Consideration Sets and Competitive Marketing

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We study a market model in which competing firms use costly marketing devices to influence the set of alternatives which consumers perceive as relevant. Consumers in our model are boundedly rational in the sense that they have an imperfect perception of what is relevant to their decision problem. They apply well-defined preferences to a “consideration set”, which is a function of the marketing devices employed by the firms. We examine the implications of this behavioural model in the context of a competitive market model, particularly on industry profits, vertical product differentiation, the use of marketing devices, and consumers’ conversion rates.

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1. INTRODUCTION

We present a model of competitive marketing based on the notion that consumers are boundedly rational and that firms use marketing tactics in an attempt to influence consumers’ decision process. The standard model of consumer behaviour assumes that the consumer applies well-defined preferences to a perfectly perceived set of available alternatives. We retain the assumption that consumers have stable preferences but relax the assumption that they have a perfect perception of what is relevant for their consumption problem, thus allowing firms to manipulate that perception. Our aim is to explore the market implications of this departure from the standard model, especially for the way firms deploy marketing strategies in competitive environments.

The cornerstone of our model is the observation that in the modern marketplace, consumers face an overwhelmingly large variety of products and therefore often use screening criteria (deliberate as well as unconscious) in order to reduce the number of “relevant” alternatives. As a result, consumers apply their preferences not to the set of objectively feasible alternatives but to a potentially smaller set which they construct at an earlier stage of the decision process. Borrowing a term from the marketing literature, we refer to this set as the “consideration set”. The basic idea underlying this term is that consumers may be unaware of some of the feasible products, and even when they become aware of a new product, they still need to be persuaded to consider it as a potential substitute to their currently consumed product.
Our model of consumer behaviour attempts to capture the idea that consumers do not automatically perceive all available options as relevant for their consumption problem and that they resist considering new alternatives. The role of marketing is to overcome this resistance. Whether or not it succeeds depends on the competing products and how they are marketed. The framework we propose accommodates a variety of ways in which marketing influences the formation of consideration sets. Here are a few examples.

Example 1.1. Advertising content. An ad that highlights a drawback in a group of rival products may attract the attention of consumers who currently consume those products (“tired of hours of waiting for customer service?” “having trouble keeping track of the fees your credit company charges you?”). Similarly, an ad for a product that highlights one of its good features may give consumers of other products lacking this feature a sufficient reason to consider the advertised product. The effect of ad content in these situations can go beyond mere information transmission. In particular, an ad that points out a flaw in a product the consumer regularly buys hardly tells him something he does not already know, and it is not likely to modify his beliefs about the quality of alternative products. However, it may have the rhetorical effect of persuading him to look for a substitute.

Example 1.2. Argumentation by a salesperson. Think of a consumer who enters a car dealership with the intention to buy a new car. The consumer has a particular car model in mind. As he inspects it on the display podium, he is approached by a salesperson who tries to convince him to consider a different car model, using arguments (which may or may not be factually true) that point out similarities and differences between the two models.

Example 1.3. Positioning. Economists have extensively studied the way firms strategically differentiate their products in the space of product attributes. However, marketing a product often involves locating it in the more amorphous space of images and consumer perceptions. This type of product differentiation is known in the marketing literature as “positioning”. For instance, a yogurt with given objective characteristics can be marketed with an emphasis on hedonic features such as taste and texture, or on health-related features, real or imaginary. Two yogurt brands may be differentiated in terms of their positioning even when their objective characteristics are very similar. Although this type of product differentiation may be pay-off irrelevant, it can affect the consumer’s decision whether to consider a new brand.

Example 1.4. Search engine optimization. The Internet has given rise to new marketing devices with which sellers try to expand consumers’ consideration sets. Think of a consumer who wishes to spend a weekend in a quiet place out in the countryside. To find such a place on the Internet, the consumer needs to enter keywords in a search engine. However, there is a variety of keywords he can use: “country inn”, “lodge”, “bed and breakfast”, “cottage”, etc. The consumer’s choice of keyword is likely to be guided by the labels he encountered in past vacations. Different keywords will elicit different lists, and suppliers can manipulate the list by bidding for keywords in sponsored-links auctions or by employing a variety of “search engine optimization” techniques.

We propose a simple model of consideration set formation and embed it in a market environment in which firms employ marketing techniques to manipulate consideration sets. In our market model, there are two identical firms and a continuum of identical consumers. Each firm chooses a pair, a product $x$ and a marketing strategy $M$, and incurs a fixed cost associated with its choice $(x, M)$. Each consumer is initially assigned (randomly) to one of the firms.
The consumer’s initial consideration set includes only the product offered by that firm. This is interpreted as the consumer’s status quo or default product.

Whether or not the consumer also considers the competing firm’s product will depend on a primitive of his decision procedure, called the consideration function. This is a function that determines whether a consumer who initially considers a product \( x \) accompanied by a marketing strategy \( M \) will also consider a new product \( x' \) when the latter is accompanied by the marketing strategy \( M' \). If the consumer ends up including both firms’ products in his consideration set, he chooses his most preferred product, according to a strict preference relation defined on the set of products. Preferences are stable and impervious to marketing.

In general, the consideration function can depend on all four variables, \( x, M, x', M' \). We begin, however, with a more special case in which the consideration set is only a function of the default product \( x \) and the marketing strategy \( M' \) that accompanies the new product. Each of the examples 1.1–1.4 includes situations that fit this specification. However, in order to fix ideas, we will use advertising content as the main “story” behind the class of consideration functions under study. We impose an additional structure by assuming that a marketing strategy is a collection of marketing devices and that the consideration function is “separable” in some sense with respect to these devices. In particular, if a firm employs all available marketing devices, it guarantees that consumers will include its product in their consideration set.

Thus, the consumers’ choice procedure determines the firms’ market shares as a function of their products and marketing strategies. We analyse symmetric Nash equilibria of the game played by the two firms, under the assumption that the firms’ objective is to maximize market share minus fixed costs. We use this model to address the following questions:

- Does the bounded rationality of consumers—namely, their resistance to consider new products—enable firms to earn profits in excess of what they would earn if consumers were rational? Or does market competition (which includes marketing) eliminate this potential source of exploitation?
- What is the link between firms’ marketing strategy and their product quality choice?
- Do firms’ profits necessarily decrease as consumers become “more rational” in the sense of being more likely to consider new products?
- How does the fraction of consumers, who switch a supplier in equilibrium, depend on the fundamentals of the market model, particularly the consumers’ consideration function? What is the probability of switching to the new product, conditional on having been persuaded to consider it?

Our main results can be summarized as follows.

1.1. \textit{Equilibrium profits}

We show that as long as costs are not too high, there exists a symmetric Nash equilibrium in which firms earn the same profits they would earn if consumers were rational. We also show that when the consideration function is “partitional”, every symmetric Nash equilibrium satisfies this property. We provide a complete characterization of symmetric equilibria for this case. A notable feature of this characterization is that it jointly describes the vertical product differentiation in the market and the marketing strategies that firms use to promote their products.

1.2. \textit{Consumer conversion}

In any symmetric equilibrium with rational-consumer profits, marketing is “effective” in the following sense. Consumers who add a new product to their consideration set always end up buying
the new product. Thus, symmetric equilibria with rational-consumer profits exhibit perfect correlation between persuading a consumer to consider a new product and persuading him to buy it, even though the two are \textit{a priori} independent.

1.3. \textit{Does greater consumer rationality make the market outcome more competitive?}

Finally, we discuss the effect of “enhancing consumer rationality” on industry equilibrium profits. We begin with a particular consideration function that generates rational-consumer profits in all symmetric Nash equilibria. We examine two independent, closely related perturbations. First, we introduce a group of rational consumers into the population. We show that firms’ equilibrium profits increase as a result, as long as the group of rational infiltrators is not too large. Second, we retain the homogeneity of the consumer population but modify their consideration function so as to make it “more rational”, in the sense that the consideration set coincides with the feasible set in a larger set of consumption problems. This turns out to give rise to new symmetric equilibria with higher, “collusive” industry profits. These two examples demonstrate that industry equilibrium profits are not necessarily monotonically decreasing with the degree of consumer rationality.

Our final piece of analysis extends the model of consumer behaviour by allowing the consideration function to depend on the entire profile of the firms’ products and marketing strategies. This extension widens the array of marketing phenomena that our model can capture, including product positioning, packaging, display, and the use of “irrelevant” products as attention grabbers. We demonstrate this potential with two market applications of this extended model.

The main contribution of our paper is the introduction of a framework for modelling the “persuading to consider” role of marketing and the demonstration that it can illuminate aspects of competitive marketing. Our framework is flexible; it can incorporate a large variety of marketing methods, and in particular, it enables us to address aspects of advertising content that are typically hard to capture with standard models.

The paper proceeds as follows. Section 2 describes the consumer’s choice procedure. Section 3 presents the market model in which we embed the consideration-sets procedure and analyses its symmetric Nash equilibria. Section 4 presents the extended model and analyses two market applications. Section 5 discusses several choice-theoretic aspects of our model. Section 6 contains a detailed discussion of the paper’s relation to the marketing literatures, as well as the economic literature on advertising. Section 7 concludes.

2. CONSUMER CHOICE

Let $X$ be a finite set of \textit{products}. Let $\mathcal{M}$ be a finite set of \textit{marketing strategies}. An extended \textit{product} is a pair $(x, M) \in X \times \mathcal{M}$—i.e. a product and a marketing strategy that accompanies it. Consumers in this paper will face choice problems that involve \textit{ordered pairs} of extended products, $((x^s, M^s), (x^n, M^n)) \in \mathcal{A}^2$, where $\mathcal{A} \subseteq X \times \mathcal{M}$. The ordering has significance for us as we will interpret $(x^s, M^s)$ as the consumer’s \textit{status quo} or default, while $(x^n, M^n)$ represents a new alternative. Given a pair $((x^s, M^s), (x^n, M^n))$, the feasible set of products is taken to be $\{x^s, x^n\}$.

Consumer choice follows a two-stage procedure, which is based on two primitives: a \textit{linear ordering} $\succ$ over $X$ and a \textit{consideration function} $\phi$, which assigns the value 0 or 1 to every pair $(x^s, M^n)$. In the first stage, the consumer constructs a \textit{consideration set}, which can take two values: $\{x^s, x^n\}$ if $\phi(x^s, M^n) = 1$ or $\{x^s\}$ if $\phi(x^s, M^n) = 0$. In the former case, the consideration set coincides with the objectively feasible set. In the latter case, it consists of the default option alone and thus forms a strict subset of the objectively feasible set. In the second stage of the choice procedure, the consumer chooses the $\succ$-maximal product in the consideration set.
We interpret the linear ordering $\succ$ as the consumer’s “true” preferences over $X$. The preference ranking $x \succ y$ is a genuine reflection of the consumer’s taste, which is applied after serious consideration or actual experience with the products. This taste is stable and impervious to marketing. Put differently, if the consumer always considered all feasible products, then his revealed choices of products would be rationalized by $\succ$, which is also the preference criterion we adopt for welfare analysis. Throughout the paper, $x^*$ and $x_*$ denote the $\succ$-maximal and $\succ$-minimal products in $X$, respectively.

The consideration function $\phi$ enriches our description of the consumer’s psychology. In addition to his preferences over products, the consumer is also characterized by his willingness (or ability) to consider $x^n$ as a potential substitute to $x^s$ and by how this willingness depends on the way the new product is marketed. Indeed, personality psychologists often regard “openness to experience” as one of the basic traits that define an individual’s personality (see Goldberg, 1993). The consideration function may be viewed as a representation of this trait: $\phi'$ represents a more “open” personality than $\phi$ if $\phi(x^s, M^n) = 1$ implies $\phi'(x^s, M^n) = 1$.

We say that $(y, N)$ beats $(x, M)$ if $\phi(x, N) = 1$ and $y \succ x$. Denote this binary relation on $\mathcal{A}$ by $\succ^*$. This is the (strict) revealed preference relation induced by the $(\succ, \phi)$ procedure. This binary relation may violate transitivity. To see why, consider the following example. Assume $x'' \succ x' \succ x$, $\phi(x, M') = 1$, $\phi(x', M'') = 1$, and $\phi(x, M'') = 0$. Then $(x', M') \succ^* (x, M)$ and $(x'', M'') \succ^* (x', M')$, yet $(x'', M'') \not\succ^* (x, M)$. Likewise, it can be shown that the weak revealed preference relation induced by the choice procedure may be incomplete as well as intransitive.

The beating relation does satisfy certain rationality properties. First, although $\succ^*$ may violate transitivity, it does not contain cycles of any length. In addition, $\succ^*$ satisfies the following property: $(y, N) \succ^* (x, M)$ implies $(x, M') \not\succ^* (y, N')$ for all $M', N' \subseteq D$. That is, marketing cannot reverse the consumer’s revealed preferences over products. In particular, when the two extended products are simply the same product in two different guises, the consumer never strictly prefers one extended product to another. The reason is that in our model, marketing can manipulate consumers’ perception of the feasible set, but it does not manipulate their preferences. We elaborate more on the choice-theoretic aspects of our model in Section 5.

Note that our choice model displays a status quo bias. Given $(x^s, M^s)$ and $(x^n, M^n)$, the consumer chooses the default/status quo product $x^s$ whenever $x^s \succeq x^n$. However, the consumer may continue to choose $x^s$ even when $x^n \succ x^s$, if it happens to be the case that $\phi(x^s, M^n) = 0$. This is a status quo bias of a different kind than the one usually referred to in the literature (see Masatlioglu and Ok, 2005), which is a preference bias that assigns an implicit switching cost to any departure from the status quo option. In our choice model, the bias in favour of the status quo inheres in an earlier stage of the decision process, in which the consumer constructs the set of alternatives he will later consider for choice. Thus, the alternative to the status quo is at a disadvantage not because the consumer tends to find it inferior to the status quo but because he does not always take it into serious consideration.¹

2.1. Comment: can we distinguish between “product” and “marketing”? The model draws a distinction between the product $x$ and the marketing strategy $M$ that is employed to promote it. In reality, the boundary between the two is often blurred. For example, is the packaging of a product a pure marketing strategy or is it part of the product’s description? Any application of the consideration-sets model involves a modelling judgement as to which aspects of the product are pay-off relevant and which are viewed as pure marketing.

¹. There is also a formal difference between the two notions of status quo bias, which we discuss in Section 5.
3. A MARKET MODEL

The heart of this paper is a market model that incorporates the choice procedure introduced in Section 2. Our market consists of two identical firms and a continuum of identical consumers. The firms play a symmetric simultaneous-move game. The strategy space is \( A \subseteq X \times M \), which is assumed to be sufficiently rich in the sense that \((x, \emptyset) \in A\) for every \( x \in X \) and \((x^*, M) \in A\) for every \( M \in M \). One reason for restricting the set of strategies is that a particular marketing strategy \( M \) may be inherently infeasible for promoting a given product \( x \). For instance, when marketing involves highlighting certain product features, it is natural to assume that a firm cannot highlight a feature that its product lacks. The assumption that firms have identical strategy spaces is not innocuous as it rules out firm-specific brand names as marketing devices.

For expository purposes, we will consistently interpret \( M \) as a set of advertising strategies. We assume that a firm builds an advertising strategy by putting together a number of elements that serve to attract the consumer’s attention. These are viewed as the “building blocks” that a firm can use to advertise its product, such as possible slogans, images, or tunes that may accompany ads, commercials, jingles, etc. Formally, let \( D \) be a finite set of advertising messages, where a generic element in \( D \) is denoted \( m \). Let \( M \equiv 2^D \). That is, an advertising strategy is a collection of advertising messages.

Each consumer is initially assigned to one of the firms (where each firm receives half the population of consumers). The extended product chosen by this firm plays the role of the default in the consumer’s choice procedure. Thus, a firm’s extended product is the default for half the consumer population and the contender for the other half. The consumer’s decision whether to switch to the rival firm’s product is governed by the consideration-sets procedure described in Section 2. The primitives of this procedure are the preference relation \( \succ \) and the consideration function \( \phi \). We interpret \( \succ \) as representing product quality.

Choosing a strategy \((x, M) \in A\) entails a fixed cost for the firm, denoted \( c(x, M) \). We assume that firms aim to maximize market share minus costs, where costs are normalized so that they are expressed in terms of market shares. The tuple \((A, c, \succ, \phi)\) thus fully defines the simultaneous-move game played between the firms, where \( A \) is the strategy space and firm \( i \)'s pay-off function is as follows:

\[
\pi_i((x_1, M_1), (x_2, M_2)) = \begin{cases} 
\frac{1}{2}[1 + \phi(x_j, M_i)] - c(x_i, M_i) & \text{if } x_i \succ x_j, \\
\frac{1}{2}[1 - \phi(x_i, M_j)] - c(x_i, M_i) & \text{if } x_j \succ x_i, \\
\frac{1}{2} - c(x_i, M_i) & \text{if } x_i = x_j.
\end{cases}
\]  

(1)

We assume that \( c(x, M) = c_x + \sum_{m \in M} c_m \). All \( c_x \) and \( c_m \) are strictly positive. Since we interpret preferences as representing product quality, it makes sense to assume that \( x \succ y \) implies \( c_x \geq c_y \), with strict inequality for \( x = x^* \). We also assume that \( c(x, M) < \frac{1}{2} \) for all \((x, \cdot) \in A \) and \((\cdot, M) \in A \).

The pay-off function (1) captures a non-trivial strategic dilemma. On one hand, firm \( i \) has an incentive to save costs by lowering its product quality. In this case, it will aim to choose a suitable low-quality product \( x_i \) for which \( \phi(x_i, M_j) = 0 \), so that consumers who are initially assigned to firm \( i \) will fail to consider firm \( j \)'s higher-quality product. On the other hand, firm \( i \) has an incentive to increase its market share by offering a better product than firm \( j \). In this case, it will aim to choose a suitable advertising strategy \( M_i \) for which \( \phi(x_j, M_i) = 1 \), so that consumers who are initially assigned to firm \( j \) will consider firm \( i \)'s higher-quality product.

Throughout the paper, we use \( \sigma \) to denote a mixed strategy (namely, a probability distribution over \( A \)), and \( \text{Supp}(\sigma) \) to denote its support. We favour the population interpretation of symmetric mixed-strategy equilibrium: there is a “sea of firms”, from which two are randomly selected to
play the roles of a default and a contender. Finally, \( \beta_\sigma(x) = \sum_M \sigma(x, M) \) is the probability that the product \( x \) is offered under \( \sigma \).

An important benchmark for this model is the case of a rational consumer. This case is subsumed into our model by letting \( \phi(x, M) = 1 \) for all \( (x, M) \in A \) and \( (\cdot, M) \in A \). A consumer with such a consideration function always considers all objectively feasible products and therefore always chooses according to \( \succ \). Under consumer rationality, each firm plays the pair \((x^*, \emptyset)\) in Nash equilibrium and consequently earns a pay-off of \( \frac{1}{2} - c_{x^*} \). We refer to the latter as the rational-consumer pay-off.

Note that the game played by the two firms is akin to a generalized all-pay auction with limited comparability of bids, where ties are broken by a lottery. In the rational-consumer benchmark, the two firms compete to win control of a market by offering different quality levels. Each firm offers a product \( x \) and pays a cost of \( c_x \) to make this offer (such that higher quality corresponds to a higher cost). The firm that offers the highest possible quality wins, and if both firms offer the same quality, one is randomly chosen to be the winner. Allowing for boundedly rational consumers—in the sense that \( \phi(x, M) = 0 \) for some pairs \((x, M)\)—is equivalent to assuming that some pairs of products cannot be compared. However, by investing in marketing, a firm can enable a comparison. Each firm then faces the trade-off we alluded to above: it can either invest in advertising messages or it can lower its quality, save on costs, and try to reach a draw by preventing a comparison. Thus, our assumption that \( c(\cdot) < \frac{1}{2} \) means that a firm would “do anything” to force a comparison when it offers the better product.

In analysing the case of boundedly rational consumers, we impose two conditions on the consideration function \( \phi \):

(P1) \( \phi(x, M) = 1 \) if and only if there exists \( m \in M \) such that \( \phi(x, \{m\}) = 1 \).

(P2) For every \( x \neq x^* \), there exists \( m \in D \) such that \( \phi(x, \{m\}) = 1 \).

Property (P1) means that the effects of different advertising messages on consumer attention are “separable”: whether or not a particular message persuades the consumer to consider a new product is independent of the other messages that promote the new product. In particular, active marketing (i.e. \( M^n \neq \emptyset \)) is necessary for the new product \( x^n \) to enter the consumer’s consideration set. Henceforth, we will say that the message \( m \) is effective against \( x \) whenever \( \phi(x, \{m\}) = 1 \). Let \( X_\phi(m) \) denote the set of products against which \( m \) is effective, i.e.

\[
X_\phi(m) \equiv \{x \in X : \phi(x, \{m\}) = 1\}. \tag{2}
\]

Property (P2) ensures that as long as the default product is not \( x^* \), it is always possible to persuade the consumer to consider the new product. Therefore, by Property (P1), even if a firm is uncertain of the consumer’s default, it can ensure being considered by employing the grand set of advertising messages \( D \). Note that consideration functions that satisfy Properties (P1)–(P2) typically induce a beating relation that violates transitivity.

We also assume that \( \phi(x^*, \{m\}) = 0 \) for every \( m \in D \). This assumption is made purely for future notational convenience and entails no loss of generality. To see this, note if a consumer’s default is the best product, \( x^* \), then this consumer will never switch to the product of the rival firm, even if he ends up considering it. Hence, from equation (1) it follows that when firm \( i \) offers \((x^*, M_i)\), the expression \( \phi(x^*, M_j) \) will not enter its pay-off function.

These two properties, together with the assumption that \( c(x, M) < \frac{1}{2} \) for all \((x, M)\), imply that \((x^*, \emptyset)\) is the max–min strategy in this game. Consequently, \( \frac{1}{2} - c_{x^*} \) is the max–min pay-off. Recall that we already observed that these are the Nash equilibrium strategy and Nash equilibrium pay-off, respectively, under the rational-consumer benchmark. Thus, all the tuples \( (A, c, \succ, \phi) \) share the same max–min outcome, and this outcome coincides with the Nash equilibrium outcome when \( \phi \) corresponds to the case of rational consumers.
3.1. Comment: limitations of the market model

The biggest limitation of our model is that it abstracts from price setting. This is primarily for the sake of analytic convenience: given the central role that fixed costs play in the model, it is simpler to analyse the model when the value of attracting a consumer is held fixed. Spatial competition models provide a precedent for this research strategy. When teaching Hotelling’s “main street” model, say, it is both easier and illuminating to begin analysis by assuming that firms care only about market share and defer the incorporation of prices into the model.

Nevertheless, this assumption does fit a variety of competitive environments where marketing plays a key role. In media markets such as broadcast television or Internet portals, prices do not play a strategic role and revenues are directly linked to the number of viewers or users. In addition, in line with our model, consumers or users in these markets typically have one “default” supplier (e.g. one television network they are used to watching in a particular time slot or one Internet portal that serves as a home page in their browser—see Meyer and Muthaly, 2008; Bucklin and Sismeiro, 2003, and the references therein). The role of marketing is thus to persuade consumers to switch away from their default. Non-profit organizations are another example in which pricing is irrelevant and marketing is important. For example, think of the way fund-raisers for charity organizations compete for donors (for more examples, see Kotler and Levy, 1969). We discuss further the issue of prices in the concluding section.

Another limitation of our market model is that it treats the likelihood that a given firm plays the role of a default for a given consumer as exogenous, thus independent of the firm’s marketing strategy. We make this assumption because we are primarily interested in the role of marketing in attracting consumer attention away from competing products. However, the assumption entails some loss of generality. If a firm’s marketing strategy is good at attracting consumers’ attention away from the rival firm, it would probably also be good at attracting consumers’ initial attention, thereby increasing the fraction of consumers for whom the firm is the default option. In addition, marketing determines not only the allocation of consumer attention within the industry but also the level of consumers’ awareness of the industry as a whole. Extending our framework in this direction is left for future work.

3.2. Equilibrium analysis

In this subsection, we analyse symmetric Nash equilibria in the market model. We begin with the following simple example that captures an advertising technology in the manner of Butters (1977): consumers become aware of a new product if and only if it is advertised (note that unlike our model, in Butters (1977) consumers are not initially attached to any firm: if no firm advertises, consumers stay out of the market).

**Proposition 1.** Suppose that \( D \) consists of a single message \( m \). Then there is a unique symmetric Nash equilibrium given by

\[
\sigma(x^*, \emptyset) = 2c_m, \\
\sigma(x^*, \{m\}) = 2(c_{x^*} - c_{x_0}), \\
\sigma(x^*, \emptyset) = 1 - 2(c_{x^*} - c_{x_0} + c_m).
\]

We omit the proof of this result since it is a special case of Proposition 4, which is proven below. The equilibrium has several noteworthy properties:
1. The equilibrium strategy is mixed and consumers end up buying an inferior product with positive probability. However, the most preferred product $x^*$ is offered with positive probability as well.

2. Firms advertise with positive probability.

3. Although the equilibrium outcome departs from the rational-consumer benchmark, firms earn the rational-consumer (max–min) pay-off $\frac{1}{2} - c_{x^*}$. This follows directly from the observation that $(x^*, \emptyset) \in \text{Supp}(\sigma)$.

4. The equilibrium exhibits a strong correlation between advertising and product quality: the only product that is advertised in equilibrium is the most preferred product.

5. Vertical product differentiation is extreme: the only products offered in equilibrium are $x^*$ and $x_*$.

Our task in this subsection is to investigate the extent to which these properties are general. Let us begin with two lemmas that demonstrate the generality of the first two properties.

**Lemma 1.** Let $\sigma$ be a symmetric Nash equilibrium strategy. Then $\beta_\sigma(x^*) \in (0, 1)$.

**Proof.** Assume that $\beta_\sigma(x^*) = 0$. Let $y$ denote the $\succ$-minimal product for which $\beta_\sigma(y) > 0$. The market share that any $(y, M) \in \text{Supp}(\sigma)$ generates in equilibrium is at most $\frac{1}{2}$. If a firm deviated to $(x^*, D)$, it would ensure a market share of 1. By the assumption that $c(x, M) < \frac{1}{2}$ for all $(x, M)$, this deviation is profitable.

Now assume that $\beta_\sigma(x^*) = 1$. Since it is impossible to beat any strategy $(x^*, M)$, the unique best reply to $\sigma$ is $(x^*, \emptyset)$. Hence, $\sigma(x^*, \emptyset) = 1$. Thus, firms earn $\frac{1}{2} - c_{x^*}$ under $\sigma$. But then it is profitable for any firm to deviate to the strategy $(x_*, \emptyset)$ since it generates a pay-off of $\frac{1}{2} - c_{x_*} > \frac{1}{2} - c_{x^*}$ against $\sigma$. $\blacksquare$

**Lemma 2.** Let $\sigma$ be a symmetric Nash equilibrium strategy. Then there exist $x \in X$ and $M \neq \emptyset$ such that $\sigma(x, M) > 0$.

**Proof.** If $M = \emptyset$ for every $(x, M) \in \text{Supp}(\sigma)$, then by Lemma 1, $(x^*, \emptyset) \in \text{Supp}(\sigma)$. Since this strategy does not beat any other strategy, firms earn a pay-off of $\frac{1}{2} - c_{x^*}$ under $\sigma$. If a firm deviates to strategy $(x_*, \emptyset)$, it will generate a pay-off of $\frac{1}{2} - c_{x_*} > \frac{1}{2} - c_{x^*}$ against $\sigma$. The reason is that since there is no active advertising in $\sigma$, the strategy $(x_*, \emptyset)$ is not beaten by any strategy in $\text{Supp}(\sigma)$. $\blacksquare$

Our next result demonstrates that the third and fourth properties are general in the sense that there always exists a symmetric equilibrium that satisfies them.

**Proposition 2.** There exists a symmetric Nash equilibrium strategy $\sigma$ such that

(i) firms earn a pay-off of $\frac{1}{2} - c_{x^*}$ under $\sigma$;

(ii) for every $(x, M) \in \text{Supp}(\sigma)$, $x = x^*$ or $M = \emptyset$.

**Proof.** We construct a mixed strategy $\sigma$ and show that it constitutes a symmetric Nash equilibrium strategy. Let us first construct $\text{Supp}(\sigma)$. The first element in $\text{Supp}(\sigma)$ is $(x^*, \emptyset)$. Let $y^1$ be the $\succ$-minimal product in $X$. By assumption, $(x^*, \emptyset)$ fails to beat $(y^1, \emptyset)$. Add $(y^1, \emptyset)$ to $\text{Supp}(\sigma)$. Let $m^1$ be the least costly message $m$ for which $\phi(y^1, \{m\}) = 1$. By Property (P1), such a message must exist. Add $(x^*, \{m^1\})$ to $\text{Supp}(\sigma)$. This concludes the first step of the construction.
The rest of the construction proceeds iteratively. For some \( k \in \{1, \ldots, |X| - 2\} \), suppose that \( \text{Supp}(\sigma) \) contains the pairs \((y^1, \emptyset), \ldots, (y^k, \emptyset)\) and \((x^*, \emptyset), (x^*, \{m^1\}), \ldots, (x^*, \{m^k\})\). If \( \phi(y, \{m^1, \ldots, m^k\}) = 1 \) for all \( y \neq x^* \), then the construction of \( \text{Supp}(\sigma) \) is complete. Otherwise, let \( y^{k+1} \) be the \( \succ \)-minimal product \( y \) for which \( \phi(y, \{m^1, \ldots, m^k\}) = 0 \) and add \( (y^{k+1}, \emptyset) \) to \( \text{Supp}(\sigma) \). Let \( m^{k+1} \) be the least costly message \( m \) for which \( \phi(y^{k+1}, \{m\}) = 1 \). By Property \( (P1) \), there must exist such a message, and by construction, \( m^{k+1} \notin \{m^1, \ldots, m^k\} \). Add \( (x^*, \{m^1, \ldots, m^k, m^{k+1}\}) \) to \( \text{Supp}(\sigma) \). Property \( (P1) \) guarantees that the iterative process must be terminated after \( K \leq |X| - 1 \) steps such that for every \( k \leq K \), the strategy \( (y^k, \emptyset) \) is beaten by all strategies \( (x^*, \{m^1, \ldots, m^l\}) \) with \( l \geq k \), and—given our assumption that \( \phi \) is independent of \( x_2 \)—by no other strategy in \( \text{Supp}(\sigma) \).

It remains to assign probabilities to each member of \( \text{Supp}(\sigma) \). For every \( k \in \{1, \ldots, K\} \), let

\[
\sigma(y^k, \emptyset) = 2c_{m^k}
\]

and

\[
\sum_{l=k}^{K} \sigma(x^*, \{m^1, \ldots, m^l\}) = 2(c_{x^*} - c_{y^k}).
\]

In addition, let

\[
\sigma(x^*, \emptyset) = 1 - 2 \left[ c_{x^*} - c_{y^k} + \sum_{k=1}^{K} c_{m^k} \right].
\]

By our assumptions on costs, all values of \( \sigma(\cdot) \) are between 0 and 1.\(^2\) By construction, the values of \( \sigma(\cdot) \) add up to 1.

Note that by construction, \( \sigma \) satisfies properties (i) and (ii). First, for every \((x, m) \in \text{Supp}(\sigma)\), \( x = x^* \) or \( M = \emptyset \). Second, since \((x^*, \emptyset) \in \text{Supp}(\sigma)\), firms earn a pay-off of \( \frac{1}{2} - c_{x^*} \) under \( \sigma \). It thus remains to show that \( \sigma \) constitutes a symmetric Nash equilibrium strategy.

To show this, we first claim that if \((y, M)\) is a best reply to \( \sigma \), then so is \((y, \emptyset)\). The expected gain in market share from playing \((y, M)\) instead of \((y, \emptyset)\) is

\[
\sum_{m \in M} \sum_{y^k \in \chi_{\phi}(m)} \frac{1}{2} \beta_{\sigma}(y^k).
\]

By construction, \( \phi(y^k, M) = 1 \) if and only if \( m^k \in M \), where \( m^k \neq m^l \) for \( k \neq l \). In addition, \( \beta_{\sigma}(y^k) = 2c_{m^k} \), where \( m^k \) is the least costly message \( m \) for which \( \phi(y^k, \{m\}) = 1 \). This means that the expected gain in market share from \( M \) cannot be lower than the cost of \( M \). This in turn implies that the expected pay-off from \((y, \emptyset)\) cannot be lower than the expected pay-off from \((y, M)\).

It follows that in searching for profitable deviations from \( \sigma \), it suffices to check for strategies of the form \((y, \emptyset)\). By construction, all strategies in \( \text{Supp}(\sigma) \) generate a pay-off of \( \frac{1}{2} - c_{x^*} \) against \( \sigma \). Furthermore, by construction, \( x^* \succ y^k \succ \cdots \succ y^1 \), and for every \( y \) for which \( y^{k+1} \succ y \succ y^k \), \( c_y \geq c_{y^k} \) and \( \phi(y, \{m^k\}) = 1 \). This means that \((y, \emptyset)\) generates the same market share as \((y^k, \emptyset)\) and costs no less. Therefore, \((y, \emptyset)\) cannot be a profitable deviation. This concludes the proof.

The result that firms earn rational-consumer pay-offs in equilibrium is of interest for several reasons. First, it shows that although consumers’ bounded rationality initially creates an

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\(^2\) Note, however, that if \( c_{y^k} = c_{y^{k+1}} \) for some \( k \), then \( \sigma(x^*, \{m^1, \ldots, m^k\}) = 0 \), and therefore, strictly speaking, \((x^*, \{m^1, \ldots, m^k\})\) does not belong to \( \text{Supp}(\sigma) \).
opportunity for firms to earn pay-offs above the rational-consumer benchmark, competitive forces (which include marketing) eliminate this potential. Second, the equilibrium outcome is Pareto inferior to the rational-consumer benchmark: firms earn the same profits in both cases, while consumers are strictly worse off in the bounded rationality case. Third, rational-consumer pay-offs imply that firms are indifferent between any advertising strategy that is employed in equilibrium and the option of no advertising. The reason is that if a firm strictly prefers some \((x, M) \in \text{Supp}(\sigma)\) to the strategy \((x, \emptyset)\), then the strategy \((x^*, \emptyset)\) would generate pay-offs strictly above \(\frac{1}{2} - c_{x^*}\).

Finally, rational-consumer pay-offs turn out to have strong implications for the equilibrium correlation between product quality and marketing and consequently on consumer conversion rates. We will explore these implications in greater detail in Subsection 3.4. At this point, it will suffice to point out that part (ii) of Proposition 2 is not general: there exist equilibria in which firms earn rational-consumer pay-offs above the rational-consumer benchmark, competitive forces (which include marketing) eliminate this potential. Second, the equilibrium outcome is Pareto inferior to the rational-consumer benchmark: firms earn the same profits in both cases, while consumers are strictly worse off in the bounded rationality case. Third, rational-consumer pay-offs imply that firms are indifferent between any advertising strategy that is employed in equilibrium and the option of no advertising. The reason is that if a firm strictly prefers some \((x, M) \in \text{Supp}(\sigma)\) to the strategy \((x, \emptyset)\), then the strategy \((x^*, \emptyset)\) would generate pay-offs strictly above \(\frac{1}{2} - c_{x^*}\).

Proposition 3. Let \(\sigma\) be a symmetric Nash equilibrium strategy in which firms earn rational-consumer pay-offs. For every \((x, M) \in \text{Supp}(\sigma)\) and every \(m, m' \in M\), the sets \(\{x \in X_\phi(m): \beta_\sigma(x) > 0\}\) and \(\{x \in X_\phi(m'): \beta_\sigma(x) > 0\}\) are disjoint.

Proof. Assume the contrary—i.e. that there exist \((x, M) \in \text{Supp}(\sigma)\) and two messages \(m, m' \in M\) such that the two sets \(\{x \in X_\phi(m): \beta_\sigma(x) > 0\}\) and \(\{x \in X_\phi(m'): \beta_\sigma(x) > 0\}\) have a non-empty intersection. Then the marginal contribution of \(m'\) to the market share generated by \((x, M)\) is strictly below \(\frac{1}{2} \sum_{y \in X_\phi(m')} \beta_\sigma(y)\). Since \((x, M)\) is a best reply to \(\sigma\), this implies that \(\frac{1}{2} \sum_{y \in X_\phi(m')} \beta_\sigma(y) > c_{m'}\). By the assumption that firms earn rational-consumer pay-offs in equilibrium, the strategy \((x^*, \emptyset)\) is a best reply to \(\sigma\). It follows that if one of the firms deviates from \((x^*, \emptyset)\) to \((x^*, \{m'\})\), it would earn a pay-off in excess of the rational-consumer level, a contradiction.

Suppose that the partitional property described in Proposition 3 holds not only with respect to the products that are offered in equilibrium but also with respect to the grand set of products. That is, assume that the collection \(\{X_\phi(m)\}_{m \in D}\) is a partition of \(X \setminus \{x^*\}\). This case fits situations in which there is a pre-existing natural categorization of products (e.g. health versus non-health food products) such that an individual message is effective against a specific category of target products. Under this special case, we are able to provide a complete characterization of the set of symmetric equilibria.

For every \(m \in D\), let \(y^*(m)\) denote the \(\succ\)-minimal product in \(X_\phi(m)\). Given a mixed strategy \(\sigma\), let \(a_\sigma(m) = \sum_{M \ni m} \sigma(x, M)\) be the probability that the message \(m\) is played under \(\sigma\).

Proposition 4. Assume that \(\{X_\phi(m)\}_{m \in D}\) is a partition of \(X \setminus \{x^*\}\). In any symmetric Nash equilibrium \(\sigma\).

3. In the constructive proof of Proposition 2, it is easy to see that when \(K > 1\), some of the weight that is assigned to \((x^*, \{m^1\})\), say, can be shifted to a new strategy \((y^2, \{m^1\})\), without upsetting any of the equilibrium conditions.
(i) Firms earn a pay-off of \( \frac{1}{2} - c_{x^*} \); 
(ii) For every \( m \in D \),

\[
\alpha_\sigma (m) = 2(c_{x^*} - c_{y^*(m)}), \\
\beta_\sigma (x) = \begin{cases} 
2c_m & \text{if } x = y^*(m), \\
1 - 2 \sum_{m \in D} c_m & \text{if } x = x^*, \\
0 & \text{otherwise}. 
\end{cases}
\]

Proof. (i) Assume the contrary—i.e. that firms earn more than the rational-consumer pay-off \( \frac{1}{2} - c_{x^*} \) under some symmetric equilibrium strategy \( \sigma \). By Lemma 1, \( \text{Supp}(\sigma) \) contains a strategy of the form \((x^*, M)\). The strategy \((x^*, \emptyset)\) generates the rational-consumer pay-off against any strategy. Therefore, it must be the case that \( M \neq \emptyset \) for every \((x^*, M) \in \text{Supp}(\sigma)\). For every \((x^*, M) \in \text{Supp}(\sigma)\) and every \( m \in M \), \( \frac{1}{2} \sum_{x \in X_\phi(m)} \beta(x) - c_m \geq 0 \), with at least one strict inequality for some \( m^* \)—otherwise the strategy \((x^*, M)\) could not generate a pay-off above the rational-consumer level.

It follows that if \( m^* \notin M^* \) for some \((x^*, M^*) \in \text{Supp}(\sigma)\), it is profitable to deviate to \((x^*, M^* \cup \{m^*\})\). Hence, \( m^* \in M \) for all \((x^*, M) \in \text{Supp}(\sigma)\). Moreover, because \( \frac{1}{2} \sum_{x \in X_\phi(m^*)} \beta(x) - c_m > 0 \), there exists \( y_{m^*} \neq x^* \) such that \( \beta_\sigma(y_{m^*}) > 0 \) and \( y_{m^*} \in X_\phi(m^*) \). It must therefore be the case that \((x^*, M)\) beats \((y_{m^*}, M')\) for every \((x^*, M), (y_{m^*}, M') \in \text{Supp}(\sigma)\).

Let \( y^* \) denote the \( \succ \)-minimal product among all these products \( y_{m^*} \). If \((y^*, \emptyset)\) is not a best reply to \( \sigma \), then there must exist \( m \) such that \( \frac{1}{2} \sum_{x \in X_\phi(m)} \beta(x) - c_m > 0 \) and \( m \in M \) for every \((y^*, M) \in \text{Supp}(\sigma)\). But this implies that \( m \in M \) for every \((x^*, M) \in \text{Supp}(\sigma)\), which means that there is a product \( y' \) such that \( y^* \succ y' \) and \((x^*, M)\) beats \((y', M')\) for every \((x^*, M), (y', M') \in \text{Supp}(\sigma)\), contradicting the definition of \( y^* \). It follows that \((y^*, \emptyset)\) is a best reply to \( \sigma \).

If a firm deviates from \((y^*, \emptyset)\) to \((x^*, D)\), it will increase its market share by at least \( \frac{1}{2} \beta_\sigma(x^*) + \frac{1}{2} (1 - \beta_\sigma(x^*)) = \frac{1}{2} \), since by assumption \( c(x^*, D) < \frac{1}{2} \), the deviation is profitable, a contradiction. It follows that firms cannot earn more than \( \frac{1}{2} - c_{x^*} \) in \( \sigma \). Since this is the rational-consumer pay-off, firms must earn exactly \( \frac{1}{2} - c_{x^*} \) in \( \sigma \).

(ii) First, we claim that for every \( m \in M \), \( \alpha_\sigma (m) > 0 \) and \( \beta_\sigma (x) > 0 \) for some \( x \in X_\phi(m) \). If \( \alpha_\sigma (m) = 0 \), then when a firm plays \((x, \emptyset)\), where \( x \neq x^* \) and \( x \in X_\phi(m) \), it earns a pay-off \( \frac{1}{2} - c_x > \frac{1}{2} - c_{x^*} \) since by assumption \( x \notin X_\phi(m') \) for every \( m' \neq m \). If \( \beta_\sigma (x) = 0 \), then it is optimal to set \( \alpha_\sigma (m) = 0 \), a contradiction.

Second, part (i) implies that for every \( m \in M \), \( \frac{1}{2} \sum_{x \in X_\phi(m)} \beta(x) \leq c_m \)—otherwise, a firm could play \((x^*, \{m\})\) and earn a pay-off above the rational-consumer level. However, if the inequality is strict, firms will find it optimal to set \( \alpha_\sigma (m) = 0 \). Therefore, \( \frac{1}{2} \sum_{x \in X_\phi(m)} \beta(x) = c_m \) for every \( m \in D \). In particular, this means that if \( \beta_\sigma (x) > 0 \), the strategy \((x, \emptyset)\) must be a best reply against \( \sigma \). Denote by \( m(x) \) the message that is effective against \( x \). Then, the pay-off from \((x, \emptyset)\) is \( \frac{1}{2} - c_x - \frac{1}{2} \alpha_\sigma (m(x)) \). Consider a product \( x \) satisfying \( \beta_\sigma (x) > 0 \) and \( x \in X_\phi(m) \). If \( x \succ y \) for some \( y \in X_\phi(m) \), then if a firm deviated to \((y, \emptyset)\) it would earn a pay-off of

\[
\frac{1}{2} - c_y - \frac{1}{2} \alpha_\sigma (m(x)) > \frac{1}{2} - c_x - \frac{1}{2} \alpha_\sigma (m(x))
\]

in contradiction to the assumption that \( \beta_\sigma (x) > 0 \). It follows that the only strategy \( x \in X_\phi(m) \) for which \( \beta_\sigma (x) > 0 \) is \( y^*(m) \), namely, the \( \succ \)-minimal product in \( X_\phi(m) \). And since the pay-off from \((y^*(m), \emptyset)\) must be the rational-consumer pay-off, it must be the case that \( \alpha_\sigma (m) = 2c_{x^*} - 2c_{y^*(m)} \).
Thus, when $\{X_\phi(m)\}_{m \in D}$ is a partition of $X \setminus \{x^*\}$, all symmetric Nash equilibria induce rational-consumer pay-offs. Apart from $x^*$, the only products that are offered in equilibrium are the inferior products in each cell of the induced partition. The more costly the message, the higher the probability with which its inferior target product is offered. The higher the cost of the target product, the lower the probability with which the message is employed.

3.3. Targeted advertising

The case of partitional consideration functions allows us to explore the notion of targeted advertising. Most discussions of targeting focus on the way advertising campaigns are tailored to particular groups of consumers that differ in their preferences. In our model, consumers are homogeneous. However, given a mixed-strategy equilibrium, different consumers have different default products, and therefore different marketing strategies may be required in order to persuade them to consider new products. A finer partition $\{X_\phi(m)\}_{m \in D}$ represents a situation in which there is greater advertising targeting.

It is interesting to examine how $(\alpha_\sigma(m))_{m \in D}$ and $(\beta_\sigma(x))_{x \in X}$ behave with respect to the coarseness of the partition induced by $\phi$. For simplicity, let us fix $\sum_{m \in D} c_m$ and compare two extreme cases: (1) the “Butters” example analysed in Proposition 1 and (2) the case in which for every $x \neq x^*$ there is a unique message $m(x)$ that is effective against $x$ (i.e. $X_\phi(m) = \{x\}$). The difference between the two cases is that in case (2) messages are specifically tailored to a particular target product, while in case (1), advertising is not targeted at all such that a single message attracts every consumer’s attention.

Since $\sum_{m \in D} c_m$ is held fixed, $\beta_\sigma(x^*)$ is the same in the symmetric equilibria of both cases. However, in case (2), relative to case (1), some of the weight that the equilibrium strategy assigns to the least preferred product in $X$ is shifted to intermediate quality products. This is a general corollary of Proposition 4: greater advertising targeting results in an upward shift in the equilibrium distribution of product quality.

3.4. Consumer conversion

In the example analysed in Proposition 1, we saw that in symmetric equilibrium, firms use active marketing only to promote the most preferred product $x^*$. However, as already mentioned, this is not a general property. Instead, there is a weaker property that captures the correlation between product quality and advertising in symmetric equilibria that induce rational-consumer pay-offs.

**Definition 1 (Effective Marketing Property).** A mixed strategy $\sigma$ satisfies the effective marketing property if for every $(x, M), (x', M') \in \text{Supp}(\sigma)$, $\phi(x, M') = 1$ implies $x' \succ x$.

The effective marketing property means that whenever a consumer considers a new product thanks to the marketing strategy that accompanies that product, he ends up buying it. (Note that when $x = x^*$, $\phi(x, M) = 0$ by assumption for all $M$. However, since this assumption was introduced merely as a notational convenience, it is perhaps more appropriate to rule out the case of $x = x^*$ in the definition of the effective marketing property.)

**Proposition 5.** Let $\sigma$ be a symmetric Nash equilibrium strategy that induces rational-consumer pay-offs. Then $\sigma$ satisfies the effective marketing property.

**Proof.** Let $(x, M), (x', M') \in \text{Supp}(\sigma)$, $\phi(x, M') = 1$, and yet $x \succeq x'$. By assumption, $x \neq x^*$, hence $x^* \succ x'$. For every strategy $(x', M')$, let $B(x', M')$ denote the set of strategies in $\text{Supp}(\sigma)$ that $(x', M')$ beats. Recall that the set of strategies that beat $(x', M')$ is independent...
of \( M' \). In order for \((x', M')\) to be a best reply to \( \sigma \), it must be weakly preferred to \((x', \emptyset)\) and therefore satisfy the following inequality:

\[
\sum_{(y,N) \in B(x', M') \setminus B(x', \emptyset)} \sigma(y,N) \geq 2 \sum_{m \in M'} c_m.
\]

By the assumption that firms earn rational-consumer pay-offs under \( \sigma \), the strategy \((x^*, \emptyset)\) is a best reply to \( \sigma \). Note that \( B(x^*, \emptyset) = \emptyset \), hence \( B(x', M') \setminus B(x', \emptyset) = B(x', M') \). Suppose that a firm deviates to \((x^*, M')\). In order for this deviation to be unprofitable, the following inequality must hold:

\[
\sum_{(y,N) \in B(x^*, M')} \sigma(y,N) \leq 2 \sum_{m \in M'} c_m.
\]

Because \( x^* \succ x' \), it must be the case that \( B(x', M') \subseteq B(x^*, M') \). Moreover, since \( \phi(x, M') = 1 \), the inclusion is strict. Therefore,

\[
\sum_{(y,N) \in B(x^*, M')} \sigma(y,N) > \sum_{(y',N') \in B(x', M')} \sigma(y',N'),
\]

which contradicts the combination of the preceding pair of inequalities.

The effective marketing property is a result that characterizes consumer conversion rates—that is, the probability that a consumer will switch to a new product conditional on having considered it. Off equilibrium, persuading a consumer to consider a product does not guarantee that he will buy it because he may fail to find it superior to the default. However, competitive forces imply that in equilibrium, persuading to consider leads to a sale (as long as firms earn rational-consumer pay-offs). Of course, the result that the conversion rate is 100% is extreme and clearly relies on several unrealistic features of the model, e.g. the assumption of consumer homogeneity. We view the result as a useful theoretical benchmark for richer, more pertinent theories of conversion rates that incorporate consumer heterogeneity, among other things.

For some specifications of the model, we can use the effective marketing property to characterize the unconditional probability that consumers switch a supplier. Consider, for instance, the case where the collection \( \{X_\phi(m)\}_{m \in D} \) is a partition of \( X \setminus \{x^*\} \). Recall that \( (\alpha_\sigma(m))_{m \in D} \) and \( (\beta_\sigma(x))_{x \in X} \) denote the probability that a message \( m \) is employed and the probability that a product \( x \) is offered under \( \sigma \). Proposition 4 characterized these quantities. By the effective marketing property, the probability that a consumer whose default is \( x \neq x^* \) will switch a supplier is \( \alpha_\sigma(m(x)) \), where \( m(x) \) denotes the unique message \( m \) for which \( \phi(x, [m]) = 1 \). Therefore, by Proposition 4, the overall switching rate is

\[
\sum_{x \neq x^*} \beta_\sigma(x) \alpha_\sigma(m(x)) = \sum_{m \in D} \beta_\sigma(y^*(m))\alpha_\sigma(m) = 4 \sum_{m \in D} c_m \cdot (c_{x^*} - c_{y^*(m)}),
\]

where \( y^*(m) \) denotes the least preferred product in \( X_\phi(m) \).

Thus, the switching rate increases with advertising costs, as well as with the cost difference between the most preferred product and inferior products. The intuition for these comparative statics is familiar from mixed-strategy equilibrium analysis. When advertising costs go up, a higher probability that inferior products are offered is required to restore the firms’ indifference between advertising and no advertising. Similarly, when the cost of offering \( x^* \) goes up, this product needs to be advertised more intensively in order to restore the firms’ indifference between offering \( x^* \) and offering an inferior product. Both changes raise the switching rate.

Note that the switching rate is equal to the expected cost of messages under \( \sigma \). This follows from the observation that the probability that a message is employed by a given firm is \( \alpha_\sigma(m) \) and the cost of the message is \( c_m = \frac{1}{2} \beta_\sigma(y^*(m)) \).
3.5. Can firms attain collusive profits in equilibrium?

Imagine a scale that measures consumers’ resistance to considering new alternatives. At one end of the scale, we have the fully rational consideration function which always yields the feasible set. Suppose that at the other end of the scale we place the consideration functions for which \( \{X_\phi(m)\}_{m \in D} \) constitutes a partition of \( X \setminus \{x^*\} \). At both ends of this scale, the fully rational one and the boundedly rational one, we saw that firms necessarily earn the rational-consumer pay-off in any symmetric Nash equilibrium. Intuitively, one would expect the competition between firms to be fiercer, the closer we move to the rational end of the scale. According to this intuition, firms would not be able to make collusive profits when the consideration set becomes more likely to coincide with the objectively feasible set.

This intuition turns out to be false as the following pair of examples demonstrates. Our first example tampers with the assumption that the consumer population is homogenous. Suppose that originally, the consumers’ consideration function is such that firms earn rational-consumer pay-offs in symmetric equilibrium. Now assume that a small group of rational consumers enters the market. The rational-consumer pay-off continues to be \( \frac{1}{2} - c_{x^*} \). However, firms necessarily earn a higher pay-off in equilibrium. The reason is that if there are not too many rational consumers, inferior products will continue to be offered with positive probability in equilibrium. But thanks to the presence of rational consumers, the strategy \( (x^*, \emptyset) \) generates a market share strictly above 50% and therefore a pay-off above the rational-consumer level. Thus, the population of consumers “more rational” can cause industry profits to go up!

To illustrate this effect, revisit Proposition 1, and let \( X = \{x^*, x_+\} \). The fraction of rational consumers is \( \lambda > 0 \). If it is sufficiently small, the (unique) symmetric Nash equilibrium strategy \( \sigma \) is given by the following system of equations, which simply state the indifference conditions that characterize the mixed-strategy equilibrium:

\[
\frac{1}{2} + \frac{1}{2} \cdot \sigma(x_+, \emptyset) - c_{x^*} = \frac{1}{2} + \frac{1}{2} \sigma(x_+, \emptyset) - c_{x^*} - c_m,
\]

\[
\frac{1}{2} + \frac{1}{2} \cdot \sigma(x_+, \emptyset) - c_{x^*} = \frac{1}{2} - \frac{1}{2} \lambda \cdot \sigma(x^*,\emptyset) - \frac{1}{2} \sigma(x^*, \{m\}) - c_{x^*},
\]

\[
1 = \sigma(x^*, \{m\}) + \sigma(x^*, \emptyset) + \sigma(x_+, \emptyset),
\]

where \( \sigma(x^*, \{m\}), \sigma(x^*, \emptyset), \) and \( \sigma(x_+, \emptyset) \) are all strictly positive. It is easy to see that equilibrium pay-offs exceed \( \frac{1}{2} - c_{x^*} \).

Our second example respects the assumption of consumer homogeneity that runs through this paper. Let \( X = \{111, 100, 010, 001\} \) and \( x^* = 111 \). Let \( D = \{1, 2, 3\} \) and assume the following consideration function \( \phi : X_\phi(1) = \{001\}, X_\phi(2) = \{100\}, \) and \( X_\phi(3) = \{010\} \). Thus, \( \{X_\phi(m)\}_{m \in D} \) constitutes a partition of \( X \setminus \{x^*\} \). Let \( c_{111} = \frac{1}{3} \) and let \( c_m = c_x = \bar{c} < \frac{1}{30} \) for all \( m \in D \) and \( x \neq x^* \). By Proposition 4, in every symmetric Nash equilibrium, firms earn the rational-consumer pay-off. Now consider modifying the consumers’ consideration function into \( \phi' \) such that \( X_{\phi'}(1) = \{010, 001\}, X_{\phi'}(2) = \{100, 001\}, \) and \( X_{\phi'}(3) = \{100, 010\} \). This modification has a natural interpretation. Each product may have up to three attributes. The most preferred product has all three attributes. A message \( m \) is interpreted as an ad that focuses on the \( m \)-th attribute. If the consumer’s default product lacks that attribute, the ad persuades him that he should consider the new product. Note that \( \cup_{m \in D} X_{\phi'}(m) = X \setminus \{x^*\} \), but \( \{X_{\phi}(m)\}_{m \in D} \) is not a partition of \( X \setminus \{x^*\} \).

It can be shown that the modified consideration function generates a continuum of symmetric equilibria, in which the support of the equilibrium strategy consists of the strategies \( (111, \{1\}), (111, \{2\}), (111, \{3\}), (100, \emptyset), (010, \emptyset), \) and \( (001, \emptyset) \), and firms earn pay-offs above...
the rational-consumer level. This example is a counterpart to Proposition 2: it demonstrates that our market model may have symmetric Nash equilibria in which firms attain collusive profits. However, although the consideration functions that give rise to the counterexample is natural, the restriction on the cost function is non-generic. Is it true that for generic cost functions, any symmetric Nash equilibrium induces rational-consumer pay-offs? This is an open problem. At any rate, our final result in this section demonstrates that when costs are sufficiently low, equilibrium pay-offs are equal to the rational-consumer level.

**Proposition 6.** If \( c(x^*, D) < 1/(2|D| + 2) \), then firms earn the rational-consumer pay-off in any symmetric Nash equilibrium.

**Proof.** Assume, by contradiction, that \( c(x^*, D) < 1/(2|D| + 2) \), and yet firms earn pay-offs above the rational-consumer level in some symmetric Nash equilibrium \( \sigma \).

We first claim that

\[
\frac{1}{2} \sum_{x \prec x^*} \beta(\sigma, x) < c(x^*, D).
\]

(6)

To see why this is true, consider some \((x^*, M) \in \text{Supp}(\sigma)\). By Lemma 1, \( \text{Supp}(\sigma) \) must contain such a strategy. By our assumption that firms earn an expected pay-off above the rational-consumer level, \((x^*, M) \) must beat some other strategy \((x, M') \in \text{Supp}(\sigma)\). Define \( B_v(M) \equiv \{x < x^*: \phi(x, M) = v\} \). Note that \( B_0(M) \cup B_1(M) = X \setminus \{x^*\} \). For any \((x^*, M) \in \text{Supp}(\sigma)\), it must be the case that

\[
\frac{1}{2} \sum_{x \in B_0(M)} \beta(\sigma, x) \leq c(x^*, D) - c(x^*, M).
\]

Otherwise, it is profitable to deviate from \((x^*, M) \) to \((x^*, D)\). In addition, it must be the case that

\[
\frac{1}{2} \sigma(x^*, M) + \frac{1}{2} \sum_{x \in B_1(M)} \beta(\sigma, x) \leq c(x^*, M) - c(x', M'),
\]

for some \((x', M') \in \text{Supp}(\sigma)\) that is beaten by \((x^*, M)\). Otherwise, it is profitable to deviate from \((x', M') \) to \((x^*, M)\). Summing over the last two inequalities, we obtain inequality (6).

Since every \((x^*, M) \in \text{Supp}(\sigma)\) must beat some \((x', M') \in \text{Supp}(\sigma)\), it must be the case that

\[
\frac{1}{2} \sigma(x^*, M) \leq c(x^*, M) - c(x', M') < c(x^*, D).
\]

Otherwise, it would be profitable to deviate from \((x', M') \) to \((x^*, D)\). It must be the case that \((x^*, \varnothing) \notin \text{Supp}(\sigma)\) — otherwise, firms earn the rational-consumer pay-off in \( \sigma \), a contradiction. It follows that the number of strategies of the form \((x^*, M) \) in \( \text{Supp}(\sigma) \) is at most \( 2|D| - 1 \). Summing over all these strategies, we obtain

\[
\frac{1}{2} \sum_{M} \sigma(x^*, M) = \frac{1}{2} \beta(\sigma, x^*) < 2|D| - 1 \cdot c(x^*, D).
\]

Combined with the inequality (6), we obtain

\[
1 < (2|D| + 2) \cdot c(x^*, D),
\]

a contradiction. ||

Note that Proposition 6 does not rely on Properties (P1)–(P2), but on a weaker condition that there exists \( M \in \mathcal{M} \) such that \( \phi(x, M) = 1 \) for all \( x \neq x^* \). Still, this result is somewhat
unsatisfactory for the following reason. When costs are small, the probability that \( x^* \) is offered is close to 1, as can easily be seen from inequality (6). Thus, a max–min pay-off result that holds only when costs are very small takes some of the sting out of the distinction between the coincidence of the market outcome with the rational-consumer benchmark and the coincidence of industry profits with the rational-consumer benchmark.

4. AN EXTENDED MODEL

In this section, we extend the consideration-sets model so as to encompass a greater range of marketing effects. As in the basic model of Section 2, a choice problem that the consumer faces is an ordered pair of extended alternatives \(((x^s, M^s), (x^n, M^n)) \in \mathcal{A}^2\), where \( \mathcal{A} \subseteq X \times M \). The consumer goes through the same two-stage procedure. In the first stage, he constructs a consideration set. The extension is that the consideration function \( \phi \) is now defined over the set of all ordered pairs of extended alternatives. That is, let \( \phi: \mathcal{A}^2 \rightarrow \{0, 1\} \). The consideration set is \( \{x^s, x^n\} \) if \( \phi(x^s, M^s, x^n, M^n) = 1 \) or \( \{x^s\} \) if \( \phi(x^s, M^s, x^n, M^n) = 0 \). In the second stage, the consumer chooses the \( \succ \)-maximal product in the consideration set that he constructed in the first stage.

By extending the domain of the consideration function, we can capture additional marketing phenomena. We develop some of these applications in separate papers. 4

**Example 4.1.** Packaging. Consumers’ decision to add a new product to their consideration set can also be influenced by the way it is packaged. For instance, a consumer is more likely to note a new brand of flavoured water with added vitamins if its bottle is designed so that it looks like the brand the consumer regularly buys. 5 A similar phenomenon known in the marketing profession as “knock-offs” or “benchmarking” occurs when a firm attempts to associate its brand with a competing brand by mimicking the latter’s advertising campaign. 6

**Example 4.2.** Products as attention grabbers. When firms design their product lines and retailers decide which products to put on display, they may take into account the possibility that certain products can help draw consumers’ attention to other products on offer. For instance, think of a consumer who wants to buy a new laptop computer. He initially considers a particular model \( x \), possibly because it shares some features with his current machine. The consumer may then note that a computer store offers a model \( y \) that is significantly cheaper or lighter than \( x \). This gives the consumer a sufficient reason to consider \( y \) in addition to \( x \). Upon closer inspection, the consumer realizes that he does not like \( y \) as much as he does \( x \). However, since he is already inside the store, he may browse the other laptop computers on offer and find a model \( z \) that he ranks above both \( x \) and \( y \). Thus, although few consumers may actually buy \( y \), this model functions as a “door opener” that attracts consumers’ serious attention to the other products offered by the store. 7

In the remainder of this section, we will analyse market implications of two examples of consideration functions that depend only on \( M^s \) and \( M^n \), using the same kind of model of competitive marketing analysed in Section 3. The following structure is common to the two examples.

4. Eliaz and Spiegler (forthcoming) include a detailed analysis of Example 4.2. Piccione and Spiegler (2009) elaborate on Example 4.1 in the context of a different market model—see a discussion in Section 7.

5. For instance, compare the brand “Vitaminwater” by Glaceau (http://www.glaceau.com) with the competing brand “Antioxidant Water” by Snapple (http://www.snapple.com).

6. One recent example is the “Beauty is . . . ” campaign of Nivea, which is almost identical to the “Real Beauty” campaign of Dove. Compare http://www.nivea.de/beauty_is/ with www.campaignforrealbeauty.com/.

7. A vivid example of this effect involves a soda company that issues a “limited holiday edition” including absurd flavours such as Christmas ham or latke—see http://www.jonessoda.com/files/limited_editions.php
Let $\mathcal{M} = \{0, 1, \ldots, K\}$ and $\mathcal{A} = X \times \mathcal{M}$. Two firms facing a continuum of identical consumers simultaneously choose an extended product $(x, M) \in \mathcal{A}$. Each consumer is initially assigned to one of the firms (where each firm receives half the population of consumers). The extended product chosen by this firm plays the role of the default in the consumer’s choice procedure. Each firm aims to maximize its market share minus the fixed cost of its strategy $c(x, M)$. The two examples will differ only in the specification of $c$ and $\phi$. As in previous sections, $x^*$ and $x_s$ denote the $\succ$-maximal and $\succ$-minimal products in $X$, and for every mixed strategy $\sigma \in \Delta(\mathcal{A})$, $\beta_\sigma(x) = \sum_M \sigma(x, M)$.

4.1. Advertising intensity

Most models of advertising in the literature represent this activity by a scalar interpreted as “advertising intensity” (see Bagwell, 2007). The standard view in this literature is that higher advertising intensity signals higher quality (see, e.g. Ackerberg, 2003). In this subsection, we offer an alternative view. The more intensely a product is advertised, the more likely it is to attract the consumer’s attention and the more likely it is to be remembered by the consumer. (A similar perspective is developed in Chioveanu, forthcoming—see Section 6.) For example, a number of studies (e.g. Janiszewski, 1993; Shapiro, MacInnis and Heckler, 1997) indicate that advertisements, even if not explicitly recalled or recognized, may influence consumers especially with regard to the inclusion of a brand in a consideration set. This suggests that advertising intensity has a “defensive role”. A firm may crank up its advertising intensity in an attempt to prevent its consumers from considering a superior competing product. As a result, higher advertising intensity need not be associated with higher quality, in contrast to the conventional view.

To explore this intuition, we define the consumers’ consideration function as follows: $\phi(x, M^s, x^n, M^n) = 1$ if and only if $M^n \geq M^s$. An element in $\mathcal{M}$ represents advertising intensity. The consumer considers the new product $x^n$ if and only if it is advertised at least as intensively as the status quo product $x^s$. The firms’ cost function is as follows: $c(x, M) = c_x + d_M \in (0, \frac{1}{2})$, where $c_x > c_y$ if and only if $x \succ y$ and $d_M > d_N$ if and only if $M > N$. Let $d_0 = 0$, and assume that $c_{x^*} - c_{x^*} > d_1$.

Recall that the beating relation is the revealed strict preference relation over extended products induced by the consideration-sets procedure. In the present sub-section, $(y, N)$ beats $(x, M)$ if and only if $N \geq M$ and $y \succ x$. It is easy to see that this relation is transitive (unlike the typical beating relation in Section 3). However, the consumer’s observed choice behaviour is not rational because the revealed indifference relation over extended products violates transitivity. For example, if $x \succ y \succ z$, then we would observe that the consumer does not switch from $(z, 2)$ to $(x, 1)$ and also does not switch from $(x, 1)$ to $(y, 3)$. If the consumer were rational and had a complete preference relation over extended products, then we would infer that he weakly prefers $(z, 2)$ to $(x, 1)$ and weakly prefers $(x, 1)$ to $(y, 3)$. Hence, we would not expect him not to switch from $(z, 2)$ to $(y, 3)$. However, in our model, the consumer does indeed switch.

Let us extend an important observation from Section 3. The max–min strategy for firms is $(x^*, 0)$, and the max–min pay-off is $\frac{1}{2} - c_{x^*}$. This is exactly the outcome that would emerge in Nash equilibrium if consumers were rational—i.e. firms would offer the best product and choose zero advertising intensity.

**Proposition 7.** In every symmetric Nash equilibrium, $\beta_\sigma(x^*) \in (0, 1)$ and firms earn the rational-consumer pay-off $\frac{1}{2} - c_{x^*}$.

**Proof.** First, let us show that in every symmetric Nash equilibrium, $\beta_\sigma(x^*) \in (0, 1)$. Let $\sigma$ be a symmetric Nash equilibrium strategy. Suppose that $\beta_\sigma(x^*) = 0$. Note that Supp($\sigma$) must
contain a strategy that does not beat any strategy in \( \text{Supp}(\sigma) \). This strategy generates a market share weakly below \( \frac{1}{2} \). If a firm deviates to \((x^*, K)\), this strategy beats every strategy in \( \text{Supp}(\sigma) \) and thus generates a market share of 1. By the assumption that \( c(x, M) < \frac{1}{2} \) for all \((x, M)\), the deviation is profitable. Now suppose that \( \beta \sigma(x^*) = 1 \). Then, no pair of strategies in \( \text{Supp}(\sigma) \) beat one another. It follows that \( \sigma \) assigns probability 1 to \((x^*, 0)\). But this means that it is profitable to deviate into \((x^*, 1)\).

Now assume that firms earn pay-offs strictly above \( \frac{1}{2} - c_{x^*} \) under \( \sigma \). Define \( M^* \) as follows: \((x^*, M^*) \in \text{Supp}(\sigma) \) and \( M > M^* \) for every other \((x^*, M) \in \text{Supp}(\sigma) \). There must exist such \( M^* \), by the previous step. Define \( B(x^*, M^*) \) as the set of strategies in \( \text{Supp}(\sigma) \) that are beaten by \((x^*, M^*) \). This set is non-empty—otherwise, \((x^*, M^*) \) would fail to generate a pay-off strictly above \( \frac{1}{2} - c_{x^*} \). Let \((y, N) \in B(x^*, M^*) \) have the property that \( y' \succeq y \) for every \((y', N') \in B(x^*, M^*) \).

The strategy \((y, N) \) has two important properties. First, it does not beat any strategy in \( \text{Supp}(\sigma) \). Assume the contrary—i.e. that \((y, M) \) beats some \((y', N') \) in \( \text{Supp}(\sigma) \). Then it must be the case that \( N \succeq N' \) and \( y \succeq y' \), hence \( M^* \succeq N' \) and \( x^* \succeq y' \), which means that \((x^*, M^*) \) beats \((y', N') \), contradicting the definition of \((y, N) \). Second, it is beaten by every \((x^*, M) \in \text{Supp}(\sigma) \) because \( M \succeq M^* \) for every such strategy. It follows that if a firm deviates from \((y, N) \) into \((x^*, K) \), it gains a market share of \( \frac{1}{2} \beta \sigma(x^*) + \frac{1}{2} (1 - \beta \sigma(x^*)) = \frac{1}{2} \), hence the deviation is profitable.

Thus, when the consideration set is determined by advertising intensity, firms earn the rational-consumer pay-off in any symmetric Nash equilibrium, even though the equilibrium outcome itself departs from the rational-consumer benchmark. Firms offer inferior products with positive probability in equilibrium. Note that expected advertising intensity is strictly above zero in equilibrium. The reason is simple. If no firm advertised in equilibrium, then firms could play the strategy \((x^*, 1) \) and avoid being beaten at all, thus generating a pay-off of \( \frac{1}{2} - c_{x^*} - d_1 > \frac{1}{2} - N_{x^*} \), contradicting our result.

However, unlike the model of Section 3, the result that firms earn rational-consumer pay-offs in equilibrium does not imply the effective marketing property. For instance, let \( X = \{1, 2, 3\} \) and \( M = \{0, 1\} \). Assume \( 1 > 2 > 3, c_2 > \frac{1}{2} (c_1 + c_3) \), and \( c_2 - c_3 > d_1 \). Then, it is easy to construct a symmetric Nash equilibrium strategy \( \sigma \) such that \( \text{Supp}(\sigma) = \{(1, 0), (1, 1), (2, 1), (3, 1)\} \). Note that when the realization of this equilibrium is that one firm plays \((2, 1) \) while the other firm plays \((3, 1) \), half the consumers will be assigned to the former and consider the latter without switching to it.

The reason that the effective marketing property does not hold in this model is precisely the defensive role of intensive advertising: firms can use high advertising intensity not only to attract the attention of the competitor’s clientele, but also to block the firm’s own clientele from paying attention to the rival firm. Thus, it is possible for the support of an equilibrium strategy to include two strategies, \((x, M) \) and \((y, N) \), such that \( y \succeq x \) (and \( x, y \not= x^* \)) and \( M \succeq N \). This means that a consumer for whom \((y, N) \) is the status quo will consider \((x, M) \) because of the high advertising intensity that accompanies \( x \), yet fail to switch because \( x \) is inferior to \( y \). The rationale for accompanying \( x \) with high advertising intensity is to prevent consumers for whom \((x, M) \) is the status quo from considering better market alternatives such as \((y, N) \).

### 4.2. Product display

Product display in supermarkets and other stores is an important component of a firm’s marketing strategy as it plays a big role in generating shoppers’ attention to brands. In many cases, big retailers demand “slotting fees” to put suppliers’ goods on their shelves, and these vary according
to which positions are considered to be prime space. For example, many stores consider eye-level shelves to be the top spot, while others charge more for goods placed on “end caps”—displays at the end of the aisles which is believed to have the greatest visibility. To be on the right-hand-side of an eye-level selection is also considered a prime location because most people are right-handed and most people’s eyes drift rightward.\(^8\) In addition, the retailer often has considerable bargaining power in setting these display costs.\(^9\) In this subsection, we develop a simple example that illustrates the implications of display costs for competition among manufacturers.

Let \( K \geq 2 \). Assume that \( \phi(x, M, y, N) = 1 \) if and only if one of the following conditions holds: (i) \( N = K \) and (ii) \( N = M \in \{1, \ldots, K - 1\} \). The interpretation is that an element in \( M \) represents a particular product display option. The option 0 represents no display, or hidden display, such that when consumers enter the store they can find the product only if they are already familiar with it from prior shopping experience—\( i.e. \) when it is their default. The option \( K \) represents prominent display (\( e.g. \) near the entrance or the cashiers). The options \( 1, \ldots, K - 1 \) represent “isle” display. A consumer can note a new product on isle display only if the product is already familiar with it from prior shopping experience—\( i.e. \) when it is their default. The option \( K \) represents prominent display (\( e.g. \) near the entrance or the cashiers). The options \( 1, \ldots, K - 1 \) represent “isle” display. A consumer can note a new product on isle display only if the product is already familiar with is placed on the same isle.

Assume that \( c(x, M) = c_x + d_M \in (0, \frac{1}{2}) \), where \( 0 = d_0 < d_1 = \cdots = d_{K - 1} < d_K \). That is, isle display costs more than no display and less than prominent display. These assumptions imply that as in the previous subsection, the max–min strategy is \( (x^*, 0) \) and the max–min pay-off is \( \frac{1}{2} - c_{x^*} \), and these coincide with the Nash equilibrium outcome when consumers are rational.

**Proposition 8.** There is a unique symmetric Nash equilibrium given by

\[
\begin{align*}
\sigma(x^*, 0) &= 2d_K, \\
\sigma(x^*, K) &= 2(c_{x^*} - c_x), \\
\sigma(x^*, 0) &= 1 - 2(c_{x^*} - c_x + d_K).
\end{align*}
\]

**Proof.** Let \( \sigma \) be a symmetric Nash equilibrium strategy. We first show that there exist no \( x \in X \) and \( M \in \{1, \ldots, K - 1\} \) such that \((x, M) \in \text{Supp}(\sigma)\). Assume the contrary, and let \((x, M) \in \text{Supp}(\sigma)\) for some \( M \in \{1, \ldots, K - 1\} \). In particular, let \( x \) have the property that for every other \((x', M') \in \text{Supp}(\sigma), x' \geq x \) or \( k \notin \{1, \ldots, K - 1\} \). By the finiteness of the support of \( \sigma \), there exists such \( x \). By the specification of \( \phi \), the strategy \((x, M)\) does not beat any other strategy in \( \text{Supp}(\sigma) \). Suppose that a firm deviates from \((x, M)\) into \((x, 0)\), and let us compare the pay-off that this strategy generates with the pay-off that \((x, M)\) generates. First, the cost of \((x, 0)\) is lower. Second, neither strategy beats any strategy in \( \text{Supp}(\sigma) \). Third, if some strategy in \( \text{Supp}(\sigma) \) beats \((x, 0)\), it necessarily beats \((x, M)\). Thus, the deviation is profitable, a contradiction. By eliminating the use of the marketing strategies \( 1, \ldots, K - 1 \), we have reduced the model to that of Propositions 1 and 4, where we establish the existence of a unique symmetric Nash equilibrium given by equations (3)–(5).

This equilibrium is structurally the same as in Propositions 1 and 4: only the best and worst products are offered in equilibrium, and the former is sometimes accompanied by prominent display. Isle display does not exist in equilibrium. Thus, in the context of our simple example, if the retailer charges for isle display more than it does for no/hidden display, manufacturers will not employ isle display at all, and they will only choose between no/hidden and prominent

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display. It would be interesting to embed the consideration-sets model in a larger, more pertinent model that endogenizes the retailers’ determination of display costs for manufacturers.

5. CHOICE-THEORETIC ASPECTS OF THE MODEL

In this subsection, we return to the basic consideration-sets model of Section 2. We examine some of revealed preference properties of the consideration-sets model and compare it to related models in the choice theoretic literature.

Recall our definition of the revealed strict preference relation (a.k.a the beating relation) induced by the consideration-sets procedure: \((y, N) \succ^* (x, M)\) if \(\phi(x, N) = 1\) and \(y \succ x\). A natural question that arises is, which properties of \(\succ^*\) characterize the consideration-sets procedure? That is, can we state axioms on \(\succ\) that will make the consumer compare a given default product with a given potential substitute.

Quasi-transitivity is a weakening of conventional transitivity, which reflects two key features of our model: (1) whether or not a consumer considers a substitute to his default depends only on the default product and the marketing strategy of the substitute and (2) the choice between “bare” products (i.e. elements in \(X\)) in the consumer’s consideration set is rational. Thus, if a consumer switched from \((y, N)\) to \((x, M)\) and from \((z, M')\) to \((y, N')\), this must mean that \(x\) is preferred to \(y\) and \(y\) is preferred to \(z\). Furthermore, the fact that the consumer was observed switching from \((z, M')\) to \((y, N')\) means that \(N'\) attracts attention from \(z\). It follows that when the consumer’s default is \((z, M')\), he would also switch to \((x, N')\).

Definition 2. A binary relation \(\succ\) on \(\mathcal{A}\) is quasi-complete (QC) if for every \(x, y \in X\), exactly one of the following is true:

(i) \((x, M) \succ (y, N)\) for some \(M, N \in \mathcal{M}\) or
(ii) \((y, N') \succ (x, M')\) for some \(M', N' \in \mathcal{M}\).

Definition 3. A binary relation \(\succ\) on \(\mathcal{A}\) is quasi-transitive (QT) if \((x, M) \succ (y, N)\) and \((y, N') \succ (z, M')\) imply \((x, N') \succ (z, M')\).

Quasi-completeness means that it is always possible to find a profile of marketing strategies that will make the consumer compare a given default product with a given potential substitute. Quasi-transitivity is a weakening of conventional transitivity, which reflects two key features of our model: (1) whether or not a consumer considers a substitute to his default depends only on the default product and the marketing strategy of the substitute and (2) the choice between “bare” products (i.e. elements in \(X\)) in the consumer’s consideration set is rational. Thus, if a consumer switched from \((y, N)\) to \((x, M)\) and from \((z, M')\) to \((y, N')\), this must mean that \(x\) is preferred to \(y\) and \(y\) is preferred to \(z\). Furthermore, the fact that the consumer was observed switching from \((z, M')\) to \((y, N')\) means that \(N'\) attracts attention from \(z\). It follows that when the consumer’s default is \((z, M')\), he would also switch to \((x, N')\).

Remark 1. A binary relation \(\succ^*\) on \(\mathcal{A}\) is irreflexive, QC, and QT if and only if there exists a linear ordering \(\succ\) on \(X\) and a consideration function \(\phi: X \times \mathcal{M} \to [0, 1]\) such that \((y, M) \succ^* (x, N)\) for some \(N \in \mathcal{M}\) iff \(y \succ x\) and \(\phi(x, M) = 1\).

Proof. Necessity is trivial, so we will only show sufficiency. For every \(x, y \in X\), define \(y \succ x\) if there exist \(M, N \in \mathcal{M}\) such that \((y, M) \succ^* (x, N)\) and let \(x \succ y\) if there exist \(M', N' \in \mathcal{M}\) such that \((x, N') \succ^* (y, M')\). By irreflexivity, QC, and QT, \(\succ\) is irreflexive, complete, and transitive. Consider some \((x, M) \in X \times \mathcal{M}\). Suppose \(x\) is not the \(\succ\)-maximal product in \(X\) — i.e. there exists \(y \in X\) such that \(y \succ x\). If \((y, M) \succ (x, N)\) for some \(N\), define \(\phi(x, M) = 1\). By QC and irreflexivity, there is no \(N'\) such that \((x, N') \succ (y, M)\). Otherwise, let \(\phi(x, M) = 0\). Suppose next that \(x\) is \(\succ\)-maximal product in \(X\). Then, by the definition of \(\succ\), there are no \(y\) and \(N\) such that \((y, M) \succ (x, N)\). Hence, we are free to choose \(\phi(x, M) = 0\) or \(\phi(x, M) = 1\). Suppose that \(y \succ x\). Then there exist some \(M', N'\) such that \((y, M') \succ^* (x, N')\). If, in addition,
\(\phi(x, M) = 1\), then \((y, M) \succ^* (x, N')\) for some \(N'\). Conversely, suppose \((y, M) \succ^* (x, N)\), then it follows from our construction that \(y \succ x\) and \(\phi(x, M) = 1\). ||

Masatlioglu and Nakajima (2008) independently conduct a choice-theoretic analysis of a more general choice procedure than ours, which they call “Choice by Iterative Search” (CIS). A consumer who follows this procedure begins with some exogenously given default option \(r\), taken from the feasible set \(B\). Given this default, the consumer constructs a consideration set \(\Omega(B, r) \subseteq B\). The consumer chooses the best alternative in \(\Omega(B, r)\) according to a complete preference relation \(\succ\) defined on the grand set of alternatives \(X\). If \(\max \succ \Omega(B, r) = r\), the procedure is terminated and the consumer chooses \(r\). If \(\max \succ \Omega(B, r) \neq r\), then the consumer constructs another consideration set \(\Omega[B, \max \succ \Omega(B, r)]\) and picks his most preferred alternative from this set. The procedure is iterated until the consumer picks some alternative \(y\) that satisfies \(\max \succ \Omega(B, y) = y\). The CIS procedure is characterized by the mapping \(\Omega\) and the preference relation \(\succ\).

Our choice procedure is a special case of the CIS model. Given a pair of extended products \((x^s, M^s), (x^n, M^n)\), let \(B = \{(x^s, M^s), (x^n, M^n)\}\), \(r = (x^s, M^s)\), \(\Omega(B, r) = B\) if \(\phi(x^s, M^s) = 1\), and \(\Omega(B, r) = \{r\}\) if \(\phi(x^s, M^s) = 0\). The strict part of the preference relation \(\succ\) coincides with our \(\succ^*\). Masatlioglu and Nakajima (2008) show that the CIS model induces an extended choice function (a mapping from pairs, consisting of a set \(B\) and a default \(r\), to an element in \(B\)) which is fully characterized by two properties, which they call “Anchor Bias” and “Dominating Anchor Bias”. Masatlioglu, Nakajima and Ozbay (2009) develop further the choice-theoretic analysis of general consideration-set procedures without default options, focusing on the problem of identifying the preferences and the mechanism that generates the consideration set from the consumer’s observed choices.10

The consideration-sets procedure is also related to the idea of “short-listing”. A decision maker who faces a large choice set may simplify his decision problem by first eliminating a subset of alternatives that are dominated according to some incomplete preference relation and then applying a complete preference relation to the remaining set. Manzini and Mariotti (2007) provide a choice-theoretic characterization of this procedure. The intuitive difference between the two models is that although they both apply a pair of binary relations in sequence, the short-listing model uses the first stage to shrink the choice set, whereas the consideration-sets model uses the first stage to expand it. Thus, when the binary relation of the first stage gets closer to being complete, the set of options on which the decision-maker applies his preferences becomes smaller in the short-listing model, whereas in our model it becomes bigger.

Unlike the consideration-sets procedure, the short-listing model does not involve an explicit default alternative. For this reason, a straightforward comparison between the two models is impossible. However, one partial comparison, which is feasible, concerns the special case in which the binary relations that are employed in both stages of the short-listing model are complete and transitive. In this case, the short-listing model is reduced to standard rational choice.

Compare this with the advertising intensity example of Subsection 4.1. In that example, the consideration function induces a complete and transitive binary relation \(P\) on extended products, defined as follows: \((y, N)P(x, M)\) if \(\phi(M, N) = 1\). In this case, the consumer chooses his default \(x^s\), unless \((x^n, M^n)P(x^s, M^s)\) and \(x^n \succ x^s\), in which case he chooses \(x^n\). Thus, in order for the consumer to switch from the default \((x^s, M^s)\) to the new alternative \((x^n, M^n)\), the latter

10. Masatlioglu and Nakajima (2008) also provide a choice-theoretic characterization when the default is not observed but has to be inferred from observations. A choice correspondence satisfies a property called “Bliss-Point” if, and only if, there exist a preference relation over alternatives \(\succ^*\) and a consideration set mapping \(\Omega\) such that for every \(B \subseteq X\), each element chosen from \(B\) is selected by the CIS procedure \((\Omega, \succ^*)\) for some default.
must be ranked above the former according to two preference relations (and strictly so according to at least one of them). As Masatlioglu and Ok (2005) showed, this sort of behaviour is consistent with choosing according to an incomplete preference relation over \( \mathcal{A} \), where a new product is chosen over the default only if it is strictly better according to this incomplete preference relation. Hence, the observed switching behaviour of the consumer may be inconsistent with rational behaviour (recall our discussion of this point in Sub-section 4.1).

A crucial difference between the consideration-sets procedure and both the CIS and the short-listing models is that our model imposes more structure on the set of outcomes, in the form of the distinction between products and marketing strategies. Salant and Rubinstein (2008) study a choice model that involves a related distinction between “alternatives” and “frames”. In their model, the frame accompanies the entire choice set rather than an individual alternative. Of course, one can translate our concept of a frame into theirs by taking the profile of marketing strategies to be the frame that accompanies the choice set. Salant and Rubinstein provide necessary and sufficient conditions for rationalizing a choice function (defined over framed choice problems) with a (possibly incomplete) preference relation defined over the set of alternatives.

6. RELATION TO THE LITERATURE ON ADVERTISING AND MARKETING

In this section, we discuss the relation between our consideration-sets model and two branches of literature: the economic literature on advertising and the marketing literature on consideration sets.

6.1. Persuasive, complementary, and informative advertising

Models of advertising in economics typically make one of the following assumptions (see Bagwell, 2007): (i) advertising changes the utility function from consumption (advertising is “persuasive”); (ii) advertising enters into the utility function as an argument (advertising is “complementary” to consumption); and (iii) advertising does not affect the utility function but it affects the consumer’s beliefs (advertising is “informative”). In this subsection, we try to relate our model to this categorization.

6.1.1. Persuasive and complementary advertising. Recall from Section 2 that if a consumer in our model switches from a default product \( x \) to a competing product \( y \) as a result of the marketing of \( y \), then no set of messages would cause the consumer to switch from \( y \) to \( x \). Hence, our framework cannot accommodate any model of persuasive or complementary advertising that allows such preference reversals. This raises the converse question: can consumer behaviour in our framework always be modelled as some form of persuasive or complementary advertising?

In order to answer this question with regards to persuasive advertising, let us fix the same marketing strategy \( M \) for both firms. Persuasive advertising then means that the consumer is characterized by a profile of weak preference relations \( (\succeq_M)_{M \subseteq D} \) over \( X \). The question is, can we find a preference relation \( \succ_M \) such that the choice from \( \{ (x, M), (y, M) \} \) is \( \max_{\succ_M} \{ x, y \} \)? The answer is no, since it is possible that \( \phi(x, M) = 1, \phi(y, M) = 0 \), and \( x \succ y \). Thus, the consumer will choose \( y \) over \( x \) when \( y \) is the default and \( x \) over \( y \) when \( x \) is the default. No preference relation \( \succeq_M \) can rationalize this choice behaviour.

Chioveanu (2008) analyses an extension of Varian’s model of sales (Varian, 1980), in which some consumers rationally perform price comparisons at no cost, while other consumers are loyal to firms they are initially assigned to, where loyalty means that they do not perform any price comparison. Chioveanu assumes that the fraction of consumers who are loyal to a given firm in this sense is a function of the profile of advertising expenditures in the industry.
Although Chioveanu refers to this advertising technology as “persuasive”, it does not fall into the definition of persuasive advertising given above. Instead, the way Varian and Chioveanu model customer loyalty and persuasive advertising fits our model: a consumer is “loyal” to a firm if his consideration set consists of the firm’s product only.

Advertising is complementary if the revealed choices of the consumer can be rationalized by a single preference relation over the extended set of alternatives $A$. As emphasized repeatedly in this paper, the consideration-sets model can induce choice behaviour that cannot be rationalized by standard preferences over $A$. Hence, our model accommodates choice behaviour that cannot be captured by a model of complementary advertising.

6.1.2. Informative advertising. Informative advertising typically takes two forms. First, in a search-theoretic environment, advertising can reduce the search costs that the consumer needs to incur in order to add a product to his choice set (in extreme cases, such as in Butters, 1977, costs fall from being infinitely high to being zero). Second, advertising can cause the consumer to update his beliefs about the quality of the product either because the advertising message contains verifiable data or because it acts as a Spencian signalling device.

The behavioural comparison between our model and informative advertising is subtle because the latter approach assumes that the consumer has rational expectations about the distribution of alternatives he is facing, a component that is absent from our model. However, any model of informative advertising would necessarily display the following monotonicity property.

If an advertising message convinces the consumer to consider a new product when his default is $x$, it should also convince him to consider the new product when his default is inferior to $x$ according to his preferences. The model analysed in Section 3 typically violates this type of monotonicity.

6.1.3. Conclusion. The consideration-sets model departs from the trinity of persuasive, comparative, and informative advertising. In our model, the role of marketing is “persuading to consider”, and this role is related to but distinct from these three conventional theories. Finally, recall that our model incorporates other marketing activities than advertising, including packaging, determination of pay-off-irrelevant product characteristics, search engine optimization, and design of product lines.

6.2. Related marketing literature

The marketing literature has long recognized that the consumption decision follows a two-step decision process (for extensive surveys of this literature, see Alba, Hutchinson and Lynch, 1991; Roberts and Lattin, 1997). Consumers first form a small set of options that they will consider for their consumption decision. They then evaluate the options in this set and choose the one they prefer the most. Whether or not an alternative is included in the consideration set may depend on factors other than the consumer’s preferences.

Empirical evidence for this two-stage procedure is not trivial to gather because the first stage is hard to observe. In a study of laundry detergent purchases, Hoyer (1984) reports that the median number of packages that consumers closely examined, as they browsed the relevant supermarket shelf, was 1. Thus, even if new, superior brands were displayed on the shelf, it is unlikely that they would have been considered by the consumer, unless they were promoted.

Shum (2004) presents evidence that is consistent with the view that marketing attempts to weaken consumers’ reluctance to consider new products. He carries out counterfactual experiments that demonstrate that uninformative advertising may be at least as effective as price discounts in stimulating a purchase of a new brand.
Alba, Hutchinson and Lynch (1991) emphasize the important role that memory plays in the formation of consideration sets. First, many purchasing decisions are made without having the feasible alternatives physically present (e.g. deciding on a restaurant for dinner). Second, even when the available options are displayed to the consumer, the display is often complex (e.g. financial products, sophisticated electrical appliances) or overwhelmingly varied (e.g. breakfast cereals or salad dressing in a supermarket). In these circumstances, consumers rely on memory to a large extent. This implies that a preferred option may be ignored if it is not easily retrieved from memory.

For example, Nedungadi (1990) studied the effect of uninformative advertising on choice of fast food restaurant. Subjects were told that they would be given a coupon for a fast food restaurant of their choice. On the premise that the experimenter had only a limited variety of coupons available to him, subjects were asked to name their most preferred restaurant and list all other restaurants for which they would accept a coupon. In one treatment, before subjects provided the names, they were exposed to an ad that mentioned a local sandwich shop (without any information on this shop’s menu). Subjects in the control treatment were not exposed to this ad. Nedungadi found that while most subjects in the control treatment listed mainly hamburger restaurants, a significant proportion of subjects in the advertising treatment named a well-known sandwich chain—different than the one which was advertised—as their most preferred choice. Thus, even though some subjects preferred sandwiches to hamburgers, the former was unlikely to be chosen simply because it was not easily recalled when the task was to choose a fast food restaurant.

Memory also plays a role in the choice between an existing brand of an incumbent firm and a new competing brand of an entrant. The likelihood of choosing the new product depends on the ease with which this product will be retrieved whenever the consumer considers making a purchase from the product class to which it belongs. Zhang and Markman (1998) propose that the likelihood of remembering a new brand is influenced by the way its attributes compare with those of the incumbent brand. Specifically, the authors provide experimental evidence suggesting that consumers are more likely to recall a new brand if its advertised attributes are comparable with the attributes of the incumbent brand along a common dimension (i.e., the differences between the two brands are alignable). Moreover, the authors demonstrate that a superior new brand may not be chosen if its good attributes are hard to align with those of the incumbent brand. In a similar vein, a recent study by Chakravarti and Janiszewski (2003) presents experimental evidence suggesting that when people are asked to select an alternative from a large set of heterogeneous alternatives, they tend to simplify their decision problem by focusing on a small subset of “easy-to-compare” options having alignable attributes.

7. CONCLUDING REMARKS

This paper introduces the concept of consideration sets into economic modelling and develops its implications in the context of a competitive market model. As such, it contributes to a growing theoretical literature on market interactions between profit-maximizing firms and boundedly rational consumers. Rubinstein (1993) analyses monopolistic behaviour when consumers differ in their ability to understand complex pricing schedules. Piccione and Rubinstein (2003) study intertemporal pricing when consumers have diverse ability to perceive temporal patterns. Spiegler (2006a,b) analyses markets in which profit-maximizing firms compete over consumers who rely on naive sampling to evaluate each firm. Shapiro (2006) studies a model in which firms use advertising to manipulate the beliefs of consumers with bounded memory. DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006, 2008), and Gabaix and Laibson (2006) study interaction with consumers having limited ability to predict their future tastes. Mullainathan,
Schwartzstein and Shleifer (2008) study the role of uninformative advertising when consumers apply “coarse reasoning”. For a field experiment that quantifies the effects of various marketing devices in terms of their price-reduction equivalent, see Bertrand et al. (2010).

We hope to extend our market model in various directions. An important challenge is to incorporate price setting into the model. One way of doing so (suggested to us by Dino Gerardi) is to add a pricing stage to the game analysed in this paper. Thus, after the profile \( (x_i, M_i) \) is realized at the end of the first stage of the game, firms make simultaneous pricing decisions in the second stage. This reflects the idea that designing the product and planning its marketing campaign is a long and costly process, while changing the product’s price can be done in a relatively short time and with little or no costs. To illustrate the workings of such a model, suppose that \( X = \{x\} \) and \( D = \{m\} \)—i.e. there is a single quality level and firms choose only whether to advertise or not. Then, if both firms advertise, they charge zero prices in the second-stage equilibrium and thus earn a net profit of \(-c_x - c_m\); if both firms refrain from advertising, each firm charges the monopoly price (normalized to one) and thus earns a net profit of \(\frac{1}{2} - c_x\); while if only firm 1, say, advertises, then the second-stage pricing game is an asymmetric variant on the price competition model due to Varian (1980). This subgame has a unique Nash equilibrium in which firms mix over prices and thus earn positive profits (ignoring the sunk costs of the first stage). While this extension of our model would clearly complicate the analysis, we believe that some of our insights (e.g. regarding the equilibrium conversion rate) would survive in some form. Since the extension dramatically changes the max–min outcome, we do not expect our results concerning the relation between the equilibrium and max–min pay-offs to survive.

Piccione and Spiegler (2009) pursue the “pricing challenge” in a different direction, by applying the consideration-sets model to a market environment that includes price setting but treats all fixed costs as sunk, thus ignoring the firms’ strategic choice of marketing costs. In the Piccione–Spiegler model, firms make simultaneous pricing and “framing” decisions, and the consideration function is an arbitrary function of the latter. Although the Piccione–Spiegler model differs from the current one in a number of crucial dimensions, the observation made here that greater consumer rationality (in the sense of increased propensity to consider new alternatives) can lead to a less competitive outcome emerges, in an even stronger form, in Piccione and Spiegler (2009).

It would be interesting to introduce prices as arguments of the consideration function itself. For example, extreme prices are often good attention grabbers (e.g. an ultra-expensive dish at a restaurant or an ultra-cheap laptop at a computer store). Thus, the trade-off that a multi-product firm faces is that the attention grabber will make a direct loss (either because it is sold at a price below marginal cost, or because it is so expensive that no one buys it and therefore it fails to cover its fixed costs) but it may generate indirect revenues by drawing consumers’ attention to other products. The question of how firms resolve this trade-off in a competitive environment is left for future research.

We treat the consideration function \( \phi \) as exogenous. An interesting extension would be to derive this function as a result of some prior optimization that is carried out by a consumer who takes into account cognitive constraints. Still, there is some justification for treating \( \phi \) as exogenous in our framework. The consideration function captures basic principles of attention grabbing that do not seem to involve any deliberation on the consumer’s part.11 For example, we will almost instinctively note an ad printed on a bright coloured paper. Another example is the principle of similarity: we are more likely to compare products or to perceive them as substitutes, if they have similar packages or similar advertising campaigns (recall the packaging and benchmarking examples from Section 4). This principle lies at the heart of what is known as subliminal attention manipulation.”

11. Indeed, the ways in which advertising can manipulate the automatic or even subconscious nature of human attention has been the subject of the famous Vance Packard, The Hidden Persuaders.
to marketing practitioners as “associative positioning strategies”, whereby a brand will use in its advertising campaign features or slogans that are typically associated with a dominating brand (e.g. see Dröge and Darmon, 1987). Likewise, people have a tendency to note a statement about some product flaw when they themselves have consumed the product and experienced its flaw.

Whatever optimization lies behind the consumer’s heightened attention to a product in these examples, it does not appear to be market specific. Instead, it takes place on a much larger, “general equilibrium” or “evolutionary” scale, where the consideration function is designed to be optimal on average across a large variety of market situations. Therefore, as long as the focus of our analysis is on a specific market, it makes sense to treat the consideration function as exogenous.

Another important extension of the model is in the direction of consumer heterogeneity. Since consumers in our model are characterized by two primitives, $\phi$ and $\phi$, heterogeneity may exist in both dimensions. We already saw in Subsection 3.5 an implication of heterogeneity in $\phi$ for industry profits. Heterogeneity in both dimensions is needed for a pertinent theory of consumer conversion.

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