think our framework suggests an interesting testable prediction on this point.

Two, in a recent empirical study, Sorensen and Yosha (2002) find that greater portfolio holdings across countries are associated with more risk sharing, whereas greater FDI holdings across countries are not associated with more risk sharing. In our framework, the main risk that is associated with foreign direct investments is the risk of a liquidation shock, following which the investment has to be sold at a low price. Since the liquidation shock has the same effect on all the direct investments that are held by the same investor, investors cannot diversify this risk by holding more direct investments. Thus, our framework can explain why greater FDI holdings are not associated with more risk sharing. As for portfolio investments, our framework suggests that liquidation shocks have a smaller effect on the returns from portfolio investments, and that these returns are affected mainly by the technology shock that is specific to the investment. Thus, our framework suggests that greater portfolio holdings across countries may be associated with more risk sharing. We believe that a thorough examination of these issues is an interesting direction for future research.

Three, our model can be extended to include debt flows. As is well known in the theory of corporate finance, the price of debt is less sensitive to problems of asymmetric information. Thus, in our framework, the return on debt is expected to be less sensitive to liquidation shocks, and thus debt is expected to attract investors with high expected liquidation needs. Thus our framework can also explain the high volatility of international debt flows.

Four, in our model, portfolio investments occur sometimes a result of a coordination failure among investors with high expected liquidation needs. When this happens, all the investors can be better-off if they invest in direct investments. This point may have interesting policy implications. Thus, the government can eliminate the bad equilibrium by creating better conditions for direct investments. As our analysis suggests, these better conditions can be in the form of greater transparency, but also in other forms.

8 Appendix

Characterization of $\varepsilon_D(\lambda)$

In order to find $\varepsilon_D(\lambda)$, we need to solve equations (5) and (6). Then, $\varepsilon_D(\lambda)$ is given by the
following equation:
\[
\frac{(1 + \varepsilon_D)^2}{2A} = \frac{(1 - \lambda) \int_{-1}^{\varepsilon_D} \frac{(1+\varepsilon)^2}{2A} g(\varepsilon) d\varepsilon + \lambda \int_{-1}^{1} \frac{1 + 2\varepsilon}{2A} g(\varepsilon) d\varepsilon}{(1 - \lambda) G(\varepsilon_D) + \lambda},
\]
which can be written as follows:
\[
F(\lambda, \varepsilon_D) = (1 - \lambda) \left[ \int_{-1}^{\varepsilon_D} \left[ (1 + \varepsilon_D)^2 - (1 + \varepsilon)^2 \right] g(\varepsilon) d\varepsilon \right] + \lambda \left[ (1 + \varepsilon_D)^2 - 1 \right] = 0.
\]

Analyzing this equation, we can see that when \(0 < \lambda < 1\), then \(-1 < \varepsilon_D(\lambda) < 0\). Moreover, as \(\lambda\) approaches 0, \(\varepsilon_D(\lambda)\) approaches \(-1\), and as \(\lambda\) approaches 1, \(\varepsilon_D(\lambda)\) approaches 0.

Analyzing the derivatives of \(F(\lambda, \varepsilon_D)\), we get:
\[
\frac{\partial F(\lambda, \varepsilon_D)}{\partial \lambda} = (1 + \varepsilon_D)^2 - 1 - \int_{-1}^{\varepsilon_D} \left[ (1 + \varepsilon_D)^2 - (1 + \varepsilon)^2 \right] g(\varepsilon) d\varepsilon < 0,
\]
and
\[
\frac{\partial F(\lambda, \varepsilon_D)}{\partial \varepsilon_D} = 2(1 + \varepsilon_D) \left[ (1 - \lambda) G(\varepsilon_D) + \lambda \right] > 0.
\]

Thus, for every \(\lambda\) in the range \((0, 1)\), we have a unique \(\varepsilon_D(\lambda)\) in the range \((-1, 0)\). Moreover, by the Implicit Function Theorem, we know that \(\varepsilon_D(\lambda) > 0\). \(\textbf{QED.}\)

**Proof of Proposition 1**

We write \(\text{Diff}(\lambda)\) as it is defined by (8) and by (9):
\[
\text{Diff}(\lambda) = \lambda \frac{(1 + \varepsilon_D(\lambda))^2}{2A} + (1 - \lambda) \left( \int_{-1}^{\varepsilon_D(\lambda)} \frac{1 + \varepsilon_D(\lambda))^2}{2A} g(\varepsilon) d\varepsilon + \int_{\varepsilon_D(\lambda)}^{1} \frac{(1+\varepsilon)^2}{2A} g(\varepsilon) d\varepsilon \right) - \frac{1}{2A}.
\]

We start by computing \(\text{Diff}(\lambda)\) as \(\lambda\) approaches 0. We already know that as \(\lambda\) approaches 0, \(\varepsilon_D(\lambda)\) approaches \(-1\). Plugging this in the expression for \(\text{Diff}(\lambda)\), we get that as \(\lambda\) approaches 0, \(\text{Diff}(\lambda)\) approaches \(\int_{-1}^{1} \frac{\varepsilon^2}{2A} g(\varepsilon) d\varepsilon\). We can easily see that \(\int_{-1}^{1} \frac{\varepsilon^2}{2A} g(\varepsilon) d\varepsilon > 0\).

We now compute \(\text{Diff}(\lambda)\) as \(\lambda\) approaches 1. We already know that as \(\lambda\) approaches 1, \(\varepsilon_D(\lambda)\) approaches 0. Plugging this in the expression for \(\text{Diff}(\lambda)\), we get that as \(\lambda\) approaches 1, \(\text{Diff}(\lambda)\) approaches 0.

Finally, we analyze \(\text{Diff}'(\lambda)\). We write \(\text{Diff}(\lambda)\) as follows:
\[
\text{Diff}(\lambda) = (1 - \lambda) \left( \int_{-1}^{\varepsilon_D(\lambda)} \frac{(1+\varepsilon_D(\lambda))^2}{2A} g(\varepsilon) d\varepsilon + \int_{\varepsilon_D(\lambda)}^{1} \frac{(1+\varepsilon)^2}{2A} g(\varepsilon) d\varepsilon \right) - \lambda \frac{1 - (1 + \varepsilon_D(\lambda))^2}}{2A}.
\]
Then, we compute $Di f' (\lambda)$:

$$Di f' (\lambda) = - \left( \frac{\int_{-1}^{0} (1 + \varepsilon)(1 + \varepsilon^2 - 1 + \varepsilon^2) g(\varepsilon) d\varepsilon}{2A} + \int_{-1}^{1} \varepsilon^2 g(\varepsilon) d\varepsilon \right) \cdot \left( \frac{1 - (1 + \varepsilon)F(\lambda)}{2A} + \varepsilon dL(\lambda) \right) + \int_{-1}^{1} \varepsilon g(\varepsilon) d\varepsilon .$$

Thus, in order to show that $Di f' (\lambda) < 0$, we need to show that

$$\varepsilon dL(\lambda) = \frac{\left( 1 - (1 + \varepsilon)F(\lambda) \right)^2 + \int_{-1}^{1} (1 + \varepsilon^2) g(\varepsilon) d\varepsilon + \left( 1 - (1 + \varepsilon)F(\lambda) \right)^2}{2 \left( 1 + \varepsilon dL(\lambda) \right) \left[ \lambda + (1 - \lambda) G(\varepsilon dL) \right]}.$$ 

Using equations (5) and (6), we can see that:

$$\varepsilon dL(\lambda) = \frac{\left( 1 - (1 + \varepsilon)F(\lambda) \right)^2 + \int_{-1}^{1} (1 + \varepsilon^2) g(\varepsilon) d\varepsilon + \left( 1 - (1 + \varepsilon)F(\lambda) \right)^2}{2 \left( 1 + \varepsilon dL(\lambda) \right) \left[ \lambda + (1 - \lambda) G(\varepsilon dL) \right]}.$$ 

Since $\int_{-1}^{1} \varepsilon^2 g(\varepsilon) d\varepsilon > 0$, we see that $Di f' (\lambda) < 0$. QED.

**Proof of Proposition 3**

We start by showing that Case 2 and Case 4 cannot be equilibria.

One condition that is required for Case 2 to be an equilibrium is:

$$(1 - \lambda L) \left[ \int_{-1}^{0} \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 g(\varepsilon) d\varepsilon + \int_{0}^{1} \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 g(\varepsilon) d\varepsilon \right] + \lambda L \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 \leq \frac{1}{2A}.$$ 

However, from Proposition 1, we know that:

$$(1 - \frac{1}{2}) \left[ \int_{-1}^{0} \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 g(\varepsilon) d\varepsilon + \int_{0}^{1} \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 g(\varepsilon) d\varepsilon \right] + \frac{1}{2} \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 > \frac{1}{2A}.$$ 

Since $\lambda L < \frac{1}{2}$, and $\int_{-1}^{0} \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 g(\varepsilon) d\varepsilon + \int_{0}^{1} \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 g(\varepsilon) d\varepsilon > \frac{(1 + \varepsilon D(\frac{1}{2}))^2}{2A}$, we get:

$$(1 - \lambda L) \left[ \int_{-1}^{0} \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 g(\varepsilon) d\varepsilon + \int_{0}^{1} \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 g(\varepsilon) d\varepsilon \right] + \lambda L \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 > \frac{1}{2A}.$$ 

and thus:

$$(1 - \lambda L) \left[ \int_{-1}^{0} \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 g(\varepsilon) d\varepsilon + \int_{0}^{1} \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 g(\varepsilon) d\varepsilon \right] + \lambda L \left( 1 + \varepsilon D(\frac{1}{2}) \right)^2 > \frac{1}{2A}.$$ 

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As a result, Case 2 is never an equilibrium.

Using the same line of argument, we can show that

$$(1 - \lambda_L) \left[ \int_{-1}^{\lambda_1^H} \left(1 + \varepsilon_D(\lambda_H)\right)^2 g(\varepsilon) \, d\varepsilon + \int_{\lambda_1^H}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) \, d\varepsilon \right] + \lambda_L \frac{(1 + \varepsilon_D(\lambda_H))^2}{2A} > \frac{1}{2A},$$

and thus, Case 4 cannot be an equilibrium.

We now turn to analyze Case 1 and Case 3. Following the analysis above, we know that the second condition to establish Case 1 as an equilibrium (the condition that refers to $\lambda_L$ investors) always holds. Thus, Case 1 will be an equilibrium if and only if the first condition holds, that is, if and only if:

$$(1 - \lambda_H) \left[ \int_{-1}^{\lambda_1^H} \left(1 + \varepsilon_D(\lambda_H)\right)^2 g(\varepsilon) \, d\varepsilon + \int_{\lambda_1^H}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) \, d\varepsilon \right] + \lambda_H \frac{(1 + \varepsilon_D(\lambda_H))^2}{2A} \geq \frac{1}{2A},$$

which can be simplified to the following condition:

$$(1 - \lambda_H) \left[ \int_{-1}^{\lambda_1^H} \left(1 + \varepsilon_D(\lambda_H)\right)^2 g(\varepsilon) \, d\varepsilon + \int_{\lambda_1^H}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) \, d\varepsilon \right] + \lambda_H \left(1 + \varepsilon_D(\lambda_H)\right)^2 \geq 1. \quad (18)$$

Similarly, following Proposition 1, we know that the second condition to establish Case 3 as an equilibrium (the condition that refers to $\lambda_L$ investors) always holds. Thus, Case 3 will be an equilibrium if and only if the first condition holds, that is, if and only if:

$$(1 - \lambda_H) \left[ \int_{-1}^{\lambda_1^L} \left(1 + \varepsilon_D(\lambda_L)\right)^2 g(\varepsilon) \, d\varepsilon + \int_{\lambda_1^L}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) \, d\varepsilon \right] + \lambda_H \left(1 + \varepsilon_D(\lambda_L)\right)^2 \leq 1. \quad (19)$$

When we analyze (18) and (19), we can see that the LHS in (18) is higher than the LHS in (19). This is because $\lambda_L < \frac{1}{2}$, and because $\varepsilon_D(\lambda)$ is increasing in $\lambda$.

When $\lambda_H$ approaches 1 the LHS in (18) approaches $\left(1 + \varepsilon_D(\frac{1}{2})\right)^2$, which is lower than 1. Then, in this case, the LHS in (19) is also lower than 1. When $\lambda_H$ approaches $\frac{1}{2}$, the LHS in (19) approaches

$$\frac{1}{2} \left[ \int_{-1}^{\lambda_1^H} \left(1 + \varepsilon_D(\lambda_H)\right)^2 g(\varepsilon) \, d\varepsilon + \int_{\lambda_1^H}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) \, d\varepsilon \right] + \frac{1}{2} \left(1 + \varepsilon_D(\frac{1}{2})\right)^2,$$

which, following Proposition 1, is higher than 1. Then, in this case, the LHS in (18) is also higher than 1.

The derivative of the LHS in (18) with respect to $\lambda_H$ is given by:

$$\left(1 + \varepsilon_D(\frac{1}{2})\right)^2 - \left[ \int_{-1}^{\lambda_1^H} \left(1 + \varepsilon_D(\lambda_H)\right)^2 g(\varepsilon) \, d\varepsilon + \int_{\lambda_1^H}^{1} \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) \, d\varepsilon \right] < 0.$$
The derivative of the LHS in (19) with respect to \( \lambda_H \) is given by: (recall that \( \lambda_L = (1 - \lambda_H) \))

\[
(1 + \varepsilon_D(\lambda_L))^2 \left[ \int_{-1}^1 (1 + \varepsilon_D(\lambda_L))^2 g(\varepsilon) \, d\varepsilon + \int_{-1}^1 (1 + \varepsilon)^2 g(\varepsilon) \, d\varepsilon \right] \\
- 2 \frac{\partial \varepsilon_D(1 - \lambda_H)}{\partial (1 - \lambda_H)} \cdot (1 + \varepsilon_D(\lambda_L)) \cdot [\lambda_H + (1 - \lambda_H)G(\varepsilon_D(\lambda_L))] < 0.
\]

Thus, both the LHS in (18) and the LHS in (19) are higher than 1 when \( \lambda_H \) approaches \( \frac{1}{2} \), lower than 1 when \( \lambda_H \) approaches 1, and monotonically decreasing in \( \lambda_H \). As a result, there exists a unique \( \lambda_H^* \) between \( \frac{1}{2} \) and 1, at which the LHS in (19) equals 1, and a unique \( \lambda_H^{**} \) between \( \frac{1}{2} \) and 1, at which the LHS in (18) equals 1. Given that the LHS in (18) is higher than the LHS in (19), we know that \( \lambda_H^{**} > \lambda_H^* \).

Then, when \( \frac{1}{2} < \lambda_H < \lambda_H^* \), only Case 1 is an equilibrium; when \( \lambda_H^* \leq \lambda_H \leq \lambda_H^{**} \), both Case 1 and Case 3 are equilibria; when \( \lambda_H^{**} < \lambda_H < 1 \), only Case 3 is an equilibrium. QED.

**Proof of Proposition 4**

The conditions required to establish each one of the four cases as an equilibrium are as follows:

**Case 1:**

\[
(1 - \beta) \left[ (1 - \lambda_H) \left[ \int_{-1}^1 \frac{\varepsilon_D(\frac{1}{2})(1 + \varepsilon_D(\frac{1}{2}))^2}{2A} g(\varepsilon) \, d\varepsilon + \int_{-1}^1 \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) \, d\varepsilon \right] + \lambda_H \cdot \frac{(1 + \varepsilon_D(\frac{1}{2}))^2}{2A} \right] \\
+ \beta \left[ (1 - \lambda_H) \cdot \int_{-1}^1 \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) \, d\varepsilon + \lambda_H \cdot \frac{1}{2A} \right] \geq \frac{1}{2A}
\]

and

\[
(1 - \beta) \left[ (1 - \lambda_L) \left[ \int_{-1}^1 \frac{\varepsilon_D(\frac{1}{2})(1 + \varepsilon_D(\frac{1}{2}))^2}{2A} g(\varepsilon) \, d\varepsilon + \int_{-1}^1 \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) \, d\varepsilon \right] + \lambda_L \cdot \frac{(1 + \varepsilon_D(\frac{1}{2}))^2}{2A} \right] \\
+ \beta \left[ (1 - \lambda_L) \cdot \int_{-1}^1 \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) \, d\varepsilon + \lambda_L \cdot \frac{1}{2A} \right] \geq \frac{1}{2A}.
\]

**Case 2:**
\[(1 - \beta) \left[ (1 - \lambda_H) \left[ \int_{-1}^{1} \frac{\beta^2}{2A} g(\varepsilon) d\varepsilon + \int_{1}^{1} \frac{\beta^2}{2A} g(\varepsilon) d\varepsilon \right] + \lambda_H \cdot \frac{(1+\varepsilon \lambda_H)^2}{2A} \right] \\
+ \beta \left[ (1 - \lambda_H) \cdot \int_{-1}^{1} \frac{(1+\varepsilon)^2}{2A} g(\varepsilon) d\varepsilon + \lambda_H \cdot \frac{1}{2A} \right] \leq \frac{1}{2A},\]

and

\[(1 - \beta) \left[ (1 - \lambda_L) \left[ \int_{-1}^{1} \frac{\beta^2}{2A} g(\varepsilon) d\varepsilon + \int_{1}^{1} \frac{\beta^2}{2A} g(\varepsilon) d\varepsilon \right] + \lambda_L \cdot \frac{(1+\varepsilon \lambda_L)^2}{2A} \right] \\
+ \beta \left[ (1 - \lambda_L) \cdot \int_{-1}^{1} \frac{(1+\varepsilon)^2}{2A} g(\varepsilon) d\varepsilon + \lambda_L \cdot \frac{1}{2A} \right] \leq \frac{1}{2A}.

Case 3:

\[(1 - \beta) \left[ (1 - \lambda_H) \left[ \int_{-1}^{1} \frac{(1+\varepsilon \lambda_H)^2}{2A} g(\varepsilon) d\varepsilon + \int_{1}^{1} \frac{(1+\varepsilon \lambda_H)^2}{2A} g(\varepsilon) d\varepsilon \right] + \lambda_H \cdot \frac{(1+\varepsilon \lambda_H)^2}{2A} \right] \\
+ \beta \left[ (1 - \lambda_H) \cdot \int_{-1}^{1} \frac{(1+\varepsilon)^2}{2A} g(\varepsilon) d\varepsilon + \lambda_H \cdot \frac{1}{2A} \right] \leq \frac{1}{2A},\]

and

\[(1 - \beta) \left[ (1 - \lambda_L) \left[ \int_{-1}^{1} \frac{(1+\varepsilon \lambda_L)^2}{2A} g(\varepsilon) d\varepsilon + \int_{1}^{1} \frac{(1+\varepsilon \lambda_L)^2}{2A} g(\varepsilon) d\varepsilon \right] + \lambda_L \cdot \frac{(1+\varepsilon \lambda_L)^2}{2A} \right] \\
+ \beta \left[ (1 - \lambda_L) \cdot \int_{-1}^{1} \frac{(1+\varepsilon)^2}{2A} g(\varepsilon) d\varepsilon + \lambda_L \cdot \frac{1}{2A} \right] \geq \frac{1}{2A}.

Case 4:

\[(1 - \beta) \left[ (1 - \lambda_H) \left[ \int_{-1}^{1} \frac{(1+\varepsilon \lambda_H)^2}{2A} g(\varepsilon) d\varepsilon + \int_{1}^{1} \frac{(1+\varepsilon \lambda_H)^2}{2A} g(\varepsilon) d\varepsilon \right] + \lambda_H \cdot \frac{(1+\varepsilon \lambda_H)^2}{2A} \right] \\
+ \beta \left[ (1 - \lambda_H) \cdot \int_{-1}^{1} \frac{(1+\varepsilon)^2}{2A} g(\varepsilon) d\varepsilon + \lambda_H \cdot \frac{1}{2A} \right] \geq \frac{1}{2A},\]
and
\[
(1 - \beta) \left[ (1 - \lambda_L) \left[ \int_{-1}^1 \left( \frac{\beta \lambda_H}{2A} \right) g(\varepsilon) d\varepsilon + \int_{\lambda_H}^1 \left( \frac{\beta \varepsilon D(\lambda_H)}{2A} \right)^2 g(\varepsilon) d\varepsilon \right] + \lambda_L \frac{(1 + \varepsilon D(\lambda_H))^2}{2A} \right] + \beta \left[ (1 - \lambda_L) \cdot \int_{-1}^1 \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) d\varepsilon + \lambda_L \cdot \frac{1}{2A} \right] \leq \frac{1}{2A}.
\]

Using the arguments in the proof of Proposition 3, and the fact that \( \int_{-1}^1 (1 + \varepsilon)^2 g(\varepsilon) d\varepsilon > 1 \), we get that Case 2 and Case 4 can never be equilibria, that Case 1 is an equilibrium if and only if
\[
(1 - \beta) \left[ (1 - \lambda_H) \left[ \int_{-1}^1 \left( \frac{\beta \lambda_H}{2A} \right) g(\varepsilon) d\varepsilon + \int_{\lambda_H}^1 \left( \frac{\beta \varepsilon D(\lambda_H)}{2A} \right)^2 g(\varepsilon) d\varepsilon \right] + \lambda_H \frac{(1 + \varepsilon D(\lambda_H))^2}{2A} \right] + \beta \left[ (1 - \lambda_H) \cdot \int_{-1}^1 \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) d\varepsilon + \lambda_H \right] \geq 1
\]
and that Case 3 is an equilibrium if and only if
\[
(1 - \beta) \left[ (1 - \lambda_H) \left[ \int_{-1}^1 \left( \frac{\beta \lambda_H}{2A} \right) g(\varepsilon) d\varepsilon + \int_{\lambda_H}^1 \left( \frac{\beta \varepsilon D(\lambda_H)}{2A} \right)^2 g(\varepsilon) d\varepsilon \right] + \lambda_H \frac{(1 + \varepsilon D(\lambda_H))^2}{2A} \right] + \beta \left[ (1 - \lambda_H) \cdot \int_{-1}^1 \frac{(1 + \varepsilon)^2}{2A} g(\varepsilon) d\varepsilon + \lambda_H \right] \leq 1
\]

We know that \( (1 - \lambda_H) \cdot \int_{-1}^1 (1 + \varepsilon)^2 g(\varepsilon) d\varepsilon + \lambda_H \) is decreasing in \( \lambda_H \), higher than 1 when \( \lambda_H \) approaches \( \frac{1}{2} \), and approaches 1 when \( \lambda_H \) approaches 1. Then, using the arguments in the proof of Proposition 3, we get that both the LHS of (20) and the LHS of (21) are decreasing in \( \lambda_H \), higher than 1 when \( \lambda_H \) approaches \( \frac{1}{2} \), and lower than 1 when \( \lambda_H \) approaches 1. We can also see that the LHS of (20) is higher than the LHS of (21).

As a result, for every \( 0 < \beta < 1 \), there exists a unique \( \lambda^*_H(\beta) \) between \( \frac{1}{2} \) and 1, at which the LHS in (21) equals 1, and a unique \( \lambda^{**}_H(\beta) \) between \( \frac{1}{2} \) and 1, at which the LHS in (20) equals 1. Given that the LHS in (20) is higher than the LHS in (21), we know that \( \lambda^*_H(\beta) > \lambda^{**}_H(\beta) \). Then, when \( \frac{1}{2} < \lambda_H < \lambda^*_H(\beta) \), only Case 1 is an equilibrium; when \( \lambda^*_H(\beta) \leq \lambda_H \leq \lambda^{**}_H(\beta) \), both Case 1 and Case 3 are equilibria; when \( \lambda^{**}_H(\beta) < \lambda_H < 1 \), only Case 3 is an equilibrium.

In order to show that \( \lambda^*_H(\beta) \) and \( \lambda^{**}_H(\beta) \) are increasing in \( \beta \), we need to show that the LHS in (21) and the LHS in (20) are increasing in \( \beta \).
The derivative of the LHS in (21) with respect to $\beta$ is given by:

$$
\lambda_H \left[ 1 - (1 + \xi_D(\lambda_L))^2 \right] - (1 - \lambda_H) \left[ \int_{-1}^{\xi_D(\lambda_L)} \left( (1 + \xi_D(\lambda_L))^2 - (1 + \varepsilon)^2 \right) g(\varepsilon) \, d\varepsilon \right].
$$

From (5) and (6), we know that:

$$
\lambda_L \left[ 1 - (1 + \xi_D(\lambda_L))^2 \right] - (1 - \lambda_L) \left[ \int_{-1}^{\xi_D(\lambda_L)} \left( (1 + \xi_D(\lambda_L))^2 - (1 + \varepsilon)^2 \right) g(\varepsilon) \, d\varepsilon \right] = 0.
$$

Then, since $\lambda_H > \lambda_L$, $1 - (1 + \xi_D(\lambda_L))^2 > 0$ and $\int_{-1}^{\xi_D(\lambda_L)} \left( (1 + \xi_D(\lambda_L))^2 - (1 + \varepsilon)^2 \right) g(\varepsilon) \, d\varepsilon < 0$, we know that the expression in (22) is positive.

Applying similar arguments, we can also show that derivative of the LHS in (20) with respect to $\beta$ is positive. QED.

**Characterization of $\xi_P(\alpha, \lambda_m)$**

Equation (16) can be written as:

$$
F_P(\lambda_m, \alpha, \xi_P) = (1 - \lambda_m) \alpha \int_{-1}^{\xi_P} \left[ (1 + \varepsilon)^2 - (1 + \xi_P)^2 \right] g(\varepsilon) \, d\varepsilon + \lambda_m \int_{-1}^{1} \left[ 1 - (1 + \xi_P)^2 \right] g(\varepsilon) \, d\varepsilon = 0.
$$

The partial derivatives of $F_P(\lambda_m, \alpha, \xi_P)$ with respect to $\lambda_m$, $\alpha$ and $\xi_P$ are:

$$
\frac{\partial F_P(\lambda_m, \alpha, \xi_P)}{\partial \lambda_m} = \int_{-1}^{1} \left[ 1 - (1 + \xi_P)^2 \right] g(\varepsilon) \, d\varepsilon - \alpha \int_{-1}^{\xi_P} \left[ (1 + \varepsilon)^2 - (1 + \xi_P)^2 \right] g(\varepsilon) \, d\varepsilon > 0,
$$

$$
\frac{\partial F_P(\lambda_m, \alpha, \xi_P)}{\partial \alpha} = (1 - \lambda_m) \int_{-1}^{\xi_P} \left[ (1 + \varepsilon)^2 - (1 + \xi_P)^2 \right] g(\varepsilon) \, d\varepsilon < 0,
$$

and

$$
\frac{\partial F_P(\lambda_m, \alpha, \xi_P)}{\partial \xi_P} = -2 \left( 1 + \xi_P \right) \left[ (1 - \lambda_m) \alpha G(\xi_P) + \lambda_m \right] < 0.
$$

Then, using the Implicit Function Theorem, we get that $\frac{\partial \xi_P(\alpha, \lambda_m)}{\partial \alpha} < 0$, and $\frac{\partial \xi_P(\alpha, \lambda_m)}{\partial \lambda_m} > 0$. Moreover, when $\lambda_m$ approaches 1, $\xi_P$ approaches 0; when $\lambda_m$ approaches 0, $\xi_P$ approaches $-1$; when $\alpha$ approaches 1, $\xi_P$ approaches $\xi_D$; and when $\alpha$ approaches 0, $\xi_P$ approaches 0. Thus, for every $\lambda_m$ and $\alpha$ between 0 and 1, there is a unique solution for $\xi_P$ between $-1$ and 1. QED.
Proof of Proposition 5

The conditions for Case 3 to be an equilibrium are:

\[
\lambda_H (1 + \bar{\varepsilon}_D (\lambda_L))^2 + (1 - \lambda_H) \left( \int_{-1}^{\varepsilon^p_d(\lambda_L)} (1 + \bar{\varepsilon}_D (\lambda_L))^2 g(\varepsilon) d\varepsilon + \int_{\varepsilon^p_d(\lambda_L)}^1 (1 + \varepsilon)^2 g(\varepsilon) d\varepsilon \right) \geq 0,
\]

and

\[
\lambda_L (1 + \bar{\varepsilon}_D (\lambda_L))^2 + (1 - \lambda_L) \left( \int_{-1}^{\varepsilon^p_d(\lambda_L)} (1 + \bar{\varepsilon}_D (\lambda_L))^2 g(\varepsilon) d\varepsilon + \int_{\varepsilon^p_d(\lambda_L)}^1 (1 + \varepsilon)^2 g(\varepsilon) d\varepsilon \right) \leq 0.
\]

The derivative of the LHS in (24) with respect to \( \alpha \) is:

\[
-\left(1 - \lambda_L\right) \cdot \left[ \int_{-1}^{\varepsilon^p_d(\alpha,\lambda_H)} (1 + \varepsilon_P (\alpha, \lambda_H))^2 g(\varepsilon) d\varepsilon + \int_{\varepsilon^p_d(\alpha,\lambda_H)}^1 (1 + \varepsilon)^2 g(\varepsilon) d\varepsilon \right] - 1
\]

\[
-2 \frac{\partial \bar{\varepsilon}_P (\alpha, \lambda_H)}{\partial \alpha} (1 + \bar{\varepsilon}_P (\alpha, \lambda_H)) \left[ \lambda_L + (1 - \lambda_L) \alpha G (\alpha, \bar{\varepsilon}_P (\lambda_H)) \right],
\]

which is equal to

\[
-\lambda_H \cdot \left[ \int_{-1}^{\varepsilon^p_d(\alpha,\lambda_H)} (1 + \varepsilon_P (\alpha, \lambda_H))^2 g(\varepsilon) d\varepsilon + \int_{\varepsilon^p_d(\alpha,\lambda_H)}^1 (1 + \varepsilon)^2 g(\varepsilon) d\varepsilon \right] - 1
\]

\[
+ \frac{(1 - \lambda_H) + \lambda_H \alpha G (\varepsilon_P (\alpha, \lambda_H))}{(1 - \lambda_H) \alpha G (\alpha, \varepsilon_P (\lambda_H)) + \lambda_H} \cdot (1 - \lambda_H) \int_{-1}^{\varepsilon^p_d(\alpha,\lambda_H)} \left[ (1 + \varepsilon_P (\alpha, \lambda_H))^2 - (1 + \varepsilon)^2 \right] g(\varepsilon) d\varepsilon,
\]

which is lower than
\begin{align*}
-\lambda_H \cdot & \left[ \int_{-1}^{1} \xi_P(\alpha, \lambda_H) (1 + \xi_P(\alpha, \lambda_H))^2 g(\varepsilon) d\varepsilon - 1 \right] \\
& + \lambda_H \int_{-1}^{1} \xi_P(\alpha, \lambda_H) \left[ (1 + \xi_P(\alpha, \lambda_H))^2 - (1 + \varepsilon)^2 \right] g(\varepsilon) d\varepsilon \\
& = \lambda_H \left( 1 - \int_{-1}^{1} (1 + \varepsilon)^2 g(\varepsilon) d\varepsilon \right) < 0
\end{align*}

Thus, the derivative of the LHS in (24) with respect to \( \alpha \) is negative. From Proposition 3 we know that the LHS in (24) is positive when \( \alpha = 0 \), and since \( \xi_P(\alpha, \lambda_H) > \xi_D(\lambda_L) \), we know it is negative when \( \alpha = 1 \). As a result, for every \( \lambda_H \), there is a unique \( \alpha^{**}(\lambda_H) \) in the range \((0, 1)\), for which the LHS in (24) is equal to 0. Condition (24) is satisfied if and only if \( \alpha \leq \alpha^{**}(\lambda_H) \).

Similarly, we can show that the derivative of the LHS in (23) with respect to \( \alpha \) is negative. From Proposition 3 we know that the LHS in (23) is positive when \( \alpha = 0 \) and \( \lambda_H < \lambda_H^* \), and negative when \( \lambda_H > \lambda_H^* \). Since \( \xi_P(\alpha, \lambda_H) > \xi_D(\lambda_L) \), we know that the LHS in (23) is negative when \( \alpha = 1 \). Thus, we can define a function \( \alpha^*(\lambda_H) : \forall \lambda_H \in [\lambda_H^*, 1) \), \( \alpha^*(\lambda_H) = 0 \); \( \forall \lambda_H \in (\frac{1}{2}, \lambda_H^*) \), \( \alpha^*(\lambda_H) \in (0, 1) \), such that: Condition (23) is satisfied if and only if \( \alpha \geq \alpha^*(\lambda_H) \).

Since the LHS in (24) is higher than the LHS in (23), we know that \( \alpha^*(\lambda_H) < \alpha^{**}(\lambda_H) \). As a result, Case 3 is an equilibrium if and only if \( \alpha \in [\alpha^*(\lambda_H), \alpha^{**}(\lambda_H)] \). QED.

References


