INDUCED BIAS IN ATTENUATION MEASUREMENTS TAKEN FROM COMMERCIAL MICROWAVE LINKS

Jonatan Ostrometzky*, Adam Eshel†, Pinhas Alpert†, and Hagit Messer* *Fellow, IEEE*
*School of Electrical Engineering, Tel Aviv University
†Department of Geosciences, Tel Aviv University

***Abstract*—Cellular backhaul networks usually consist of commercial microwave links, known to be sensitive to weather conditions. The management network systems usually provide records of measurements of the transmitted and the received signals levels from the different microwave links for monitoring and analyzing the network performance. Many of them log only the minimum and the maximum levels of the transmitted and the received signals in pre-set intervals (usually 15-minute). Moreover, only quantized version of these measurements are logged. In the last decade it has been proposed to use these existing measurements for rainfall monitoring. In this paper we analyze the effects of the quantizer and the min/max operators on commercial microwave links signals levels measurements. We show that the quantization process, in combination with the min/max operators, adds bias to the measurements which can be significant. We then propose a method to calculate this bias, and demonstrate our findings using measurements from actual commercial microwave links.**

***Index Terms*—Quantization Noise, Microwave Networks, Precipitation Attenuation**

I. INTRODUCTION

Current cellular communication networks are based, at least partly, on Commercial Microwave Links (CMLs). In order to inspect and analyse the performance of these networks, current Network Management Systems (NMS) constantly monitor the CMLs Transmitted Signal Level (TSL) and Received Signal Level (RSL).

The NMS help in monitoring the Link Budget (LB), as it is critical that the LB will not breach the Fading Margin limits of the CML. Thus, many NMS will log only the minimum and the maximum values of the observed RSL and TSL values, usually in 15-minute intervals, using a rough quantizer [5], [16], as these relatively low-resolution datasets are sufficient for the LB monitoring purposes [9].

Furthermore, since CMLs signals are known to be sensitive to rain [11], it was suggested back in 2006 to use the TSL and RSL available datasets for rain monitoring [17]. Since then, a vast number of studies suggested different methods and techniques which use the TSL and RSL available measurements for rain monitoring [7], [14], [19], rainfall maps plotting [15], [22], classification and estimation of snow and sleet [1], [18], fog monitoring [4], the detection of dew [10], and recently, the detection of air-pollution [3].

On the other hand, although sufficient for network monitoring, quantized min/max TSL and RSL values given at 15-minute intervals have proven to be sub-optimal for environmental monitoring purposes. Thus, it was suggested to deal with the non-linear min/max operators either by performing calibration of different model parameters [20], by implementing weighted average of the minimum and the maximum values [21], or by bypassing this limitation by directly accessing the CML hardware, and retrieve the instantaneous RSL/TSL measurements themselves [2]. It is worth noting, that unlike the min/max operators, the quantizer is usually a property of the hardware itself, and thus, even accessing the CML directly and retrieving the instantaneous measurements sill results in quantized data [2].

The fact that the available RSL and TSL measurements pass a quantizer was generally ignored. Indeed, *Goldstein et. al.* [6] discussed the normalization of the quantization error in regard to the CMLs length, and suggested to include the normalized quantization error within the covariance matrix of the noise. And, although it was shown that the quantization process might introduce errors [13], these errors were considered to be unavoidable and relatively small for the purpose of rain monitoring using CMLs attenuation measurements [16], [23], and thus, did not attract special interest.

In this paper we show that the combination of a quantizer with a non-linear min/max operator induces a non-negligible bias to the output value. We show that this bias, unless compensated, may introduce a bias to the LB calculations, and may cause an over-estimation of the rain. Furthermore, we show that the expected value of this bias can be calculated using the available minimum and maximum RSL and TSL measurements during dry periods. We demonstrate our findings using actual measurements taken from CMLs.

The rest of the paper is organized as follows: *Section II* presents the theory and establishes the bias calculation methodology. *Section III* describes the experimental demonstration, and discusses the results. Lastly, *Section IV* concludes this paper.

II. THEORY AND METHODOLOGY

The nearest-neighbour (or a *"round"*) quantizer $q(x)$ is defined by:

$$y = q(x) = L \cdot round\left(\frac{x}{L}\right) \tag{1}$$

where x is the input signal, y is the (quantized) output, and $0 < L \in \mathbb{R}$ is the quantization interval. Note, that $q(x)$ is said to be both *uniform* and *symmetric* quantizer [8].

A. Combination of the Quantizer q(x) and the Min/Max Operators

First, note that the specific order of the operations in regard to the min or max operators, and the quantizer $q(x)$, does not change the outcome, as described in *Lemma 1*:

Lemma 1. *For any given* $\{x_i \in \mathbb{R}\} : i \in [1, 2, \cdots, n]$,

$$max\left(q(x_1), q(x_2), \cdots, q(x_n)\right) = q\left(max(x_1, x_2, \cdots, x_n)\right) \tag{2}$$

Proof.

$$\begin{aligned} &max\left(q(x_1), \cdots, q(x_n)\right) = \\ &= L \cdot max\left(round\left(\frac{x_1}{L}\right), \cdots, round\left(\frac{x_n}{L}\right)\right) = \\ &= L \cdot round\left(\frac{(max(x_1, \cdots, x_n))}{L}\right) = \\ &= q\left(max\left(x_1, \cdots, x_n\right)\right) \end{aligned} \tag{3}$$

□

Similarly, *Lemma 1* applies in regard to the min operator.

In order to inspect the combined effect of applying a min or max operator in combination with a quantizer $q(x)$, a simple illustration is presented **Fig. 1**: C is a known signal which lies between two consecutive quantization levels, marked by $Q0$ and $Q1$. $w(t)$ is an unbiased additive noise, such that the sampled signal, $x(t)$, equals to $x(t) = Q0 + C + w(t)$. From this illustration, it is obvious that given enough samples of $x(t)$, (i.e., $\{x(t_i)\} : i \in [1, 2, \cdots, n]$ where $n >> 1$), the followings hold:

$$q\left(min(x(t_1), x(t_2), \cdots, x(t_n))\right) = q(A) = Q0 \tag{4a}$$
$$q\left(max(x(t_1), x(t_2), \cdots, x(t_n))\right) = q(B) = Q1 \tag{4b}$$

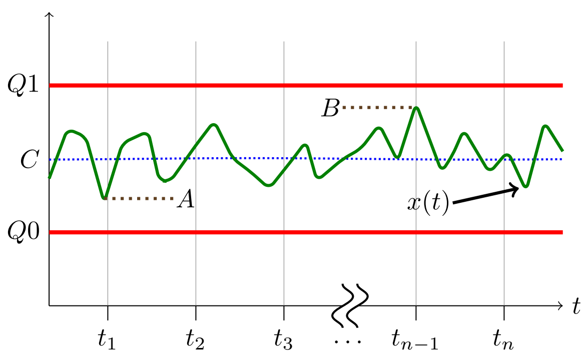

Fig. 1. Illustration of $x(t)$: The two consecutive quantization levels are plotted in RED (marked by $Q0$ and $Q1$), and $x(t)$ is plotted in GREEN for $C = Q0 + 0.5 \cdot L$, which is marked by a dotted BLUE line. A represents $min(x(t_1), x(t_2), \cdots, x(t_n))$, and B represents $max(x(t_1), x(t_2), \cdots, x(t_n))$.

We formalize the insights shown in **Fig. 1** in the following *Proposition*:

Proposition 1. C is a constant signal which value lies between two quantization levels, defined by $Q0$ and $Q1$, such that $Q0 = k \cdot L$, and $Q1 = Q0 + L$ (where $k \in \mathbb{N}$). Given $x(t)$, such that:

$$x(t) = Q0 + C + w(t) \tag{5}$$

where $0 < C < L$, and $w(t)$ is an unbiased additive white noise whose samples are bounded such that $\forall i$:

$$Q0 - 0.5 \cdot L < x(t_i) < Q1 + 0.5 \cdot L \tag{6a}$$
$$0 < \epsilon \to 0 : Pr[x(t_i) = Q0 + (0.5 - \epsilon) \cdot L] > 0 \tag{6b}$$
$$0 < \epsilon \to 0 : Pr[x(t_i) = Q1 - (0.5 - \epsilon) \cdot L] > 0 \tag{6c}$$

then the followings hold:

$$min\left(q(x(t_1)), \cdots, q(x(t_n))\right) \xrightarrow[n\to\infty]{w.p.\ 1} Q0 \tag{7}$$

$$max\left(q(x(t_1)), \cdots, q(x(t_n))\right) \xrightarrow[n\to\infty]{w.p.\ 1} Q1 \tag{8}$$

A proof of *Proposition 1* is given in the *APPENDIX*. Note, that *Proposition 1* is presented for the case where the noise samples are bounded (see eq. (6)), so that the output of the quantizer is bounded between two consecutive quantization levels. However, the same conclusions can be expanded for the case where the noise samples follow the normal distribution ($\forall i : w(t_i) \sim \mathcal{N}(0, \sigma^2)$), by replacing the quantization interval, L, in *Proposition 1* and its proof with a quantization gap (defined by g), where $g/L \in \mathbb{N}$, such that g is large enough so that the noise is practically bounded ($g >> \sigma^2$). For simplicity, in the sequel, we assume that the noise is bounded such that $-l \leq w(t) \leq l$ using the noise profile shown in the *APPENDIX*. Furthermore, equations will be written using g.

B. Induced Bias

Based on the proof of *Proposition 1*, we show that the direct quantization of $x(t_i)$, $q(x(t_i))$, is a biased estimator of $x(t_i)$:

$$\begin{aligned} &E\left[q(x(t_i)) - x(t_i)\right] = E\left[q(x(t_i))\right] - Q0 - C = \\ &= Q0\left(\frac{1}{2} + \frac{g}{4l} - \frac{C}{2l}\right) + Q1\left(\frac{1}{2} - \frac{g}{4l} + \frac{C}{2l}\right) - Q0 - C = \\ &= \frac{g}{2} - \frac{g^2}{4l} + \frac{gC(1-2l)}{2l} \end{aligned} \tag{9}$$

Although the estimator $q(x(t_i))$ is biased, without any prior information regarding the value of C, it can be shown that this estimator is unbiased on the average:

$$\frac{1}{g}\int_0^g \left(\frac{g}{2} - \frac{g^2}{4l} + \frac{gC(1-2l)}{2l}\right) dC = 0 \tag{10}$$

This property of the quantizer justifies the fact that the quantization error was usually ignored.

However, implementing a quantizer on the minimum or maximum values of $x(t_i)$ induces bias to the estimation process also on the average. Based on *Proposition 1* and its proof, the bias of $min\left(q(x(t_1)), \cdots, q(x(t_n))\right)$ and $max\left(q(x(t_1)), \cdots, q(x(t_n))\right)$ in regard to $x(t_i)$ can be directly calculated:

$$\begin{aligned} &E\left[min\left(q(x(t_1)), \cdots, q(x(t_n))\right) - x(t_i)\right] = \\ &= E\left[min\left(q(x(t_1)), \cdots, q(x(t_n))\right) - x(t_i)\right] - Q0 - C = \\ &= g\left(\frac{1}{2} - \frac{g}{4l} + \frac{C}{2l}\right)^n - C \xrightarrow[n\to\infty]{} -C \end{aligned} \tag{11}$$

$$\begin{aligned} &E\left[max\left(q(x(t_1)), \cdots, q(x(t_n))\right) - x(t_i)\right] = \\ &= E\left[max\left(q(x(t_1)), \cdots, q(x(t_n))\right) - x(t_i)\right] - Q0 - C = \\ &= g - C - g\left(\frac{1}{2} + \frac{g}{4l} - \frac{C}{2l}\right)^n - C \xrightarrow[n\to\infty]{} g - C \end{aligned} \tag{12}$$

Unlike $q(x(t_i))$, both $min\,(q(x(t_1)),\cdots,q(x(t_n)))$ and $max\,(q(x(t_1)),\cdots,q(x(t_n)))$ are biased also on the average:

$$min(q(x(t_i))): \frac{1}{g}\int_0^g (-C)\,dC = -\frac{g}{2} \qquad n>>1 \tag{13}$$

$$max(q(x(t_i))): \frac{1}{g}\int_0^g (g-C)\,dC = \frac{g}{2} \qquad n>>1 \tag{14}$$

From equations (13) and (14) two important conclusions arise: First, a combination of a min/max operator and a quantizer introduces bias to the original measurements of $x(t_i)$, which may not be negligible. Second, this bias depends on the quantization gap, g.

C. Maximum and Minimum TSL and RSL Measurements

NMS monitor the TSL and the RSL of the CMLs. The specific sampling and logging protocols varies between the hardware vendors. For instance, Ericsson™ systems usually sample the signal level at 10-seconds intervals, and save the minimum and the maximum values every 15 minutes, using a standard quantization intervals of $1[dB]$ for the TSL, and $0.3[dB]$ for the RSL [23].

During dry periods, the LB is considered to remain relatively constant [12], meaning that the transmitted power (defined by Tx) and the received power (defined by Rx) fluctuations are also limited. Thus, under the assumption that the minimum and the maximum TSL and RSL values are extracted from large enough instantaneous samples series (i.e., $n>>1$), *Proposition 1* is valid, and can be used in order to connect the CML path-loss (which equals to $Tx-Rx$) with the minimum channel attenuation (defined by A_{min}) and the maximum channel attenuation (defined by A_{max}), which yields:

$$\begin{aligned} A_{min} &= TSL_{min} - RSL_{max} = \\ &= (Tx - \frac{g_T}{2}) - (Rx + \frac{g_R}{2}) = \\ &= (Tx - Rx) - (\frac{g_T}{2} + \frac{g_R}{2}) \end{aligned} \tag{15}$$

$$\begin{aligned} A_{max} &= TSL_{max} - RSL_{min} = \\ &= (Tx + \frac{g_T}{2}) - (Rx - \frac{g_R}{2}) = \\ &= (Tx - Rx) + (\frac{g_T}{2} + \frac{g_R}{2}) \end{aligned} \tag{16}$$

where g_T is the quantization gap of the TSL values, and g_R is the quantization gap of the RSL values.

Next, subtracting eq. (15) from eq. (16), yields:

$$A_{diff} \equiv A_{max} - A_{min} = g_T + g_R \tag{17}$$

which connects the extreme attenuation measurements with the expected value of the bias.

And, although this calculation is made during dry periods, it can be assumed that the same expected value of the bias remains during rainy periods, as the rain does not affect the quantization intervals and levels. Thus, on average, the same bias occurs.

III. Experimental Demonstration

In order to demonstrate our findings, we gained access to a chain of four CMLs provided by the Israeli cellular operator *Cellcom*™. The four CMLs are located in the south of Israel, near the Dead-Sea. A map of the experiment area is presented in **Fig. 2**, and the specific CMLs properties are summarized in TABLE I.

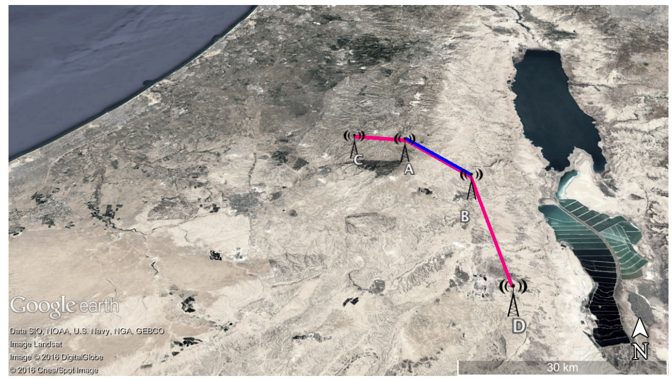

Fig. 2. Map of the experiment location (captured from *Google Earth*), showing the four different CMLs (Colored in RED and BLUE). The sites which hold the CMLs antennas are marked by A,B,C,D, where B is the city of *Arad* near the Dead-Sea.

As the CMLs hardware was manufactured by *Ericsson*^{TM}, the minimum and the maximum values of both the TSL and the RSL measurements, in 15-minute intervals, were logged. The TSL values were logged with a quantization interval of $L_T \equiv 1[dB]$, whereas the RSL values where logged using a quantization interval of $L_R \equiv 0.3[dB]$. The internal sampling rate (from which the minimum and the maximum values were obtained) was 10-seconds [23], which validates *Proposition 1*, as $n = 90 >> 1$.

TABLE I

Detailed properties of the four available CMLs: The specific location (Path), path-length (Length), Frequency (Freq.) and polarization (Pol.) are summarized.

CML No. #	Path	Length (km)	Freq. (GHz)	Pol.
1	$A \leftrightarrow C$	10.3	18.6	hor
2	$A \leftrightarrow B$	16.0	18.6	hor
3	$A \leftrightarrow B$	16.0	18.73	ver
4	$B \leftrightarrow D$	26.4	18.6	ver

A. Results

The four available CMLs were monitored during a dry period of 24 hours, from November 5^{th} 2015 at 00:00 until November 6^{th} 2015 at 00:00. During these 24 hours, each of the CMLs produced the four relevant data series: TSL_{min}, TSL_{max}, RSL_{min}, and RSL_{max}.

For each CML, these four data series were used to calculate A_{min} and A_{max}, of eq. (15) and (16), which in turn were used to calculate A_{diff} of eq. (17). The four series of A_{diff} for the four CMLs are plotted in **Fig. 3**. In addition, the average and the median values of A_{diff} for each of the four CMLs are summarized in TABLE II.

It is worth noting, that although no rain was detected throughout the duration of the demonstration[1], two unknown environmental phenomena probably affected the LB of the CMLs: Between 00:00 and 03:00 of November 5^{th}, CML 1 suffered from increased attenuation (suggesting that the phenomenon responsible is located near site C, as no other CMLs were affected), wheras CMLs 2,3, and 4 suffered from increased attenuation around 17:00 of the same day (suggesting that the phenomenon responsible is located near site B, as all CMLs connected to site B were affected).

Nonetheless, the results presented in **Fig. 3** and TABLE II are significant. Apart from these two unexplained phenomenon, the A_{diff} series for all CMLs are consistently "resting" at a value of $1.6[dB]$. This "resting point" of $1.6[dB]$ suggests two important conclusions: First, it validates the fact that a bias is indeed introduced to the measurements. Second, by looking back at eq. (15) and (16), it can be concluded that for every CML in this setup:

$$g_T = L_T = 1 \quad (dB) \tag{18}$$

$$g_R = 2 \cdot L_R = 0.6 \quad (dB) \tag{19}$$

from which, it can be concluded (based on eq. (13) and (14)) that each measurement logged in this setup (regardless of the specific CML) is subject to the following induced biases:

$$RSL_{min} : -0.3 \quad (dB) \tag{20a}$$

$$RSL_{max} : +0.3 \quad (dB) \tag{20b}$$

$$TSL_{min} : -0.5 \quad (dB) \tag{20c}$$

$$TSL_{max} : +0.5 \quad (dB) \tag{20d}$$

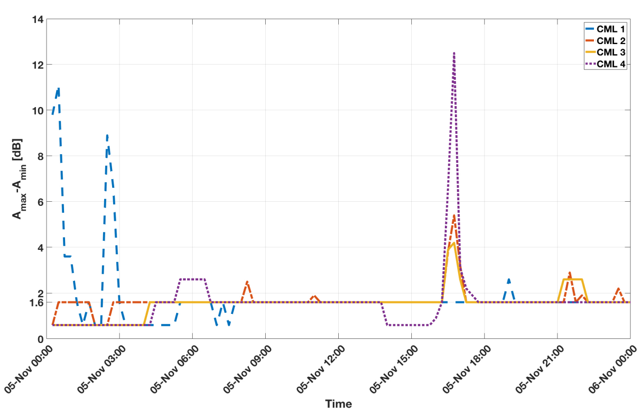

Fig. 3. Calculation of $A_{diff} \equiv A_{max} - A_{min}$ based on the measurements taken from CMLs 1,2,3 and 4, during November 5^{th} 2015.

It is worth noting that we were able to extract additional measurements from this setup intermediately during 2015, which yielded similar results consistently.

[1]The fact that no rain was detected throughout the duration of the demonstration and that this duration was indeed "dry", was validated using two rain-gauges operated and monitored by the Israeli Meteorological Services, located near sites A and B.

TABLE II

AVERAGE (MEAN) AND MEDIAN VALUES OF $A_{diff} \equiv A_{max} - A_{min}$ AS MEASURED BY CMLS 1,2,3 AND 4, FROM 05-NOV-2015, 00:00 UNTIL 06-NOV-2015, 00:00.

CML No. #	mean(A_{diff}) (dB)	median(A_{diff}) (dB)
1	1.8187	1.6000
2	1.6729	1.6000
3	1.5365	1.6000
4	1.5750	1.6000

IV. CONCLUSION

This paper deals with CMLs TSL and RSL measurements which are produced by current NMS. We show that the measurements which are being produced by these systems, which are generally logged after passing a rough quantizer and a min or max operators, include an inherited bias. This bias can cause an error in the LB monitoring procedures, as well as in the estimation of rain, which may not be neglected.

This paper presents a theory which explains the origin of this bias, and suggests a methodology which can be used to calculate this induced bias using only the available measurements themselves. We demonstrate our approach using four CMLs, and show that a bias is indeed induced into the measurements produced by these CMLs, and that our suggested methodology is capable of determining its value.

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APPENDIX

For simplifying the proof, we define $\{w(t_i)\} : i \in [1, 2, \cdots, n]$ to be identically and independent uniformly distributed between $(-l, +l)$, where $|C - L/2| < l < L - |C - L/2|$. However, the same proof can be easily repeated for different noise profiles, as long as the conditions of eq. (6) are met.

Proof. Subject to the definition of $\{w(t_i)\}$, the probabilities of a single quantized sample, $q(x(t_i))$, is given by:

$$q(x(t_i)) = \begin{cases} Q0 & w.p. \ \ \frac{1}{2} + \frac{L}{4l} - \frac{C}{2l} \\ Q1 & w.p. \ \ \frac{1}{2} - \frac{L}{4l} + \frac{C}{2l} \end{cases} \tag{21}$$

from which, the following probabilities emerge:

$$\begin{aligned} &min\,(q(x(t_1)), \cdots, q(x(t_n))) = \\ &= \begin{cases} Q0 & w.p. \ \ 1 - (\frac{1}{2} - \frac{L}{4l} + \frac{C}{2l})^n \\ Q1 & w.p. \ \ (\frac{1}{2} - \frac{L}{4l} + \frac{C}{2l})^n \end{cases} \end{aligned} \tag{22}$$

$$\begin{aligned} &max\,(q(x(t_1)), \cdots, q(x(t_n))) = \\ &= \begin{cases} Q0 & w.p. \ \ (\frac{1}{2} + \frac{L}{4l} - \frac{C}{2l})^n \\ Q1 & w.p. \ \ 1 - (\frac{1}{2} + \frac{L}{4l} - \frac{C}{2l})^n \end{cases} \end{aligned} \tag{23}$$

noting that $(\frac{1}{2} \pm \frac{L}{4l} \mp \frac{C}{2l}) < 1$ with the fact that $n \to \infty$ (i.e., $n >> 1$), completes this proof. $\square$

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