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## The Mundell-Fleming Model: Stochastic Dynamics

The Mundell-Fleming model, which is still the workhorse model of international macroeconomics, can now be cast in a stochastic framework. Such a framework assumes a set of exogenous stochastic processes (e.g., money supply) which drives the dynamics of the equilibrium system. Since economic agents are forward looking, each short term equilibrium is based on expectations about future shocks and the resulting future short term equilibria.

### 4.1 The Stochastic Framework

Let us begin with a description of the stochastic version of the Mundell-Fleming model. For simplicity, we express all variables in logarithmic forms (except for the interest rates) and assume all behavioral relations are linear in these log variables. This linear system (similar to the ones in Clarida and Gali (1994)) can be viewed as an approximation from an original nonlinear system.

Aggregate demand in period  $t$ ,  $y_t^d$ , specified as a function of an exogenous demand component,  $d_t$ , the real exchange rate,  $q_t$ , and the domestic real rate of interest,  $r_t$ , is given by

$$y_t^d = d_t + \eta q_t - \sigma r_t. \quad (4.1)$$

where  $\eta$  and  $\sigma$  are positive elasticities. This equation is an analogue of equation (3.5) of the previous chapter. As is usual, the real variables are derived from the following nominal variables:  $s_t$ , the spot exchange rate (the domestic value of foreign currency);  $p^*$ , the foreign

price level;  $p_t$ , the domestic price level; and  $i_t$ , the domestic nominal rate of interest. More specifically,  $q_t = s_t + p^* - p_t$  and  $r_t = i_t - E_t(p_{t+1} - p_t)$ . For simplicity, we assume the foreign price level,  $p^*$ , to be constant over time.

Aggregate demand is positively related to the exogenous demand shock, capturing external, fiscal expansion and other internal shocks. The real exchange rate affects positively aggregate demand by stimulating the traded sector (exportables and domestic production of importables). The real interest rate affects negatively aggregate demand by discouraging investment and consumption.

Money market equilibrium is specified as:

$$m_t^s - p_t = Y_t - \lambda i_t, \quad (4.2)$$

where  $m_t^s$  is the money supply at time  $t$ , and  $\lambda (> 0)$  the interest semi-elasticity of the demand for money. This equation is an analogue of equation (3.7) of the previous chapter. As usual, the domestic nominal rate of interest ( $i_t$ ) has a negative effect on the demand for money, while domestic output ( $y_t$ ) has a positive effect. To simplify matters, the output demand elasticity is assumed to be unity.

Price setting is based on a mix of auction markets and long term contract markets. The market clearing price in the auction market is  $p_t^e$ . The price in the long term contract market is set one period in advance according to expectations of the future market clearing price in that market,  $E_{t-1}p_t^e$ . Accordingly, the general price level in the domestic economy,  $p_t$ , is given by a weighted average of these two prices:

$$p_t = (1-\alpha)E_{t-1}p_t^e + \alpha p_t^e, \quad (4.3)$$

where  $0 < \theta < 1$  is the share of the auction market in domestic output. The long term contract element is akin to Taylor (1981) and Fischer (1981). This introduces an element of price rigidity into the system.

Due to free capital mobility, interest parity prevails. Assuming risk neutrality, uncovered interest parity should hold. That is,

$$i_t = i^* + E_t(s_{t+1} - s_t), \quad (4.4)$$

where  $i^*$  is the world rate of interest, assumed for simplicity to be constant over time. Through costless arbitrage, the return on investing one unit of domestic currency in domestic security,  $i_t$ , is made equal to the expected value of the domestic currency return on investing the same amount in foreign security, which yields a foreign currency return,  $i^*$ , plus an expected depreciation of domestic currency,  $E_t(s_{t+1} - s_t)$ .

The equilibrium system consists of the four equations (4.1)–(4.4) at each point in time. Observe that domestic output is demand-determined, as in all models with price rigidity.

The shock (or forcing stochastic) processes that drive the dynamics of the equilibrium system are:

$$y_t^s = g_y + y_{t-1}^s + \epsilon_{yt}, \quad (4.5a)$$

$$d_t = g_d + d_{t-1} + \epsilon_{dt}, \quad (4.5b)$$

$$m_t^s = g_m + m_{t-1}^s + \epsilon_{mt}, \quad (4.5c)$$

where  $g_y$  and  $g_m$  are the deterministic growth rates of output and money, and  $\epsilon_{yt}$ ,  $\epsilon_{dt}$ ,  $\epsilon_{mt}$  are independently and identically distributed (i.i.d.) *supply*, *demand*, and *money* shocks with zero means and constant variances.<sup>1</sup> Accordingly, our specification assumes that the system is bombarded by permanent shocks (in a random walk fashion).<sup>2</sup>

## 4.2 Flex-Price Equilibrium

Since our stochastic framework is both forward and backward looking, a systematic procedure is required to obtain a solution. We thus apply a two-stage procedure for solving the equilibrium system (4.1)–(4.5). In the first stage, we solve for a flexible price equilibrium that corresponds to this system. In the second stage, we use the flex-price equilibrium to arrive at a full-fledged solution for the mixed fix-flex-price system.

Using superscript 'e' to denote flex-price equilibrium values, we can express the first stage solution in the following form.

$$Y_t^e = Y_t^s. \quad (4.6)$$

$$q_t^e = \frac{1}{\mathbf{0}} (Y_t^s - d_t + \mathbf{F}i^*). \quad (4.7)$$

$$p_t^e = m_t^s - Y_t^s + \mathbf{8}(i^* + g_m - g_y). \quad (4.8)$$

$$\mathbf{B}_t^e (\equiv E_t P_{t+1}^e - p_t^e) = g_m - g_y. \quad (4.9)$$

$$r_t^e = i^*. \quad (4.10)$$

$$i_t^e = i^* + g_m - g_y. \quad (4.11)$$

$$s_t^e = m_t^s + \left( \frac{1}{\mathbf{0}} - 1 \right) Y_t^s - \frac{1}{\mathbf{0}} d_t + \left( \frac{\mathbf{F}}{\mathbf{0}} + \mathbf{8} \right) i^* - p^* + \mathbf{8}(g_m - g_y). \quad (4.12)$$

The flex-price equilibrium is economically intuitive. When prices are flexible and the supply of output is exogenous, output must be supply-determined, hence (4.6). With constant money demand elasticities, the expected rate of inflation (which turns out also to be the actual inflation rate) must be equal to the difference between money growth and output growth, hence (4.9). Since world prices are constant in the foreign country (hence, zero world inflation), the world real and nominal rates of interest must be equal to  $i^*$ . Under the assumption of free capital mobility, the domestic real rate of interest must be equal to  $i^*$  as well, hence (4.10). From (4.9) and the Fisher equation linking the nominal rate of interest to the real rate and the expected rate of inflation, we can obtain the corresponding domestic nominal interest rate as (4.11). Using the domestic real interest rate expression in (4.10), the real exchange rate that equates output demand to the exogenous supply of output can be solved from the aggregate demand equation (4.1) to yield (4.7). Given the domestic nominal rate of interest (4.11) and output (4.6), the domestic price level which is consistent with money market equilibrium (4.2) can be expressed as in (4.8). Finally, we can derive the nominal exchange rate from (4.7) and (4.8) together with the definition of real exchange rate in terms the nominal exchange rate and domestic and foreign price levels, hence (4.12).

As an application, consider an expansionary fiscal policy indicated by a positive  $\Delta(d_t)$ , where  $\Delta(\cdot)$  is a difference operator. From (4.7) and (4.12), one can verify that the real and nominal exchange rates will appreciate, without any effects on output, prices, and interest rates. This should be familiar to the reader from the result established in the previous chapter that, under a flexible exchange rate system with perfect capital mobility, fiscal policies are neutral.

Consider next an expansionary monetary policy indicated by a positive  $\Delta(m_t^s)$ . From (4.8), the domestic price level will go up. From (4.12), the domestic nominal exchange rate will depreciate. Output, interest rates, and the domestic real exchange rate will not be affected. This is obviously consistent with the classical dichotomy between real and nominal magnitudes associated with monetary policy under flexible prices, in addition to the familiar Mundell-Fleming effects of monetary policy on the nominal exchange rate discussed in the previous chapter.

### 4.3 Full-fledged Equilibrium

Following our two-stage solution procedure, we can now use the flex-price equilibrium values obtained in the first stage to solve for the full-fledged equilibrium in this second stage.

The equilibrium, derived in Appendix A, is as follows.

$$y_t = y_t^e + \left( \frac{\mathbf{F}+\mathbf{0}}{\mathbf{8}+\mathbf{F}+\mathbf{0}} \right) (1+\mathbf{8})(1-\mathbf{2}) (,_{mt^-}, y_t) . \quad (4.13)$$

$$q_t = q_t^e + \left( \frac{1}{\mathbf{8}+\mathbf{F}+\mathbf{0}} \right) (1+\mathbf{8})(1-\mathbf{2}) (,_{mt^-}, y_t) . \quad (4.14)$$

$$P_t = P_t^e - (1-\theta) (\Delta m_t - \Delta y_t) . \quad (4.15)$$

$$B_t = B_t^e + (1-\theta) (\Delta m_t - \Delta y_t) . \quad (4.16)$$

$$r_t = r_t^e - \left( \frac{1}{\theta + \mathbf{F} + \mathbf{O}} \right) (1+\theta) (1-\theta) (\Delta m_t - \Delta y_t) . \quad (4.17)$$

$$i_t = i_t^e + \left( \frac{\mathbf{F} + \mathbf{O} - 1}{\theta + \mathbf{F} + \mathbf{O}} \right) (1-\theta) (\Delta m_t - \Delta y_t) . \quad (4.18)$$

$$s_t = s_t^e - \left( \frac{\mathbf{F} + \mathbf{O} - 1}{\theta + \mathbf{F} + \mathbf{O}} \right) (1-\theta) (\Delta m_t - \Delta y_t) . \quad (4.19)$$

The full-fledged equilibrium values in equations (4.13)-(4.19) reveal interesting features:

(1) *Price Rigidity and the Classical Dichotomy*

Price rigidity is reflected in (4.15) since a positive excess money shock generates a price increase which falls short of the market clearing price. With pre-set prices, the classical dichotomy no longer holds. Accordingly, in (4.13), one can observe that output responds to the innovation in the money supply in excess of the innovation in domestic output supply. The real exchange rate is positively affected, and the domestic real rate of interest, negatively affected by the difference in innovations. The magnitudes of these effects depend on the degree of price flexibility, indicated by  $\theta$ . Indeed, in the extreme case of complete price flexibility ( $\theta = 1$ ), these real effects of monetary policy will vanish (as shown also in the previous chapter).

(2) *The Phillips Curve*

Define excess output capacity (which is directly related to the rate of unemployment) by  $u_t$  as  $y_t^e - y_t$ . Then we can obtain an expectations-augmented Phillips curve relation between inflation ( $\pi_t$ ) and excess capacity ( $u_t$ ) as follows:

$$\mathbf{B}_t = \mathbf{B}_t^e - \left( \frac{\mathbf{8}}{(1+\mathbf{8})(\mathbf{F}+\mathbf{0})} + \frac{1}{1+\mathbf{8}} \right) u_t . \quad (4.20)$$

The flatter line in Figure 4.1 portrays the open-economy Phillips curve under free capital mobility. Equation (4.20) shows that the Phillips curve is flatter when the aggregate demand elasticities  $\eta$  (with respect to the real exchange rate) and  $\sigma$  (with respect to the domestic real rate of interest) are larger. The effect of the interest semi-elasticity of money demand ( $\lambda$ ) on the slope of the Phillips curve is, however, ambiguous, depending on whether  $\sigma + \eta$  exceeds or falls short of unity. The source of this ambiguity is derived from the more fundamental ambiguous effects of excess innovations on the domestic nominal interest rate (4.18) and spot exchange rates (4.19).

### (3) *Real Exchange Rate and Real Rate of Interest*

Substituting (4.17) into (4.14) yields a contemporaneous negative relation between the real exchange rate and the domestic real interest rate as follows:

$$q_t = q_t^e - (r_t - r_t^e) . \quad (4.21)$$

This unambiguous prediction has been subject to a large body of empirical studies (see Campbell and Clarida (1987), Meese and Rogoff (1988), and Edison and Pauls (1993)), with mixed



results. It thus seems that the Mundell-Fleming model should account for this inconsistency with data before it will be used with great confidence for policy advice.

(4) *Expected Long Run Values*

Applying the expectation operator as of period  $t$  to the system of equations (4.13)–(4.19) in periods  $t+1$  and on reveals that the expected long run equilibrium values are equal to the flex-price solution. Equation (4.19) then shows that an excess money innovation will lead to exchange rate overshooting *à la* Dornbusch (1976) if the sum of demand elasticities  $(\sigma + \eta)$  falls short of unity.

#### 4.4 Capital Controls

The hallmark of the Mundell-Fleming model is the distinct role played by international capital movements on the effectiveness of policies. Thus, restricting capital flows should have a significant effect on the working of the international macro system.

Consider the extreme case where capital flows are completely restricted. In this case, the interest parity (4.4) will no longer hold, and trade balance will be equilibrated fully by the market clearing exchange rate.

The final form of the aggregate demand equation (4.1), derived from the structural equation, will have to be modified. We can write the original structural equation as:

$$Y_t^d = (\tilde{d}_t^A + A_y Y_t^d + A_x r_t) + (d_t^X + X_y Y_t^d + X_q q_t) .$$

where the first parenthetical expression refers to domestic absorption (A), and the second to net trade balance (X),  $d_t^A$  denotes the autonomous component of absorption,  $A_y > 0$ ,  $A_r < 0$ ,  $d_t^X$  denotes the autonomous component of trade balance,  $X_y < 0$ , and  $X_q > 0$ . To arrive at the final form (4.1), we simply solve for  $y_t^d$  as a function of  $r_t$  and  $q_t$ . Define the sum of marginal propensities to save and import,  $1 - A_y - X_y$ , as  $\alpha$ . Notice that  $d_t = (d_t^A + d_t^X)/\alpha$ ,  $\eta = X_q/\alpha > 0$ , and  $\sigma = -A_r/\alpha > 0$ .

In the presence of full capital controls, the net trade balance (X) is zero. Hence,  $d_t^X + X_y y_t^d + X_q q_t = 0$ , which can be rewritten as

$$d_t^X - \mu y_t^d + \alpha q_t = 0, \quad (4.1)'$$

where  $\mu = -X_y$  and  $\alpha \eta = X_q$ . Substituting this into the structural equation for aggregate demand, we can modify the final form as

$$y_t^d = d_t^A - F(r_t), \quad (4.1)''$$

where  $d_t^A = d_t^A/(1 - A_y)$  and  $\gamma = (1 - A_y - X_y)/(1 - A_y) < 1$ . As an analogue to (4.5b), we specify the stochastic process for  $d_t^A$  as

$$d_t^A = g_y + d_{t-1}^A + \epsilon_{dt}^A, \quad (4.5b)'$$

where  $\epsilon_{dt}^A$  is assumed to have similar properties as  $\epsilon_{dt}$ .

We lay out the solutions for the flex-price and full-fledged equilibria in Appendix B. Here, we focus on the effect of capital controls on output  $y_t$ , the real exchange rate  $q_t$ , and the

inflation rate  $\pi_t$ . The full-fledged equilibrium output, real exchange rate, and inflation rate are given respectively by

$$y_t^e = y_t^e + \left( \frac{F(\cdot)}{\mathbf{8}+F(\cdot)} \right) (1+\mathbf{8})(1-2) \left[ ,_{mt} - \left( \frac{\mathbf{8}+F(\cdot)}{F(\cdot)} \right),_{yt} + \left( \frac{\mathbf{8}}{F(\cdot)} \right),_{dt}^A \right], \quad (4.13)'$$

where  $y_t^e = y_t^s$ .

$$= q_t^e + \left( \frac{\mu}{\alpha} \right) \left( \frac{F(\cdot)}{\mathbf{8}+F(\cdot)} \right) (1+\mathbf{8})(1-2) \left[ ,_{mt} - \left( \frac{\mathbf{8}+F(\cdot)}{F(\cdot)} \right),_{yt} + \left( \frac{\mathbf{8}}{F(\cdot)} \right),_{dt}^A \right], \quad (4.14)'$$

where  $q_t^e = (\mu y_t^s - d_t^X) / \alpha \eta$ .

$$\mathbf{B}_t = \mathbf{B}_t^e + (1-2) \left[ ,_{mt} - \left( \frac{\mathbf{8}+F(\cdot)}{F(\cdot)} \right),_{yt} + \left( \frac{\mathbf{8}}{F(\cdot)} \right),_{dt}^A \right], \quad (4.16)'$$

where  $\pi_t^e = g_m - g_y$ .

Comparing these equilibrium values with those under free capital flows (4.13) and (4.14), we can highlight three main differences.

### 1. *Demand Shocks*

Under capital controls, the absorption shock  $\epsilon_{dt}^A$  has a positive effect on  $y_t - y_t^e$ ,  $q_t - q_t^e$ , and  $\pi_t - \pi_t^e$ , through its negative effect on the domestic real rate of interest  $r_t$ . In contrast, it has no effect in the case of free capital flows since the real rate of interest there is nailed down by the world rate of interest.

### 2. *Monetary Shocks*

Since  $\sigma\gamma/(\lambda + \sigma\gamma) < (\sigma + \eta)/(\lambda + \sigma + \eta)$ , the monetary shock  $\epsilon_{mt}$  has a smaller effect on  $y_t - y_t^e$  (through a stronger negative effect on  $r_t$ ) under capital controls. The relative sensitivity of  $q_t - q_t^e$  to the shock under the two capital mobility regimes is ambiguous in general, depending on the relative magnitudes of  $\mu(\sigma\gamma)/\alpha\eta(\lambda + \sigma\gamma)$  and  $1/(\lambda + \sigma + \eta)$ . Finally, the shock has the same effect on  $\pi_t - \pi_t^e$  as in the free capital mobility case.

### 3. *Supply Shocks*

Since  $(\sigma + \eta)/(\lambda + \sigma + \eta) < 1$ , the productivity shock  $\epsilon_{yt}$  has a bigger negative effect on aggregate demand  $y_t - y_t^e$  (through a stronger positive effect on  $r_t$ ) under capital controls. The relative sensitivity of  $q_t - q_t^e$  to the shock under the two capital mobility regimes is again ambiguous. The shock will nonetheless produce a more pronounced effect on  $\pi_t - \pi_t^e$  under capital controls.

### 4. *The Phillips Curve*

Substituting equation (4.13)' into equation (4.16)' and defining  $u_t = -(y_t - y_t^e)$  as before, we can express the Phillips curve under capital controls as follows:

$$\mathbf{B}_t = \mathbf{B}_t^e - \left( \frac{\mathbf{8}}{(\mathbf{1} + \mathbf{8})\mathbf{F}} + \frac{\mathbf{1}}{\mathbf{1} + \mathbf{8}} \right) u_t. \quad (4.20)'$$

The steeper line in Figure 4.1 portrays the open-economy Phillips curve under capital controls. In other words, fluctuations in inflation rates will be associated with smaller variations in unemployment.

The intuition behind the steeper slope has to do with the impact effect of capital controls on aggregate demand. Comparing the aggregate demand functions under capital controls (4.1)" and under free capital mobility (4.1), we observe that in the former case the interest semi-elasticity becomes smaller ( $\sigma\gamma < \sigma$  since  $\gamma = 1 - X_y/(1 - A_y) < 1$ ) and the real exchange rate effect disappears ( $0 < \eta$ ) from the reduced form equation for aggregate demand due to the zero net trade balance restriction under capital controls.

On the other hand, capital controls do not alter the mechanisms underlying the determination of prices (i.e., the price setting equation (4.3) and the money market equation (4.2)). Combined with the structural change in aggregate demand, restrictions on capital flows will generate less variations in unemployment rates (excess output capacity) at the expense of more variations in inflation rates.

Indeed, a comparison between equations (4.20) and (4.20)' reveals that the difference in the slopes of the Phillips curve under free capital mobility and capital controls depends solely on the aggregate demand parameters  $\sigma + \eta$  versus  $\sigma\gamma$ , and not on the money market parameter  $\lambda$  and the price setting parameter  $\theta$ .

Naturally, the natural rate of unemployment ( $= 0$  in our case) and the expected rate of inflation ( $\pi^e$ ) are unaffected by structural changes such as capital controls. This is reflected by the intersection of the two Phillips curves at the point  $(0, \pi^e)$ . While the various shocks will move economy away from this point along the respective Phillips curve (depending on the capital mobility regime), changes in the expected rate of inflation due to permanent changes in the relative money-output growth rates ( $g_m - g_y$ ) will shift the Phillips curve around.

#### **4.5 Some Policy Implications**

There exist three types of gains from trade: trade of goods/services for goods/services; trade of goods/services for assets (intertemporal trade); and trade of assets for assets (for diversification of risk). Evidently, capital controls limit the potential gains from the last two types. However, there are second-best and other suboptimal (due to, say, non-market-clearing) situations where capital controls can improve efficiency.

When, for example, taxation of foreign-source income from capital is not enforceable, it proves efficient to 'trap' capital within national borders so as to broaden the tax base and to alleviate other tax distortions (see Razin and Sadka (1991)).

This chapter introduces another argument in favour of constraints on capital mobility. We show that capital controls alter the inflation-unemployment tradeoffs. In particular, output/employment variations are reduced at the expense of bigger variations in inflation rates. When the policy maker puts heavier weight on stable employment than on stable inflation, his/her objective can be attained more easily under capital controls.

## Appendix A: Derivation of the Full-fledged Equilibrium Solution

This appendix derives the solution for the full-fledged equilibrium (4.13)–(4.19), taking as given the flex-price equilibrium.

### 1. Derivation of $p_t$ (4.15)

To get the solution for the domestic price level (4.15), we simply substitute (4.5c) and (4.8) into (4.3).

### 2. Derivation of $q_t$ (4.14)

Substituting (4.4) and (4.7) into (4.1), using the definitions of real exchange rate  $q_t = s_t + p^* - p_t$  and real interest rate  $r_t = i_t - E_t(p_{t+1} - p_t)$ , and subtracting  $(\lambda + \sigma)E_t q_{t+1}$  and adding  $(\lambda + \sigma)q_t^e$ , we get

$$q_t^e = (\mathbf{8} + \mathbf{F}) [E_t(q_{t+1} - q_{t+1}^e) - (q_t - q_t^e)] + (1 + \mathbf{8})(1 - \mathbf{2})(,_{mt} -$$
 (A.1)

Observe, from (4.5a), (4.5b), and (4.7) and the properties of  $\epsilon_{yt+1}$  and  $\epsilon_{dt+1}$ , that  $E_t q_{t+1}^e = q_t^e$ .

We guess a solution of the form  $q_t = q_t^e + \kappa(\epsilon_{mt} - \epsilon_{yt})$  and apply it to (A2.1).

$$0\mathbf{6}(,_{mt}, ,_{yt}) = (\mathbf{8} + \mathbf{F}) E_t [0\mathbf{6}(,_{mt+1}, ,_{yt+1}) - 0\mathbf{6}(,_{mt}, ,_{yt})] + (1 + \mathbf{8})(1 - \mathbf{2})(,_{mt}, ,_{yt}).$$

Since  $E_t(\epsilon_{mt+1} - \epsilon_{yt+1}) = 0$ , we can obtain  $\kappa = (1 + \lambda)(1 - \theta)/(\lambda + \sigma + \eta)$  from the above equation.

This value of  $\kappa$  in our guess solution yields the solution for the real exchange rate (4.14).

3. *Derivation of  $y_t$  (4.13) and  $i_t$  (4.18)*

Substituting the solutions for  $p_t$  and  $q_t$  from (4.15) and (4.14) just derived above into the aggregate demand equation (4.1) while using the interest parity (4.4) yields the solutions for  $y_t$  (4.13) and  $i_t$  (4.18).

4. *Derivation of  $\pi_t$  (4.16)*

Applying the definition of the expected rate of inflation,  $\pi_t = E_t(p_{t+1} - p_t)$ , to (4.15) derived in step 1 and (4.9) yields the solution for the inflation rate (4.16).

5. *Derivation of  $r_t$  (4.17)*

Using  $i_t$  derived in step 3 and  $\pi_t$  in step 4 and the Fisher equation yields the solution for the domestic real rate of interest  $r_t$ .

6. *Derivation of  $s_t$  (4.19)*

Using the solutions for  $p_t$  derived in step 1 and  $q_t$  in step 2 and applying them to the definition of the real exchange rate  $q_t = s_t + p^* - p_t$  yields a solution for the nominal exchange rate  $s_t$ .



## Appendix B: Solutions for the Capital Controls Model

This appendix provides the solution to (4.1)', (4.1)", (4.2) and (4.3), subject to the stochastic processes (4.5) and (4.5b)'. The flex-price equilibrium conditions are

$$y_t^e = y_t^s. \quad (\text{B.1})$$

$$q_t^e = \frac{1}{\pi \mathbf{0}} (\mu y_t^s - d_t^x). \quad (\text{B.2})$$

$$p_t^e = \mathbf{8} \left[ \frac{1}{\mathbf{F}(\cdot)} (d_t^A - y_t^s) + g_m - g_y \right] + m_t^s - y_t^s. \quad (\text{B.3})$$

$$\mathbf{B}_t^e = g_m - g_y. \quad (\text{B.4})$$

$$r_t^e = \frac{1}{\mathbf{F}(\cdot)} (d_t^A - y_t^s). \quad (\text{B.5})$$

$$i_t^e = \frac{1}{\mathbf{F}(\cdot)} (d_t^A - y_t^s) + g_m - g_y. \quad (\text{B.6})$$

$$= m_t^s + \left( \frac{\mu}{\pi \mathbf{0}} - \frac{\mathbf{8} + \mathbf{F}(\cdot)}{\mathbf{F}(\cdot)} \right) y_t^s - \left( \frac{1}{\pi \mathbf{0}} d_t^x - \frac{\mathbf{8}}{\mathbf{F}(\cdot)} d_t^A \right) - p^* + \mathbf{8} (g_m - g_y) \quad (\text{B.7})$$

To solve the full-fledged equilibrium, we use the flex-price solution to obtain the equilibrium price  $p_t$  and inflation rate  $\pi_t$ . We then use the Fisher equation along with the aggregate demand and money market equilibrium equations (4.1)" and (4.2) to get the solutions for the real interest rate  $r_t$  and output  $y_t$  simultaneously. From the trade balance equation (4.1)', we can calculate the real exchange rate  $q_t$ . The nominal interest rate  $i_t$  and the nominal exchange

rate  $s_t$  are then derived from the Fisher equation and the definition of the real exchange rate respectively. Below, we lay out the solution for the full-fledged equilibrium.

$$Y_t = Y_t^e + \left( \frac{F(\cdot)}{\mathbf{8}+F(\cdot)} \right) (1+\mathbf{8})(1-\mathbf{2}) \left[ ,_{mt} - \left( \frac{\mathbf{8}+F(\cdot)}{F(\cdot)} \right),_{yt} + \left( \frac{\mathbf{8}}{F(\cdot)} \right),_{dt}^A \right]. \quad (\text{B.8})$$

$$= \alpha_t^e + \left( \frac{\mu}{\mathbf{0}} \right) \left( \frac{F(\cdot)}{\mathbf{8}+F(\cdot)} \right) (1+\mathbf{8})(1-\mathbf{2}) \left[ ,_{mt} - \left( \frac{\mathbf{8}+F(\cdot)}{F(\cdot)} \right),_{yt} + \left( \frac{\mathbf{8}}{F(\cdot)} \right),_{dt}^A \right] \quad (\text{B.9})$$

$$P_t = P_t^e - (1-\mathbf{2}) \left[ ,_{mt} - \left( \frac{\mathbf{8}+F(\cdot)}{F(\cdot)} \right),_{yt} + \left( \frac{\mathbf{8}}{F(\cdot)} \right),_{dt}^A \right]. \quad (\text{B.10})$$

$$\mathbf{B}_t = \mathbf{B}_t^e + (1-\mathbf{2}) \left[ ,_{mt} - \left( \frac{\mathbf{8}+F(\cdot)}{F(\cdot)} \right),_{yt} + \left( \frac{\mathbf{8}}{F(\cdot)} \right),_{dt}^A \right]. \quad (\text{B.11})$$

$$r_t = r_t^e - \left( \frac{1}{\mathbf{8}+F(\cdot)} \right) (1+\mathbf{8})(1-\mathbf{2}) \left[ ,_{mt} - \left( \frac{\mathbf{8}+F(\cdot)}{F(\cdot)} \right),_{yt} + \left( \frac{\mathbf{8}}{F(\cdot)} \right),_{dt}^A \right]. \quad (\text{B.12})$$

$$i_t = i_t^e + \left( \frac{\mathbf{8}(1-F(\cdot))}{\mathbf{8}+F(\cdot)} \right) (1-\mathbf{2}) \left[ ,_{mt} - \left( \frac{\mathbf{8}+F(\cdot)}{F(\cdot)} \right),_{yt} + \left( \frac{\mathbf{8}}{F(\cdot)} \right),_{dt}^A \right]. \quad (\text{B.13})$$

$$s_t^e + \left[ \left( \frac{\mu}{\mathbf{0}} \right) \left( \frac{F(\cdot)}{\mathbf{8}+F(\cdot)} \right) (1+\mathbf{8}) - 1 \right] (1-\mathbf{2}) \left[ ,_{mt} - \left( \frac{\mathbf{8}+F(\cdot)}{F(\cdot)} \right),_{yt} + \left( \frac{\mathbf{8}}{F(\cdot)} \right),_{dt}^A \right] \quad (\text{B.14})$$

Comparing the full-fledged equilibrium under capital controls (B.8)-(B.14) with the corresponding equilibrium under free capital flows (4.13)-(4.19), we can assess the significant role that capital mobility plays in the Mundell-Fleming model.

## Problems

1. Consider the stochastic dynamic version of the Mundell-Fleming model with perfect capital mobility. Introduce transitory shocks to the money supply process by adding an extra term  $-\phi\epsilon_{mt-1}$  ( $\phi > 0$ ) to the right hand side of (4.5c). Decompose the variance of the real exchange rate  $q_t$  into transitory and permanent components of the monetary shock.
  
2. Consider the stochastic dynamic version of the Mundell-Fleming model with perfect capital mobility. Introduce a correlation between the money supply process  $m_t^s$  and aggregate demand process  $d_{t-1}$  by adding  $\rho d_{t-1}$  to the right hand side of (4.5c). One can view this as a feedback rule whereby current monetary policy is conditioned on fiscal impulse in the previous period. What value of  $\rho$  will minimize output variance? Inflation variance?
  
3. Consider the stochastic dynamic version of the Mundell-Fleming model with and without capital controls.
  - (a) Compare the sensitivity of the following economic indicators to the various shocks between the two capital mobility regimes:  $p_t$ ,  $\pi_t$ ,  $r_t$ ,  $i_t$ ,  $s_t$ .
  - (b) Compare the slopes of the Phillips curves under the two regimes.
  - (c) Check whether the negative relation between the real exchange rate and the domestic real rate of interest under perfect capital mobility holds also under capital controls.

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## **ENDNOTES**

1. To guarantee the existence of a long run (steady state) equilibrium for our system, the deterministic growth rates of output on both the supply and demand sides ( $g_y$ ) are assumed to be identical.
2. The problem set at the end of the chapter considers also effects of transitory shocks.